MIMO wireless power transfer based on magnetic beamforming

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Abstract: Wireless power transfer (WPT) promises to provide sufficient power for multiple users. However, majority of prior works focused on systems with single transmitter or receiver. In this paper, we base our MIMO wireless power transfer (MIMO-WPT) system with N transmitters and two receivers on magnetic beamforming which is similar to that in a MIMO system of wireless communication. From the perspective of signal processing and practical application, we formulate an optimisation problem to minimise total power consumed by all transmitters, subject to the minimum power required by both receivers. Furthermore, we demonstrate that the semi-definite relaxation (SDR) of such a quadratically constrained quadratic programming (QCQP) problem is tight, thus can be solved by standard interior-point algorithm. Simulation results show that compared with the uncoordinated baseline scheme of identical current allocation, our MIMO-WPT system has boost power efficiency prominently. In the end, we consider a more general model with N transmitters and M receivers, and the characteristic of the optimal solution is deduced.

Keywords: wireless power transfer; WPT; magnetic beamforming; MIMO; semi-definite relaxation; SDR; convex optimisation.

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1 Introduction

With the continuous development and popularisation of mobile and wearable devices, wireless power transfer (WPT) has drawn increasing attention since it brings consumers a better user experience than traditional charging with cord. Lu et al. (2015) have described the huge advantage and potential of WPT. Shi et al. (2015) have reported that several techniques such as RF radiation, magnetic coupling, lasers as well as ultrasound have been adopted to accomplish the task of WPT. However, Kurs et al. (2007) have considered the issue of safety and application degree of those techniques, their research has demonstrated that magnetic coupling is much more safe and efficient than other techniques. Thus, we base our design on magnetic coupling. Besides, we employ magnetic resonant coupling (MRC) rather than inductive magnetic coupling in view of its higher efficiency.

When it comes to technique of traditional MIMO, as is known to all, by using multi-antenna, beamforming greatly enhances the power efficiency of a MIMO system in the field of wireless communication. Although traditional MIMO works in the far field, which hints the wave length is far less than the distance between transmitter and receiver while WPT works in the near field. Jadidian and Katabi (2014) have verified that such a principle defined as 'magnetic MIMO' is still applicable in WPT by the way of theoretical analysis and experimental investigations.

However, as far as we know, majority of prior works on WPT focused on systems with single transmitter or receiver. Moghadam and Zhang (2016) have investigated a SIMO-WPT system while Yang et al. (2015) have reported a MISO-WPT system. What are worse, extremely limited researches on multiuser merely pay their attention to adjusting circuit components parameters. Lu et al. (2015) have focused on studying how circuit impedances of transmitters and receivers influence total power consumed by all transmitters in a MIMO-WPT system.

In this article, in view of actual application scenarios, we come up with a MIMO-WPT model with N transmitters and two receivers rather than the model equipped with only one receiver. Furthermore, based on magnetic beamforming, this paper derives and analyses series of conclusions from the perspective of controlling signal instead of adjusting parameters of circuits.

First, we build our MIMO power transfer system (MIMO-WPT) on the basis of magnetic beamforming. To enhance efficiency, both transmitters and receivers are composed of MRC circuits which resonate at the same resonant frequency. Then, we consider an optimisation beamforming problem for such a MIMO system with N transmitters and two receivers. For the purpose of consuming the least energy drawn from the voltage sources, we deduce an optimisation problem, which is restricted by the minimum receiver power assigned to each receiver. We find that such an optimisation problem satisfies the form of quadratically constrained quadratic programming (QCQP) problem. As the usual method, we recast the QCQP problem to a form of semi-definite programming (SDP) problem.

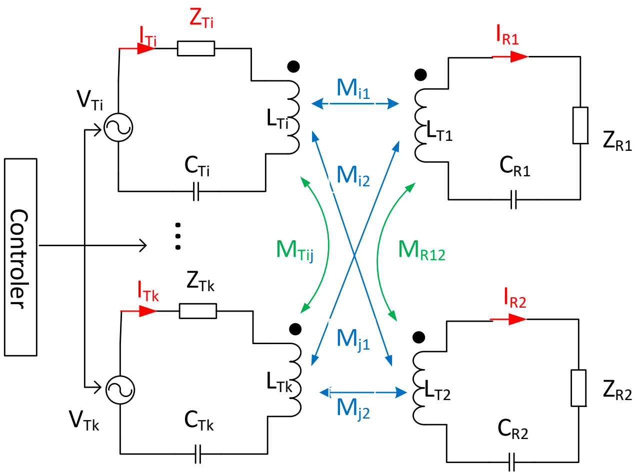
Furthermore, we show that this SDP problem has a tight semi-definite relaxation (SDR), so an optimal solution for such a scale of problem with only two receivers can be smoothly obtained by applying traditional interior-point algorithm.

