A new simple formulation for instantaneous coil diameter of a SMA helical spring

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Abstract: Helical coil tension springs made of shape memory alloy (SMA) materials generally undergo large deflection under loading during which their mean coil diameter changes noticeably. In the design of these helical coil springs, it is necessary to identify the real behaviour which may be affected by the variation in actual coil diameter. Therefore, a simple formulation is proposed in this paper for predicting instantaneous coil diameter. The predictions from present formulation match very closely with experimental measurements. The proposed formulation is relatively easy to adopt for design calculations. The effects of varying coil diameter on the spring characteristics are also discussed. This is very general and can be used for any helical spring which undergoes small or large deflections, although the proposed formulation is derived for SMA helical spring.

Keywords: shape memory alloy; SMA; helical coil tension springs; instantaneous coil diameter; large deflection.

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1 Introduction

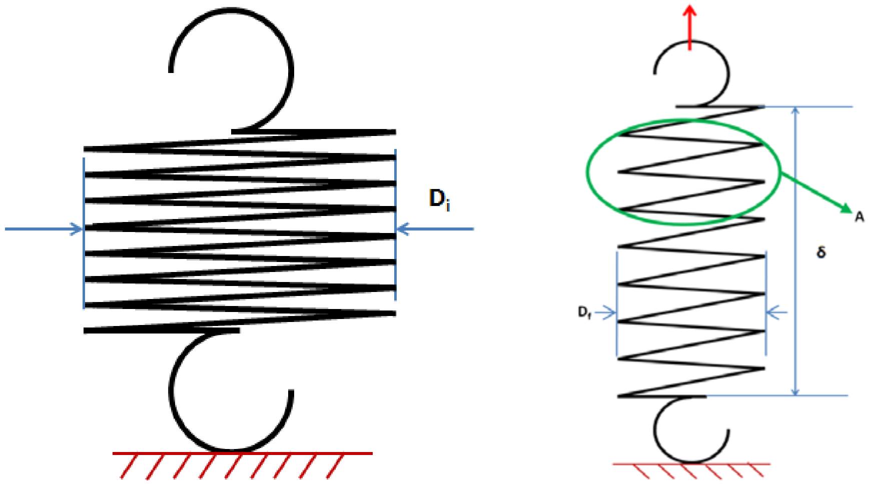
Helical coil tension springs made of shape memory alloy (SMA) materials are widely used in many actuator applications. These springs due to their smaller wire diameter and grain structure generally undergo large deflection. During this large deflection, the mean coil diameter of springs decreases considerably with increase in deflection. Many researchers (Tobushi and Tanaka, 1990; Liang and Rogers, 1993; Toi et al., 2004; Lee and Kim, 2008; Yu et al., 2008; Yang and Gu, 2008; Lee et al., 2009; Aguiar et al., 2010; Mirzaeifar et al., 2011) have thus studied various characteristics and responses of these SMA springs based on theoretical and experimental approaches. Tobushi and Tanaka (1990) designed SMA helical spring based on stress-strain-temperature relation which in turn derived from load-deflection relation, apparent shear modulus of elasticity of the spring material. They found that for certain maximum deflection the recoverable force and recoverable strain energy increases with temperature and dissipated strain energy takes a maximum value at certain temperature. Liang and Rogers (1993) presented one dimensional thermo-mechanical multi-dimensional model of SMA springs briefly and unique characteristics of SMAs, i.e., the stress-strain-temperature relations and shape memory effects. They found that recovery force may also be used to increase the effective stiffness of a structure utilising the concept of active strain energy tuning first developed for SMA hybrid composites. Some possible applications of SMA springs in vibration control have also been discussed. Toi et al. (2004) analysed superelastic behaviours of SMA helical spring. They formulated the incremental finite element method using linear Timoshenko beam elements by the total Lagrangian approach for the superelastic, large deformation analysis of SMA helical springs. Brinson's one-dimensional constitutive modelling for SMA has been extended to consider the asymmetric tensile and compressive behaviour and the torsional deformation. The incremental finite element analysis program has been developed by using the layered linear Timoshenko beam element equipped with the extended Brinson's constitutive Modelling for SMA. Lee and Kim (2008) designed