Explicit optimal hysteretic damper design in elastic-plastic structure under double impulse as representative of near-fault ground motion

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Abstract: A closed-form maximum elastic-plastic response is derived for a structure with hysteretic dampers under the critical double impulse as a representative of near-fault ground motion. It is shown that the classification into nine cases is possible depending on the combination of elastic-plastic responses of the structure and the hysteretic damper and the double impulse enables the derivation of the closed-form maximum elastic-plastic critical response of the controlled structure by using the energy balance in each case. It is demonstrated that the closed-form maximum elastic-plastic critical response is quite useful for finding an optimal hysteretic damper quantity depending on the input level of the double impulse.

Keywords: optimal design; hysteretic damper; elastic-plastic response; near-fault ground motion; structural control; double impulse.

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1 Introduction

Structural control using passive dampers is becoming a popular and promising strategy in the structural design of infrastructures in earthquake-prone countries where earthquake resilience and continuing use of buildings are strongly desired (Hanson, 1993; Aiken et al., 1993; Nakashima et al., 1996; Soong and Dargush, 1997; Hanson and Soong, 2001; Takewaki, 2009; Lagaros et al., 2013). In the 2016 Kumamoto earthquake in Japan, severe shaking over the Japan Meteorological Agency (JMA) seismic intensity six was repeated during a short time span and the control of responses within a limited plastic deformation is strongly desired.

Most structural designers are looking for sophisticated methods for finding the most efficient location and quantity of passive dampers, see for example Xia and Hanson (1992), Inoue and Kuwahara (1998), Uetani et al. (2003), Takewaki (2009), Aydin et al. (2007), Lavan and Levy (2010), Adachi et al. (2013a, 2013b) and Fujita et al. (2014). In the design using linear and nonlinear viscous dampers, many useful methods have been developed (Uetani et al., 2003; Takewaki, 2007; Aydin et al., 2007; Attard, 2007; Lavan and Levy, 2010; Adachi et al., 2013a, 2013b). On the other hand, in the design using hysteretic dampers, a limited number of methods has been proposed due to its complex characteristics (Uetani et al., 2003; Murakami et al., 2013). Inoue and Kuwahara (1998) proposed a formula for the optimal hysteretic damper quantity by introducing the concept of equivalent viscous damping (Jacobsen, 1960; Caughey, 1960). Lavan and Levy (2010) developed an optimal design method based on the optimality condition. Murakami et al. (2013) presented a general sensitivity-based approach which is applicable to various kinds of dampers.

In contrast to viscous dampers, hysteretic dampers exhibit residual deformation and complex hysteretic rules have to be considered in the response evaluation. The nonlinear properties of hysteretic dampers are similar to those of friction-damped ones (Pall and Marsh, 1982; Austin and Pister, 1985; Filiatrault and Cherry, 1990; Cherry and Filiatrault, 1993; Ciampi et al., 1995).
Most researches on hysteretic dampers are using numerical optimisation methods because of the necessity of time-history response analysis for response evaluation, and a lot of computational works are required in order to make clear the intrinsic properties of the optimal damper location and quantity. In contrast to these approaches, the present paper derives a closed-form solution of the maximum elastic-plastic response of a single-degree-of-freedom (SDOF) system with hysteretic dampers under the critical near-fault ground motion modelled by ‘double impulse’ (Kojima and Takewaki, 2015). Then, an explicit optimisation is performed using this closed-form solution. Through the comparison with the conventional optimal design formula by Inoue and Kuwahara (1998), the property of the design derived from the proposed optimisation method is clarified.

2 Frame model with hysteretic damper and its modelling into SDOF system with hysteretic damper

Consider a SDOF main frame of stiffness $K_F$ and mass $m$ as shown in Figure 1(a). The frame includes a hysteretic damper in parallel. The frame and the hysteretic damper possess different bilinear hysteretic properties as shown in Figure 2 and the total shear force-deformation relation can be expressed as shown in Figure 1(b). The damper initial stiffness is expressed by $kK_F$ with the stiffness ratio $k$. The yield deformation of frame is denoted by $d_{Fy}$ and that of damper is expressed by $d_{Dy}$. $Q_u$ is the total strength of the frame and damper and $\beta$ is the ratio of the damper strength to $Q_u$.

