Integrated application of network traffic and intelligent driver models in the test laboratory analysis of autonomous vehicles and electric vehicles

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Abstract: The aim of the research is to develop a laboratory model-based diagnostic procedure that performs tests of the motion processes of autonomous electric vehicles in a particular city, on a transport network or track. The test consists of a laboratory based generation of the corresponding speed and steering angle signals, being in accordance with real driving and traffic conditions, which are also used in the test procedure. The procedure takes into account the real trajectory tracking process as well (Péter and Lakatos, 2017).

Keywords: autonomous electric vehicles; laboratory model-based diagnostic procedure; corresponding speed and steering angle signals; laboratory based generation; real traffic environment.

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1 Introduction

The vehicle movement on the designated trajectory is executed in such a way that the autopilot or driver can even commit limited arbitrary errors in trajectory tracking during driving. When generating the speed of each vehicle, account must be taken of the particular traffic conditions, i.e., the composition and movement of vehicles in the environment of the network section (Csiszár and Földes, 2018; Koryagin, 2018; Lakatos and Mándoki, 2017), as well as the complex transport environment typical (Farooq et al., 2018; Ghadi et al., 2018; Iordanopoulos et al., 2018) of the time of day and seasonality determined by processes of the large-scale transport network (Péter et al., 2015, 2016). The model-based method subject to this study is therefore a theoretical foundation that takes into account and examines the drivers' (Szabó et al., 2004, 2009) characteristics during the movement along the trajectory as well as the relationship between the microscopic vehicle environment and the macroscopic traffic environment (Busznyák and Lakatos, 2017; Lakatos et al., 2016).

2 Traffic related application of the intelligent driver model (IDM)

Adaptive cruise control (ACC) is a vehicle system that allows the vehicle to adjust its speed to the environment. The IDM is an ACC model that is widely used in transportation research to model longitudinal movement. Treiber, Hennecke and Helbing developed the IDM, which is being used by the car company BMW, in 2000 at the transport laboratory of Dresden Technical University.

The IDM model is used for modelling continuous traffic flows in simulations of highway and city traffic (Kovács et al., 2016). As a vehicle tracking model, IDM describes the dynamics of the position and speed of each vehicle. In the case of multi-model open source traffic simulator (Treiber and Helbing, 2002) use IDM to simulate the longitudinal movement of the vehicle and this simulator also introduces a lane change strategy. Model-based single-lane traffic inhomogeneity is studied by Treiber et al. (2004).

Treiber et al. (2006) study vehicle stability and IDM parameter sensitivity. Kesting et al. (2008) propose to extend the driver parameters of the IDM model. They study the impact of vehicles equipped with IDM on traffic flow and travel times as bottlenecks. Jerath (2010) also uses the IDM model and examines the impact of ACC on traffic flows. The results of the above work show that increasing the proportion of ACC vehicles will result in increased traffic efficiency by reducing travel times. Treiber and Kesting (2011) used IDM to study instability in congested traffic.

The IDM has many advantages over other ACC models from calibration and intuitive parameters point of view, and the also modelling requires simple simulation. However, there are also disadvantages in respect of assuring the proper features of the vehicle and the driver. The IDM is a collision-free model, therefore, in critical accident situations, the desired minimum distance is no longer sufficient to guarantee driver safety and, in the event of an emergency braking, it tends to overshoot the actual deceleration of the vehicle.

Derbel et al. (2012, 2013) developed a proposal for a more accurate operation of the IDM and studied possible modifications to IDM, taking into account the driver's safety and the real capabilities of the vehicle. As a result of this amendment, the driver has to take into account the behaviour of the following vehicles, and thus a modified IDM has been developed and tested with a microscopic simulator considering string stabilisation. This modified IDM already highlights the proper vehicle capabilities.

In our present work we rely on the model joint-developed with French researchers to overcome the disadvantages. Based on this, the IDM is already providing greater performance in driver security by following real reactions in near-collision critical situations. The paper shows the modification and the state-of-the-art operation of the intelligent driver model in connection with the proper capabilities of the vehicle. Modelling and research work encompasses a complex area and includes approaches of both microscopic and macroscopic modelling (Derbel et al., 2017).

The complex macroscopic traffic environment is generated by the large-scale network model, in which the microscopic traffic simulation model provides the individual vehicle movement in traffic on the sections of the defined trajectories. However, this microscopic model must properly reproduce dynamic traffic processes and must also be validated. Accordingly, at this stage of our work we rely on the IDM research and development of Treiber et al. (2000a, 2000b) and Derbel et al. (2017).

The features of the classical IDM are the following: a single system of differential equations that analyses the case of *n* vehicles travelling on a single lane; the microscopic model describes a chain model-like longitudinal dynamics; each driver looks only forward and aims to keep an appropriate distance; there is no overtaking, the vehicles keep their order and the first vehicle has a dominant role, as do the slow-moving vehicles in the group.

The classical IDM is written with separate differential equations member by member. In our study this is summarised in the following system of differential equations (1), where the current position of the *i*th vehicle is described by function $x_i(t)$. The parameters and functions used in the model are as follows:

a_i is the maximal acceleration of the *i*th vehicle

v_i is the desired speed of the *i*th vehicle

s_i is the required distance between the *i*th and the preceding vehicle ($i = 1,2, ..., n$).

$$\langle \underline{\underline{A}} \rangle^{-1} \ddot{x}(t) + \langle \underline{\underline{V}} \rangle^{-1} \underline{f_1}(\dot{x}(t)) + \langle \underline{\underline{S}} \rangle \underline{f_2}(x(t)) = \underline{1} \tag{1}$$

The notation used in differential equation system (1) is the following

$$\langle \underline{\underline{A}} \rangle^{-1} = \left\langle \frac{1}{a_1}, \frac{1}{a_2}, ..., \frac{1}{a_n} \right\rangle ; \langle \underline{\underline{V}} \rangle^{-1} = \left\langle \frac{1}{v_1^4}, \frac{1}{v_2^4}, ..., \frac{1}{v_n^4} \right\rangle ; \langle \underline{\underline{S}} \rangle = \langle s_1, s_2, ..., s_n \rangle$$