and fabricated an in-pipe moving mechanism using the SMA spring type actuator. They used SMA spring and a bias spring to measure the dynamic characteristics of the bias type actuator and the differential type actuator. Yu et al. (2008) designed and manufactured a changeable aero-foil model using SMA springs with the help of stop structures. They measured deformation of the skins actuated by SMA springs and the results of the experiment and simulation are compared and analysed. Yang and Gu (2008) developed a silicon rubber rod with three SMA springs embedded off-axially such that spatial bending is accomplished by controlling the heating of SMA springs by suitable current and investigated experimentally. Lee et al. (2009) investigated the change of the dynamic characteristics of the transversely loaded SMA helical spring due to the martensite-austenite transformation using Castigliano's first theorem. They used the derived spring constant to define the equivalent flexural stiffness of the spring. They derived consequently the natural frequency of a stepped composite beam with the assembly of SMA spring and other beam-like component by using transfer matrix method. They validated their theoretical formulation through experimental measurements. Aguiar et al. (2010) analysed the quasi-static response of SMA helical springs. They evaluated evolution equations using the implicit Euler method combined with an orthogonal projection algorithm and a constitutive model that includes four macroscopic phases in the formulation. Mirzaeifar et al. (2011) proposed two new strategies for analysis of SMA helical springs subjected to an axial load with one based on an exact solution for the pure torsion of a straight SMA bar whereas the other considers a curvature correction and uses the torsion of a curved SMA bar. All their studies (Tobushi and Tanaka, 1990; Liang and Rogers, 1993; Toi et al., 2004; Lee and Kim, 2008; Yu et al., 2008; Yang and Gu, 2008; Lee et al., 2009; Aguiar et al., 2010; Mirzaeifar et al., 2011), have considered invariant mean coil diameter which is derived based on small deflection approach. Careful observations of their theoretical studies reveal a perennial difference with experimental validations. Kim et al. (2009) were the first to consider the change in coil diameter of the spring in their model and presented a micro muscle fibre crafted from SMA coiled springs. They described an enhanced spring NiTi model considering the combination of martensite deformation and spring effect due to its geometry. This paper also describes a manufacturing process and characterisation for micro scale NiTi coil actuators in various annealing temperature. They developed a soft robotic platform that deform its body dimension significantly and realise locomotion using the body deformation. The change in coil diameter of the spring was also noticed by An et al. (2012) who provided a formulation based on large deflection approach for prediction of instantaneous mean coil diameter. An et al. proposed an engineering design framework for an SMA coil spring actuator using two-state model. They have fabricated the coil spring actuator by annealing an SMA wire wound on a rod, conducted set of experiments to obtain the properties to verify their design. However, the formulation for prediction of instantaneous coil diameter was not validated with experiments or any other methods. Further, this is complex to adopt for design calculations.