In the end, to show the excellent performance in energy efficiency of our designed MIMO-WPT system, we design a baseline scheme called uncoordinated scheme with equal current allocation in all transmitter coils for comparison. Simulation results indicate our MIMO-WPT scheme prominently boost the performance in energy efficiency.

2 System model

As shown in Figure 1, our MIMO-WPT system is composed of N transmitters and two receivers, for the benefit of efficiency; we adopt the scheme of magnetic resonance rather than inductive magnetic coupling. In this advanced technique, for the purpose of resonating, a capacity is added to both transmitter and receiver circuits. We assume our MIMO power transfer system resonates at the frequency ω.

Figure 1 System model of MIMO-WPT with N transmitters and two receivers (see online version for colours)



Each transmitter possesses one induction coil, indexed as T_n while each receiver owns one induction coil, indexed as Rm. We denote the resistance of transmission circuit and receiver circuit as Z_{Tn} and Z_{Rn}, respectively.

Besides, we apply the sinusoidal voltage source as energy source for each transmitter. By applying steady-state analysis for this system, we have the following basic equations.

For each voltage source, we have:

$$\widetilde{v_n}(t) = \text{Re}\{v_n e^{j\omega t}\} \tag{1}$$

Thus, for each receiver circuit, we have the steady-state current

$$\widetilde{i_m}(t) = \text{Re}\{i_m e^{j\omega t}\} \tag{2}$$

The complex v_n and i_m respectively represent an adjustable voltage and current, and represents the resonating angular frequency of this system.

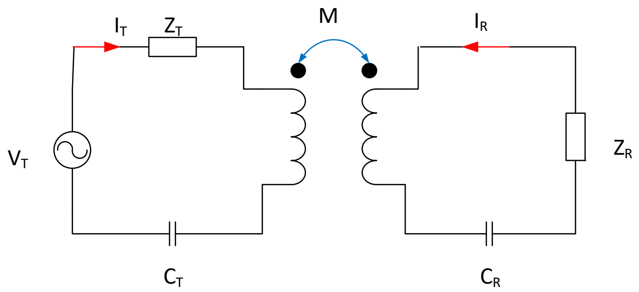
Apparently, compared with the MISO power transfer system which has two kinds of mutual inductance – the one among different transmitters and the one between a transmitter and a receiver, there also exists a kind of mutual inductance among different receivers in our MIMO system. For convenience of analysis, we denote the mutual inductance between transmitter i and transmitter j as M_{Tij}, the one between receiver i and receiver j as M_{Rij}, and the one between transmitter i and receiver j as M_{ij}.

First, in order to facilitate the analysis, we take a basic simplified system with a single transmitter and a single receiver resonating at frequency ω into consideration. The system is shown in Figure 2.

For the transmitter, we denote I_R as the current flowing through the transmit circuit, while I_T as the current flowing through the transmit circuit. Besides, Z_R represents receiver circuit's impedance and M reflects the magnetic coupling existing between such two different induction coils. Take advantage of Kirchhoff's circuit theorem, we deduce such an equation:

$$Z_R I_R = j\omega M I_T \tag{3}$$

Figure 2 Basic model of MRC (see online version for colours)



This equation describes the influence exerted on the receiver coil by the transmitter coil, which is the effect of electromagnetic induction.

For the receiver, however, since we have added a sinusoidal voltage source into the transmitter circuit to supply energy source, such a voltage source should be taken into consideration. Similarly, by applying Kirchhoff's circuit theorem, we have the following equation:

$$VT = Z_T I_T - j\omega M I_R \tag{4}$$

where V_T is the voltage offered by the sinusoidal voltage source, and Z_T denotes transmitter circuit's impedance. In a sense, equation (4) reflects the fact that the receiver coil has a reaction on the transmitter one. When we replace I_R in equation (4) with that in equation (3), we have equation (5).

$$V_T = \left(Z_T + \omega^2 M^2 / Z_R\right) I_T \tag{5}$$

Equation (3) and equation (5) provide an overall description of this basic simplified system. In the following analysis, we will extend this conclusion to our MIMO power transfer system.