![Figure 1](a) SDOF system (b) Hysteretic relation (see online version for colours)
3 Conventional formula for optimal quantity of hysteretic damper

The well-known comparative work is one by Inoue and Kuwahara (1998). They proposed an optimal hysteretic damper quantity $\beta_{\text{opt}}$ by introducing the concept of equivalent viscous damping. They derived the optimal damper quantity $\beta_{\text{opt}}$ by requiring the maximisation of damping ratio.

$$\beta_{\text{opt}} = 1 - \frac{1}{\sqrt{k+1}}$$  \hspace{1cm} (1)

In their theory, it is assumed that the maximum damping ratio minimises the deformation under an earthquake ground motion and corresponds to the optimal damper quantity. It can be understood that their formula does not depend on the input level. This aspect will be discussed in detail in this paper.

4 Modelling of near-fault ground motion into double impulse

In this paper, a principal part of a near-fault ground motion is modelled into a one-cycle sinusoidal wave (Kalkan and Kunnath, 2006) and then simplified into a double impulse (Kojima and Takewaki, 2015) as shown in Figure 3. The ground motion in Figure 3 is the fault-normal component at the Renaldi station during the Northridge earthquake in 1994. This is because the double impulse in the form of shock has a simple characteristic and a straightforward expression of the maximum response can be expected even for nonlinear responses based on an energy balance in free vibrations.

Consider a ground acceleration $\ddot{u}_g(t)$ as a double impulse, as shown in Figure 3(a), expressed by
\[ \ddot{u}_g(t) = V\dot{\delta}(t) - V\dot{\delta}(t - t_0) \]  

(2)

where \( V \) is the given initial velocity (also the second velocity with an opposite sign) and \( t_0 \) is the time interval between two impulses. The time derivative is denoted by an over-dot. The comparison with the corresponding one-cycle sinusoidal wave is plotted in Figure 3(b). The corresponding velocity and displacement of such double impulse and sinusoidal wave can also be found in the reference (Kojima and Takewaki, 2015). It has been confirmed that the double impulse is a good approximation of the corresponding sinusoidal wave even in the form of velocity and displacement.

The Fourier transform of the acceleration \( \ddot{u}_g(t) \) of the double impulse can be derived as

\[
\hat{\ddot{u}}_g(\omega) = \int_{-\infty}^{\infty} \left\{ V\dot{\delta}(t) - V\dot{\delta}(t - t_0) \right\} e^{-i\omega t} dt = V(1 - e^{-i\omega t_0})
\]

(3)

Figure 3  Modelling of principal part of near-fault ground motion into one-cycle sinusoidal wave and re-modelling into double impulse (see online version for colours)

5  Closed-form maximum elastic-plastic response of SDOF system with hysteretic damper under critical double impulse

Since the consideration of response under resonance provides a safer design, the critical double impulse is taken into account in this paper. A closed-form maximum elastic-plastic response of an SDOF elastic-perfectly plastic model under the critical double impulse has been derived (Kojima and Takewaki, 2015). In that model, the timing of the second impulse in the critical input has been proved to be the time with zero restoring force after the first impulse. It can be shown by the energy balance that this property is also retained in the present model of the trilinear hysteretic model. In this section, the closed-form maximum elastic-plastic response of a normal trilinear hysteretic SDOF model (without structural damping) under the critical double impulse will be derived.
Nine response cases can be posed depending on the input level as shown in Figure 4.

- **Case 1** shows the elastic response both for the frame and damper.
- **Case 2** is the inelastic response (damper) only after the second impulse.
- **Case 3** is the inelastic response (damper and frame) only after the second impulse.
- **Case 4-1** is the inelastic response (damper) after the first and second impulses and the second impulse acts at the damper unloading stage.
- **Case 4-2** is the inelastic response (damper) after the first and second impulses and the second impulse acts at the damper re-yielding stage.
- **Case 5-1** is the inelastic response (damper) after the first impulse and the inelastic response (damper and frame) after the second impulse. The second impulse acts at the damper unloading stage.
- **Case 5-2** is the inelastic response (damper) after the first impulse and the inelastic response (damper and frame) after the second impulse. The second impulse acts at the damper re-yielding stage.
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- [Case 6-1] is the inelastic response (damper and frame) after the first impulse and the inelastic response (damper and frame) after the second impulse. The second impulse acts at the damper unloading stage.

- [Case 6-2] is the inelastic response (damper and frame) after the first impulse and the inelastic response (damper and frame) after the second impulse. The second impulse acts at the damper re-yielding stage.