Therefore, a simple formulation for instantaneous coil diameter is derived in this paper. This formulation is generic in nature and applicable to any helical coil spring undergoing small or large deflections. But only the SMA springs can undergo such large deflection than any other springs made up of conventional metallic materials. This is compared with carefully conducted experimental measurements and An et al. (2012) and found to yield a better agreement with experiments compared to the latter. Furthermore, the conventional design formulae for helical coil springs have been modified to account the change in coil diameter based on proposed formulation. The variation of some of the spring characteristics such as stiffness, shear strain, torque and angle of twist due to change in coil diameter are brought out.

2 Formulation

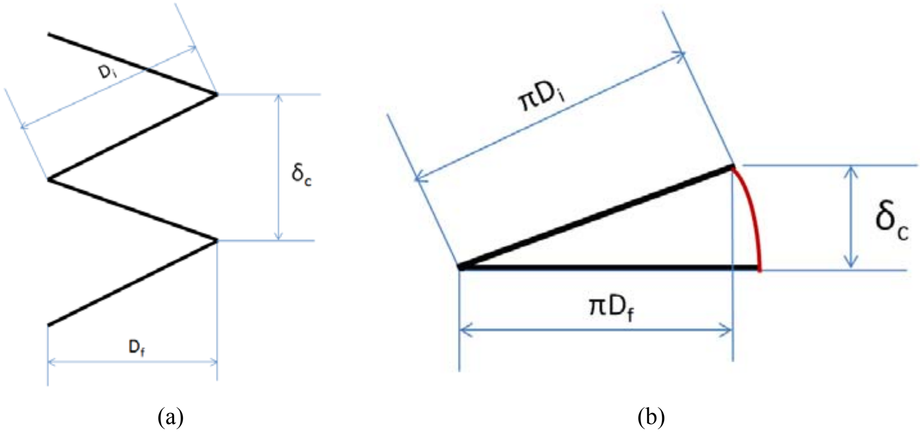
The proposed formulation is derived based on general observation on kinematics/geometry of deflection per unit coil of a tension spring. Figure 1(a) shows a tension spring in unstretched condition. Figure 1(b) shows a tension spring subjected to a large deflection and Figure 2(a) shows a closer view of few adjacent coils representing the final or instantaneous mean coil diameter D_f and the initial coil diameter D_i which is close to the slant length of a coil for reasonably large deflections. Figure 2(b) shows a closer view of development of one expanded coil. Let n, p, δ and δc respectively represent the number of coils, pitch of the coil, total deflection and the deflection per unit coil of the spring. In small deflection theory it is always assumed that D_i is equal to D_f which is not true when springs undergo large deflections.

Figure 1 (a) Spring in unstretched condition (b) Spring subjected to large deflection (see online version for colours)



(a) (b)

Figure 2 (a) Enlarged view ‘A’ of few coils (b) Development of one expanded coil (see online version for colours)



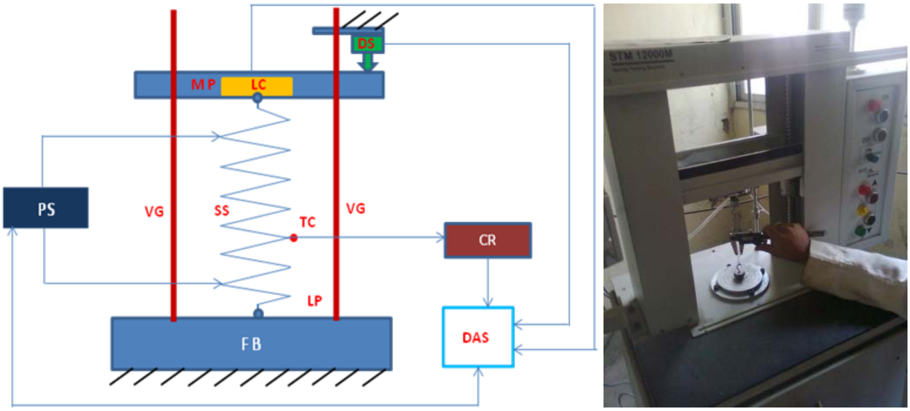
A closer observation on geometry of development of one expanded coil in Figure 2(b) shows that, equation (3) is relatively complex compared to the present formulation given in equation (2) which is simple and straight forward. In order to use equation (3) to find out D_f, α_i and α_f should be known which can be calculated from equation (4) and equation (5).

Pythagoras theorem has been used in the formulation for identification of instantaneous coil diameter of SMA helical coil spring. Pythagoras theorem is valid if diameter is greater than zero. Here initial coil diameter D_i and instantaneous coil diameter D_f are greater than zero (D_i and $D_f > 0$). One helical coil is uncoiled and represented as a ramp going up from the base circle as it is a projection for varying coil diameter, i.e., development of one expanded coil is shown in Figure 2(b). The deflection per coil δ_c for both 2(a) and 2(b) are same. It can be noted that although this proposed formulation is derived for loading of SMA spring in martensite phase, this is applicable to martensite unloading and loading and unloading of austenite phase also. This is because unloading from detwinned state do not lead to gross change in geometry and during unloading, only the elastic part of deflection is recovered which is a very small percentage of the total deflection. Also during elastic loading, it is observed that the change or decrease in coil diameter is not significant. Only heating of SMA spring to A_f temperature brings back its initial geometry. In austenite state, more force is required to deflect the spring to a larger deflection of the order of say 250 mm. But if it is deflected to 250 mm, then D_f (martensite) is equal to D_f (austenite) at that deflection. The proposed formulation for D_f for a given δ is applicable to martensite, austenite and mixed state. Hence the coil diameter variation depends on induced deflection and is independent of its phase/state. Here in this proposed formulation, small or large deformation does not affect the results since the direct geometry without approximation is used. Also the proposed formulation is applicable to predict the coil diameter irrespective of any initial pitch angle of the spring coil.