From the above analysis mentioned, we know that there exists three types of mutual inductance, i.e., M_{Tij}, M_{Rij} and M_{ij}. Similar to the basic simplified system, when we consider an arbitrary receiver μ (μ = 1, 2), compared to equation (3), we have:

$$Z_{R\mu} I_{R\mu} + \sum_{\upsilon \neq \mu} j\omega M_{R\mu\upsilon} I_{R\mu} = \sum_{i} j\omega M_{i\mu} I_{Ti} \tag{6}$$

For an arbitrary transmitter i (i = 1, 2), compared to equation (5), we have:

$$V_{Ti} = Z_{Ti} I_{Ti} + \sum_{k \neq i} j\omega M_{Tik} I_{Tk} - \sum_{\mu} j\omega M_{i\mu} I_{R\mu} \tag{7}$$

Note that we rewrite the above two equations into matrix form for the convenience of subsequent analysis. And all variables needed are clarified in detail in Table 1.

$$\overrightarrow{i_R} = j\omega Z_R^{-1} M \overrightarrow{i_T} \tag{8}$$

$$\overrightarrow{V_T} = \left(Z_T + \omega^2 M^T Z_R^{-1} M \right) \overrightarrow{i_T} \tag{9}$$

Table 1 Definition and remark of vector/matrix

Term	*Definition*	*Remark*
$\overrightarrow{V_T}$	$[V_{T1}, V_{T2}, \ldots, V_{Tn}]_{\mathrm{N}\times 1}$	Transmitter voltages, N represents the number of transmitters
$\overrightarrow{i_T}$	$[I_{T1}, I_{T2}, \ldots, I_{Tn}]_{\mathrm{N}\times 1}$	Tx currents, N represents the number of transmitters
$\overrightarrow{i_R}$	$[I_{R1}, I_{R2}]$	Rx currents, our MIMO-WPT system has two receivers
Z_R	$\begin{bmatrix} Z_{R1} & j\omega M_{R12} \\ j\omega M_{R21} & Z_{R2} \end{bmatrix}$	Rx impedance and inter-receiver magnetic couplings between the two receivers
Z_T	$\begin{bmatrix} Z_{T1} & j\omega M_{T12} & \cdots & j\omega M_{T1n} \\ j\omega M_{T21} & Z_{T2} & \cdots & j\omega M_{T2n} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega M_{Tn1} & j\omega M_{T21} & \cdots & Z_{Tn} \end{bmatrix}$	Transmitter impedance and inter-transmitter magnetic couplings, N represents the number of transmitters
M	$\begin{bmatrix} M_{11} & M_{21} & \cdots & M_{n1} \\ M_{12} & M_{22} & \cdots & M_{n2} \end{bmatrix}_{2\times N}$	Magnetic couplings between transmitter and receiver, N represents the number of transmitters

From Table 1, we know that $\overrightarrow{i_T}$ and $\overrightarrow{i_R}$ are vectors representing transmitter currents and Rx currents respectively.

Thus, we obtain the total power provided by all transmitter source voltages as follows

$$P_{sum} = \frac{1}{2} \overrightarrow{i_t^H} \left(Z_T + \omega^2 M^T Z_R^{-1} M \right) \overrightarrow{i_T} \tag{10}$$

To obtain the power consumed by an arbitrary receiver, denoted by P_{Rm}, we define an M-order (M represents the number of receivers) diagonal matrix R_m whose diagonal element m represents the resistance of receiver m and all the other elements are zeros. Then we can obtain:

$$P_{Rm} = \frac{1}{2} \overrightarrow{i_R^H} R_m \overrightarrow{i_R} \tag{11}$$

We substitute in equation (11) with equation (8), then, we derive another expression of equation (11).

$$PRm = \frac{1}{2}\overrightarrow{i_T^H} M^H \left(Z_R^{-1}\right)^H R_m Z_R^{-1} M \overrightarrow{i_T} \tag{12}$$

For convenience, we define the N-order square matrix

$$B = Z_T + \omega^2 M^T Z_R^{-1} M \tag{13}$$

Clearly, B is an N-order Hermitian matrix, and N represents the number of receivers.