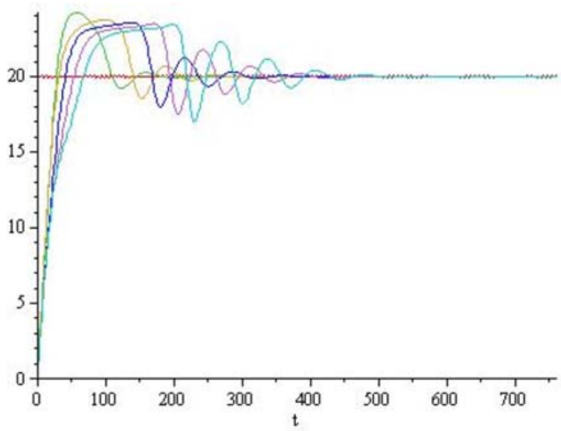
$s_i = s_{0i} = const.$, or $s_i = s_i(\dot{x}_{i-1}, \dot{x}_i)$ $(i = 1,2,\ldots,n)$.

$$\underline{f_1}(\dot{x}(t)) = \begin{bmatrix} \dot{x}_1^4 \\ \dot{x}_2^4 \\ \ldots \\ \dot{x}_n^4 \end{bmatrix}, \; \underline{f_2}(x(t)) = \begin{bmatrix} \dfrac{1}{(x_0 - x_1)^2} \\ \dfrac{1}{(x_1 - x_2)^2} \\ \ldots \\ \dfrac{1}{(x_{n-1} - x_n)^2} \end{bmatrix}, \; \underline{1} = \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix}$$

The modified IDM applied in this work is represented by formula (2). Detailed description of the above is provided by Derbel et al. (2012, 2013). This model, using a function $h(t)$ also takes into account the fact that drivers monitor the movement of the following vehicles as well, Figure 1.

$$\langle \underline{\underline{A}} \rangle^{-1} \ddot{x}(t) + \underline{\underline{V}}\, \underline{f_1}(\dot{x}(t)) + \underline{\underline{S}}\, \underline{f_2}(x(t)) = \underline{1}(t) + \underline{h}(t) \tag{2}$$

Figure 1 Setting of the relative distances at a stabilised speed state after a vehicle group consisting of five elements is started (see online version for colours)



The notation used in differential equation system (2) is as follows:

$$\langle \underline{\underline{A}} \rangle^{-1} = \left\langle \frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n} \right\rangle; \quad \underline{\underline{V}} = \begin{bmatrix} \frac{1}{v_1^4} & \frac{h_1}{v_2^4} & & & \\ & \frac{1}{v_2^4} & \frac{h_2}{v_3^4} & & \\ & & \frac{1}{v_i^4} & \frac{h_i}{v_{i+1}^4} & \\ - & - & - & - & - \\ & & & & \frac{1}{v_n^4} \end{bmatrix};$$

$$\underline{\underline{S}} = \begin{bmatrix} s_1^2 & h_1 s_2^2 & & & \\ & s_2^2 & h_2 s_3^2 & & \\ & & s_i^2 & h_i s_{i+1}^2 & \\ - & - & - & - & - \\ & & & & s_n^2 \end{bmatrix}$$

$$h(t) = \begin{bmatrix} h_1(t) \\ h_2(t) \\ \dots \\ h_n(t) \end{bmatrix}; \; h_i(t) = hf_i(t) \cdot \frac{a_{i+1}}{a_i}; \; (i = 1,2, \dots, n-1); \; h_n(t) = 0 .$$

$$s_i = s_{0i} = const., \text{ or } s_i = s_i(\dot{x}_{i-1}, \dot{x}_i) \; (i = 1,2,\dots,n).$$

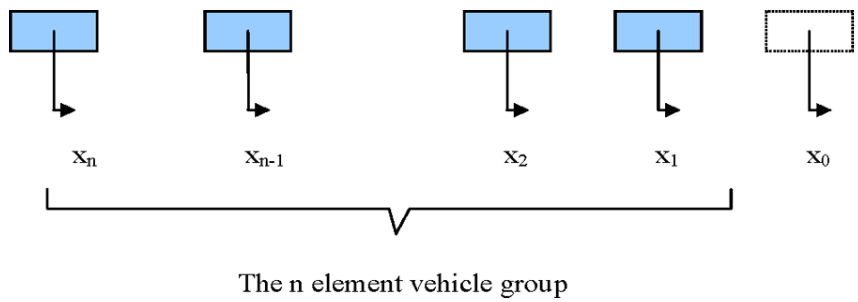
$$\underline{f_1}(\dot{x}(t)) = \begin{bmatrix} \dot{x}_1^4 \\ \dot{x}_2^4 \\ \dots \\ \dot{x}_n^4 \end{bmatrix}, \; \underline{f_2}(x(t)) = \begin{bmatrix} \frac{1}{\varepsilon_1^2 + (x_0 - x_1)^2} \\ \frac{1}{\varepsilon_2^2 + (x_1 - x_2)^2} \\ \dots \\ \frac{1}{\varepsilon_n^2 + (x_{n-1} - x_n)^2} \end{bmatrix}, \; \underline{1}(t) = \begin{bmatrix} 1(t) \\ 1(t) \\ \dots \\ 1(t) \end{bmatrix}$$

3 Relationship between the IDM and the large-scale network

The speed of a given vehicle and the distance kept are determined by the driver. Their decision, however, depends on their own perceptions, on signals that are transmitted by the physical environment and received by the vehicle and on the local and general effects of network traffic (Lakatos et al., 2016; Pokorádi, 2018).

Physical impacts resulting from road quality, meteorological and visibility conditions at a given vehicle density determine a selectable speed range. The modified IDM discussed in the previous section can be used to describe the dynamic traffic connections originating from forward-moving vehicle-vehicle effects in a given section.

Figure 2 The n-element vehicle group and the environment determining their movement (see online version for colours)



At the same time, the dynamics of the movement of the IDM group is not arbitrary. It is determined by control speeds formed in the large-scale network or network sections. The vehicles slow down if a congestion occurs, and stop when the traffic light switches to red, but after the reaction delay time, they will accelerate to the maximum permitted speed if the road section ahead is free. This is indicated in Figure 2 by the control speed function x_0(t) defined by the large-scale macroscopic network processes for each trajectory.

4 Application of the network traffic model

For this research we apply the reduced network traffic model (Péter and Bokor, 2011, 2010a, 2010b; Péter, 2012; Dömötörfi et al., 2016) which contains internal network elements of n sections located in a domain characterised by state vector x. The model incorporates m external sections that are directly related to an internal section or sections. The state vector s of the latter is considered known by measurement. In this model, out of the matrices that form the link hyper-matrix, only matrices $\boldsymbol{K}_{11}$ and $\boldsymbol{K}_{12}$ play a role, because they represent each transfer that applies to the internal sections.

$$\dot{x} = \langle L \rangle^{-1} [K_{11}(x,s)x + K_{12}(x,s)s] \tag{3}$$

where $x \in \Re^n$, $\dot{x} \in \Re^n$, $s \in \Re^m$, $L = \text{diag}\{l_1, ..., l_n\}$, l_i is the length of the internal sectors in the main diagonal ($\forall l_i > 0$, $i = 1,2, \ldots, n$), $K_{11} \in \Re^{nxn}$, $K_{12} \in \Re^{nxm}$.

