Multi-objective methods in development planning

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Abstract: The purpose of this article is to investigate the applicability of multi-objective decision making methods in development planning. A multi-objective, project selection, linear programming model is developed as a planning tool for hypothetical economy. The model is solved in different versions using the method of Geoffrion et al. Assumed logarithmic, additively separable utility functions, differing in their degree of nonlinearity, have been developed and utilised to provide simulated decision maker responses needed by the method. The article emphasises that for development plans to be optimal, realistic, logical, and meaningful, multi-objective, mathematical programming models, based on projects rather than sectors, are needed in formulating these plans.

Keywords: linear programming; development planning; mathematical programming; multi-objective decision making; the method of Geoffrion et al.

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1 Introduction

1.1 Planning and mathematical models

Development planning represents an attempt to coordinate economic decision making over the long run, in order to give direction to and accelerate a country's development. It is a complex process involving choosing social objectives, setting various targets, disseminating information, as well as organising a framework for the implementation, coordination, and monitoring the plan. Development planning has become the chief hope of less developed countries for ultimately achieving parity with advanced countries in living standards and social progress. As countries have developed and their experience has increased, approaches to policy making have become more complex. Although debate continues over the role of formal planning and the relationship between planning and implementation, there is a widespread acceptance of the usefulness of some form of a systematic economic analysis as a basis for government policy. The use of optimisation techniques (linear programming techniques in particular, LP) as a device for making the most efficient use of resources has made economists look with favour at the significance of using them in national development planning. Quite a few linear programming models have been put forward in the field of development planning, the most important of which are those of Adelman and Sparrow (1966), Blyth and Crothall (1965), Bruno (1966), Chenery and Kretschmer (1956), Chenery and MacEwan (1966), Hjerppe (1976), Kornai (1967), Kornai and Liptak (1965), Manne (1963, 1966, 1974), Morva (1970, 1975), Nugent (1970), and Sandee (1960). Literature in this area makes use of some variations of the LP models with a' single' welfare objective function. These previous models are strictly 'sectoral' models working with input-output ratios for different sectors. Our criticism of sectoral modelling is that, while the consistency framework of a multi-sector model highlights gross inter-relationships between sectors and it may guarantee consistency at the 'sectoral' level, it seems in many cases not to give directly operational information at the 'project' level within the sector. When the economy becomes more complex and the model thus becomes bigger, the identification of projects within a sectoral aggregate becomes very difficult and the pertinence of multi-sector forecasts to project appraisal becomes too far-reaching.

1.2 Multi-objective decision making

Previous models have ignored 'multiplicity' of objectives and, in many cases, it is casually assumed that a well-established, well-identified and agreed upon single objective function is readily available for insertion into any model that happens to need it. Unfortunately, this is far from the truth. In practice, planners are constantly faced with the fact that more than one objective function should be considered simultaneously if they seek to formulate a closer-to-reality plan. Certainly, in complex industrial or governmental problems, the decision maker (DM) may repeatedly have to select among alternative courses of action on the basis of 'multiple' and conflicting objectives. This multiple and heterogeneous nature of objectives in such situations has called for an approach that can take this nature into consideration. That is the multiple objective approach in decision making as opposed to the single- objective approach.

If we consider a decision situation like development planning, planners are faced with a number of objectives in which they would like to see improvements: for instance, national income, balance of payments, regional development, level of investment, level of consumption, level of employment and so on. Meanwhile, conventional optimisation techniques like LP cannot simultaneously optimise more than a unique single objective function, say, national income. National income here is really a surrogate for a number of complex alternatives like consumption, investment, savings, employment, balance of payments, social services, etc. Such a compression of several alternatives in one surrogate will reduce their information content and the result will turn out to be misleading.

The problem of multiple objectives can be formally stated as:

$$\left.\begin{array}{l} \text{Max } C_1(X) = \Omega_1 X \\ \text{Max } C_2(X) = \Omega_2 X \\ \\ \text{Max } C_p(X) = \Omega_p X \\ \text{Subject to } \; AX \le b \\ X \ge 0 \end{array}\right| \qquad (1)$$

where X is the decision variables vector, $C_1(X)$, …, $C_p(X)$ are the multiple objectives considered, Ω_r is the contribution vector of X related to objective function r where $r = 1, 2, …, p$. A is an $m \times n$ matrix of technical coefficients and b is an $n \times 1$ vector of available resources.