Let \( \omega_h = \sqrt{K_p / m} \) denote the fundamental natural circular frequency of the frame. Then the fundamental natural circular frequency of the total SDOF system is given by \( \sqrt{(k+1)K_p / m} \). The structural damping is not considered here. \( V_c \) denotes the velocity of the SDOF system at the stage of zero restoring force after the first impulse. The reference input velocity \( V_y \) is defined as the level at which the frame without damper just yields after the first impulse and is expressed by \( V_y = \omega_1 d_{y1} \).

Using the energy balance twice (kinetic energy = dissipated energy + strain energy) after the first impulse and after the second impulse, the maximum deformation \( u_{max1} \) after the first impulse and that \( u_{max2} \) after the second impulse can be obtained. As an example, a detailed explanation is shown only for Case 6-2.

- [Case 1]
  
  \[
  u_{max1} = \frac{V}{\omega_h \sqrt{k+1}}, \quad u_{max2} = \frac{V + V_c}{\omega_h \sqrt{k+1}} \quad (V_c = V)
  \]  

- [Case 2]
  
  \[
  u_{max1} = \frac{V}{\omega_h \sqrt{k+1}}
  \]
  
  \[
  u_{max2} = -kd_{y1} + \sqrt{k(k+1)d_{y1}^2 + \left\{ \left( V + V_c \right) / \omega_h \right\}^2} \quad (V_c = V)
  \]  

- [Case 3]
  
  \[
  u_{max1} = \frac{V}{\omega_h \sqrt{k+1}}
  \]
  
  \[
  u_{max2} = \frac{\left\{ (V + V_c) / \omega_h \right\}^2 + kd_{y1}^2 + d_{y2}^2}{2\left( kd_{y1} + d_{y2} \right)} \quad (V_c = V)
  \]
\[ u_{\text{max}1} = -kd_y + \sqrt{k(k+1)d_y^2 + \left(V^2/\omega_t^2 \right)} \]
\[ u_{\text{max}2} = -kd_y \]
\[ + \frac{k^2d_y^2 - (u_{\text{max}1} - 2d_y)^2}{\sqrt{\left(2u_{\text{max}1} - 2d_y - \frac{kd_y + u_{\text{max}1}}{k+1}\right)}} \left\{ (k + 2)d_y - u_{\text{max}1} \right\} \]
\[ + \left\{ \left( V_c + V \right)/\omega_t \right\}^2 \]
\[ \left( V_c = \omega_t \sqrt{kd_y^2 + \left[V^2/\left(k+1\omega_t^2\right)\right]} \right) \quad (7) \]

\[ u_{\text{max}1} = -kd_y + \sqrt{k(k+1)d_y^2 + \left(V^2/\omega_t^2 \right)} \]
\[ u_{\text{max}2} = \frac{V + V_c}{\omega_t} - kd_y \]
\[ \left( V_c = \omega_t \sqrt{u_{\text{max}1}^2 - 2kd_y u_{\text{max}1} + k(k+4)d_y^2 } \right) \quad (8) \]

\[ u_{\text{max}1} = -kd_y + \sqrt{k(k+1)d_y^2 + \left(V^2/\omega_t^2 \right)} \]
\[ u_{\text{max}2} \]
\[ + \frac{\left\{ \left(V_c + V \right)/\omega_t \right\}^2}{\left(2u_{\text{max}1} - 2d_y - \frac{kd_y + u_{\text{max}1}}{k+1}\right)} \left\{ (k + 2)d_y - u_{\text{max}1} \right\} \]
\[ - \left( u_{\text{max}1} - d_y - 2d_y \right) \left\{ u_{\text{max}1} + 2d_y \right\} \]
\[ = \frac{2\left( kd_y + d_y \right)}{\left( kd_y + d_y \right)} \]
\[ \left( V_c = \omega_t \sqrt{kd_y^2 + \left[V^2/\left(k+1\omega_t^2\right)\right]} \right) \quad (9) \]

\[ u_{\text{max}1} = -kd_y + \sqrt{k(k+1)d_y^2 + \left(V^2/\omega_t^2 \right)} \]
\[ u_{\text{max}2} = \frac{(V_c + V)^2}{2\omega_t^2} - \frac{kd_y - d_y}{2} \]
\[ \left( V_c = \omega_t \sqrt{u_{\text{max}1}^2 - 2kd_y u_{\text{max}1} + k(k+4)d_y^2 } \right) \quad (10) \]
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• [Case 6-1]

\[ u_{\text{max}1} = \frac{(1-\beta)V^2}{2d_F \omega_T^2} + \frac{1-\beta}{2} d_F + \frac{\beta}{2} d_D \]

\[ u_{\text{max}2} = \left[ \frac{m(V_c + V)^2 / K_F}{4d_F - 2d_D} \right] \left( u_{\text{max}1} - d_F - d_D \right) \]

\[ = \left( \frac{2u_{\text{max}1} - 2d_D - \frac{k d_D + d_F}{k+1}}{2(k+1)K_F} \right) \left( kd_D + 2d_D - d_F \right) \]

\[ \left( V_c = \omega_t \left( kd_D + d_F \right) / \sqrt{k+1} \right) \]