Then equation (10) can be rewritten as follows

$$P_{sum} = \frac{1}{2}\overrightarrow{i_R^H} B \overrightarrow{i_R} \tag{14}$$

Together, equation (8) and equation (9) supply a comprehensive description of our MIMO power transfer system. The first equation reflects changes on current in receiver circuit when transmitters exert their influence. And the second relational expression gives tips on what voltage we really need in order to supply such a MIMO power transfer system. Above all, we will base our subsequent optimisation problem on these two equations in the following part of this paper.

3 Problem formulation

In field of engineering, engineers tend to concern more about energy utilisation. Thus, when it comes to technology of power transfer, how to advance the efficiency as much as possible should be given first priority.

As a matter of course, in this part, we consider such a common scene where each receiver load has to obtain a minimum power, such as β_m, i.e., $p_m \geq \beta_m$, to ensure the device to work properly. Practically, we set our objective function to make all source voltage provide power as less as possible. Thus, our optimisation problem can be established as follows:

$$\text{(P1)} \quad \min_{\overrightarrow{i_T} \in C^N} \frac{1}{2}\overrightarrow{i_T^H} B \overrightarrow{i_T} \tag{15a}$$

$$\frac{1}{2}\overrightarrow{i_T^H} M^H \left(Z_R^{-1}\right)^H R_m Z_m^{-1} M \overrightarrow{i_T} > \beta_m (m = 1, 2) \tag{15b}$$

Equation (15a) represents the objective function which aims at consuming all transmitters' power as less as possible while equation (15b) denotes the constraint condition that each receiver should gain a minimum power to guarantee normal work.

According to convex optimisation theory in Boyd and Vandenberghe (2004), (P1) is a complex-valued non-convex QCQP problem since the objective function and constraint condition is all quadratical.

Traditionally, we need focus on transforming the QCQP problem into the form of SDP, then we consider the SDR of this SDP, if the SDR is tight, then we can apply standard interior-point method to obtain the optimal solution of this problem.

4 Optimal solutions

In this section, we make our efforts to get the optimal solution to P1. For convenience, we define $X = \overrightarrow{i_T}.\overrightarrow{i_T^H}$, $C = (Z_R^{-1}M)^H R_m Z_R^{-1}M$, then we transform the QCQP problem into a SDP problem.

(P1-SDP)

$$\min_{X \in C^{N*N}} \frac{1}{2} Tr(BX) \tag{16a}$$

$$Tr(CX) \geq \frac{2\beta_m}{\omega^2} \tag{16b}$$

$$X \succ 0, \text{rank}(x) = 1 \tag{16c}$$

Generally, we ignore the rank constraint, which is non-convex, then we get the SDR of this SDP problem, which is defined as P1-SDR, according to convex optimisation theory in Boyd and Vandenberghe (2004), P1-SDR is a convex problem.

Luo et al. (2010) have investigated that when we define X^* as the optimal solution to SDR, and m as the number of linear constraints, then we can obtain the following empirical formula:

$$\frac{rank(X^*)(rank(X^*)+1)}{2} \leq m \tag{17}$$

Obviously, in P1-SDR, m is equal to the number of receivers, i.e., $m = 2$. Then we substitute $m = 2$ into equation (17), we get the conclusion

$$rank(X^*) = 1 \tag{18}$$

Equation (18) means P1-SDR is tight, then we can apply an interior-point method to solve P1-SDR with the help of the convex optimisation toolbox CVX. Luo et al. (2010) have demonstrated that P1 has the same optimal solution with P1-SDR, thus, we can obtain the optimal solution to P1 without any more processing.

5 Numerical results

As the distribution location of all coils shown in Figure 3, we build our MIMO system whose operating angular frequency is $\omega = 6.78 \times 2\pi \times 10^6$ rad/second with $N = 4$ transmitters and $M = 2$ receivers, and all coils are made up of copper materials. The physical parameters of these coils are shown in Table 2.