Taking into account the delays in practice, which largely originate from the time lags that can be derived from the reaction time (perception, decision, action: 0.6 ... 0.7 s) and the time from the actuation to its effect (0.15–0.3 s), results in a mathematical model describing reality more precisely. In this case, we assume that internal automatisms $S(x)$ and $E(x)$ are functions continuously differentiable with x, while traffic control light functions $u_{i,j}(t)$ are continuously differentiable with t. This can be accomplished in the model without special restrictions.

Similarly, the traffic light signal $u_{i,j}(t)$ can be made continuously differentiable in its domain if the above method is applied within every ε_t-radius of each t_0 breakpoint where values change either from 1 to 0 or from 0 to 1. In this way we use a continuous dynamic traffic model considering also the actual deceleration and reaction delay time phenomenon in it. In case of S a deceleration phenomenon occurs, as drivers become cautious when they realise that the section they want to drive over is already heavily loaded. There is no delay effect in case of function E, however, since when a section becomes empty (which takes place at the time the last vehicle leaves it in a given interval) it determines a continuous vehicle density function in this section, thus there is no contradiction to the above model paradigm applied to E. In the case of traffic lights, the reaction delay phenomenon occurs in two ways.

On the one hand, the vehicles do not start immediately when the light changes to green, and on the other hand, there could be irregular vehicle crossings at road intersections when the light changes to yellow (or to red). These real-life phenomena can be taken into consideration in the transfer processes with the application of a continuously differentiable light signal.

4.1 Validation of the model

In Budapest, the validation was performed on the Grand Boulevard, in the section northwards from the Pest-side bridgehead of Petőfi Bridge to the Nyugati Square, at the traffic light junctions on the basis of the actual traffic light program data provided to the BME (Budapest University of Technology and Economics) by the Traffic Engineering Directorate of FKF ZRt. (Metropolitan Public Domain Maintenance Ltd.) and the traffic counts performed on-site. The test section was also surveyed at each simulation time with vehicles equipped with a GPS device. The actual speed profiles were recorded during the vehicle measurements. This Boulevard model is a typical line model where the typical speed process is determined by the traffic light programs. We used the PannonTraffic software for the validation. The simulation took into account the actual traffic light programs, so it followed the timely formation of measured speed processes well. In the validation, the best approximation of speed limits was achieved by the software by adjusting the vehicle densities most appropriate to reality in each section of the Boulevard. The applied speed-density law was the Greenshields (linear) function in all cases. Comparison of the speed profiles obtained from the simulation and vehicle measurements naturally showed that speed-time functions should be considered as single realisations of a stochastic process and should be analysed accordingly with probability theory and statistical analysis methods. The analysis was carried out in urban environment as described above, in heavy traffic passing through and crossing several traffic light junctions. A large number of non-parametric statistical probes for speed profiles and engine performances were used for homogeneity testing. The subject of the study was that if the sample set consisting of two independent probability variables (the one measured by the GPS device and the other simulated by the traffic model) came from a population of equal distributions, so in practice can the two samples be considered as having the same distribution. In the analysis it was found that in both cases the two samples could be considered homogeneous at 95%. The results of our study reassuringly demonstrated that the model allows the extraction of custom speed processes and derived engine performance processes that are true to reality (Peter et al., 2011).

In Győr the validated model included the Szent István Road (Highway no. 1) and its surroundings, with one of the busiest traffic regions in the city centre. We validated the model based on the cross-sectional traffic counts conducted by the City in 2012. The main features of the network are 228 road sections, nine traffic light junctions and 38 other junctions. The network domain had 18 input sections and 15 output sections. The phase-plans of traffic lights were provided by the Győr Directorate of Hungarian Public Road Non-profit Plc and the Municipality of Győr. In the model 63 cross-sectional data were available for the validation. The simulation ran for a 24-h real-time period with a computer run-time of 2 min and 14 s. The software can be restarted from any point in time by taking into account the state parameters that are valid at that time as initial values. During the validation the software reviewed the actual distributions and factors influencing the transfers quarter-hourly to approximate the measured cross-sectional traffic data as good as possible. During this progress correlation analysis was performed hourly by taking into account the 63 measurement points. At the 63 examined cross-sections the correlation coefficient of values between the measured and modelled hourly cross-sectional traffic data was very close to 1, e.g., $r_{xy} = 0.993$ was obtained for peak hours between 7am and 8am, which in practice is 100% correlation (Péter and Fazekas, 2014).

4.2 Analysis of speed processes

A model assumption is that the speed value $v_i \geq 0$ can also be assigned to the state parameter $\forall x_i$, ($x_i \in [0,1]$, $i = 1, 2, \ldots, n$) using function f_i which is continuously differentiable with x_i:

$$v_i = f_i(x_i(t)) \tag{4}$$

By getting the individual speed processes from the macroscopic network model and by the application of a driver-vehicle model engine performance demands and emissions of individual vehicles can be analysed as well (Csonka and Csiszár, 2016). Speed processes are also suitable for model validation. In addition, this macroscopic model can be used to integrate the control signal $x_0(t)$ and to produce speed processes generated by the modified IDM along the designated trajectories.

4.3 Methods and instruments used in the measurement in Gyor

The advantage of the instruments with a GPS receiver is that it helps the connection to the database system and the automatic processing of data in a manner consistent with the additional tasks (Figures 3 and 4). Thus, not only the speed values but also the GPS location and time coordinates are recorded during the measurement. (Data of at least three working satellites is required for the measurements) (Szauter et al., 2014; Istenes et al., 2017). The data files of the GPS measurements were stored according to the GPS coordinates. The above method assigned the results of the speed measurements in the domain to the corresponding road sections, so we obtained a unique speed function for all the road sections involved in the measurement, Figures 5 and 6.

