Reliability estimation of photovoltaic system using Markov process and dynamic programming approach

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Abstract: Adoption of solar energy is still at early stage and has failed to reach expected levels because of presence of several impeding factors lying in its diffusion path. One of the major impediments in adoption of Photovoltaic (PV) systems is low efficiency due to frequent failures. With the worldwide growth of renewable energy, the importance of the aspect of the reliability and stability is getting close attention. Present study is oriented towards deriving the reliability measures of a PV system made up of three subsystems, i.e. PV array, inverter and transformer connected in series. The behaviours of Mean Time to System Failure (MTSF), availability and cost–benefit function have been analysed graphically. Present investigation reveals that inverter failure affects the performance of PV system in significant manner. To fix this issue, Dynamic Programming Approach (DPA) has been applied and 95% reliability has been achieved.

Keywords: photovoltaic system; exponential distribution; Markov process; reliability measures; MTSF; availability; cost–benefit function; regenerative point technique; dynamic programming approach.


Biographical notes: Sonal Sindhu is pursuing PhD in adoption of photovoltaic systems in rural areas. She received her master’s (ECE) and bachelor’s degree (ECE) from MDU Rohtak, Haryana, India, in 2007 and 2011, respectively. She has published many papers in peer reviewed journals. Her area of interest is solar energy and reliability modelling. She is lifetime member of professional societies such as ISTE.

Vijay Nehra obtained BTech (ECE) from JMIT, Radaur, Kurukshetra University and ME (Electronics) from PEC, Chandigarh, in 2000 and 2002.
1 Introduction

PV generation system may be designed as a separate system known as stand-alone system or it may be tied to a grid to form an integrated system (Kumar and Padma, 2014). Grid connected PV system is the application of solar energy that has been adopted widely across the globe in significant numbers. Since 1997, the total number of grid tied PV systems installed per annum exceeds other applications of PV technology (Guerrero-Perez et al., 2014). But still a long path is ahead to reconcile grid connected PV system to be at par with traditional energy sources. It is fact that solar market has achieved tremendous growth and cost reductions in recent time yet many technical obstacles need to be overcome (Obi and Bass, 2016; Sindhu et al., 2017a; Sindhu et al., 2017b). The presence of various barriers such as low efficiency, low reliability, intermittent nature, need of storage devices, etc., impede the diffusion of solar energy (Sindhu et al., 2016a; Saheb-Koussa et al., 2009). At present, efforts are under way to foster the numerous installations of PV systems worldwide. With the penetration of renewable energy into the conventional energy field, reliability and stability concerns are arising (Ge et al., 2008).

Many potential users resist investing in solar technologies because of perception of low reliability (de Leon et al., 2016; Sindhu et al., 2016b). Two key factors impart a vital role in maintaining the exponential growth of PV systems: (1) reasonable cost and (2) minimum risk to facilitate adequate investment. Reliability is the integral component of both the factors (Kurtz et al., 2009). Therefore, it is mandatory to estimate the reliability of the PV system.

The estimation of reliability and availability may help in facilitating cost trade-off studies in relation to competing PV systems. Reliability estimation is helpful in projecting the maintenance cost over the entire lifetime of the system. Similarly estimation of availability of the system may help in providing projection of energy
generation over the year (Collins et al., 2009). Literature survey reveals that studies concerned with reliability modelling of PV system are missing. So, the present investigation is oriented towards estimating the reliability of PV systems.

Various objectives of present study have been framed which are as follows:

- To derive the expressions for various reliability and economic measures such as mean sojourn times, Mean Time to System Failure (MTSF), availability, busy period of the server and expected number of visits of the server due to repair and maintenance of the PV system.
- To obtain numerical results for various worthy measures of system effectiveness providing particular values to the involved parameters.
- To obtain cost–benefit analysis or profit analysis of the PV system.
- To enhance the reliability of the PV system using Dynamic Programming Approach (DPA).

A reliable tool is necessary to be incorporated to predict energy production from a PV system so that a sound decision on this technology may be taken. Various models have been developed in the past to predict energy production but a large amount of input data are required for implementation which are not available (De Soto, 2004). Therefore, present investigation has been carried out to perform cost–benefit analysis with needing any prior information.

2 Reliability modelling of a system

To predict performance of PV system, it is indispensible to obtain reliable knowledge and understanding of the system accurately (Zhou et al., 2010). The estimation of performance of PV system before its installation may prove to be an aid. Different techniques have been executed by scholars for estimating reliability of the complex systems, viz. Markov process method, Monte-Carlo Simulation (MCS), state enumeration, fault tree analysis, reliability block diagram, reliability indices for PV system, supplementary variable technique, and Failure Modes and Effects Analysis (FMEA) (Zhang et al., 2013; Colli, 2015; Mellit et al., 2006). Moreover, a new framework on the basis of Markov Reward Models (MRM) for integrating reliability and performance aspects of grid tied PV system has also been proposed (Zhang et al., 2013).

