Alternative capital requirement for insurers: possibilities and issues

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Abstract: Solvency II framework regulates how much capital the insurance companies of the European Union must hold. Although the framework lasts four years, there is still place for improvements considering experience. The goals of the research are to propose an alternative capital model methodology using copulas for reserve risk and to show the case study of potential capital shift impact. To conduct the research, the authors have used the extensive literature review, analytical methods and modelling. Research scope is non-life insurance companies under the Solvency II framework with a focus on reserve risk. The research will help avoid that alternative models are only a modern risk management tool and add risk management reality. Higher capital surplus can be achieved if a copula approach is used for risk aggregation in the Baltic non-life market. The Baltic market does not use alternative capital requirement and internal models. Using an alternative model is the right insurer's approach in modern risk management.

Keywords: solvency II; value-at-risk; risk management; insurance; internal model; reserve risk; best estimate; copula; solvency capital requirement; alternative capital requirement; *t* copula; normal copula; economic modelling.

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1 Introduction

The Solvency II regime came into effect in 2016. It has redesigned the capital adequacy regime for the European Union insurers and reinsurers. It regulates how much capital the insurer should hold to run business and takes control of the situation where a company is unable to pay liabilities to customers during one year, which is no more than 1-in-200 (0.5%). To calculate the necessary capital, insurers can use either a standard formula or their own full internal model as approved by the local supervisory authorities. Although the framework lasts four years, there is still room for improvement in risk management, considering case studies and issues discussed in numerous research papers. It will avoid the situation, in which alternative models should not only achieve modern risk management but also add risk management reality. The European insurance groups should continuously resubmit and introduce internal models and prove model accurateness with model back-testing result reports. For example, the Dutch supervisory authority has just approved the expansion of a partial internal model for NN Group (2018). Therefore, the uniform risk measurement framework and capital management framework are achieved among the largest solo companies in the group.

On the one hand, insurance customers do not have to pay more due to their insurer’s unproductive capital and high capital costs. On the other hand, regulators should monitor financial stability in the European Union (EU) market and make a capital model, in which the entire underlying capital amount is sufficient, sensitive to a certain risk as much as possible. Risk measurement framework must ensure that companies’ management boards have a proper understanding of the risks to which companies are exposed. Insurance penetration rate can be calculated as a share of premium in GDP. The Financial Stability Report of the European Insurance and Occupational Pensions Authority (EIOPA) represents that share is high for such countries as the UK, which accounts for around 15%, in France – around 16%, in Ireland – around 28% and in Malta close to 30% (EIOPA, 2018). The failure of insurance companies in these specific countries would negatively affect a large proportion of financial service customers.

The alternative capital requirement and regulators' capital requirement must be as close as possible instead of running two models – one for internal decision-making, the other for financial reporting. The 2008 economic recession provoked a regulatory onslaught against the use of internal models that is still ongoing (Embrechts, 2017). The Basel committee for the Basel III regime has started to permit the restricted use of internal modelling approaches (Bank for International Settlements, 2017) for specific risk categories as an argument that internal models are non-transparent (Gillespie et al., 2008). There have not been alike discussions in the EU financial regulatory institutions (EIOPA) yet, but national regulators can disallow the use of an alternative model. The UK regulator has started discussions on the UK insurers' capital models that might be underestimating the risks they face (Financial Times, 2019). Alternative capital modelling is essential due to an increase in capital costs, the low return on investment, and the low-interest rate setting in the EU. Spread between the cost of capital rate and the EU government bond yields is increasing. Alternative capital modelling also helps implement new upcoming risks that have not been implemented yet by the EIOPA, such as cybercrime, accurate natural catastrophe risk and risk arising from the process of using digital technologies. Accenture's (2019) research results show that cyber risk could lead to additional costs amounting to 4.6 trillion EUR and a lost revenue drop could be significant in the next five years. Only 30% of listed companies are confident of the internet security (Accenture, 2019). System for the accumulation risk control of natural hazards (CRESTA, 2013) is changing as well and could be different compared to the Solvency II framework.

Over the period of 1960–2005, the Canadian insurance market found evidence that inadequate pricing and reserve deficiency were the leading causes of insurer insolvency (Leadbetter and Dibra, 2008; Kleffner and Lee, 2009). The published working paper by Massey et al. (2001) summarises the main causes of 214 insurers' insolvency in the USA. The main cause is under-reserving in 34% of defaults; in 20% of defaults, it is rapid growth, while in 10% of defaults, it is due to allegedly fraudulent claims, and in 9% of defaults – investment failure (Massey et al., 2001; Buckham et al., 2017). For the insurance sector in Canada and the USA, the historically main risk that has caused insolvency for insurers is reserve risk and growth that are too fast and uncontrolled. The authors suggest performing a full risk assessment for these risks during own risk solvency assessment (ORSA) process, including valuation of risk aggregation techniques.

The present study focuses on non-life insurance reserve risk issues and possibilities, using a standard formula or an alternative model. The issues are identified by summarising published papers. The object of the research is non-life insurance companies, and the subject is an alternative capital requirement and its possibilities and issues. The goals of the research are to show how to determine capital needs and to propose an internal capital model methodology by using a copula approach, as well as demonstrate the case study of potential capital decrease impact by using the

Baltic non-life insurance market data. The paper also provides the discussion of internal capital models under the Solvency II framework, investigates what should be done in order to create a model that would be empirically consistent and predictively precise, and demonstrates the way of obtaining a lower capital requirement through the theory of the copulas. The framework is not strict in the area of internal modelling. The regime is based on the idea that an insurer should know more than a national regulator about the main risks in respect of company's goals and own risk profile.