If the need for multiple objective decision making is acknowledged, a continuing problem is how to choose a course of action (or a policy) when more than one objective are to be optimised. A typical definition of optimality is not particularly workable in such a situation. It is impossible to simultaneously optimise all the objectives considered in problem (1) above, since contradictions will arise between them. These contradictions occur because the individual objectives assume optimal values at different points within the common feasible region. Another problem which arises in a typical multi-objective situation is the problem of measurability since each objective might have to be measured using a different yardstick.

To resolve these problems, many approaches have been put forward in the literature. These approaches have been tracked in a number of studies. The most comprehensive of them are Roy (1971), Cochrane and Zeleney (1973), Starr and Zeleney (1977), Zeleney (1973, 1976, 1982), Thiriez and Zionts (1976), Zionts (1978), Cohon and Marks (1975, 1975), Bell et al. (1977), Farquahar (1984), Sfeir-Younis and Bromley (1977), White (1983, 1984), Belenson and Kapur (1973), Johnson (1968), Chankong and Haimes (1983), Evans (1984), French (1983, 1984), Ignizio (1982), ReVelle et al. (1980), Rietveld (1980), Hwang and Masud (1979), MacCrimmon (1973), Rothermel and Schilling (1984), Miettinen et al. (2008), Steuer (1985), Steuer and Na (2003), and Weistroffer et al. (2005).

One major stream of the suggested approaches to which this work is related, assumes that the final reconciliation between the conflicting objectives is a matter of value judgement that only the DM can make. In other words, it is asserted that the problem of conflict between objectives is due to the incomplete ordering of the objectives. Such incomplete ordering, which is a characteristic of a multi-objective optimisation problem, signifies that without preference information, an optimal solution for this problem cannot be found since all feasible solutions are not ordered and thus not comparable, Cohon and Marks (1975). A complete ordering, which is a characteristic of a scalar, single-objective optimisation problem, can be obtained for a multi-objective problem by introducing value judgement in the solution process. With this in mind, this type of approach suggests that the principle of preference or utility is the answer. Bearing the principle of utility in mind, problem (1) above becomes as

$$\left.\begin{array}{l}\text{Max } U\left[C_1(X)\ldots C_p(X)\right] \\ \text{S.t. } AX \le b \\ X \ge 0\end{array}\right| \quad (2)$$

where U is the overall utility (preference) function defined on the values of the objectives, X is the constrained set of feasible decisions, and $C_1, \ldots, C_p$ are p distinct linear objective functions of the decision vector X. It is assumed that the DM has in mind, consciously or unconsciously the principle of utility when he ranks his objectives. Thus, the adoption of this principle of utility can help resolving the conflicts inherited in a multiple-objective situation. In such a situation, the final decision is one that maximises the DM's utility and the concept of an 'optimal' solution is replaced by that of an 'efficient' solution (Belenson and Kapur, 1973; Geoffrion et al., 1972), or the best compromise solution (Belenson and Kapur, 1973; Benayoun et al., 1971).

Let D be the feasible region defined by the constraints, an efficient solution is a feasible solution, $X' \in D$, for which there exists no other feasible solution $X \in D$ such that

$$\begin{array}{ll} C_1(X') > C_1(X) & \\ \text{For some } i = 1, 2, \ldots, p, \text{ and} & \\ C_k(X') \ge C_k(X) & \text{for all } k \ne i \end{array} \quad (3)$$

Each efficient solution implies values for each of the p objectives and the collection of all efficient solutions is 'the set of efficient solutions'. Without preference information, not one of the efficient solutions is preferable to any other efficient solution, but when preferences are known, as represented by an indifference surface, for example, and then one of the efficient solutions can be identified as the 'best-compromise' solution. In other words, the solution of the problem will be a point at which the efficient set of solutions and the indifference curves, which represent the contours of the DMs utility functions, are tangent to each other. Thus, the solution will have the highest utility for the DM, and it will also be efficient. Since U is not explicitly known, certain information is needed about it from the DM. This cloud be done either by uncovering the DMs overall utility function over the set of objectives or by inferring certain characteristics of this function from information provided by the DM. A number of methods for doing this has been put forward (Farquahar, 1984; Geoffrion et al., 1972; Keeney and Raiffa, 1976). A good representative of these methods is the method of Geoffrion et al. (1972).