The present investigation focuses on the estimation of reliability of PV systems using Markov process and regenerative point technique. The brief overview of the same is as follows:

1. **Stochastic process:** It is simply a statistical process involving a number of random variables, the state space parameter (which is usually time) and the index set denoted by \( t \) and \( T \), respectively. Some basic types of stochastic processes include Markov processes, Poisson processes (such as radioactive decay), and time series, with the index variable referring to time. This indexing can be either discrete or continuous.

2. **Markov Model:** A Markov process is a stochastic process that satisfies the Markov property. In this, the state of the system is associated with the probability \( P_{ij} \) that indicates the probability to move from state \( i \) to state \( j \) which is called as
transition probability. Moreover, it is assumed that the transition probability from the state ‘\(i\)’ to ‘\(j\)’ depends entirely on these two states and is independent to all previous states except state ‘\(i\)’.

3 **Semi-Markov process**: It is stochastic process in which change of state occurs according to Markov chain and the time interval between two successive transitions is a random variable whose distribution may depend upon the state from where the transition has taken place and the state where the next transition will take place.

4 **Regenerative Process**: It is a stochastic process with time points at which, from a probabilistic point of view, the process restarts itself. A time point at which the system history prior to it is irrelevant to the system conditions is called a regenerative point. A process in which each point is regenerative is called regenerative process. The technique which is used to analyse a system with regenerative points is called regenerative point technique.

For obtaining cost–benefit analysis of the PV system, the methodology adopted has been depicted in Figure 1.

**Figure 1** Framework of present investigation

State transition diagram demonstrates all the states and its transitions precisely. Failures and repairs of PV components cause transitions between the states. This method is beneficial in modelling the outages of concerned subsystems. To obtain steady states, state transition matrix is solved. Markov model provides the steady-state probability and the duration of every state as preliminary outputs (Zhang et al., 2013). Various assumptions have been considered to carry out the present investigation such as follows:
• System consists of three subsystems, i.e. PV array as $X_1$, inverter as $X_2$ and transformer as $X_3$ unit. First two units are the main units while the third one is supporting unit. Failure of either of the first two units may lead to collapse of the system.

• There is a single server that is involved in repair activities and preventive maintenance of the system.

• Failure time distributions of the subsystems have been considered as negative exponential while the repair time distributions have been assumed as arbitrary in nature. Subsystems act as just like new ones after getting repaired.

The various possible transition states in system model have been depicted in Figure 2.

**Figure 2** State transition diagram

3 **Materials and Methodology**

Research methodology adopted in the present investigation is to calculate the various relevant parameters mathematically using standard formulae following the steps sequentially shown in framework. The various relevant parameters are presented as follows.
3.1 Transition probabilities

Simple probabilistic considerations yield the following expressions for the non-zero elements as:

\[ P_i = Q_i (x) = \int_0^\infty q_i (t) dt \]  \hfill (1)

The various transition probabilities have been presented in Appendix 1.

3.2 Mean sojourn times

The mean sojourn times \( \mu_i \) in the state \( S_i \) is given using equation (2) as follows:

\[
\begin{align*}
\mu_0 &= m_{01} + m_{02} + m_{03} + m_{07}, \\
\mu_1 &= m_{10}, \\
\mu_2 &= m_{20}, \\
\mu_3 &= m_{30} + m_{31} + m_{35} + m_{36}, \\
\mu_4 &= m_{31}, \\
\mu_5 &= m_{32}, \\
\mu_6 &= m_{33}, \\
\mu_7 &= m_{30}, \\
\mu'_5 &= m_{30} + m_{34} + m_{36} + m_{32.5}, \\
\mu'_6 &= m_{31} + m_{36} + m_{31.4}, \\
\mu'_7 &= m_{30} + m_{31.4} + m_{36} + m_{32.5}
\end{align*}
\]  \hfill (2)

3.3 Reliability

Let \( \varphi_i (t) \) denotes the CDF of first two passage times from regenerative state \( S_i \) to a failed state. Considering the failed state as absorbing state, the various recursive relations may be drawn for \( \varphi_i (t) \) using equation (3) as follows:

\[
\begin{align*}
\varphi_0 (t) &= Q_{00} (t) + Q_{01} (t) + Q_{02} (t) + Q_{07} (t) + Q_{03} (t) + Q_{05} (t) + Q_{06} (t) \\
\varphi_1 (t) &= Q_{10} (t) + Q_{01} (t) + Q_{02} (t) + Q_{07} (t) + Q_{13} (t) + Q_{16} (t)
\end{align*}
\]  \hfill (3)

After taking LST of equation (3) and solving for \( \varphi'' (s) \), yields:

\[ R' (s) = \frac{1-\varphi'' (s)}{s} \]  \hfill (4)

For obtaining reliability of the system model, LT of equation (4) is taken.