The paper is organised as follows. First, the analysis of key performance and financial stability indicators is presented for the Baltic (Latvia, Lithuania, Estonia) and EU non-life insurance market (Section 2). Second, the extended literature review is provided in respect of standard formula weaknesses, potential solutions for reserve risk, risk aggregation and loss reserving techniques, as well as the discussion is held regarding the internal model under the Solvency II and other regimes (Section 3). Next, the description of the research methodology is provided for the case study and its results are discussed for reserve risk that will help improve the internal model methodology and avoid unproductive capital (Sections 4 and 5). The last section concludes the paper and indicates the future challenges.

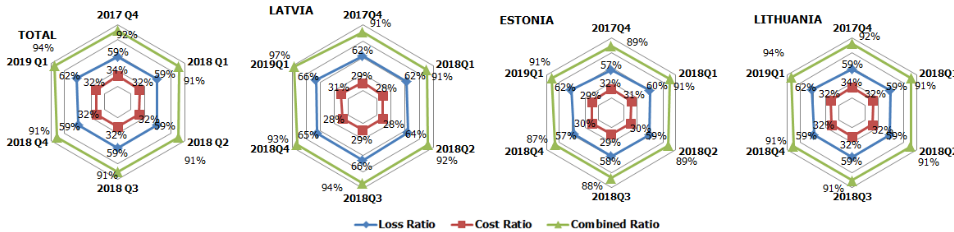
2 The Baltic and EU non-life insurance market performance

The key performance and financial stability indicators of the Baltic and EU insurance markets.

The present section summarises the key performance, solvency and financial stability indicators for the Baltic non-life insurance market. This will help us understand the key performance measures in the Baltic insurance market where the case study will be applied further. In 2018, the Baltic insurance market had a 23% market share for the life insurance business and 77% for the non-life insurance business (EIOPA, 2019). The top key performance and financial stability indicators are cost ratio, expense ratio and the sum of both ratios: the combined ratio. Figure 1 illustrates the three key profitability performance indicators of the Baltic non-life insurance market. It shows overall market results during the fourth quarter (Q4) of the year 2017 to the first quarter (Q1) of the year 2019 (2017Q4–2019Q1) with a total of 15 insurers. As can be observed, the loss ratio has small fluctuations during the period under consideration. Cost ratios in this period for all countries have been between 28% and 34% with the highest average cost level for companies registered in Lithuania.

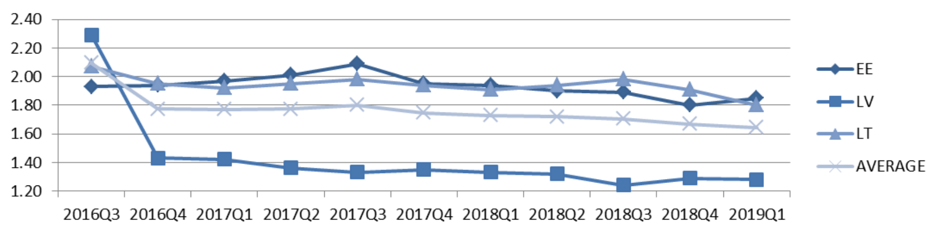
Latvia has the lowest cost ratio but the highest loss ratio in the Baltic region. Lithuania has the highest cost ratio but the lowest loss ratio. Estonia has the best overall performance and has the least combined ratio. Overall, the Baltic performance ratios are stable in the period under consideration. The EU non-life insurance market had very close average indicators during the same period – combined ratio (92.3%) and cost ratio (32.3%) – that coincided with the Baltic indicators. Solvency margin is the ratio of eligible own funds (EOF) to solvency capital requirement (SCR), and results can be seen in Figure 2.

Figure 1 The key financial performance indicators of the Baltic non-life insurers (see online version for colours)



Source: Created by the authors based on EIOPA Insurance Statistics (2019)

Figure 2 Solvency margins (ratio of eligible own funds to SCR) (see online version for colours)

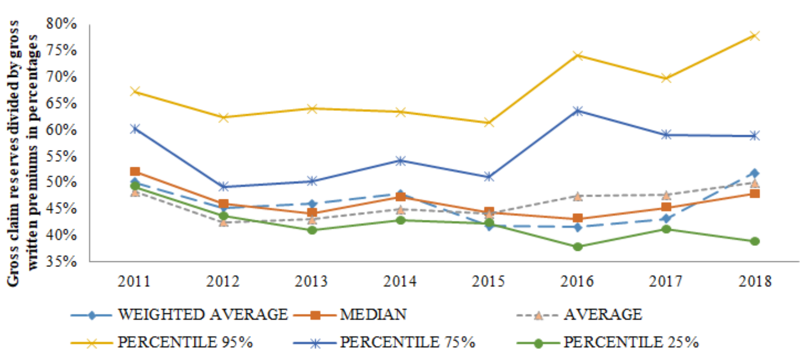


Source: Created by the authors based on EIOPA Insurance Statistics (2019)

For the last quarter 2019Q1, we cannot see significant fluctuations in solvency margins but the negative ratio trend for market average can be seen during the period from 180% in 2017Q3 to 164% in 2019Q1. Comparing 2019Q1 Baltic average solvency margin and EU all countries' margins, the Baltic margin falls below the EU 10th percentile.

Reserving level can be calculated as gross claim reserves divided by gross written premiums. It shows market reserving methodology practice and fluctuations. Figure 3 illustrates that there is a high deviation of reserving ratio, reserving policy in the market, and the average reserving level is increasing. The authors of the present study assume that reserve risk can be significant based on the calculated wide-ranging ratios year to year.