Figure 3 Vehicle measurement system equipped with GPS receiver (see online version for colours)

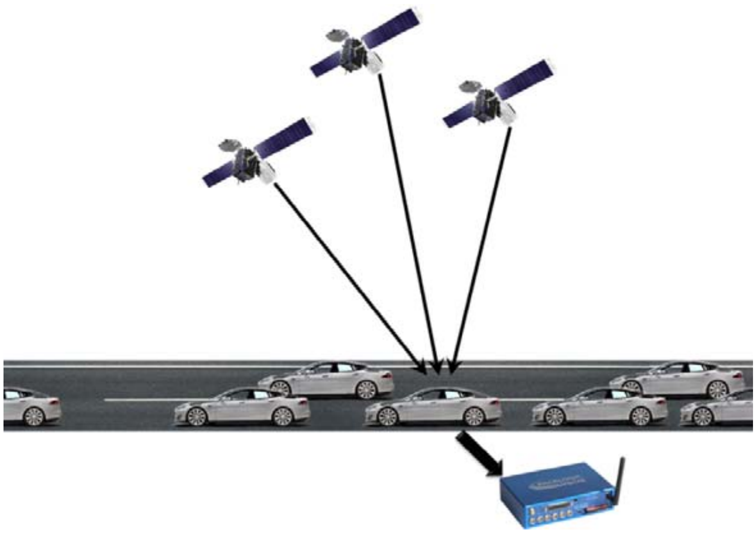


Figure 4 Routes travelled with GPS measurements in Győr (see online version for colours)

Figure 5 Speed values measured on route 004 as a function of distance (see online version for colours)

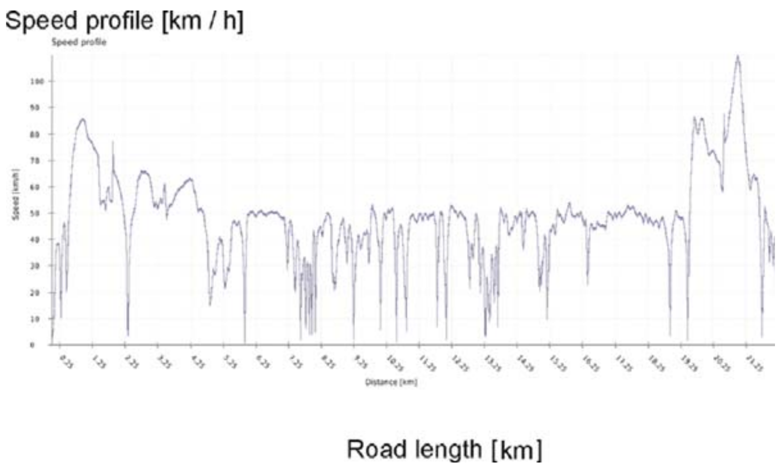
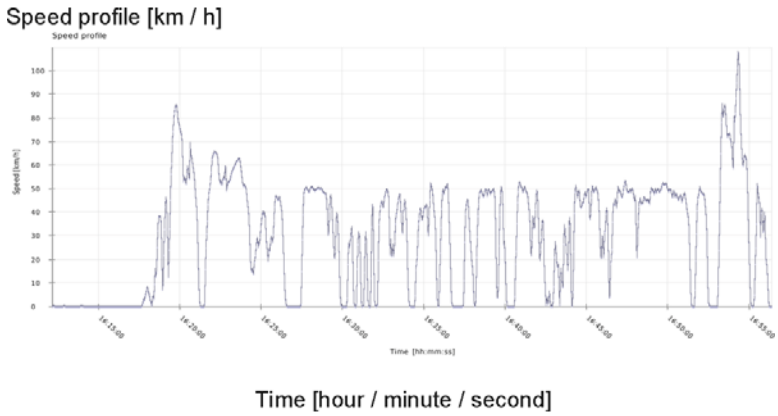


Figure 6 Speed values measured on route 004 as a function of time (see online version for colours)



5 Trajectory tracking

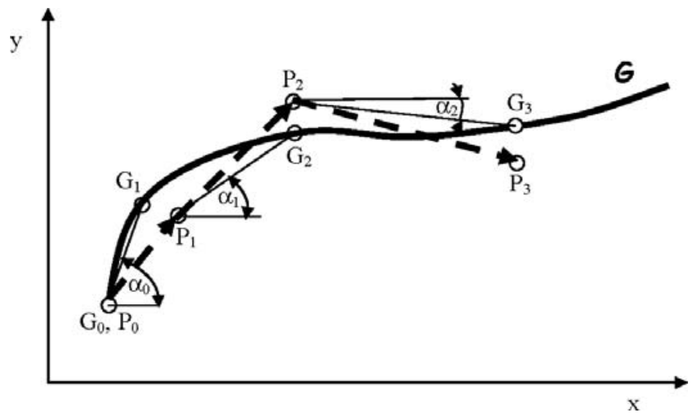
The algorithm followed by the driver or the autopilot is modelled in a fixed Descartes coordinate system as follows: The driver or the autopilot follows trajectory $\boldsymbol{G}$. At starting time t_0 they are on trajectory $\boldsymbol{G}$, so at that time $\mathbf{P}_0 = \mathbf{G}_0$. Henceforth, trajectory tracking decisions are generated at times $t_0, t_1, t_2, ..., t_n$. At time t_i they want to reach the selected point $\mathbf{G}_{i+1}$ on the trajectory from their position $\mathbf{P}_i$ with the chosen angle $\boldsymbol{\alpha}_i$ and speed $\boldsymbol{V}_i$.

However, at time t_{i+1} $\mathbf{P}_{i+1}$ is reached due to errors $\boldsymbol{\varepsilon}_\alpha$ and $\boldsymbol{\varepsilon}_s$ made in choosing the angle and the speed, respectively. This is how the geometric stability continues and the trajectory $\boldsymbol{G}$ itself provides.

5.1 The geometric stability of the trajectory tracking

Taking into account simple equidistant intervals, the motion process can be written with the following assignments, Figure 7.