3.3.1 Mean time to system failure

The MTSF may be obtained using equation (5) as follows:

\[ \text{MTSF} = \lim_{s \to 0} \frac{1-\varphi'' (s)}{s} = \frac{N}{D} \]  \hfill (5)

Here,

\[ N = P_{03} \mu_5 + \mu_6 \text{ and } D = 1 - P_{03} P_{30} \]  \hfill (6)
3.4 Steady-state availability

Let $A_i(t)$ denotes the probability that the system is in upstate at any instant ‘$t$’ provided that the system entered regenerative state $S_i$ at $t = 0$. The steady-state availability is expressed using equation (7) as follows:

$$A_i(\infty) = \lim_{s \to 0} s A_i(s) = \frac{N_i}{D_i}$$  \hspace{1cm} (7)

where:

$$N_i = \mu_0 (1 - P_{30} P_{60}) + \mu_3 P_{60}$$

$$D_i = P_{60} (\mu_i^* + P_{60} \mu_3) + (1 - P_{30}) (\mu_i + P_{60} \mu_3) + \mu_1 (P_{03} P_{354} + (1 - P_{30}) P_{00})$$

+ $\mu_2 (P_{03} P_{325} - P_{36} + (1 - P_{36}) P_{02})$  \hspace{1cm} (8)

3.5 Busy period analysis for server

The busy period of the server is calculated for two stages, i.e. whether server is busy with repair activity of the system or in performing activities related to preventive maintenance to avoid failures.

3.5.1 Busy period due to repair

Let $B^{R_r}_i(t)$ denotes the probability that the server is busy in repairing the unit at any instant ‘$t$’ provided that the system entered regenerative state $S_i$ at $t = 0$. The time for which server is busy due to repair activities is given using equation (9) as follows:

$$B^{R_r}_i(\infty) = \lim_{s \to 0} s B^{R_r}_i(s) = \frac{N_2}{D_i}$$  \hspace{1cm} (9)

where:

$$N_2 = W^{R_r}_1 (0) (P_{01} + P_{05} P_{314} - P_{01} P_{60} P_{00}) + W^{R_r}_2 (0) (P_{02} + P_{05} P_{323} - P_{02} P_{36})$$

+ $W^{R_r}_3 (0) P_{60}$ and $D_i$ is already mentioned.  \hspace{1cm} (10)

3.5.2 Busy period due to preventive maintenance

Let $B^{M}_i(t)$ refers the probability that the server is busy in preventive maintenance of the unit at any instant ‘$t$’ provided that the system entered regenerative state $S_i$ at $t = 0$. The time for which the server is busy due to maintenance is given using equation (11).

$$B^{M}_i(\infty) = \lim_{s \to 0} s B^{M}_i(s) = \frac{N_j}{D_i}$$  \hspace{1cm} (11)

where:

$$N_j = W^{M}_7 (0) (P_{30} - P_{07} P_{30} P_{30}) + W^{M}_6 (0) (P_{03} P_{36})$$ and $D_i$ is already mentioned.  \hspace{1cm} (12)
3.6 Expected number of visits of server

It is calculated in two stages, i.e. whether the server visited for repair activity of the system or for performing preventive maintenance of the system to avoid failures.

3.6.1 Expected number of visits of the server due to repair

Let \( R_i(t) \) denotes the expected number of repairs by the server in \((0, t]\) provided that the system entered the regenerative state \( S_i \) at \( t = 0 \). The expected number of repairs per unit time by the server is expressed using equation (13) as follows:

\[
R_i(\infty) = \lim_{S \to 0} sR_i^* (s) = \frac{N_i}{D_i}
\]

(13)

where:

\[
N = \left(P_{01} + P_{02}\right)(1 - P_{36}) + P_{03}\left(P_{30} + 2P_{31} + 2P_{32.5}\right) \text{ and } D_i \text{ is already mentioned.} \quad (14)
\]

3.6.2 Expected number of visits of the server due to preventive maintenance

Let \( M_i(t) \) denotes the expected number of preventive maintenance of the unit by the server in \((0, t]\) given that the system entered the generative state \( S_i \) at \( t = 0 \). The expected number of maintenance per unit time by the server is given using equation (15):

\[
M_i(\infty) = \lim_{S \to 0} sM_i^* (s) = \frac{N_i}{D_i}
\]

(15)

where:

\[
N = P_{07}(1 - P_{36}) + P_{03}P_{36} \text{ and } D_i \text{ is already mentioned.} \quad (16)
\]