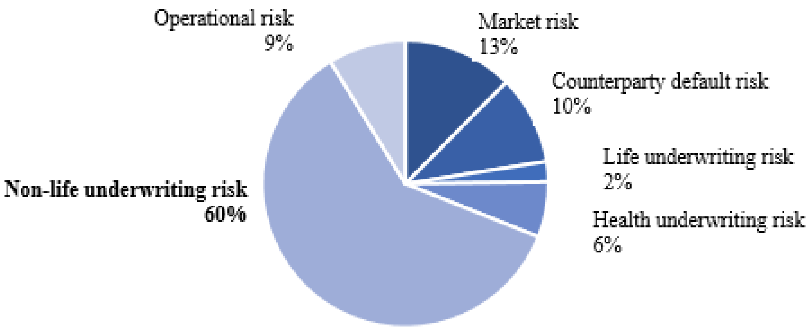
Figure 3 Reserving level development (see online version for colours)



Source: Created by the authors based on BTA, ERGO, GJENSIDIGE, AB Lietuvos Draudimas, Swedbank, IF Annual Financial Reports (2011–2018)

Public reports on the Baltic solvency and financial condition (BTA, ERGO, GJENSIDIGE, AB Lietuvos draudimas, Swedbank, IF, 2018) show no use of the partial internal or full internal model. Non-life companies were mostly driven by non-life insurance risk in the year 2018 (see Figure 4).

Figure 4 Insurers' risk profile 2018 representing top companies with 80% market share (see online version for colours)



Source: Created by the authors based on BTA, ERGO, GJENSIDIGE, AB Lietuvos Draudimas, Swedbank, IF Solvency and Financial Stability Reports (2018)

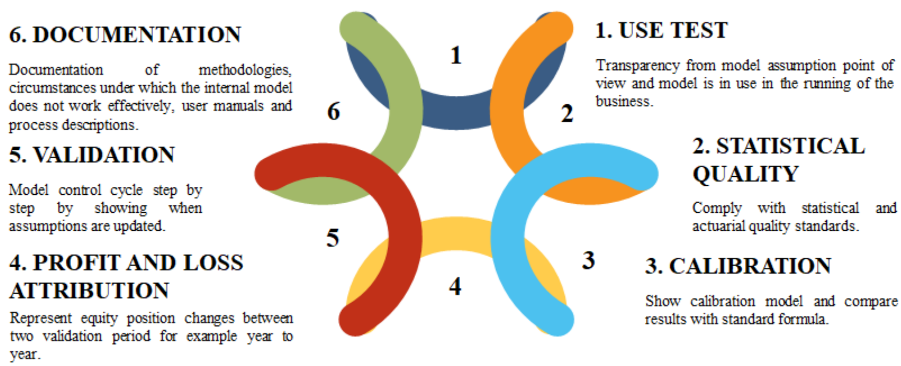
The standard formula includes diversification between risks. The diversification effect can be calculated as the difference between the sums of all risks minus final SCR after diversification. The average impact of SCR diversification was 25.5% in the Baltic non-life insurance market at the end of 2018 (minimum 17% for ERGO, maximum 37% for BTA). In the European market, the average diversification impact reveals a 16% reduction for SCR (Krzykowski and Lech, 2018). Diversification is vital to the capital calculation; such is the case even more for the Baltic insurance market. This means that companies have a wide range of exposure to market risks and health, life, and non-life underwriting risks in the Baltic countries compared with the average European market. A risk profile shows the importance of risk aggregation, especially for non-life underwriting risk, such as reserve risk in the Baltic countries.

3 Literature review

The Solvency II framework has three pillars, a forward-looking structure and a principle-based approach model. Pillar 1 is related to the calculation of capital requirements. Solvency capital requirements shall cover at least these risks (sub-risks) – market risk, health underwriting, credit default risk, life underwriting risk, non-life underwriting risk, intangible risk, operational risks, adjustment for the loss-absorbing capacity of deferred taxes, and technical provisions. In order to improve risk management and capital management, it is possible to calculate capital requirements using one of the most appropriate models: a standard formula (SF) with or without undertaking specific parameters, a partial internal model (PIM) and a full internal model (IM).

In order to implement an internal model, insurers should keep in line with requirements according to Articles 120–125 of the Solvency II Directive 2009 (European Parliament, & Council of the European Union, 2009). The internal model needs to be individually tested by a regulator. It is important that regulators regularly update knowledge of new risk assessment methods and, therefore, inappropriate mathematical models are not approved. Embrechts (2017) provides a wide-ranging discussion concerning quantitative, alternative models in the finance industry and one of the statements is that internal models play an important role for regulatory purposes as long as their use is methodologically clear, scientifically sound and ethically correct (Embrechts, 2017). It means that internal model standards must be implemented in the model methodology. Many standards and tasks must not only be included in the documentation but also must be built into the daily decision-making process and risk governance system. Figure 5 presents the summary of the main standards and the aim of each standard.