Figure 7 The algorithm followed by the autopilot



Let P_i denote the points affected by the driver or the autopilot. G_{i+1} the targeted trajectory points, t_{i+1} discrete times, $tg(\alpha_i)$ direction tangents and s_i the distance between P_i and G_{i+1}.

$$P_i = [x_i, y_i] \rightarrow G_{i+1} = [X(t_{i+1}), Y(t_{i+1})]\,;\; t_{i+1} = (i+1)\Delta t\,;\; tg(\alpha_i) = m_i\,;\; s_i = \overline{P_i G_{i+1}}\,;$$
$(i = 0, 1, 2,\ldots)$

Taking into account the direction vector, the section lengths and speed are:

$$\underline{r}_i = [1, m_i], \rightarrow \underline{r}^0{}_i = \left[\frac{1}{\sqrt{1+m^2{}_i}}, \frac{m_i}{\sqrt{1+m^2{}_i}}\right] \tag{5}$$

$$\left(\mathrm{m_i} = \frac{Y((i+1)\Delta t) - y_i}{X((i+1)\Delta t) - x_i}\right)$$

$$s_i = \sqrt{(X((i+1)\Delta t - x_i)^2 + (Y((i+1)\Delta t - y_i)^2} \tag{6}$$

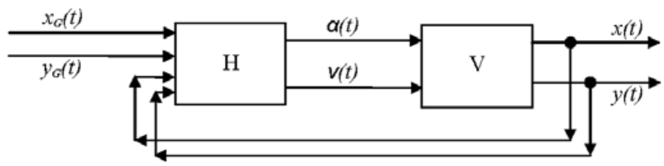
$$\left(v_i = \frac{s_i}{\Delta t}\right)$$

Taking into account the above the discrete points of trajectory tracking can be easily described with the following equation (7) recursive formula:

$$\begin{aligned} x_{i+1} &= x_i + \frac{s_i}{\sqrt{1+m_i^2}} \\ y_{i+1} &= y_i + \frac{s_i \cdot m_i}{\sqrt{1+m_i^2}} \end{aligned} \tag{7}$$

The mechanism of error-free trajectory tracking is shown in Figure 8.

Figure 8 *X*(*t*) and y(*t*) in the case of error-free trajectory tracking



h denotes the driver or the autopilot and V denotes the vehicle.

In the event that trajectory tracking is subject to errors, the following algorithm describes and Figure 9 illustrates the mechanism of the process:

$$\begin{aligned} &s_i \xrightarrow{real} s_i + Error_s = s_i \cdot \varepsilon_s \\ &\left(v_i \xrightarrow{real} v_i + Error_v = v_i \cdot \varepsilon_s\right) \end{aligned} \tag{8}$$

$$m_i \xrightarrow{real} m_i + Error_\alpha = m_i \cdot \varepsilon_\alpha \tag{9}$$

$$0.7 \le \varepsilon_s \le 1.3 \quad 0.7 \le \varepsilon_\alpha \le 1.3$$

In case of small angles:

$$\alpha \cong tg\alpha \quad \alpha \cong m$$

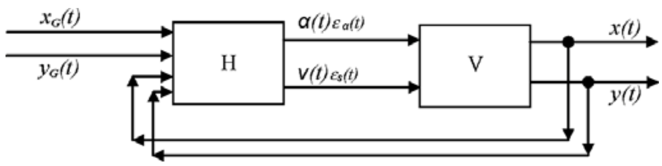
Discrete points of trajectory tracking with error:

$$x_{i+1} := x_i + \frac{s_i \varepsilon_s}{\sqrt{1 + m_i^2 \varepsilon_\alpha^2}}$$
$$y_{i+1} := y_i + \frac{m_i \varepsilon_\alpha s_i \varepsilon_s}{\sqrt{1 + m_i^2 \varepsilon_\alpha^2}} \tag{10}$$

where ε_s is the distance (or speed) estimation error, ε_α is the angle estimation error.

These are errors that are re-generated during every step in interval $[t_i, t_{i+1}]$. Their distribution largely depends on driver skills and visibility.

Figure 9 X(t) and y(t) in case of trajectory tracking with error



The geometric stability of the trajectory tracking method is well underpinned by the preliminary simulation tests, as illustrated in Figure 10(a) when the angle and distance estimations are accurate, but stability is provided even if the direction and distance estimation is subject to different rates (5–30%) of random errors, see Figure 10(a)–(d).

Further studying the trajectory tracking at discrete points, we move towards the continuous time tracking. In this case, starting from an initial value that is not necessarily a trajectory point, continuous error functions $\boldsymbol{\varepsilon}_\alpha = \boldsymbol{\varepsilon}_\alpha(t)$ and $\boldsymbol{\varepsilon}_s = \boldsymbol{\varepsilon}_s(t)$ are taken into account in the construction of the differential equations describing trajectory tracking.

5.2 The differential equation system for trajectory tracking

When applying formula (10), in order to move to a continuous case, we use the following relations and notation:

$$x_{i+1} := x_i + \frac{s_i \varepsilon_s}{\sqrt{1 + m_i^2 \varepsilon_\alpha^2}}$$
$$y_{i+1} := y_i + \frac{m_i \varepsilon_\alpha s_i \varepsilon_s}{\sqrt{1 + m_i^2 \varepsilon_\alpha^2}}$$

In order to move to a continuous case, we use the following relations and notation:

$$\Delta t \to 0$$

$$\Delta x = x_{i+1} - x_i\,;\ \Delta y = y_{i+1} - y_i\,;\ s_i = \Delta s = \sqrt{\Delta x^2 + \Delta y^2}\,;\ m_i = \frac{\Delta y}{\Delta x}$$

$$\Delta x = \frac{\sqrt{\Delta x^2 + \Delta y^2} \cdot \varepsilon_s}{\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2 \varepsilon_\alpha^2}}$$

$$\Delta y = \frac{\frac{\Delta y}{\Delta x}\sqrt{\Delta x^2 + \Delta y^2} \cdot \varepsilon_\alpha \cdot \varepsilon_s}{\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2 \varepsilon_\alpha^2}} \tag{11}$$

At equation (11) we use the difference quotient in discreet time

$$\frac{\Delta x}{\Delta t} = \frac{\sqrt{\left(\left(\frac{\Delta x}{\Delta t}\right)\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2} \cdot \varepsilon_s}{\sqrt{1 + \left(\frac{\left(\frac{\Delta y}{\Delta t}\right)}{\left(\frac{\Delta x}{\Delta t}\right)}\right)^2 \varepsilon_\alpha^2}}$$

$$\frac{\Delta y}{\Delta t} = \frac{\left(\frac{\left(\frac{\Delta y}{\Delta t}\right)}{\left(\frac{\Delta x}{\Delta t}\right)}\right)\sqrt{\left(\left(\frac{\Delta x}{\Delta t}\right)\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2} \cdot \varepsilon_\alpha \varepsilon_s}{\sqrt{1 + \left(\frac{\left(\frac{\Delta y}{\Delta t}\right)}{\left(\frac{\Delta x}{\Delta t}\right)}\right)^2 \varepsilon_\alpha^2}} \tag{12}$$