3.7 Profit analysis

The profit incurred to the system model in steady state is obtained using equation (17).

\[
P = K_0 A_0 - K_1 B_{0i}^R - K_2 B_{0i}^M - K_3 R_0 - K_4 M_0
\]

(17)

Here,

\( P = \) Profit of the system model

\( K_0 = \) Revenue per unit up-time of the system

\( K_1 = \) Cost per unit time for server being busy in performing repair activity

\( K_2 = \) Cost per unit time for server being busy in performing preventive maintenance

\( K_3 = \) Cost per unit time repair

\( K_4 = \) Cost per unit time preventive maintenance
3.8 Dynamic programming approach (DPA)

This approach is utilised to improve the reliability of the system. The reliability of the PV system may be enhanced by utilising any of the following methods (Mishra and Joshi, 1996):

- Parallel or hot redundancy may be used which facilitates the utilisation of one or more spare components, i.e. if one component fails, system still operate.
- Standby or cold redundancy, i.e. if a component gets failed, the other automatically comes into operation and system remains operational.

Hot redundancy gives better results than cold redundancy. Now, it is important to recognise how many components must be connected in parallel to each subsystem to achieve optimum reliability. This purpose may be served using DPA as it is one of the best techniques. For applying DPA, data presented in Table 1 have been utilised.

### Table 1: Data for parallel redundancy

<table>
<thead>
<tr>
<th>$i$</th>
<th>$r_1$</th>
<th>$c_1$</th>
<th>$r_2$</th>
<th>$c_2$</th>
<th>$r_3$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>3</td>
<td>0.94</td>
<td>3</td>
<td>0.95</td>
<td>5</td>
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<tr>
<td>2</td>
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<td>4</td>
<td>0.96</td>
<td>5</td>
<td>0.97</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
<td>6</td>
<td>0.98</td>
<td>7</td>
<td>0.99</td>
<td>8</td>
</tr>
</tbody>
</table>

Note: $m_i$ = Number of parallel units placed with the $i$th component; $r_i$ = reliability of the $i$th component; $c_i$ = cost of the $i$th component.

The data mentioned in Table 1 are the hypothetical data which have been obtained by taking the close approximation of grid connected PV system.

Let $x_i$ = Total capital allocated to the $i$th stage

$R(c_i)$ = Reliability $r_i$ at cost $c_i$

$f_i(x_i)$ = Reliability of the components (stages)

Since $x_i$ denotes the various stages of the system, the recursive equations are given by:

$$f_i(x_i) = \max_{r_i} \{r_i(c_i)\} \quad \forall m_i = 1, 2, 3 ; 0 \leq c_i \leq x_i$$  \hspace{1cm} (18)

$$f_i(x_i) = \max_{r_i} \{r_i(c_i).f_{i-1}(x_i - c_i)\} \quad 0 \leq c_i \leq x_i$$  \hspace{1cm} (19)

Here, $i = 2, 3 \ldots$

After utilising the specified material and methodology, several results have been obtained discussed in following section.

### 4 Results and discussion

Results and discussions are given in two segments. First segment discusses the results from the Markov process and regenerative point technique and the second one discusses the results in relation to DPA.
4.1 Interpretation of results of Markov process

The present study is oriented towards evaluation of relevant reliability measures of grid tied PV system using the ideas of preventive maintenance. Estimation of system reliability characteristics is based on the Markov state diagram developed from individual hardware component reliability. The results for particular cases \( g_1(t) = \theta e^{-\theta t} \), \( g_2(t) = \alpha e^{-\alpha t} \), \( g_3(t) = \beta e^{-\beta t} \), and \( f(t) = \gamma e^{-\gamma t} \) are obtained for depicting the behaviour of MTSF, availability and profit function graphically for arbitrary values of involved parameters and costs as depicted in Figures 3–5, respectively.

Table 2 shows values of MTSF after varying failure rate of inverter \( (\lambda_2) \) and its repair rate. It can be deduced from Figure 3 that MTSF is relatively insensitive to failure rate of inverter as compared to availability and profit function. It coincides with the decreasing trend of MTSF with respect to rate of undergoing system for preventive maintenance \( (x) \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \lambda_1 = 0.0001, \lambda_2 = 0.001, \lambda_3 = 0.00001, \theta = 4, \beta = 3, \alpha = 2, \gamma = 5 )</th>
<th>( \lambda_2 = 0.005 )</th>
<th>( \beta = 2 )</th>
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</thead>
<tbody>
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<td>0.327323</td>
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<tr>
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</tr>
<tr>
<td>0.14</td>
<td>0.318361</td>
<td>0.317956</td>
<td>0.467052</td>
</tr>
</tbody>
</table>

After varying the parameters, obtained graph for MTSF has been depicted in Figure 3.