1.3 The method of Geoffrion et al

The method of Geoffrion et al. (1972), demonstrates that a large-step gradient algorithm can be used for solving problem (2) if the DM is able to specify an overall utility function defined on the values of the objectives. However, the method never requires this function to be identified explicitly. Instead, it asks only for such local information as is needed to perform the computation. The procedure is described in the context of the Frank-Wolfe algorithm (Wolfe, 1977). To be able to solve problem (2) using this algorithm, U is assumed to be differentiable and concave on X and the constraint set is convex and compact and each C_i is linear and concave. The algorithm consist of a direction-finding sub-problem and the choice of a 'step-size' along that direction – the latter being a one dimensional optimisation problem. Geoffrion et al. (1972) have noticed that such an algorithm could be applied to problem (2) if the gradient of U, $\nabla_c U(c(x)_k)$ and an optimal solution to the step-size problem were known at each iteration k. A vector collinear with $\nabla_c U(c(x)_k)$ can be determined if the DM answered the $p - 1$ questions, "With all other objectives held constant at the point $C_k = C(X)k$ how much would you be willing to decrease the value of objective i to obtain an increase of ΔC_1^K in objective 1?" The responses ΔC_i^K, $i = 2 \dots p$, permit the making of the following approximation

$$w_i^k \triangleq \frac{(\delta U/\delta C_i)^k}{(\delta U/\delta C_1)^k} \approx \frac{\Delta C_1^k}{\Delta C_i^k}, \quad i = 1, \text{---}, p \tag{4}$$

where w_i^k is the trade- off (or the marginal rate of substitution) between objective i and objective 1 at the current point k. The selection of the first objective C_1, as a reference objective is arbitrary and the estimation of the $p - 1$ trade-off weights by the DM are necessary. The direction of the gradient of U is generally sufficient for use in the direction-finding problem of the algorithm. The interpretation of the algorithm in the spirit of the steepest ascent algorithm is that it determines the best direction of movement from the current point and the DM is asked to determine how far to move in this direction. The sequence of improving the feasible solutions to problem (2) is obtained which converges to an optimal solution.

Thus, according to the Frank-Wolfe algorithm, the following steps can be distinguished:

1 Select an initial solution $X_k \in X$ at will, and set $k = 1$.

2 Determine the 'best' direction in which to move from the current X_k. Namely, find an optimal solution y_k, of the following direction-finding problem:

$$\text{Max } \nabla_x U\left[C_1(X_k)\dots C_p(X_k)\right].y_k \tag{5}$$

Subject to $y_k \in X$

Since U is implicit, problem (5) cannot be solved. We have to know some information about U from the DM. The DMs trade-off weights w_i^k can be used to determine the gradient of U in (5) and thus the direction for the steepest ascent of U.

Since by the chain rule we have

$$\nabla_x U\left[C_1(X_k)\dots C_p(X_k)\right] = \sum_{i=1}^{p}\left[\delta U/\delta C_i\right]^k \nabla_x C_i(X_k) \tag{6}$$

where $[\delta U / \delta C_i]^k$ is the i^{th} partial derivative of U evaluated at the point $[C_1(X_k) \dots C_p(X_k)]$, and $\nabla_x C_i(X_k)$ is the gradient of C_i evaluated at X_k. Thus, problem (5) becomes

$$\text{Max } \sum_{i=1}^{p}\left[\delta U/\delta C_i\right]^k \nabla_x C_i(X_k).y_k \tag{7}$$

Subject to $y_k \in X$

Dividing the objective function in (7) by the positive coefficient $[\delta U / \delta C_1]^k$ of the reference objective C_1 and using equation (4), problem (7) becomes:

$$\text{Max} \sum_{i=1}^{p} w_i^k \nabla_x C_i \left(X_k \right) . y_k \tag{8}$$

Subject to $y_k \in X$

Problem (8) can be solved as a normal LP problem to find y_k.