By generating boundary transition and assuming that time derivatives of track trajectories exist:

$$\text{If } \lim \Delta t \xrightarrow{0} \exists \dot{x}(t), \dot{y}(t)$$

Formulas (13) and (14) are the following:

$$\dot{x}(t) = \frac{\dot{x}_G \sqrt{\dot{x}_G{}^2 + \dot{y}_G{}^2} \cdot \varepsilon_s}{\sqrt{\dot{x}_G{}^2 + \dot{y}_G{}^2 \varepsilon_\alpha^2}}$$
$$\dot{y}(t) = \frac{\dot{y}_G \sqrt{\dot{x}_G{}^2 + \dot{y}_G{}^2} \cdot \varepsilon_\alpha \varepsilon_s}{\sqrt{\dot{x}_G{}^2 + \dot{y}_G{}^2 \varepsilon_\alpha^2}} \tag{13}$$

Since equation (13) can be converted to the following equation (14) form as well

$$\frac{\dot{y}(t)}{\dot{x}(t)} = \frac{\dot{y}_G}{\dot{x}_G}\varepsilon_\alpha$$

$$\sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{\dot{x}_G{}^2 + \dot{y}_G{}^2}\,\varepsilon_s \tag{14}$$

If, for any arbitrary initial value and with zero error (ε_s:=1 ε_α:=1), we apply the result obtained, the trajectory curve provides an orthogonal and directional mapping, i.e., the trajectory mapping performs a congruent transformation, in Figure 11 a mapping of two adjacent circular routes can be seen for an arbitrary initial value: [{A∪ B }→ {A*∪B*}].

Figure 10 (a) During a 12 s period a bend is correctly tracked by the error-free algorithm at discrete times; (b) during a 12 s period a bend is well tracked by the algorithm with an error of 5% at discrete times; (c) during a 12 s period the algorithm is stable with extreme errors of 20% and (d) during a 12 s period the algorithm is stable with extreme errors of 30% (see online version for colours)

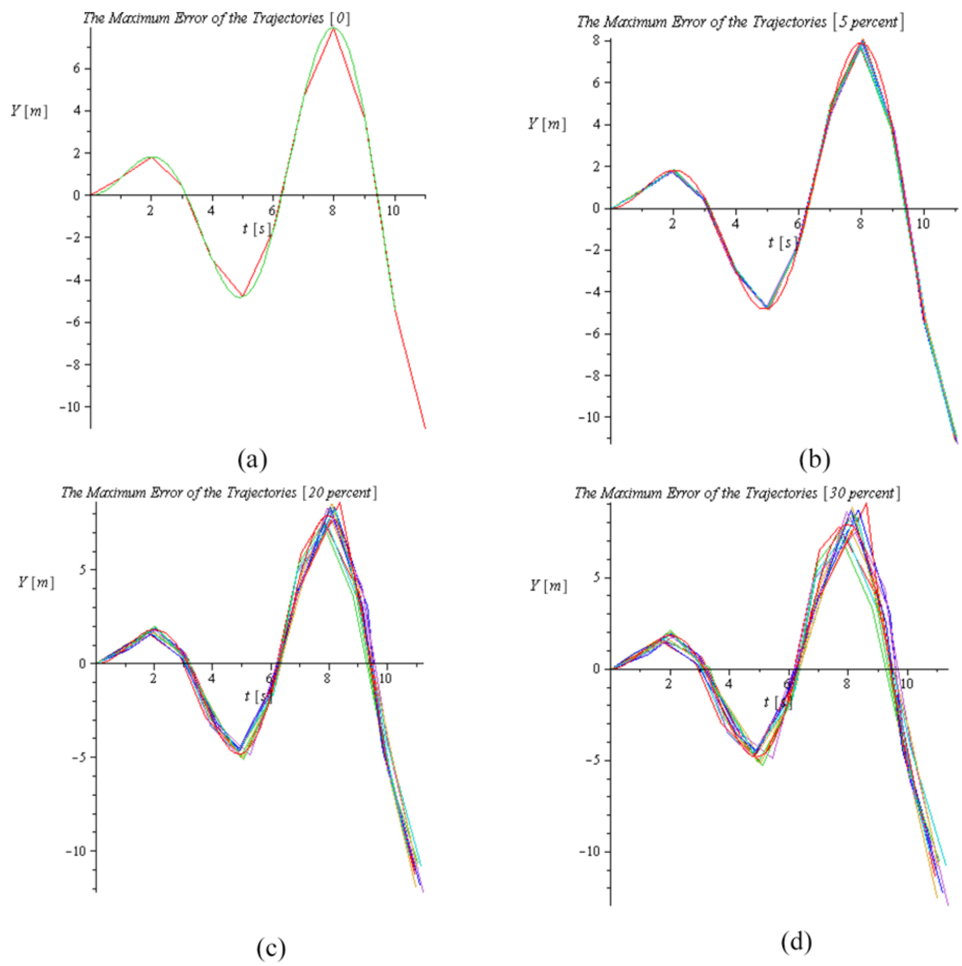
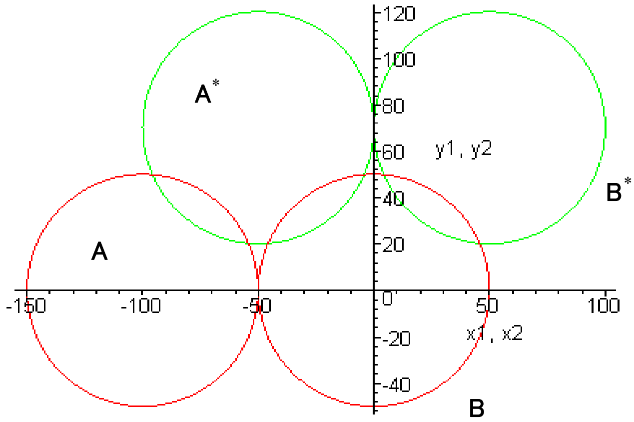


Figure 11 A mapping of two adjacent circular routes can be seen for an arbitrary initial value (see online version for colours)



6 Conclusions

Based on the system of differential equations derived from the continuous trajectory tracking it can be established that the physical considerations used for trajectory tracking can be utilised well during laboratory tests. In that case a suitable GPS-based set of points must be provided at the selected trajectory points to describe the geometry. The point to be tracked runs on this trajectory, taken into account the macroscopic environment influenced by complex traffic conditions and the microscopic environment according to the joint dynamics of vehicles, during the simulation.

This process provides the steering angle in the given conditions, which can be calculated from the simulation angle, the vehicle and steering geometry. In parallel with this, the speed of movement on the trajectory is directly provided by the simulator in a manner that is adequate for the real disordered traffic conditions. Further measurements and validations will of course be required in both urban traffic conditions and on the Zalaegerszeg test track (Szalay, 2016; Asaithambi et al., 2017; Pagliara et al., 2017; Szalay et al., 2017, 2018; Török et al., 2017; Mihály et al., 2018; Takács et al., 2018).