**Figure 3** MTSF vs. the rate of undergoing preventive maintenance \( (x) \)

It is evident that availability and profit functions show a decreasing trend with increasing failure rate of inverter and system undergoing preventive maintenance \( (x) \) rate. Figures 4
and 5 depict that PV system is sensitive to the changes in repair rate of inverter ($\beta$). It shows a significant change by decreasing the repair rate. MTSF, availability and profit function decrease with the decrease in repair rate of inverter. The repair rates of PV array and transformer do not show noticeable change until a change in significant number therefore they have not been shown. Table 3 depicts the variations of availability function due to changes in failure rate and repair rate of inverter.

Table 3  Variations of parameters for availability

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\lambda_1 = 0.0001, \lambda_2 = 0.001, \lambda_3 = 0.00001, \theta = 4, \beta = 3, \alpha = 2, \gamma = 5$</th>
<th>$\lambda_2 = 0.005$</th>
<th>$\beta = 2$</th>
</tr>
</thead>
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<td>0.973573</td>
<td>0.976955</td>
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</table>

After varying the parameters, obtained graph for availability has been illustrated in Figure 4 as follows.

Figure 4   Availability vs. the rate of undergoing preventive maintenance ($x$)

The study reveals that the performance of the system depends largely upon the failures of inverter which is the important part of the PV system. It is important to ensure the proper functioning, repair and preventive maintenance of the inverters for enhancing the reliability of the system. Table 4 depicts the variations of profit function due to changes in failure rate and repair rate of inverter.
Reliability estimation of photovoltaic system

Table 4 Variations of parameters for profit function

<table>
<thead>
<tr>
<th>X</th>
<th>$\lambda_1 = 0.0001$, $\lambda_2 = 0.001$, $\lambda_3 = 0.00001$, $\theta = 4$, $\beta = 3$, $\alpha = 2$, $\gamma = 5$</th>
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</table>

After varying the parameters, obtained graph for profit function has been depicted in Figure 5 as follows.

Figure 5 Profit vs. the rate of undergoing preventive maintenance ($x$)

The most important observation regarding PV systems that makes study of reliability and availability as integral research activity is high value of the Mean Time to Repair (MTTR) because of absence of competent workforce, spares parts and supporting tools nearby the plant (Zhang et al., 2013). Therefore, failure in PV system causes complete failure of the service for longer period of time. This in turn increases the overall payback period of the system. Therefore, it is mandatory to fix the issue as early as possible.

4.2 Interpretation of the results of dynamic programming approach

Let the total cost available be 12 units. Accordingly the cost availability of DPA has been obtained as depicted in Tables 5–7.
Table 5  First iteration ($i = 1$) for parallel redundancy

<table>
<thead>
<tr>
<th>$x_j$</th>
<th>$m_1 = 1$, $r_1 = 0.15$, $c_1 = 3$</th>
<th>$m_2 = 2$, $r_1 = 0.30$, $c_1 = 4$</th>
<th>$m_3 = 3$, $r_1 = 0.45$, $c_1 = 6$</th>
<th>$f_1(x_1)$</th>
<th>$m_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>–</td>
<td>–</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.30</td>
<td>–</td>
<td>0.30</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.30</td>
<td>–</td>
<td>0.30</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
<td>0.45</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5 illustrates that first component which is PV array must be repeated three times to increase reliability. It is also well-known fact that increasing the modules also increases the overall output.

Table 6  Second iteration ($i = 2$) for parallel redundancy

<table>
<thead>
<tr>
<th>$x_j$</th>
<th>$m_1 = 1$, $r_1 = 0.94$, $c_1 = 3$</th>
<th>$m_2 = 2$, $r_1 = 0.96$, $c_1 = 5$</th>
<th>$m_3 = 3$, $r_1 = 0.98$, $c_1 = 7$</th>
<th>$f_1(x_1)$</th>
<th>$m_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>0.94(–)</td>
<td>–</td>
<td>–</td>
<td>0.141</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.94(–)</td>
<td>–</td>
<td>–</td>
<td>0.141</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.94(–)</td>
<td>0.96(–)</td>
<td>–</td>
<td>0.141</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.94(0.15) = 0.141</td>
<td>0.96(–)</td>
<td>–</td>
<td>0.141</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.94(0.15) = 0.141</td>
<td>0.96(–)</td>
<td>0.98(–)</td>
<td>0.141</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.94(0.30) = 0.282</td>
<td>0.96(0.15) = 0.144</td>
<td>0.98(–)</td>
<td>0.282</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0.94(0.45) = 0.423</td>
<td>0.96(0.30) = 0.288</td>
<td>0.98(–)</td>
<td>0.282</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.94(0.45) = 0.423</td>
<td>0.96(0.30) = 0.288</td>
<td>0.98(0.15) = 0.147</td>
<td>0.282</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0.94(0.45) = 0.423</td>
<td>0.96(0.45) = 0.432</td>
<td>0.98(0.30) = 0.294</td>
<td>0.432</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>0.94(0.45) = 0.423</td>
<td>0.96(0.45) = 0.432</td>
<td>0.98(0.30) = 0.294</td>
<td>0.432</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6 depicts that second component must be repeated two times in order to increase the reliability. Markov process has proved that inverter failures are frequent in PV systems so repeating it two times increases the reliability of the system.
Table 7 Third iteration \((i = 3)\) for parallel redundancy