3 Experimental setup and measurements

In the present study, commercially available SMA springs made of NiTi alloy with composition of around 49.22% Ti and 50.78% Ni have been used with coil mean diameter of 5.67 mm, wire diameter of 0.78 mm and number of active coils turns of 18. The block diagram and actual experimental setup are shown in Figures 3(a) and 3(b) respectively. It consists of a fixed base as lower platform at the bottom and movable platform at the top where one end of the spring is attached to the fixed base and the other end of the spring is attached to top movable platform. This movable platform moves up and down to induce the required deflection on the spring and holds at that deflected condition. During holding time the outer coil diameter of the SMA spring is measured as shown in Figure 3(b).

Figure 3 (a) Block diagram of experimental setup (b) Experimental setup (see online version for colours)



(a) (b)

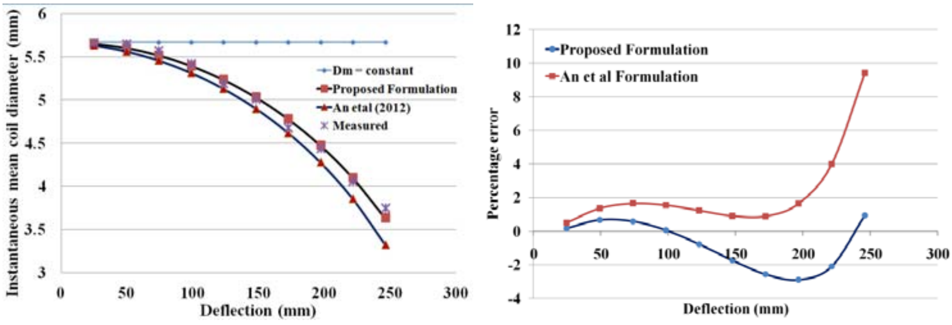
Notes: FB – fixed base
LP – lower platform
MP – movable platform
VG – vertical guide
SS – SMA spring
LC – load cell
TC – thermo-couple
CR – Controller
PS – power supply
DS – displacement sensor
DAS – data acquisition system.

The schematic representations of springs in unstretched and stretched conditions are shown in Figures 1(a) and 1(b) respectively. The outer coil diameter of the spring was measured for different deflected conditions using digital vernier calliper having a length measurement accuracy of ±0.01 mm. For every applied deflection, the average outer coil diameter was calculated using three measurements. The instantaneous mean coil diameter was obtained by subtracting the wire diameter from the measured outer coil diameter. All the above experiments were conducted at room temperature, i.e., the SMA spring was in martensite phase. Force loading corresponds to martensite reorientation. Martensite undergoes transformation from twinned condition to detwinned condition due to loading. In this process austenite phase was not involved.

4 Comparison with previous solution and experiment

The present formulation for instantaneous mean coil diameter of an SMA spring is compared with carefully conducted experimental measurements and also with the theoretical formulation of An et al. (2012) in Figure 4(a). This shows that the present formulation is in good agreement with experimental measurements. The difference between the prediction based on proposed formulation and An et al. (2012) with respect to measurements is calculated as percentage error and is plotted in Figure 4(b). This comparison shows that the present formulation exhibits less percentage error as compared to that of An et al. (2012). It also shows that the trends are same for both present and An et al. (2012) formulations up to the applied deflection of 172.2 mm beyond which the percentage error of An et al. (2012) is increasing considerably. Hence, the present model is especially able to capture large deflections more accurately than that of An et al. (2012), which is an essential requirement for SMA springs.