Figure 5 The main standards and their aim for internal model implementation (see online version for colours)



Source: The authors' interpretation based on EIOPA (2014)

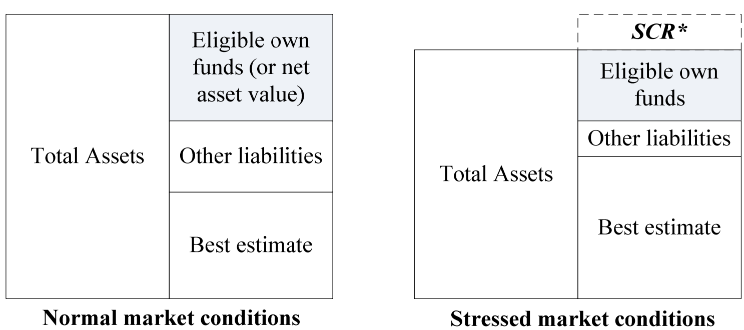
The use test is an essential lever to achieve a positive return on income (Sia Partners, 2011). The methodology of the internal model includes model components, such as model type (e.g., deterministic or stochastic), assumptions (e.g., longevity), interactions (e.g., future management actions, policyholders' behaviour), data (e.g., internal or external), IT (e.g., program used for calculations) and process (procedure, governance, internal, or external audit).

Pillar 2 is related to the system of governance requirements. Pillar 2 requires the ORSA and the standard formula evaluation. Signs of the internal model implementation potentially creating a greater proper risk profile and signs of significant changes can already be observed during the ORSA process. If that is the case, the internal model should be created, and it can also be asked for by a financial supervisor. It is one of the reasons why ORSA is regarded as the heart of Solvency II. The internal model is approved by a national regulator. These issues where standard and ORSA results have conflicts may relate not only to an economic capital model of the entire group but also to a stand-alone economic capital model of a single insurer that is part of a group. Regulators have just approved a re-submitted internal model for NewRe, a Zurich-based subsidiary of Munich Re Group, and the main reason was a fully different risk profile, risk aggregation compared with the group (NewRe, 2019). Finally, Pillar 3 is related to disclosure management and reporting.

Alternative models are currently in use in several risk-based capital systems, such as Basel III for the banking industry in the EU, Solvency II for the insurance industry in the EU, LITAC for life insurers in Canada, LAGIC's approach for insurers in Australia, NAIC's standard in the US, C-ROSS in China and Swiss Solvency Test (SST) in Switzerland. A parent and subsidiary can be located in different regions and many large insurance groups can be affected by any standard changes (Geneva Association, 2016). The US regulator NAIC admits that insurance is more heavily regulated than other types of business due to the complexity of insurance contracts (NAIC, 2011).

The exact definition of alternative model has not been provided under the Solvency II framework in the main European Union commission delegated regulations – Solvency II Directive 2009/138/EC, Solvency II Regula 2015/35. It is due to the fact that Solvency II is principle-based. Different experts can have a different view and understanding of what an internal model is. CEA and Groupe Consultatif (2007) use the definition of internal model as a risk management system developed by an insurer to analyse the overall risk position, to quantify risks and to determine the economic capital required to meet those risks. The International Actuarial Association uses the definition of internal model as a mathematical model of an insurer's operations to analyse its overall risk position, to quantify risks and to determine the capital needed to meet those risks (IAA, 2010). Internal model can also be explained by an economic balance sheet in normal (pre-stress) and post-stress (after extreme events) situation (Cadoni, 2014). Figure 6 explains a basic principle in year end, where a company had a certain net asset value or eligible own funds and after one year time several underwriting risks occurred and, therefore, SCR was the difference between net asset values in two periods.

Figure 6 Internal modelling, SCR results using an economic balance sheet



**SCR – alternative model results, changes in eligible own funds after extreme events*

Source: The authors' interpretation based on Cadoni (2014)

The used economic capital models can vary from simple deterministic approaches to extremely complex stochastic approaches. Internal model may be beneficial for reflecting a true risk situation (Gatzert and Kosub, 2016). It can lead to complex models but also complexity should have limits. Green and Armstrong (2015) compared simple and complex methods and concluded that complexity increased a forecast error by 27% on average in 25 papers with quantitative comparisons. Clients who prefer accuracy should accept forecasts only from simple evidence-based procedures (Green and Armstrong, 2015).

The exact definition of reserve risk is not provided by the Solvency II framework, but the main idea developed in research papers is the same. The only difference is that the Solvency II framework considers the distribution of the profit and loss on the estimated reserves over a one-year time horizon (England and Evans, 2018). Buckham et al. (2011) defined reserve risk as a risk where additional technical provisions might have to be raised against previous years' claims. Diers and Linde (2013) defined reserve risk as embedding future accident years that lead to an integrated approach to quantifying multi-year risk arising from the settlement of outstanding claims.

Reserves (claim best estimate) have a significant part in the economical balance sheet for non-life insurers. Different kinds of techniques have been developed to assess reserve amount in order to reach reliable, best practice results and assess potential deviation, risk. The Non-life Section of the International Actuarial Association Section Actuarial Studies in Non-Life Insurance (ASTIN) has published survey results for non-life reserving practice in the world (ASTIN, 2016). Deterministic approaches, which are used for claim best estimate calculation, are as follows, starting with the widely used technique: Chain Ladder, Bornhuetter-Ferguson, Loss ratio, Average cost, Cape Cod, Fisher Lange, Generalised linear model, Munich chain ladder and De Vylder (ASTIN, 2016). Schmidt (2007) provided extended bibliography on loss reserving.

Stochastic approaches, such as Mack (1993) and Bootstrap (Efron, 1979), have been developed using a classical Chain Ladder approach as a base model. These methods help calculate reserve's standard deviation from mean (the amount in economical balance sheet). Some countries, e.g., Germany or Finland clearly favour Mack, others, such as Australia or the Netherlands, prefer Bootstrap (ASTIN, 2016). Wütrich and Bühlmann (2009) or Merz and Wütrich (2008) developed classical approaches to assess reserve risk for one-year time horizon as it was under the Solvency II framework.