In our system, we assume that all transmitters and receivers are respectively identical. For each transmitter coil, the number of turns is 260, and its radius is 0.2 metre, while for each receiver coil, the number of turns is 150.

Figure 3 System setup, four transmitters and two receivers (see online version for colours)

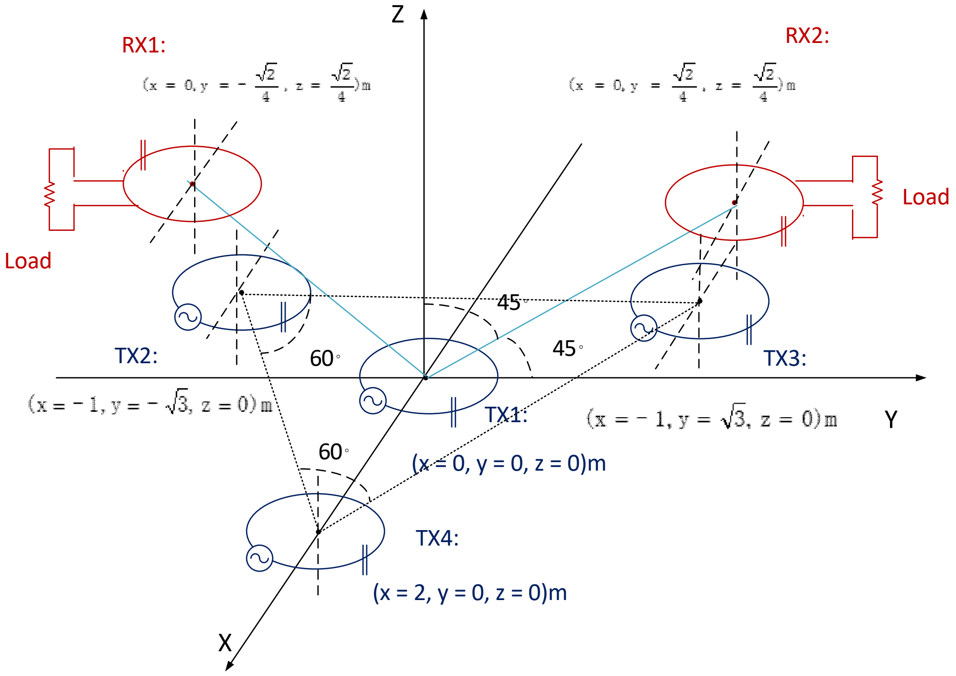


Table 2 Physical parameters of these coils

EM coil	*Inner radius (cm)*	*Outer radius (cm)*	*Average radius (cm)*	*Number of turns*	*Material of wire*	*Resistivity of wire*
Tx1	19.9	20.1	20	260	Copper	0.0168
Tx2	19.9	20.1	20	260	Copper	0.0168
Tx3	19.9	20.1	20	260	Copper	0.0168
Tx4	19.9	20.1	20	260	Copper	0.0168
Rx1	4.95	5.05	5	150	Copper	0.0168
Rx2	4.95	5.05	5	150	Copper	0.0168

Moghadam and Zhang (2016) have investigated how to compute physical parameters such as self/mutual inductances of a system with many induction coils distributed in different locations as following

$$r_i = \frac{2\sigma_i b_i e_{ave,i}}{e_{wire,i}^2} \tag{19}$$

$$l_i = b_i^2 e_{ave,i} \mu \left(\ln\left(\frac{8e_{ave,i}}{e_{wire,i}} \right) - 2 \right) \tag{20}$$

Based on equation (19) and equation (20), we get all quantities needed as follows: each transmitter has a resistance of 0.336Ω while the resistance of each receiver is 40 Ω, besides, based on equation (19), we can get self/mutual inductances as shown in Table 3.