Figure 4 (a) Coil diameter from formulations and measurements (b) Error percentage of present and An et al. (2012) formulations compared to measurements (see online version for colours)



(a) (b)

The proposed formulation requires only two parameters such as induced deflection (δ) and initial mean coil diameter (D_i) which are directly measurable and can be used directly to calculate final/instantaneous mean coil diameter. This can be verified with experiments easily. An et al. (2012) formulation requires an additional parameter called pitch angle α (in addition to above said parameters) which is not easily measurable hence it is very difficult to compare with experiment and prone to manual errors in calculated results. But α can be derived to calculate final/instantaneous mean coil diameter. As the pitch angle α_i increases with increase in deflection (δ), the difference between proposed formulation and An et al. (2012) formulation increases considerably. Hence comparatively the proposed formulation is simple and generic for both small and large deflections for any spring. Also proposed formulation is closely matching with the measured D_f of the SMA spring without using α_i. So the proposed formulation can be adopted for design calculations easily. An et al. (2012) formulation suggests α_i is always equals to zero. In actual scenario, α_i will not be zero but it will be more than zero degrees. Considering the closeness of the proposed formulation with the measurement results and its easiness in predicting instantaneous mean coil diameter without any need for pitch angle (α), it can be concluded that the proposed one is simple and accurate formula in predicting the instantaneous diameter of the SMA spring.

5 Effects of varying coil diameter on spring characteristics

Based on the proposed formulation for the mean coil diameter as given in the equation (2), the conventional spring design formulae have been modified. The comparison between the conventional design formulae and modified design formulae based on the proposed formulation are presented in Table 1.

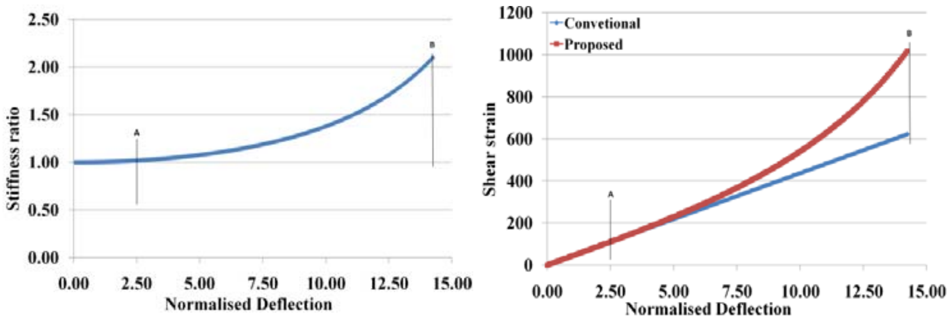
Table 1 Conventional and proposed formulae for spring design

Parameters	*Conventional formulae*	*Proposed formulae*
Force (F)	$\left(\frac{Gd^4}{8D^3n}\right)\delta$	$\frac{Gd^4\delta\pi^3}{8n\sqrt{\left[(\pi D_i)^2-(\delta_c)^2\right]^3}}$
Stiffness (K)	$\left(\frac{Gd^4}{8D^3n}\right)$	$\frac{Gd^4\pi^3}{8n\sqrt{\left[(\pi D_i)^2-(\delta_c)^2\right]^3}}$
Shear strain (γ)	$\frac{\delta d}{\pi n D^2}$	$\frac{\delta d\pi}{n\sqrt{\left[(\pi D_i)^2-(\delta_c)^2\right]}}$
Shear stress (τ)	$\frac{8FD}{\pi d^3}$	$\frac{8f\sqrt{(\pi D_i)^2-(\delta_c)^2}}{\pi^2 d^3}$
	γG	γG
Torque (T)	$\frac{FD}{2}$	$\frac{F\sqrt{(\pi D_i)^2-(\delta_c^2)}}{2\pi}$
Angle of twist (Ø)	$\frac{2\delta}{\pi n D^2}$	$\frac{2\delta\pi}{n[(\pi D_i^2)-(\delta_c)^2]}$

Notes: D_i = initial mean coil diameter; D_f = final/instantaneous mean coil diameter.
D = mean coil diameter; d = wire diameter; α_i = initial helix angle.
α_f = final/instantaneous helix angle; n = number of active turns; p = pitch.
δ = total deflection of spring; δc = deflection of one coil (or) deflection per coil = δ/n.
G = shear modulus; T= Torque; Ø = angle of twist.