The authors have not found the summary of reserve risk weaknesses if a standard formula is used. It was done in the previous research by the authors. The authors have been guided by the research question: What are the weaknesses of the EIOPA's standard formula for non-life reserve risk under the Solvency II framework? The answer to the question will help find capital influencing factors based on quantitative case studies. In the search process, the authors have used peer-reviewed papers of the Scopus database published between 2006 and 10 February 2019. The search starting year of 2006 has been chosen since it was for the first time when the quantitative impact study of Solvency II was published. Based on the authors' content analysis results and publication interpretation (Zariņa et al., 2019), factors that should be considered in the internal model methodology for reserve risk were justified and the two latest research papers were mentioned for each factor group (see Table 1).

The majority of papers mentioned the dependency problem related to risk aggregation. Non-linear risks mainly exist in the real world. Then it is mentioned that problems occur because insurance companies use only deterministic approaches for reserving. Reserve risk also depends on reserving policy, actuaries' behaviour, management actions (for example, reinsurance) and reserving sufficiency. Therefore, EIOPA's standard formula approach is too simple. Four papers state that the main problem is related to a too short time horizon for SCR calculation. Biard et al. (2008), Forte et al. (2012), and Kemaloglu and Gebizlioglu (2009) have mentioned that the standard formula fits only large companies in the case of normal market conditions.

Table 1 The overview of factors that should be implemented and considered in the internal model methodology for reserve risk

Factors	*Description of the factor*	*Latest research papers and total count of papers*
Factor 1: Risk aggregation	Can be interpreted as a formula that works appropriately till the risk diversification calculation is made and capital is calculated in an inappropriate way for line of business. Example: Hail natural catastrophe event occurs. Claims are reported in these lines of business – property and motor own damage insurance. In a standard model, correlation is rather weak but due to unique insurance event correlation can be strong, resulting in inadequate capital for portfolio	Araichi et al. (2017) Clemente and Savelli (2017) Total mentions – 14
Factor 2: Time horizon	Can be interpreted as when capital is set to an adequate amount for a one-year horizon but must be assessed in a long time horizon. Example: Claims settlement can take from minutes to years for court cases. Risk assessment may also take a longer period than one year as it is in a standard module.	England et al. (2019) Diers and Linde (2013) Total mentions – 4
Factor 3: Model type (deterministic vs. stochastic)	Can be interpreted as capital needs are calculated in a way, in which the risk is not even calculated in an appropriate way for each line of business. Example: Inappropriate actuarial method is used for assessing outstanding claim reserve. Not reliable deterministic method or no use of stochastic simulations or testing with alternative methods.	Dacorogna et al. (2018) Fröhlich and Weng (2018) Total mentions – 10
Factor 4: Profitability	Can be interpreted as when risk depends on average claim costs, which can be different when comparing different regions. Example: The same capital needs are necessary if a company has different profitably results but equal amount of outstanding claim reserve exists.	Clemente and Savelli (2017) Bermúdez et al. (2013) Total mentions – 3

Source: Zarina et al. (2019)

In order to solve the dependency problem, a copula approach is proposed. The main copulas that are mentioned include the Gaussian copula, the Clayton copula, the Farlie-Morgenstern copula and the non-specific copula.

In order to resolve the issue of a short time period, ruin theory and geometric Brownian motion are proposed. In order to resolve deterministic and time issues, the following methods are proposed: bootstrapping (Ohlsson and Lauzeningks, 2009) and stochastic reserving methods, including a Robust Chain Ladder (Peremans et al., 2017), the Fröhlich model (Fröhlich and Weng, 2018), a neural network approach (Hejazi and Jackson, 2017), and a COT method (Dacorogna et al., 2018) developed by the SCOR insurance group.

Investors, regulators and economists often assess a diversification impact and its benefits using a measure of dependence, such as correlation (Chollete et al., 2011). It is therefore vital to have an appropriate choice of measures for dependence. Measures can be the traditional correlation (Spearman, Pearson) and copulas. While each approach has advantages and disadvantages, researchers have rarely compared them in the same empirical study especially for the insurance sector. The Pearson correlation model captures linear correlation but non-linear risks mainly exist in the real world. The authors' literature review showed that it was case of reserve risk under the Solvency II framework where the Pearson correlation was used (see Table 1). Copula approach allows modelling non-linear risks. Copulas are not yet widely used in the insurance sector. Copulas are a well-known approach for risk aggregation, assessment method for the banking sector, credit risk and market risk modelling. Stolyarov (2018) has considered the gamma Poisson model of behaviour using copula. Poisson distribution is usually used for modelling the insurance number of claim frequency. Copula model is widely used in valuating collateralised debt obligations, as well as in default risk assessment where default of one asset can cause the default of another. Romano (2012) has used copula for listed equities in Italy and demonstrated how useful it could be for extreme event modelling, which is in the interest of risk managers and supervisors. Extreme event modelling must be performed by insurers if risk-based capital confidence level set by regulators is so high – value at risk 99.5% (European Parliament, & Council of the European Union, 2014). Pellecchia and Perciaccante (2019) have just published a paper where the main risks (market, counterparty, life, non-life) are aggregated using copulas, and conclusion is that a standard formula can lead to an overestimation of capital requirements. There is a different kind of copulas, such as Gaussian or normal copula, *t* copula, skew *t* copula, Frank copula (Demarta and McNeil, 2005).

The literature review clearly shows that risk aggregation and choice of correlation metrics are the main weaknesses of a standard model approach. The authors have not found the impact analysis for the Baltic countries. Therefore, the authors will further propose an approach to solve a risk aggregation aspect for reserve risk using copula and case study in the Baltic countries.