3 Let $d_k = y_k - X_k$ and let the DM determine the 'best' step size t_k, for which the objectives are most preferred, in the following step-size problem:

$$\text{Max U}\left[C_1 \left(X_k + td_k \right) \ldots C_p \left(X_k + td_k \right) \right] \tag{9}$$

$0 \le t \le 1$

Set $X_{k+1} = X_k + td_k$, $k = k + 1$ and return to step (2). Continue until $X_{k+1} \equiv X_k$, i.e., until the solutions X_{k+1} and X_k are equal. This scheme yields a sequence of improving feasible solutions to the multiple-objective problem (2) which converges to an optimum solution. Both w_i^k and X_k must be estimated by the DM at each step. The difficulty in this method is that the DM may not be able to compare all the objectives $i = 2, \ldots, p$, with the reference objective $i = 1$ and reach a sound , consistent vector of trade-off weights, w_i^k. Dyer (1973) presents a time-sharing computer routine which obtains necessary trade-off information via simple man-machine interactive dialogue. Geoffrion et al. (1972) have applied their method to the problem of resource allocation within an academic department. They were able to obtain the required information from the department administrators.

2 The description of our model

The purpose of this article is to develop a plan in the form of a mathematical multi-objective LP model for a hypothetical economy, which can be used in development planning, and to solve this model using the method of Geoffrion et al. (1972).

2.1 The projects

Our model is formulated in the framework of an assumed project selection situation. We assume that 40 public projects have been assessed, appraised and found relevant to the developmental objectives. These projects cover most activities of the economy. They include agricultural, industrial, educational, social and regional projects.

2.2 Different versions of projects

It is assumed that each project may be undertaken in three different ways. Each project will thus exist in three versions (V1, V2, and V3). Each version will require different resource flows from the other two and will generate different resource streams. The versions of a project are mutually exclusive. The data for V2 and V3 versions of each project have been generated by a simple Monte Carlo program from the original version V1. No significance should be attached to this however.

The three versions of a project (V1, V2, and V3) may be thought of as being available from different countries (e.g., V1 from the USA, V2 from an EU country, and V3 from other industrialised countries, say Japan), or as using different levels of technology, or as proposals from three different companies in response to invitations to tender. The idea of different versions for projects provides a great deal of reality and flexibility to the plan.

The model also allows a choice for each project to begin in any year in the planning period. Thus, for each and every year, the possible three versions will compete between themselves for the limited resources available in that year, and the model will select the version that is better than the other two versions according to its contribution to the objective function. The idea of different starts for projects is meant to give planners flexibility in shifting resources from one year of the planning period to another where they are optimally needed without disturbing the plan.

2.3 The developmental objectives

We assume that there are seven national objectives proposed for the plan (i.e., for the model). These objectives are:

1 the maximisation of per capita income

2 the minimisation of foreign aid used to fill the import-export gap

3 the maximisation of skilled manpower

4 the minimisation of foreign aid used to fill the investment-saving gap

5 the minimisation of the level of foreign technical expertise

6 the maximisation of the level of social services and infrastructure

7 the maximisation of the level of regional development.

It is worth noting that:

a Maximising the level of per capita income in objective no. 1 is carried out in the form of maximising the net present value (NPV) of the per capita income of the projects, not only over the planning horizon, but also for the post-horizon data starting from the year that follows the plan up to the end of the project's economic life. The project's post-horizon data (its cash flows, either outlays or income) are discounted back, using a social discount rate, to obtain their NPV at the horizon, i.e., the year that follows the planning period. For instance, if the planning period is three years, there will be four values of per capita income for each project: one at the end of each of the three years of the planning period the fourth one is the summary figure of the post-horizon values discounted at the end of year 4. All of these four values are discounted back to obtain their NPV now which is to be maximised. According to this NPV, the projects are assessed and selected. The NPV approach has also been used for objectives nos. 2 and 4.

b In order to compute the contributions of the model's projects to objective no. 5; we have used a scoring system on the scale from zero to ten, depending on the need of the projects to foreign technical expertise.

c The project that contributes to objectives nos. 6 and 7 is given a score of 1, whereas the project that does not, has a score of zero for its contribution to this objective.