<table>
<thead>
<tr>
<th>(x_3)</th>
<th>(m_1 = 1, r_1 = 0.95, c_1 = 5)</th>
<th>(m_2 = 2, r_1 = 0.97, c_1 = 7)</th>
<th>(m_3 = 3, r_1 = 0.99, c_1 = 8)</th>
<th>Max. reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>0.95(–)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>0.95(–)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>0.95(–)</td>
<td>0.97(–)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>0.95(–)</td>
<td>0.97(–)</td>
<td>0.99(–)</td>
<td>–</td>
</tr>
<tr>
<td>9</td>
<td>0.95(–)</td>
<td>0.97(–)</td>
<td>0.99(–)</td>
<td>–</td>
</tr>
<tr>
<td>10</td>
<td>0.95(–)</td>
<td>0.97(–)</td>
<td>0.99(–)</td>
<td>–</td>
</tr>
<tr>
<td>11</td>
<td>0.95(1) = 0.95</td>
<td>0.97(–)</td>
<td>0.99(–)</td>
<td>0.95</td>
</tr>
<tr>
<td>12</td>
<td>0.95(1) = 0.95</td>
<td>0.97(–)</td>
<td>0.99(–)</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 7 shows that third component is sufficient for one time. It is a supporting device and using it once is sufficient. The overall 95% reliability is obtained using PV array three times, inverter two times and transformer one time.

5 Concluding remarks

Electricity is not just a necessary commodity. In fact, it is the lifeblood of the economy and our standard of living. Now no option is left to fail to meet the expectations of society for making available power in low cost. As the world moves into the digital age, our dependency on power quality will grow accordingly. Therefore, studies related to the estimation of reliability modelling of PV systems can prove a milestone. To sum up, certain conclusions have been drawn as follows:

- The accelerating growth of the PV industry may take it to peak position in less than a decade but it needs enhanced reliability.
- To recapitulate, it is mentioned that the failure rate of inverters is the dominating cause of hindrance in penetration of solar energy. This component needs attention and focus.
- Inverters have been estimated as the weakest link in the PV system. However, it is projected that the power delivered to the grid is high over the entire life time. If inverter reliability is increased, it can lower the corrective maintenance costs of the system.
• Modern grid-interactive inverters capable of providing stabilised voltage, regulated frequency, storage abilities and utilisation of modern communications protocols at lower cost may boost the adoption of PV technology. The new generation of inverters may be known as ‘smart inverters’ (Obi and Bass, 2016).

• Present study reveals that inverter failure greatly affects the performance of the PV systems and it can be reduced by using the parallel redundancy.

• The reliability obtained in present study is 0.95 by using the dynamic approach which finds wide application in any electronics industry.

It is worth noting that several factors such as technical, economic, social, environmental, and political barriers exist in developmental path of solar energy which need to be addressed in appropriate manner. This study is an effort towards providing the technical solution to enhance the reliability of PV systems. Present study uses the hypothetical data for performing dynamic approach. If the real-time data are applied, appropriate results may be obtained for PV systems. Future work may be concentrated towards increasing the reliability of PV systems by utilising any of the following methods (Mishra and Joshi, 1996).

• Standby or cold redundancy, i.e. if a component fails, the other automatically comes into operation and system remains operational.

• The results obtained with the cold and hot redundancy may be compared to get the appropriate concept.

• Present work may be extended for standby systems for residential or commercial application.

Present study may provide a development path for the penetration of solar energy and help in achievement of solar missions. It also provides cost–benefit analysis of PV systems, which is the important factor for their adoption. Moreover, it provides cost reliability trade-off analysis using dynamic approach which is novel contribution in PV field. This study presents supporting features in agreement to the current scenario of going solar.