The proposed formulation brings out the differences in behaviour/characteristics of SMA spring in terms of stiffness, shear strain, torque and angle of twist as compared to conventional formulation are typically shown in Table 1. For any geometry of spring, stiffness is a function of coil diameter D, wire diameter d and number of turns n. Conventional spring formulae are valid only when spring deflection is small with diameter assumed to be constant or negligible change in coil diameter whereas in SMA coil diameter change is considerable as spring deflection is large. The corrections have to be applied on the conventional spring formulae considering an instantaneous coil diameter and thus the conventional spring formulae get modified. Table 1 shows only the correction applied to conventional spring formulae. Shear modulus G is a material property which is a function of phase and phase depends on temperature. Appropriate G value has to be considered in these formulae (Table 1), i.e., for austenite phase G_A has to be used and for martensite phase G_M has to be used. But the geometric change is independent of phase which is clearly explained at the end of formulation, i.e., Section 2. Figure 5(a) shows the variation of stiffness ratio (ratio between proposed stiffness and conventional stiffness) with normalised deflection (deflection of spring normalised with initial height/free length of the spring). Figure 5(b) shows the shear strain with normalised deflection. The impact of varying mean coil diameter can be clearly observed in Figures 5(a) and 5(b), beyond the region O-A where nonlinearity characteristics variation is observed on both stiffness ratio (Kr) and shear strain (γ). The sensitive zone of SMA spring is actually the region A-B which is considerably affected due to change in coil diameter and the proposed formulation could capture these variations very well. For example, considering various parameters, for the given maximum deflection of 200 mm, the maximum stiffness of 0.1021 N/mm is obtained by considering constant coil diameter where as by considering varying coil diameter the stiffness obtained is 0.2140 N/mm and this leads to an error of 52.30%. Similarly for maximum deflection of 200 mm, all the other characteristics of springs such as stiffness, shear strain and torque show an error of 38.95%. Hence it is necessary to consider the varying coil diameter in the study/formulation.

Figure 5 (a) Variation of stiffness ratio with normalised deflection (b) Variation of shear strain with normalised deflection (see online version for colours)



(a) (b)

6 Conclusions

A new simple formulation for predicting instantaneous coil diameter (D_f) of a helical coil extension spring undergoing large deflection has been presented. Model is developed to consider geometric nonlinearity without using helix angle (α) and the formulation is simple. The proposed formulation requires only two parameters namely induced deflection (δ) and initial mean coil diameter (D_i) which are directly measurable and can be used directly to calculate final/instantaneous mean coil diameter (D_f) hence this can be verified with experiments easily. The predictions from proposed formulations are in good agreement with a set of experimental measurements as compared to an already existing complex formula derived based on large deflection approach. Percentage error between the present formula and existing formula with experimental measurements shows that the present formula is having less percentage error compared to that of existing one. Proposed model is simple and generic for both small and large deflections for any spring. This formula is well suited for design applications of SMA springs due to its simple form and is independent of its phase/state. The effects of varying coil diameter on some of the spring characteristics such as stiffness, shear strain, torque and angle of twist on the sensitive zone of SMA spring are also discussed. The proposed formulation is also applicable to predict the coil diameter irrespective of any initial pitch angle of the spring coil. The predictions of instantaneous coil diameter do not get affected for small or larger deflections since the direct geometry without any approximations is used in the proposed formulation.

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Appendix

A closer observation on geometry of development of one expanded coil in Figure 2(b) shows that,

$$(\pi D_f)^2 = \left[(\pi D_i)^2 - \delta_c^2\right] \tag{1}$$

This gives

$$D_f = \frac{\sqrt{\left[(\pi D_i)^2 - \delta_c^2\right]}}{\pi} \tag{2}$$

An et al. (2012) have provided the below formulation for instantaneous coil diameter of the tension spring as:

$$D_f = D_i\left(\frac{\cos\alpha_f}{\cos\alpha_i}\right) \tag{3}$$

where

$$\alpha_i = \tan^{-1}\left[\frac{p}{\pi D_i}\right] \tag{4}$$

$$\alpha_f = \sin^{-1}\left[\left(\frac{\delta\cos\alpha_i}{\pi n D_i}\right) + \sin\alpha\right] \tag{5}$$