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References

Asaithambi, G., Sekhar Mourie, H. and Sivanandan, R. (2017) ‘Passenger car unit estimation at signalized intersection for non-lane based mixed traffic using microscopic simulation model’, *Periodica Polytechnica Transportation Engineering*, Vol. 45, No. 1, pp.12–20, doi: https://doi.org/10.3311/PPtr.8986

Busznyák, T. and Lakatos, I. (2017) 'Automotive engineering possibilities in combining global positioning and vehicle diagnostics', *5th International Scientific Conference on Advances in Mechanical Engineering (ISCAME 2017,* 12–13 October, Debrecen, Hungary, pp.1–6.

Csiszár, Cs. and Földes, D. (2018) 'System model for autonomous road freight transportation', *Promet-Traffic and Transportation*, Vol. 30, No. 1, February, pp.93–103, https://doi.org/10.7307/ptt.v30i1.2566

Csonka, B. and Csiszár, Cs (2016) 'Service quality analysis and assessment method for European carsharing systems', *Periodica Polytechnica Transportation Engineering*, [S.l.], Vol. 44, No. 2, pp.80–88, ISSN 1587-3811, https://pp.bme.hu/tr/article/view/8559, doi: https://doi.org/10.3311/PPtr.8559

Derbel, O., Péter, T., Mourllion, B. and Basset, M. (2017) 'Generalized velocity–density model based on microscopic traffic simulation', *Transport*, doi: 10.3846/16484142.2017.1292950 To link to this article: http://dx.doi.org/10.3846/16484142.2017.1292950, ISSN: 1648-4142 (Print) 1648-3480 (Online) Journal homepage: http://www.tandfonline.com/loi/tran20

Derbel, O., Peter, T., Zebiri, H., Mourllion, B. and Basset, M. (2012) 'Modified intelligent driver model', *Periodica Polytechnica Transportation* Engineering, Vol. 40, No. 2, pp.53–60, https://doi.org/10.3311/pp.tr.2012-2.02

Derbel, O., Peter, T., Zebiri, H., Mourllion, B. and Basset, M. (2013) 'Modified intelligent driver model for driver safety and traffic stability improvement', *IFAC Proceedings Volumes*, Vol. 46, No. 21, pp.744–749, https://doi.org/10.3182/20130904-4-JP-2042.00132

Dömötörfi, Á., Péter, T. and Szabó, K. (2016) 'Mathematical modeling of automotive supply chain networks', *Period. Polytech. Transp. Eng.*, Vol. 44, No. 3, pp.181–186, doi: 10.3311/PPtr.9544.

Farooq, A., Xie, M., Williams, E., Gahlot, V., Yan, D. and Yi, Z. (2018) 'Downsizing strategy for cars, Beijing for people not for cars: planning for people', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 1, pp.50–57, doi: https://doi.org/10.3311/PPtr.10851

Ghadi, M., Török, Á. and Tánczos, K. (2018) 'Study of the economic cost of road accidents in Jordan', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 3, pp.129–134, doi: https://doi.org/10.3311/PPtr.10392

Iordanopoulos, P., Mitsakis, E. and Chalkiadakis, C. (2018) 'Prerequisites for further deploying ITS systems: the case of Greece', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 2, pp.108–115, doi: https://doi.org/10.3311/PPtr.11174

Istenes, G., Szauter, F. and Rödönyi, G. (2017) 'Vibration analysis of a suspension system subject to high level of measurement noise', *2017 4th International Conference on Control, Decision and Information Technologies (CoDIT)*, 5–7 April, Barcelona, Spain, doi: 10.1109/CoDIT.2017.8102707.

Jerath, K. (2010) *Impact of Adaptive Cruise Control on the Formation of Self-Organized Traffic Jams on Highway*, Master's Thesis, The Pennsylvania State University The Graduate school. Department of Mechanical and Nuclear Engineering.

Kesting, A., Treiber, M. and Helbing, D. (2008) 'Agents for traffic simulation', *Physics and Society*, Vol. 11, pp.325–356.

Koryagin, M. (2018) 'Urban planning: a game theory application for the travel demand management', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 4, pp.171–178, doi: https://doi.org/10.3311/PPtr.9410

Kovács, T., Bolla, K., Gil, R.A., Csizmás, E., Fábián, Cs., Kovács, L., Medgyes, K., Osztényi, J. and Végh, A. (2016) 'Parameters of the intelligent driver model in signalized intersections', *Technical Gazette*, Vol. 23, No. 5, October, pp.1469–1474, ISSN 1330-3651 (Print), ISSN 1848-6339 (Online) doi: 10.17559/TV-20140702174255.

Lakatos, I. and Őri, P. (2017) 'Diagnostic measurement and data evaluation for the effective performance of motor vehicles with free acceleration', *IEEE 15th International Symposium on Intelligent Systems and Informatics: SISY 2017*, 14-16 September, Szabadka, Szerbia, New York, IEEE, pp.217–222 (ISBN: 978-1-5386-3855-2).

Lakatos, A. and Mándoki, P. (2017) 'Quality evaluation of the long-distance bus and train transportation in Hungary', *Transportation Research Procedia*, Vol. 27, pp.365–372, ISSN: 2352-1465.

Lakatos, I., Peter, T., Oberling, J., Pup, D. and Szauter, F. (2016) 'An in-depth analysis of cycling and pedestrian accidents in Hungary', *ASME/IEEE International Conference on Mechatronic and Embedded Systems and Applications (MESA),* 29–31 August, Auckland New Zealand pp.1–6.

Mihály, A., Németh, B. and Gáspár, P. (2018) 'Real-time look-ahead cruise control simulator', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 1, pp.11–16, doi: https://doi.org/10.3311/PPtr.9896

Pagliara, F., Esposito, M. and Troisi, C. (2017) 'Planning and designing transport infrastructures for sustainability: the great project NAPLEST', *Periodica Polytechnica Transportation Engineering, [S.l.]*, Vol. 45, No. 2, pp.53–57, ISSN 1587-3811, https://pp.bme.hu/tr/article/view/9812, doi: https://doi.org/10.3311/PPtr.9812

Péter, T. and Bokor, J. (2011) 'New road traffic networks models for control', *GSTF International Journal on Computing*, Vol. 1, No. 2, February, pp.227–232, doi: 10.5176_2010-2283_1.2.65.