Figure 5  Energy balance: Case 6-2 (see online version for colours)

Note: Kinetic energy imparted at impulse is in balance with the sum of dissipation and strain energies.

• [Case 6-2]

Figure 5 shows the energy balance for Case 6-2. The kinetic energy \( mV^2 / 2 \) or \( m(V + V_c)^2 / 2 \) imparted at the first or second impulse is in balance with the sum of dissipation and strain energies (area of ABCDE or area of FGI). It can be proved that the critical timing of the second impulse is the time of the stage corresponding to zero restoring force. This situation is the same as the model of elastic-perfectly plastic model (Kojima and Takewaki, 2015). Then the maximum deformation \( u_{\text{max}1} \) after the first impulse and that \( u_{\text{max}2} \) after the second impulse can be obtained as follows.
In order to investigate the response properties under the double impulse, the comparison with the response under the corresponding equivalent one-cycle sine wave is conducted. The equivalent one-cycle sine wave has been defined (Kojima and Takewaki, 2015) using the equivalence of the maximum Fourier amplitude.

Figure 6 Comparison of time histories of mass displacement between double impulse and equivalent one-cycle sine wave, (a) $\beta = 0$ (b) $\beta = 0.3$ (c) $\beta = \beta_{\text{opt}} = 0.423$ (d) $\beta = \beta_{\text{MAX}} = 0.666$ (see online version for colours)

It may be useful to introduce the maximum limit of the damper strength limit $\beta_{\text{MAX}} = k/(k+1)$. This limit corresponds to the case $d_{DY} = d_{Fy}$. 

$$u_{\text{max}1} = \frac{(1 - \beta)V^2}{2d_{Fy} \omega_0^2} + \frac{1 - \beta}{2} d_{Fy} + \frac{\beta}{2} d_{DY}$$

$$u_{\text{max}2} = \frac{(V_c + V)^2}{2 \omega_0^2 (d_{Fy} + kd_{DY})} - \frac{2u_{\text{max}1} - 3d_{Fy} + kd_{DY}}{2}$$

$$V_c = \omega_0 \sqrt{d_{Fy} - 2kd_{FY} + k(k+4)d_{FY}}$$

(12)
Consider as SDOF system of mass $m = 4.0 \times 10^5$ [kg], frame stiffness $K_F = 1.0 \times 10^8$ [N/m], frame yield displacement: $d_{Fy} = 0.04$ [m], damper stiffness ratio to frame $k = 2$, input level $V / V_y = 300$, damper strength ratio $\beta = 0, 0.3, \beta_{opt}, \beta_{MAX}$ (damper yield displacement can be obtained from this parameter setting).

Figure 6 shows the comparison of time histories of the mass displacement between the double impulse and the equivalent sine wave. Four models with different stiffness ratios are treated ($\beta = 0, 0.3, \beta_{opt}, \beta_{MAX}$). It can be observed that both responses correspond fairly well although the maximum response after the first impulse exhibits a limited discrepancy.

### 7 Comparison of optimal damper quantity between the previous method and the proposed method

In order to investigate the relation between the conventional formula $\beta_{opt}$ and the proposed one based on the minimisation of the maximum displacement in terms of the closed-form expression in Section 5, consider five and ten-story frames with different properties (Models 1–4). The parameters are shown in Table 1. The parameters are computed from the following building properties. Floor area: 400 [m²], each floor mass: $4.0 \times 10^5$ [kg], each story height: 3.5 [m], each yield interstory drift divided by story height: 1 / 150. These multi-story frames are reduced into SDOF systems as shown in Figure 7 where $H$ is the total height of the building and $H'$ is the effective height of the reduced SDOF system (approximated as $H' = 0.7H$ here).