2.4 The model's constraints

Development cannot be achieved endlessly and freely. Each economy has its own limitations on the development process. In recognition of the resource limitations, the requirements of the projects cannot exceed the available amounts of resources (either domestic or foreign). We assume that the fundamental limitations or restrictions of our hypothetical economy are on:

1 local funds

2 foreign funds

3 skilled manpower

4 other resources.

In this regard, we have chosen three of the intermediate resources which can be critical to development. The shortages in any of them may handicap development. These are energy, cement, and steel. It has to be noted that there is a certain degree of interdependence between the projects of the model since we allow for the possibility that some projects will benefit from the output of others. For example, the project 'Electricity-generating station' is expected to produce electricity in the third year of the planning period and this is included in the amount of energy available in the economy in that year (the RHS of the energy constraint).

Of course, if the planning horizon were longer than it is assumed to be, more and more projects like the Cement factory, the Steel factory, and the Electricity-generating station would start to produce such resources as cement, steel, and energy and their output would be included in the nationally available resources from which other projects may benefit.

2.5 The plan period

Development plans may be formulated in three sizes: short (annual plan), medium (between three to seven years) and long range (between ten to 25 years) [Waterston (1969), pp.120–144]. The roles and comparative importance of these plan periods vary considerably between countries. Since development planning started, ideal planning period has not been decided or agreed upon. It is likely to remain so. Our model can be used for any planning period provided that the required and relevant data are available. However, the planning period adopted in our model is three years for reasons of simplicity and manageability. It is supposed that our medium-term plan is formulated within the framework of a more long-term perspective plan (say, 20 to 25 years). This is because the perspective plan can act as a guide to policy makers by exposing bottlenecks which will emerge as the economy expands if an anticipatory action is not taken well in advance. A medium-term plan, like ours, can also be supported by an annual plan. The annual plan in this case is used as a controlling plan in the sense that it is this which, year by year, matches resources to possible achievements. It is guided by the medium-term plan, which sets its direction, but the annual plan is the operative document.

2.6 The data

The data of this model are hypothetical but they are guided by the data published in the literature with respect to less developed countries. The data given in national planning models referred to in the introduction (Section 1.1), have also been useful. In addition to this, published plans for India (Sandee, 1960), and data books related to planning in Egypt (Ikram, 1980), and Pakistan (Chenery and MacEwan, 1966), have been beneficial.

To guarantee the necessary randomness in the data of our model we have used a computer model and a Monte Carlo simulation routine. This routine will yield random numbers, taken from a normal distribution with a certain mean and certain standard deviation, if called with different initial values.

As has been mentioned, we have created three versions of the model's data using three different initial values to start the routine with. This has resulted in having the three versions V1, V2, and V3 of our model. In doing this, each of these versions has different figures for the coefficients of both the constraints and of the objective functions.

3 The mathematical formulation of our model

The objectives functions are

$$\text{Max } C_1 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \pi_{ik} \ X_{ik} \tag{10}$$

$$\text{Min } C_2 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \varnothing_{ik} \ X_{ik} \tag{11}$$

$$\text{Max } C_3 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \sigma_{ik} \ X_{ik} \tag{12}$$

$$\text{Min } C_4 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \Gamma_{ik} \ X_{ik} \tag{13}$$

$$\text{Min } C_5 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} (_{ik} \ X_{ik} \tag{14}$$

$$\text{Max } C_6 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \Upsilon_{ik} \ X_{ik} \tag{15}$$

$$\text{Max } C_7 = \sum_{i=1}^{N} \sum_{k=1}^{\tau} \Omega_{ik} \ X_{ik} \tag{16}$$