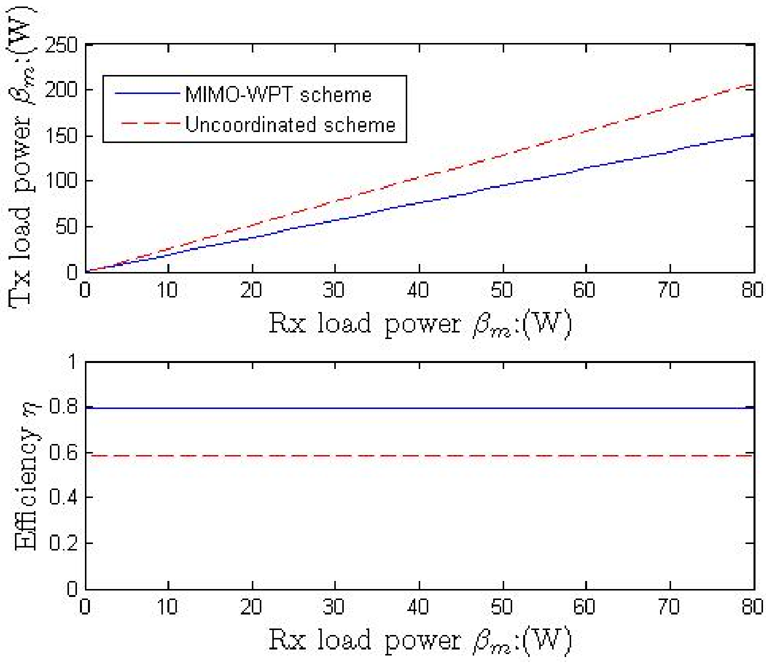
$$h = -\frac{\pi\mu b_1 b_2 e_{ave,1} 2_{eave,2}}{4d^3}\begin{pmatrix} 3\cos(\theta')\sin(\theta')\cos(\phi')n_{x,2} \\ +3\cos(\theta')\sin(\theta')\sin(\phi')n_{x,2} \\ +\left(2\cos^2(\theta') - \sin^2(\theta')\right)n_{z,2} \end{pmatrix} \tag{21}$$

Table 3 Mutual/self inductances (μH)

	Tx1	*Tx2*	*Tx3*	*Tx4*	*Rx1*	*Rx2*
Tx1	3,378.9	0.4935	0.4935	0.4935	–0.9870	–0.9870
Tx2	0.4935	3,378.9	0.0950	0.0950	0.0411	0.0179
Tx3	0.4935	0.0950	3378.9	0.0950	0.0179	0.0411
Tx4	0.4935	0.0950	0.0950	3,378.9	0.0257	0.0257
Rx1	–0.9870	0.0411	0.0179	0.0257	753.90	0.0436
Rx2	–0.9870	0.0179	0.0411	0.0257	0.0436	753.90

As a contrast, we setup an uncoordinated baseline scheme where all transmitters have identical current distribution. Since current in each transmitter is identical, according to equation (15b), we can easily get the identical current in each transmitter, apply it into equation (15b) then we obtain the transmitter power in the uncoordinated baseline scheme.

Figure 4 Power and efficiency vs. RX power *βm* (see online version for colours)



We define our MIMO-WPT system's efficiency as $\eta = \frac{M\beta_m}{p_{sum}}$, where M means the number of receivers, *βm* represents the minimum power of the m^{th} receiver (to simplify, each receiver has identical power requirements), and p_{sum} denotes the total power of all transmitters.

Figure 4 provides curves of total transmitter power p_{sum} and efficiency η versus βm. Clearly, we discover that the efficiency of MIMO-WPT scheme is 80% while for the uncoordinated baseline scheme, the value is 58%. That is to say, by using our MIMO-WPT scheme, we increase our efficiency by 22%.

6 N*M model

In the above analysis, we gain the global optimal solution of convex problem (P1) which is based on an N*2 model. In this section, we consider a more general system with N transmitters and M receivers, thus we set up an optimisation problem whose objective function aims at consuming power as less as possible as is shown in equation (22a)

$$(P2) \quad \min_{iT \in C^N} \frac{1}{2} \overrightarrow{i_T^H} B \overrightarrow{i_T} \tag{22a}$$

$$\frac{1}{2} \overrightarrow{i_T^H} M^H \left(Z_R^{-1}\right)^H R_m Z_R^{-1} M \overrightarrow{i_T} \leq \beta_m \tag{22b}$$

$$\overrightarrow{i_T^H} b_n^H b_n \overrightarrow{i_T} \leq V_n^2 \tag{22c}$$

$$\overrightarrow{i_T^H} Q_n \overrightarrow{i_T} k \leq A_n^2 \tag{22d}$$

Equation (212b) represents receiver's power constraint condition, while equation (22c) and equation (22d) represents Transmitter's current and voltage constraint condition.