Figure 8 shows the maximum displacement of the SDOF system with respect to the damper strength ratio for four input levels $V / V_y = 2, 3, 4, 5$. In Figure 8, the larger displacement between $u_{max1}$ and $u_{max2}$ is plotted and the circle indicates the minimum value in each input level. It can be observed that, when the input level is rather small, the optimal damper strength ratio corresponding to the minimum deformation is close to the conventional one $\beta_{opt}$. On the other hand, when the input level is rather large, the optimal damper strength ratio is approaching to the proposed one $\beta_{MAX}$ (simultaneous yielding of hysteretic damper and main structure). This means that, while the conventional formula cannot take into account the input level, the proposed one based on the minimisation of the maximum displacement in terms of the closed-form expression in Section 5 can consider this appropriately.

**Figure 7** Reduction of multi-story frame into SDOF system
Figure 8 Maximum displacement of SDOF system with respect to damper strength ratio for four input levels $V/V_y = 2, 3, 4, 5$, (a) [Model 1] $\beta_{\text{opt}} = 0.423$, $\beta_{\text{MAX}} = 0.666$ (b) [Model 2] $\beta_{\text{opt}} = 0.423$, $\beta_{\text{MAX}} = 0.666$ (c) [Model 3] $\beta_{\text{opt}} = 0.293$, $\beta_{\text{MAX}} = 0.500$ (d) [Model 4] $\beta_{\text{opt}} = 0.553$, $\beta_{\text{MAX}} = 0.800$ (see online version for colours)

Table 1 Parameters of multi-story frame and hysteretic damper

<table>
<thead>
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<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
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<td>Number of stories</td>
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<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Frame stiffness $K_F$ [N/m]</td>
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<td>$1.0 \times 10^8$</td>
<td>$1.0 \times 10^8$</td>
<td>$1.0 \times 10^8$</td>
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<td>Damper stiffness ratio $k$</td>
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<td>2</td>
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<td>4</td>
</tr>
<tr>
<td>Mass $m$ [kg]</td>
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<td>$4.0 \times 10^6$</td>
<td>$4.0 \times 10^6$</td>
<td>$4.0 \times 10^6$</td>
</tr>
<tr>
<td>Yield deformation of frame $d_F$ [m]</td>
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<td>0.163</td>
<td>0.163</td>
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<tr>
<td>Natural period without damper [s]</td>
<td>0.628</td>
<td>1.26</td>
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<tr>
<td>Natural period with damper [s]</td>
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<tr>
<td>$V_y$ [m/s]</td>
<td>0.817</td>
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8 Analysis under the corresponding one-cycle sine wave

In order to investigate the validity of use of the double impulse in place of the one-cycle sine wave, analysis for the one-cycle sine wave is conducted. Figure 9 shows the maximum displacement of the SDOF system (Model 2) under the corresponding one-cycle sine wave. In this figure, three input levels $V/Y = 2, 3, 5$ are considered and the input period is varied as $T_0/T_0' = 0.7, 1.0, 1.2$ where $T_0$ is the input period and $T_0'$ is the critical input period giving the maximum deformation under the double impulse. It can be concluded that, as the input level becomes larger, the optimal damper quantity moves from $\beta_{opt}$ to $\beta_{MAX}$ and the property obtained for the double impulse is retained also under the corresponding one-cycle sine wave. Furthermore, even if the criticality is not kept, such property is retained approximately.
9 Conclusions

The following conclusions have been derived.

1 A closed-form maximum elastic-plastic response has been derived for a structure with hysteretic dampers (normal trilinear hysteretic relation) under the critical double impulse as a representative of near-fault ground motion.

2 It has been shown that the classification into nine cases is possible depending on the combination of elastic-plastic responses of the structure and the hysteretic damper. The double impulse enables the derivation of the closed-form maximum elastic-plastic critical response of the controlled structure by using the energy balance in each case.

3 It has been demonstrated that the closed-form maximum elastic-plastic critical response is quite useful for finding an optimal hysteretic damper quantity depending on the input level of the double impulse.

4 At a smaller input level, the optimal damper quantity is determined by the conventional formula $\beta_{opt}$ derived from the maximisation of the equivalent damping ratio. On the other hand, at a larger input level, the optimal damper quantity is determined by the proposed one depending on the input level. For a very large input level, the optimal design is close to the maximum damper quantity $\beta_{MAX}$ (simultaneous yielding of hysteretic damper and main structure).

The present paper uses an SDOF model. The extension of the present theory to a multi-degree-of-freedom model should be discussed in the future. Furthermore, the present paper uses a double impulse as a representative of near-fault ground motions. The validity of the present theory for input ground motions except impulsive ones treated in this paper has to be investigated in more detail in the future.

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