4 Research methodology

The authors summarise the used approach to the claim best estimate, using a standard model approach for reserving risk under the Solvency II framework. The study also shows how an alternative capital requirement model approach can be developed using a copula approach. The Mack Chain Ladder is used for reserve calculation in the paper. In order to calculate aggregate distribution by combining line of business, *t* copula and normal copula are applied.

As described in the literature review, there can be other combination of methods, such as the Bornhuetter-Ferguson technique for reserve and skew *t* copula, Spearman rank correlation for aggregated loss distribution. The authors have chosen the most widely used methods. The chosen techniques and steps used in the case study are described in the following sub-sections.

4.1 Reserve calculation

The Mack chain ladder model provides not only the best estimate but also reserve standard deviation. Best estimate usually is very sensitive in the most recent accident years. Therefore, the Mack model shows reserve risk, capital sensitivity and whether there is significant uncertainty by using any other method (Mack, 1993).

Let C_{ik} denote the cumulative total claim amounts of accident year, $1 \le i \le I$, incurred up to development year k, $1 \le k \le I$. A goal is to estimate ultimate claim amount C_{iI} and reserve, best estimate R_i

$$R_i = C_{iI} - C_{i,I+1-i} \tag{1}$$

for accident years $i = 2,\ldots,I$. The basic idea is that there are development factors $f_1,\ldots,f_{I-1} > 0$ with

$$E\left(C_{i,k+1}|C_{i1},\ldots,C_{ik}\right) = C_{ik} f_k, 1 \le i \le I, 1 \le k \le I-1 \tag{2}$$

The chain ladder method consists of estimating the f_k by

$$\hat{f}_k = \sum_{j=1}^{I-k} C_{j,k+1} / \sum_{j=1}^{I-k} C_{jk}, 1 \le k \le I-1 , \tag{3}$$

and the ultimate claim amount C_{iI} by

$$\widehat{C_{iI}} = C_{i,I+1-i} \cdot \widehat{f_{I+1-i}} \cdot \ldots \cdot \widehat{f_{I-1}} , \tag{4}$$

and reserve R_i by

$$\widehat{R_i} = C_{i,I+1-i} \cdot \widehat{f_{I+1-i}} \cdot \ldots \cdot \widehat{f_{I-1}} - 1, \tag{5}$$

where all current diagonal elements are granted "hats" (e.g., f_k and $\widehat{f_k}$ or C_i and $\widehat{C_{iI}}$ or R_i and $\widehat{R_i}$) (Mack, 1993).

We assume that there are no any dependencies between accident years and we can assume that variables C_{ik} of different accident years

$$\{C_{i1}, C_{i2}, \ldots, C_{iI}\}, \{C_{j1}, C_{j2}, \ldots, C_{jI}\}, \ i \ne j , \text{ are independent.} \tag{6}$$

It is proven that the mean squared error of overall reserve estimate $\hat{R} = \hat{R}_2 + \cdots + \hat{R}_I$ can be estimated by

$$\widehat{mse(\hat{R})} = \sum_{i=2}^{I} \left\{ (s.e.(\widehat{R_i}))^2 + \widehat{C_{iI}} \left(\sum_{j=i+1}^{I} \widehat{C_{jI}} \right) \sum_{k=I+1-i}^{I-1} \frac{2\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{n=1}^{I-k} C_{nk}} \right\} , \tag{7}$$

where

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{ik} \left(\frac{C_{i,k+1}}{C_{ik}} - \widehat{f_k} \right)^2 , 1 \le k \le I-2.$$

Formula (7) is a combination of process error and parameter error (Mack, 1993).

We suggest seeing all proofs, theorems and examples in Mack's (1993) research paper. The Mack chain-ladder model can be regarded as a weighted linear regression through the origin for each development period. The best practice would be to assess reserves also with an alternative method and compare results by using, for example, the Bornhuetter-Ferguson technique, chain ladder or any other approach based on expert valuations.

4.2 Standard model approach

The Solvency II framework has provided a standard formula approach for assessing reserve risk. Non-life underwriting risk consists of 3 sub-risks: premium and reserve risk, lapse risk, catastrophic risk. Reserve risk is calculated as net outstanding claim reserve and three standard deviation multiplication for each line of business. The standard deviation for reserve risk for each line of business is set by EIOPA. It is assumed that reserve risk follows a log-normal distribution. Linear correlation matrix provided by EIOPA is used for reserve risk aggregation. More detailed calculation can be seen in the European Commission Solvency II Regulation 2015/35 (European Parliament, & Council of the European Union, 2014). In this case, the capital requirement for reserve risk $SCR_{reserve}$ is derived from the following formula:

$$SCR_{reserve} = 3 \cdot \sigma_{total} \cdot CBE_{net\ total}\ , \tag{8}$$

where: σ_{total} – measures the volatility of non-life reserve risk; $CBE_{net\ total}$ – claim best estimate after reinsurance is calculated by using one of techniques from Section 4.1 where the result represents the confidence level of 50%; $3 \cdot \sigma_{\text{total}}$ shows the confidence level of 99.5% over a one-year period.

Overall standard deviation of insurance portfolio is calculated from the following formula where standard deviation for each line of business is given by EIOPA:

$$\sigma_{total} = \sqrt{\frac{\sum_{k,l=1}^{n} Corr_{k,l} \cdot \sigma_k \cdot \sigma_l \cdot CBE_k \cdot CBE_l)}{CBE_{net\ total}^2}}, \tag{9}$$

where: l,k – line of business (LoB); $CBE_{net\ total}$ – claim best estimate after reinsurance; Corr – linear correlation matrix set by EIOPA (European Parliament, & Council of the European Union, 2014). The authors will not use formula (9) but instead apply other risk aggregation technique – copula. The risk aggregation procedure is the same as in a standard model and market practice in case if an alternative model is not accepted by regulators.