References
Reliability estimation of photovoltaic system


Nomenclature

The various notations utilised in the present study are as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_i$</td>
<td>Constant failure rate of the $X_i$ unit</td>
</tr>
<tr>
<td>$g_i(t)/G_i(t)$</td>
<td>PDF/CDF of repair time of $X_i$ unit</td>
</tr>
<tr>
<td>$f(t)/F(t)$</td>
<td>PDF/CDF of preventive maintenance time of the system</td>
</tr>
<tr>
<td>$X$</td>
<td>The rate of undergoing preventive maintenance of the system</td>
</tr>
<tr>
<td>SUm</td>
<td>The system is under preventive maintenance</td>
</tr>
<tr>
<td>Wr</td>
<td>The unit has failed and is waiting to get repaired</td>
</tr>
<tr>
<td>Ur</td>
<td>The unit has failed and is under repair activity</td>
</tr>
<tr>
<td>UR</td>
<td>The unit has failed and is under repair continuously from last state</td>
</tr>
<tr>
<td>$q_{ij}(t)/Q_{ij}(t)$</td>
<td>PDF/CDF of direct transition time from regenerative state $S_i$ to another state $S_j$ or to a failed state $S_j$ without visiting any other regenerative state in $(0, t]$</td>
</tr>
<tr>
<td>$q_{ij,k}(t)/Q_{ij,k}(t)$</td>
<td>PDF/CDF of direct transition time from regenerative state $S_i$ to a state $S_j$ or to a failed state $S_j$ visiting state $S_k$ once in $(0, t]$</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>The mean sojourn time in the state $S_i$ which is expressed as: $\mu_i = E(T) = \int_0^\infty P(T &gt; t) dt = \sum_j m_{ij}$</td>
</tr>
</tbody>
</table>

Here $T$ stands for the time to system failure.

| $m_{ij}$ | Contribution to mean sojourn time ($\mu_i$) in state $S_i$ when system transits directly to state $S_j$ so that: $\mu_i = y \sum_j m_{ij}$ and $m_{ij} = \int_0^\infty Q_{ij}(t) = -q_{ij}^*(0)$ |

$\&/©$ Laplace–Stieltjes convolution/Laplace convolution

$*/**$ Laplace Transformation (LT)/Laplace–Stieltjes Transformation (LST)
Appendix 1

1 Transition probabilities

\[ P_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + x}, \quad P_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3 + x}, \quad P_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3 + x} \]

\[ P_{07} = \frac{x}{\lambda_1 + \lambda_2 + \lambda_3 + x}, \quad P_{10} = g_1^*(0), \quad P_{20} = g_2^*(0), \quad P_{30} = g_3^*(\lambda_1 + x + \lambda_2) \]

\[ P_{34} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + x}(1 - g_3^*(\lambda_1 + x + \lambda_2)), \quad P_{35} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3 + x}(1 - g_3^*(\lambda_1 + x + \lambda_2)) \]

\[ P_{36} = \frac{x}{\lambda_1 + \lambda_2 + \lambda_3 + x}(1 - g_3^*(\lambda_1 + x + \lambda_2))(g_3^*(0)), \quad P_{41} = g_1^*(0), \quad P_{52} = g_2^*(0), \quad P_{63} = P_{30} = f^*(0) \]

(1.1)

\[ P_{32.5} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3 + x}(1 - g_3^*(\lambda_1 + x + \lambda_2))(g_3^*(0)) \]

\[ P_{31.4} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3 + x}(1 - g_3^*(\lambda_1 + x + \lambda_2))(g_3^*(0)) \]

It can be easily verified that:

\[ P_{01} + P_{02} + P_{03} + P_{07} = P_{30} + P_{34} + P_{35} + P_{36} = P_{30} + P_{34} + P_{35} + P_{32.5} = 1 \]

\[ P_{30} + P_{31.4} + P_{35} + P_{36} = P_{30} + P_{36} + P_{31.4} + P_{32.5} = 1 \]

\[ P_{30} = P_{39} = P_{35} = P_{36} = P_{30} = 1 \]

2 Availability

\[ A_0(t) = M_n(t) + q_01(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t) + q_{07}(t) \odot A_7(t) \]

\[ A_1(t) = q_{10}(t) \odot A_0(t) \]

\[ A_2(t) = q_{20}(t) \odot A_0(t) \]

\[ A_3(t) = M_2(t) + q_{30}(t) \odot A_0(t) + q_{31.4}(t) \odot A_4(t) + q_{36}(t) \odot A_6(t) + q_{32.5}(t) \odot A_7(t) \]

(1.2)

\[ A_4(t) = q_{40}(t) \odot A_0(t) \]

\[ A_5(t) = q_{50}(t) \odot A_0(t) \]

where:

\[ M_n(t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \gamma)t} \quad \text{and} \quad M_2(t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \gamma)t} \]  

(1.3)

Take LT of equations (1.2)–(1.3) and solve for \( A_0(s) \) and put in equation (7).
3 Busy period due to repair