The utility function to be maximised is

$$U\left[C_1(X), C_2(X) \ldots C_7(X)\right] \tag{17}$$

Subject to

$$\sum_{i=1}^{N} \quad \sum_{k=1}^{\tau} \quad \alpha_{ikr\theta} \ X_{ik} \leq \beta_{r\theta}, \quad r = 1,2 \ldots R \ \text{ and } \ \theta = 1,2 \ldots \tau \tag{18}$$

$$\sum_{k=1}^{\tau} \quad X_{ik} \leq 1.0, \tag{19}$$

$$0 \leq X_{ik} \leq 1.0, \quad \forall_i, \forall_k \tag{20}$$

where

X_{ik}	the level undertaken of project *ik*
π_{ik}	the NPV of the contribution of project *ik* in the objective of 'maximisation of per capita income'
k	the year in which project *i* starts
i	the reference number of a project in each year *k*, where $i = 1, \ldots, N$
N	the basic 40 projects in their three versions with a total of 120 projects
τ	the number of the final year of the planning period
$\varnothing_{ik}$	the NPV of the contribution of project *ik* in the objective of ' minimisation of foreign aid needed to fill the import-export gap'
σ_{ik}	the contribution of project *ik* to the objective of ' maximisation of skilled manpower'
Γ_{ik}	the NPV of the contribution of project *ik* in the objective of 'minimisation of foreign aid needed to fill the investment-saving gap'
$(_{ik}$	the contribution of project *ik* to the objective of 'minimisation of the level of technical expertise'
Υ_{ik}	the contribution of project *ik* to the objective of 'maximisation of the level of social services and infrastructure'
Ω_{ik}	the contribution of project *ik* to the objective of 'maximisation of the level of regional development'
$\alpha_{ikr\theta}$	the amount of resources *r* required by project *ik* in year θ of the planning period
$\beta_{r\theta}$	the amount of national resources *r* available for the projects of the current plan in year θ
R	the number of all resources considered in the model
$C_1 - C_7$	the seven linear objective functions of the model.
U	the utility function to be maximised.

Constraint (19) constrains the project from being taken more than once during the planning period. The objective functions and the constraints of the model are assumed to be linear.

It is important to emphasise that while any model should be a reflection of reality, no model is a perfect reflection. This is so for many reasons. A model is an abstraction which can only incorporate certain aspects of the real world. Many relationships cannot yet (if ever) be formulated either in quantitative or qualitative terms. Besides, all models leave out relationships and details which could, in principle, be included.

4 Assessing the DMs utility function

Many previous applications of multi-objective methods were founded on actual experiences of a single DM (or a group of DMs) who has (or have) an implicit utility function(s). The DM would supply the information needed by the method regarding his preferences. This article, however, has a different orientation. It takes the approach of simulating the DMs responses, needed by the method of Geoffrion et al. (1972), using a hypothetical explicitly known utility function as a surrogate for the DM. The use of this function makes it possible to solve our model described in Section 3. Using a utility function for the DM ensures the most satisfactory solution to the DM. The solution will be a point at which the non-dominated set of solutions and the indifference curves of the DM are tangent to each other (the indifference curves can be considered as contours of equal utility). Thus, the solution will be the highest utility for the DM and it will also be non-dominated.

Although there are different forms of aggregate utility functions (Hwang and Masud, 1979; Keeney and Raiffa, 1976), we have assumed that our DMs utility function is additively separable with respect to the objectives considered. This assumption permits the consideration of each of the objectives independently and the assessment of a single dimensional utility function defined on each. It also implies the existence of p single utility functions $u_1 \ldots u_p$ such that

$$U(C) = \sum_{i=1}^{p} u_i\left(c_i\right), \quad \text{for } C = \left(c_1 \ldots c_p\right) \tag{21}$$

where $U(C)$ is the aggregate utility of the DM related to the objectives vector C.

As for the single utility functions, it is assumed that they are logarithmic functions of the form:

$$u_i = A \log_{10}\left(k + c_i\right) + \beta \qquad i = 1, 2 \ldots p \tag{22}$$

where A, K, and β are constants, and c_i is the value of each objective i.

The single utility, u_i, is normalised, i.e., consistently scaled between 0.0 and 1.0 such that it is 0.0 when the value of the corresponding objective is minimum and it is 1.0 when the value is maximum. Both the minimum and maximum values considered depend on the project data of the model. To find the minimum and maximum values for each objective separately, we have solved our model using each of the objectives independently as an objective function for the model which is solved once as a minimisation problem to obtain the minimum value, c_m, of the objective, and another time as a maximisation problem to obtain the maximum value of the same objective, c_x. Thus, the normalised value of this objective, c_{iz} at the current point is calculated as:

$$c_{iz} = \frac{c_i - c_m}{c_x - c_m} \tag{23}$$

where c_i is the current value of objective i obtained from the multiple-objective model, and c_m and c_x are both the minimum and maximum values, respectively, that could be obtained for the same objective. Thus, equations (21) and (22), respectively, become

$$U_z(C) = \sum_{i=1}^{p} u_{iz}(c_{iz}), \qquad \text{for } C = (c_{iz} \ldots c_{pz}) \tag{24}$$

$$u_{iz} = A\log_{10}(k + c_{iz}) + \beta \qquad i = 1, 2 \ldots p \tag{25}$$

where U_z is the normalised aggregate utility and u_{iz} is the normalised i^{th} single-dimensional utility. Through this normalisation process, all objectives and utilities, irrespective of their different wide-ranging values, can be easily viewed and compared.

The logarithmic form of the single-dimensional utility function has been chosen because it is plausible to suppose that a utility increases with the corresponding objective but at a diminishing rate. A function having this property is the logarithmic function.

Furthermore, we have divided the nonlinear curvature of the utility function (25) into three different forms, each with a different degree of nonlinearity. Since nonlinearity itself differs along the curvature, it is worth investigating utility in its different degrees of nonlinearity. The difference between the three forms of nonlinearity is in the values of the constants A, β, and K used in the single dimensional utility functions. These constants ensure that the logarithms in the single utility functions are defined throughout the range values which the objectives can assume. Accordingly, we have used three forms of utility functions for our DM to correspond to the three degrees of nonlinearity. These forms are

1 The almost linear form

$$u_{iz} = 24.16 \log_{10}(10 + c_{iz}) - 24.16 \tag{26}$$

2 The ordinary nonlinear form

$$u_{iz} = 3.322 \log_{10}(1.0 + c_{iz}) \tag{27}$$

3 The highly nonlinear form

$$u_{iz} = 0.5 \log_{10}(0.01 + c_{iz}) + 1.0 \tag{28}$$

where u_{iz} and c_{iz} are the normalised values of the utility functions and the objective functions, respectively.

The classification of our utility into different types of nonlinearity is based on the power series expansion of Taylor's theorem (Tennant-Smith, 1973).

5 Application

In practice, the DM determines the trade-off weights between the objectives. But since we do not have our DM present, we used the utility functions, which we have assessed in the previous section, as a surrogate for him. We have obtained the weights needed by the method of Geoffrion et al. (1972), by making use of the derivatives of the single utility functions. These weights were used as coefficients in the direction-finding problem expressed in the LP model (8) given above to compute a new feasible operating point as has been explained in Section 1.3.

Table 1 The solution of version 'V1' of the model using the method of Geoffrion et al.

Cycle \ Step size	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*
The almost-linear utility (normalised)											
1	3.11714	3.32842	3.53835	3.74693	3.95417	4.16008	4.36467	4.56796	4.76994	4.97063	5.17004
2	5.17004	5.17007	5.17009	5.17012	5.17015	5.17017	5.17019	5.17022	5.17024	5.17026	5.17028
3	5.17028	5.17228	5.17028	5.17028	5.17028	5.17028	5.17028	5.17028	5.17028	5.17028	5.17028
The ordinary nonlinear utility (normalised)											
1	3.49981	3.74282	3.74282	4.19395	4.40367	4.60371	4.79461	4.97684	5.15081	5.31685	5.47526
2	5.47526	5.47787	5.48025	5.48240	5.48431	5.48600	5.48745	5.48868	5.48967	5.49044	5.49098
3	5.49098	5.49179	5.49250	5.49309	5.49357	5.49394	5.49421	5.49436	5.49440	5.49433	5.49415
4	5.49440	5.49456	5.49469	6.49480	5.49488	5.49493	5.49495	5.49494	5.49492	5.49486	5.49477
5	5.49495	5.49503	5.49510	5.49514	5.49517	5.49518	5.49517	5.49514	5.49509	5.49502	5.49494
6	5.49518	5.49519	5.49520	5.49520	5.49521	5.49520	5.49520	5.49519	5.49517	5.49514	5.49511
7	5.49521	5.49520	5.49520	5.49520	5.49520	5.49519	5.49518	5.49517	5.49515	5.49513	5.49511
The highly nonlinear utility (normalised)											
1	5.49737	5069273	5.83405	5.94404	6.03268	6.10509	6.16397	6.21052	6.24474	6.26519	6.26800
2	6.26800	6.31224	6.34522	6.36934	6.38601	6.39606	6.39998	6.39797	6.39004	6.37598	6.35537
3	6.39998	6.41174	6.42320	6.43437	6.44526	6.45588	6.46623	6.47634	6.48619	6.49581	6.50519
4	6.50519	6.50615	6.50686	6.50734	6.50757	6.50755	6.50729	6.50678	6.50601	6.50500	6.50372
5	6.60757	6.50794	6.5031	6.50866	6.50900	6.50932	6.50964	6.50994	6.51023	6.51052	6.51077
6	6.51077	6.51078	6.51079	6.51080	6.51080	6.51081	6.51080	6.51080	6.51079	6.51078	6.51077
7	6.51081	6.51080	6.51080	6.51080	6.51080	6.51080	6.51079	6.51079	6.51078	6.51077	6.51076