4.3 Internal model approach using copula

As mentioned in the sub-section above, alternative capital requirement should be calculated using formula:

$$SCR_{reserve\ for\ k\ LoB} = VaR_{99.5\%}^{k} - CBE_k\ , \tag{10}$$

where: $VaR_{99.5\%}^{k}$ – value at risk (*VaR*) with 99.5% confidence level for line of business k; CBE_k – the best estimate for k line of business with *VaR* 50% confidence level. Difference between value at risk 99.5% percentile and mean. Actuary, reserve risk holder do not need to risk aggregation underlying distribution for companies' portfolio reserve but should know the value at risk at certain confidence levels. In order to get multivariate distribution on an aggregate risk level taking into account all lines of business, a copula approach is used. Diversification effect can be calculated as difference between sums of all risks and aggregated risk from multivariate distribution.

Copulas are particular multivariate distribution functions. Let us recall that the distribution function H of a d-dimensional random vector $\boldsymbol{X} = (X_1, \ldots, X_2)$ is the function defined by

$$H(x) = \mathbb{P}(X \leq x) = \mathbb{P}(X_1 \leq x_1, \ldots, X_d \leq x_d), x = (x_1, \ldots, x_d) \in \mathbb{R}^d. \quad (11)$$

The distribution function F_j of X_j, $j \in \{1, \ldots, d\}$, can be recovered from the multivariate distribution function H by $F_j(x_j) = H(\infty, \ldots, \infty, x_j, \infty, \ldots, \infty)$, $x_j \in \mathbb{R}$. This is why $F_1, \ldots, F_d$ are also called the *univariate margins* of H or the *marginal distribution functions* of $\boldsymbol{X}$. Sklar's theorem can be used to create copula families from existing families of multivariate distribution functions. It is a central theorem of copula theory. Proof can be found in Sklar (1996), a probabilistic one in Rüschendorf (2009). Univariate distribution function F, $ranF = \{F(x) : x \in \mathbb{R}\}$ denotes the *range* of F and $F^{\leftarrow}$ denotes the quantile function associated with F.

Sklar's theorem (Sklar, 1959)

1 For any d-dimensional distribution function H with univariate margins $F_1, \ldots, F_d$, there is a d-dimensional copula C such that

$$H(\boldsymbol{x}) = C(F_1(x_1), \ldots, F_d(x_d)), \boldsymbol{x} \in \mathbb{R}^d. \quad (12)$$

The copula C is uniquely defined on $\prod_{j=1}^{d} ran F_j$ and there given by

$$C(\boldsymbol{u}) = H(F_1^{\leftarrow}(u_1), \ldots, F_d^{\leftarrow}(u_d)), \boldsymbol{u} \in \prod_{j=1}^{d} ran F_j. \quad (13)$$

2 Conversely, given a d-dimensional copula C and univariate distribution functions $F_1, \ldots, F_d$, H defined by equation (8) is a d-dimensional distribution function with margins $F_1, \ldots, F_d$.

Normal copula

The d-dimensional normal copula C_P^n is the copula defined by Sklar's theorem from the multivariate normal distribution $N_d(0, P)$ with correlation matrix P. If Φ_P denotes the distribution function of the latter, $C_P^n(\mathbf{u})$ is given, for any $\mathbf{u} \in [0, 1]^d$, by

$$C_P^n(\boldsymbol{u}) = \Phi_P(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)) = \int_{-\infty}^{\Phi^{-1}(u_d)} \ldots \int_{-\infty}^{\Phi^{-1}(u_1)} \frac{exp(-\left(\frac{1}{2}\right)x'P^{-1}x}{(2\pi)^{\frac{d}{2}}\sqrt{detP}} \mathrm{d}x_1 \ldots \mathrm{d}_{x_d}, \quad (14)$$

where Φ^{-1} denotes the quantile function of $N(0,1)$ (Hofert et al., 2018).

t copula

The d-dimensional *t copula* $C^t_{P,v}$ is the copula defined by Sklar's theorem from the multivariate *t* distribution with location vector **0**, scale matrix *P* and *v>0* degrees of freedom. If $t_{P,v}$ denotes the distribution function of the latter, $C^t_{P,v}(\boldsymbol{u})$ is given, for any **u** ∈ $[0, 1]^d$, by

$$C^t_{P,v}(\boldsymbol{u}) = t_{P,v}\left(t_v^{-1}(u_1),\ldots,t_v^{-1}(u_d)\right)$$

$$= \int_{-\infty}^{t_v^{-1}(u_d)} \cdots \int_{-\infty}^{t_v^{-1}(u_1)} \frac{\Gamma\left(\frac{v+d}{2}\right)}{\Gamma\left(\frac{v}{2}\right)(\pi v)^{\frac{d}{2}}\sqrt{detP}} \left(1+\frac{\boldsymbol{x}'P^{-1}\boldsymbol{x}}{v}\right)^{-\frac{v+d}{2}} \mathrm{d}x_1 \ldots \mathrm{d}x_d, \tag{15}$$

where t_v^{-1} denotes the quantile function of t_v of the univariate Student *t* distribution with v degrees of freedom (Hofert et al., 2018).