The recursive relations for $B^R_t(t)$ may be summarised as follows:

$$B^R_0(t) = q_{01}(t)\otimes B^R_0(t) + q_{02}(t)\otimes B^R_1(t) + q_{03}(t)\otimes B^R_2(t) + q_{07}(t)\otimes B^R_7(t)$$

$$B^R_1(t) = W^R_1(t) + q_{20}\otimes B^R_0(t)$$

$$B^R_2(t) = W^R_2(t) + q_{20}\otimes B^R_1(t)$$

$$B^R_i(t) = W^R_i(t) + q_{30}(t)\otimes B^R_{i-1}(t) + q_{314}(t)\otimes B^R_{i-2}(t) + q_{36}(t)\otimes B^R_{i-2}(t) + q_{325}(t)\otimes B^R_{i-3}(t)$$

(1.4)

$$B^R_t(t) = q_{31}(t)\otimes B^R_0(t)$$

$$B^R_t(t) = q_{30}(t)\otimes B^R_0(t)$$

where:

$$W^R_1(t) = \overline{G_1(t)} dt, W^R_2(t) = \overline{G_2(t)} dt$$

$$W^R_i(t) = \overline{G_i(t) + \left(\lambda e^{-\zeta_0/(\lambda + \zeta_0 + \gamma)}\right)\overline{G_i(t)} + \left(\lambda e^{-\zeta_0/(\lambda + \zeta_0 + \gamma)}\right)\overline{G_i(t)}}$$

(1.5)

Take LT of equations (1.4)–(1.5) and solve for $B^R_0(s)$ and put in equation (9).

4 Busy period due to preventive maintenance

The recursive relations for $B^M_t(t)$ may be expressed as follows:

$$B^M_0(t) = q_{01}(t)\otimes B^M_0(t) + q_{02}(t)\otimes B^M_1(t) + q_{03}(t)\otimes B^M_2(t) + q_{07}(t)\otimes B^M_7(t)$$

$$B^M_1(t) = q_{10}\otimes B^M_0(t)$$

$$B^M_2(t) = q_{20}\otimes B^M_1(t)$$

$$B^M_i(t) = q_{30}(t)\otimes B^M_{i-1}(t) + q_{314}(t)\otimes B^M_{i-2}(t) + q_{36}(t)\otimes B^M_{i-2}(t) + q_{325}(t)\otimes B^M_{i-3}(t)$$

(1.6)

$$B^M_t(t) = q_{31}(t)\otimes B^M_0(t)$$

$$B^M_t(t) = q_{30}(t)\otimes B^M_0(t)$$

where:

$$W^M_0(t) = W^M_1(t) = F(t) dt$$

(1.7)

Take LT of equations (1.6)–(1.7) and solve for $B^M_0(s)$ and put in equation (11).
5 Expected number of visits of the server due to repair

The recursive relations for \( R_i(t) \) may be written as follows:

\[
R_0(t) = Q_{00}(t) & R_1(t) + Q_{01}(t) & R_2(t) + Q_{02}(t) & R_3(t) + Q_{03}(t) & R_4(t) + Q_{04}(t) \& R_5(t)
\]
\[
R_1(t) = Q_{00}(t) & (1 + R_0(t))
\]
\[
R_2(t) = Q_{00}(t) & (1 + R_0(t))
\]
\[
R_3(t) = Q_{00}(t) & (1 + R_0(t)) + Q_{11}(t) & (1 + R_1(t)) + Q_{12}(t) & R_4(t) + Q_{13}(t)
\]
\[
R_4(t) = Q_{00}(t) & (1 + R_0(t)) + Q_{22}(t) & (1 + R_2(t))
\]
\[
R_5(t) = Q_{00}(t) & (1 + R_0(t)) + Q_{33}(t) & (1 + R_3(t)) + Q_{44}(t)
\]
\[
(1.8)
\]

Take LST of equation (1.8) and solve for \( R_s^*(s) \) and put in equation (13).

6 Expected number of visits of the server due to preventive maintenance

The recursive relations for \( M_i(t) \) may be summarised as follows:

\[
M_0(t) = Q_{00}(t) & M_1(t) + Q_{01}(t) & M_2(t) + Q_{02}(t) & M_3(t) + Q_{03}(t) & M_4(t) + Q_{04}(t) & M_5(t)
\]
\[
M_1(t) = Q_{00}(t) & M_6(t)
\]
\[
M_2(t) = Q_{00}(t) & M_7(t)
\]
\[
M_3(t) = Q_{00}(t) & (1 + M_1(t)) + Q_{11}(t) & (1 + M_2(t)) + Q_{12}(t) & M_6(t) + Q_{13}(t) & M_7(t)
\]
\[
(1.9)
\]

Take LST of equation (1.9) and solve for \( M_s^*(s) \) and put in equation (15).