Thus, the above problem is a non-convex QCQP problem. According to the usual practice, we define $X = \overrightarrow{i_T} . \overrightarrow{i_T^H}$, $B_n = b_n^H . b_n$, $Y_m = (Z_R^{-1} M)^H R_m Z_R^{-1} M$ thus, we have

(P2-SDP)

$$\min_{X \in C^{N*N}} \frac{1}{2} Tr(BX) \tag{23a}$$

$$Tr\left(Y_m X\right) \geq \frac{2\beta_m}{\omega^2} \tag{23b}$$

$$Tr\left(B_n X\right) \leq V_n^2 \tag{23c}$$

$$Tr\left(Q_n X\right) \leq A_n^2 \tag{23d}$$

$$X \succ 0, rank(X) = 1 \tag{23e}$$

By ignoring the only non-convex condition equation (22a), we get P2-SDR, and we define as the optimal solution to P1-SDR, then we have the following Lagrange dual function

$$L(X, \lambda, \rho, S) = 0.5Tr(BX) - \sum_{m=1}^{M} \lambda_m \left(\left(Y_m X\right) - \frac{\beta_m}{\omega^2} \right) + \sum_{n=1}^{N} \rho_n \left(Tr\left(B_n X\right) - A_n^2 \right) \tag{24}$$

The KKT condition is as follows

$$\nabla xL\left(X^*, \lambda^*, \rho^*, \mu^*, S^*\right) = 0.5B - \sum_{m=1}^{M} \lambda^* m Y_m + \sum_{n=1}^{N} \rho_n^* B_m + \sum_{n=1}^{N} \mu_n^* Q_n - S^* = 0 \quad (25a)$$

$$S^* X^* = 0 \quad (25b)$$

We apply equation (25a) in to equation (24), we obtain

$$0.5BX^* - \sum_{m=1}^{M} \lambda_m^* Y_m X^* + \sum_{n=1}^{N} \rho_n^* B_n X^* + \sum_{n=1}^{N} \mu_n^* Q_n X^* = 0 \quad (26)$$

Thus, we have

$$rank\left(\left(0.5B \sum_{n=1}^{N} \rho_n^* B_n + \sum_{n=1}^{N} \mu_n^* Q_n\right) X^*\right) = rank\left(\sum_{m=1}^{M} \lambda_m^* Y_m X^*\right) \leq rank\left(\sum_{m=1}^{M} Y_m\right) \quad (27)$$

$$rank\left(\sum_{m=1}^{M} Y_m\right) = rank\left(\left(Z_R^{-1} M\right)^H R_m Z_R^{-1} M\right) \leq \min\{M, N\} \quad (28)$$

We define $B = Z_T + \omega^2 M_T Z_R^{-1} M$ as positive semi-definite, then we get the conclusion that $\left(0.5B + \sum_{n=1}^{N} \rho_n^* B_n + \sum_{n=1}^{N} \mu_n^* Q_n\right) X^*$ is full rank, thus we have

$$rank\left(X^*\right) \leq \{M, N\} \quad (29)$$

Equation (29) represents that the rank of the optimal solution is less than the number of receivers and transmitters. It is not certain that we can obtain the global optimal solution with the method used in the N*2 model since we can not guarantee equation (19) is satisfied.

However, according to Luo et al. (2010), we can combine standard interior-point method with Gauss random algorithm to obtain the local optimal solution, which is valid in practice.

7 Conclusions

In this paper, based on an intensive analysis on a MIMO-WPT system with N transmitters and two receivers, we deduce a QCQP problem to obtain the minimum power consumed by all transmitters. Practically thinking, we put this optimisation problem under constraint of minimum receiver power to satisfy requirement of each receiver. As the usual method to deal with the non-convex QCQP problem, we recast it into a SDP problem, and demonstrate that its SDR is tight, thus can be solved by convex optimisation toolbox CVX. For comparison, we formulate an uncoordinated baseline scheme with identical transmitter current distribution. Simulation results show that compared with the baseline scheme, our MIMO-WPT system significantly enhances power efficiency, which has a profound significance for the actual deployment of WPT. In the end, we take a consideration on the general N*M model, and point out that standard interior-point method combined with Gauss random algorithm is still a useful method in solving the optimisation problem.

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