5 Case study

The authors have calculated the capital requirement using a standard formula and an internal model based on a copula approach by means of historical, exposure changed aggregated data from the one Baltic non-life market company over a period of 10 years. Three lines of business have been considered: property insurance, motor third partly liability (MTPL) and general third party liability (GTPL). Capital requirement is calculated using a linear correlation matrix given by EIOPA and using *normal copula* and *t copula*.

The aim of the case study is to show potential capital requirement shifts using an alternative capital requirement model or a standard model for the Baltic non-life insurance companies. The case study also intends to show the sensitivity of capital requirement if different risk aggregation, correlation methods are used, and to reveal the importance of the analysis of companies' individual reserve underlying distribution assumptions for lines of business; otherwise, significant capital surplus can be decreased, capital costs can be increased even in a one-year period.

The analysis was conducted in R 3.5 (R Core Team, 2018) and figures were produced using the package 'copula' (Yan, 2007) and package 'actuar' (Dutang et al., 2008).

5.1 Data

Claim reserve is calculated using the Mack model, assessing underlying distribution and distribution parameters for reserve risk for each line of business. In accordance with the Mack model described in Section 4.1, input data are the following:

- for each claim case it is necessary to know the claim payment year, reporting year, reserve change year

- all cases that have accident year in the period of 2008–2018 are in scope of further calculations
- triangles are created from the 10 years (2008–2018) paid claims data and reserve development based on accident year and development year for 3 lines of business
- reserve is calculated for each line of business using formulas (1)–(5)
- reserve that will be found in an economical balance sheet is 3.5 MEUR for MTPL, 0.45MEUR for property insurance, 2.60MEUR for GTPL;
- reserve mean and standard deviation can be extracted for each line of business.

Tables 2 shows all input data parameters used in aggregated loss distribution. Correlation between all lines of business is positive – 0.25 (Table 3).

Table 2 Case study input data, different correlation principle

	MTPL	*PROPERTY*	*GTPL*
Underlying distribution for best estimate	*Log-normal*	*Log-normal*	*Log-normal*
Mean	15.0784	13.0067	14.7667
Standard deviation used in an internal model and a standard model	0.09	0.1	0.11
Claim reserve, best estimate *VaR 50% (EUR)*	3 549 900	447 616	2 604 441

Table 3 Linear correlation matrix used for a standard model approach and an internal model approach

	MTPL	*Property*	*GTPL*
MTPL	1	0.25	0.25
Property	0.25	1	0.25
GTPL	0.25	0.25	1

5.2 Results and discussions

Solvency capital requirement is calculated using two copula-based approaches – *normal copula* and *t copula* if degrees of freedom are 4. All input parameters that are provided in Tables 2 and 3 are included in code – mean, standard deviation, correlation between lines of business. As shown in Table 4, difference as percentage of reserve risk is significant. Capital saving using normal copula is 12.11% and using *t copula* is 6.24%. The higher amount of degrees of freedom, the more similar results between *t copula* and *normal copula*.

There is a long settlement period for these specific lines of business and insurance loss distributions are usually skewed (Meyers, 2005). Therefore, there is tail correlation. *t copula* could be more appropriate taking into account the fact that there can also be tail correlation for these specific lines of business. Tail correlation for normal copula is 0. The financial crisis of 2007–2008 happened due to the fact that tail correlations were ignored (Balla et al., 2014). Therefore, the authors think that for this case study *t copula* would be more appropriate than *normal copula*. This would be a case for the major part of insurers, but in order to assess the appropriateness for the Baltic business data, this specific company research should be continued.

Table 4 Results of the case study – capital requirement for combined reserve risk for insurance company regarding MTPL, property, GTPL insurance line of business

Approach	*Reserve risk in EUR*	*Diversification effect after risk aggregation in EUR*	*Capital savings in EUR*	*Capital savings %*
Standard model	1,619,018	–2,402,575	0	0
Internal model using *normal copula*	1,423,034	–2,598,560	195,985	12.11%
Internal model using *t copula* degrees of freedom = 4	1,518,034	–2,503,559	100,984	6.24%

6 Conclusions

The Baltic non-life insurance market did not use alternative models in the year 2018. The Baltic market does not use them even for significant risk premium and reserve risk identified by the authors.

Results of the literature review have shown that the alternative model methodology should solve the dependency problem.

The authors conclude that each alternative model under the Solvency II should have at least five characteristics. First, the model follows the principles of the Solvency II regulation standard formula: It consists of market consistent valuation techniques, using value at risk measure with a 99.5% confidence level in a one-year horizon. Second, reserves and capital are properly set aside and allocated to each line of business; therefore, pure risk profiles of all portfolios can be observed. Next, precise capital allocation retains a good reputation. Then, there must be a balance between accuracy and simplicity, and it cannot be too costly or time-consuming. Finally, the model must avoid all hotly discussed issues in academic journals.

The methodology based on a copula approach proposed by the authors can avoid this problem and unproductive capital as well. The case study has demonstrated that capital saving using appropriate risk aggregation can be even 12% depending on the used copula for the Baltic non-life insurance market. Taking into account that there is tail correlation among the lines of business in the case study and other research papers *t* copula would be more appropriate resulting in 6% capital decrease. The internal model should be used in case of significant shifts when compared to the standard formula assessment.

Risk aggregation, diversification effect calculation and its splitting by products afterwards make up an important part of the alternative model. Improper approach can lead to incorrect return on capital calculation and return on risk adjusted capital for each insurance product and, therefore, result in bad business decisions by stopping underwriting for a certain product. The suggested topics for further research include finding an appropriate type of copula insurance sector reserve risk modelling and determining stability tests for an appropriate choice of copula model.

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