Péter, T. and Bokor, J. (2010a) 'Research for the modelling and control of traffic', *Scientific Society for Mechanical Engineering*, *33rd Fisita-World Automotive Congress: Proceedings*, Budapest, Magyarország, 30 May–4 June, Budapest: GTE. 2010, pp.66–73 (ISBN: 978-963-9058-28-6).

Péter, T. and Bokor, J. (2010b) 'Modeling road traffic networks for control', *Annual International Conference on Network Technologies and Communications: NTC. (2010*) 30 November, Thaiföld, pp.18–22, Paper 21 (ISBN: 978-981-08-7654-8).

Péter, T. and Fazekas, S. (2014) 'Determination of vehicle density of inputs and outputs and model validation for the analysis of network traffic processes', *Periodica Polytechnica Transportation* Engineering, Vol. 42, No. 1, pp.53–61 (Budapest University of Technology and Economics).

Peter, T., Fülep, T. and Bede, Z. (2011) 'The application of a new principled optimal control for the dynamic change of the road network graph structure and the analysis of risk factors', *13th EAEC European Automotive Congress,* 13–16 June, Valencia – SPAIN Society of Automotive Engineers (STA) (2011), pp.26–36 (ISBN: 978-84-615-1794-7).

Péter, T. (2012) 'Modeling nonlinear road traffic networks for junction control', *International Journal of Applied Mathematics and Computer Science (AMCS) (2012)*, Vol. 22, No. 3, pp.723–732, doi: 10.2478/v1006-012-0054-1.

Péter, T. and Lakatos, I. (2017) 'Hybrid model of vehicle and traffic for combined dynamic analysis', *Int. J. Heavy Vehicle Systems*, Vol. 24, No. 2, pp.1–14, ISSN: 1744-232X.

Péter, T., Lakatos, I. and Szauter, F. (2015) 'Analysis of the complex environmental impact on urban trajectories', *ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference: Mechatronics for Electrical Vehicular Systems*, 2–5 August, Boston, Massachusetts, USA, Paper issue DETC2015-47077, pp.V009T07A071; p.7, doi: 10.1115/DETC2015-47077, ISBN: 978-0-7918-5719-9.

Peter, T., Lakatos, I., Szauter, F. and Pup, D. (2016) 'Complex analysis of vehicle and environment dynamics', *12th IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications (MESA)*, 29–31 August, Auckland New Zealand, pp.1–7, Paper 34 (ISBN: 978-1-5090-6190-7).

Pokorádi, L. (2018) 'Graph model-based analysis of technical systems', *IOP Conf, Series: Materials Science and Engineering*, Vol. 393, No. 012007, pp.1–9, doi: 10.1088/1757-899X/393/1/012007.

Szabó, K., Szabó, G. and Renner, P. (2009) 'Emberi hibamodellezés alkalmazása a légiközlekedési kockázatelemzésekben', *Közlekedéstudományi Szemle*, Vol. 59, No. 5, pp.29–35.

Szabó, G., Szabó, K. and Zerényi, R. (2004) 'Safety management systems in transportation: aims and solutions', *Periodica Polytechnica Transportation Engineering*, Vol. 32, No. 1, pp.123–134.

Szalay Zs (2016) 'Structure and architecture problems of autonomous road vehicle testing and validation', *Proceedings of the 15th Mini Conference on Vehicle System Dynamics, Identification and Anomalies – VSDIA. (2016)*, pp.229–236, ISBN: 978-963-313-266-1.

Szalay, Z., Nyerges, A., Hamar, Z. and Hesz, M. (2017) 'Technical specification methodology for an automotive proving ground dedicated to connected and automated vehicles', *Periodica Polytechnica Transportation Engineering*, Vol. 45, No. 3, pp.168–174, doi: https://doi.org/10.3311/PPtr.10708.

Szalay, Z., Tettamanti, T., Esztergár-Kiss, D., Varga, I. and Bartolini, C. (2018) 'Development of a test track for driverless cars: vehicle design, track configuration, and liability considerations', *Periodica Polytechnica Transportation Engineering*, Vol. 46, No. 1, pp.29–35, doi: https://doi.org/10.3311/PPtr.10753

Szauter, F., Péter, T. and Lakatos, I. (2014) 'Examinations of complex traffic dynamic systems and new analysis, modeling and simulation of electrical vehicular systems', *10th IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications*, 10–12 September, Senigallia, INSPEC Accession Number: 14701078, pp.1–5, doi: 10.1109/Mesa.2014.6935613, Print ISBN: 978-1-4799-2772-2.

Takács, Á., Drexler, D.A., Galambos, P., Rudas, I.J. and Haidegger, T. (2018) 'Assessment and standardization of autonomous vehicles', *2018 IEEE 22nd International Conference on Intelligent Engineering Systems (INES)*, 21–23 June., Las Palmas de Gran Canaria, Spain, pp.185–192., ISSN: 1543-9259, doi: 10.1109/INES.2018.8523899.

Török, Á., Fedorko, G., Molnar, V., Husaková, N. and Csiszár, C. (2017) 'How to choose and when to start best ITS projects that enhance logistic performance?', *Periodica Polytechnica Transportation Engineering*, Vol. 45, No. 1, pp.8–11, doi: https://doi.org/10.3311/PPtr.9222

Treiber, M. and Helbing, D. (2002) 'Realistische mikrosimulation von straenverkehr mit einem einfachen modell', *Symposium 'Simulationstechnik', ASIM. 2002.*

Treiber, M., Hennecke, A. and Helbing, D. (2000a) 'Congested traffic states in empirical observations and microscopic simulations', *Physical Review E*, Vol. 62, No. 2, pp.1805–1824, https://doi.org/10.1103/PhysRevE.62.1805

Treiber, M., Hennecke, A. and Helbing, D. (2000b) 'Microscopic simulation of congested traffic', in Helbing, D., Herrmann, H.J., Schreckenberg, M. and Wolf, D.E. (Eds.): *Traffic and Granular Flow'99: Social, Traffic, and Granular Dynamics*, pp.365–376. https://doi.org/10.1007/978-3-642-59751-0_36

Treiber, M., Hennecke, A. and Helbing, D. (2004) 'Microscopic simulation of congested traffic', *Physical Review*, Vol. 62, pp.1805–1824.

Treiber, M., Hennecke, A. and Helbing, D. (2006) 'Delays, inaccuracies and anticipation in microscopic traffic models', *Physica A*, Vol. 360, pp.71–88.

Treiber, M. and Kesting, A. (2011) 'Evidence of convective instability in congested traffic flow: a systematic empirical and theoretical investigation', *Procedia Social and Behavioral Sciences*, Vol. 17, pp.698–716.