Notes: The optimal almost-linear utility = 5.17028.
The optimal ordinary non-linear utility = 5.49521.
The optimal highly non-linear utility = 6.51081.
The underlined value of utility is the next, revised point.
The value of the utility in a rectangle is the optimal value.

Table 2 The solution of version 'V2' of the model using the method of Geoffrion et al.

Cycle \ Step size	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
The almost-linear utility (normalised)											
1	2.33424	2.62429	2.91186	3.19697	3.47964	3.75992	4.03781	4.31336	4.58658	4.85749	5.12612
2	5.12612	5.12647	5.12682	5.12750	5.12750	5.12783	5.12815	5.12847	5.12878	5.12909	5.12939
3	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939	5.12939
The ordinary nonlinear utility (normalised)											
1	2.65647	3.01708	3.35468	3.67159	3.96972	4,25066	4.51571	4.76598	5.01239	5.22570	5.43655
2	5.43655	5.43904	5.44143	5.44372	5.44592	5.44801	5.45001	5.45191	5.45372	5.45543	5.45705
3	5.45705	5.45708	5.45711	5.45714	5.45720	5.45720	5.45723	5.45726	5.45729	5.45732	5.45734
4	5.45734	5.45734	5.45734	5.45734	5.45734	5.45734	5.4734	5.45734	5.45734	5.45734	5.45734
The highly nonlinear utility (normalised)											
1	4.90912	5.28858	5.54267	5.73845	5.89796	6.03183	6.14606	6.24436	6.32901	6.40140	6.46210
2	6.46210	6.46686	6.47072	6.47376	6.47602	6.47754	6.47837	6.47852	6.47803	6.47690	6.47516
3	6.47852	6.48001	6.48150	6.48291	6.48427	6.48558	6.48683	6.48804	6.48919	6.49029	6.49133
4	6.49133	6.49139	6.49143	6.49144	6.49143	6.49140	6.49134	6.49126	6.49116	6.49103	6.49089
5	6.49144	6.49147	6.49149	6.49152	6.49154	6.491576.49	6.49159	6.49161	6.49163	6.49165	6.49167
6	6'49167	6.49168	6.49167	6.49165	6.49161	157	6.49151	6.49145	6.49137	6.49128	6.49118
7	6.49168	6.49167	6.49167	6.49166	6.49165	6.49164	6.49162	6.49160	6.49158	6.49155	6.49152

Notes: The optimal almost-linear utility = 5.12939.
The optimal ordinary nonlinear utility = 5.45734.
The optimal highly nonlinear utility = 6.49168.
The underlined value of utility is the next, revised point.
The value of the utility in a rectangle is the optimal value.

Table 3 The solution of version 'V3' of the model using the method of Geoffrion et al.

Cycle \ Step size	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
The almost-linear utility (normalised)											
1	2.29715	2.59828	2.89664	3.19227	3.48520	3.77546	4.06309	4.34812	4.63058	4.91051	5.18792
2	5.18792	5.18795	5.18799	5.18802	5.18805	5.18807	5.18809	5.18811	5.18813	5.18814	5.18815
3	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815	5.18815
The ordinary nonlinear utility (normalised)											
1	2.56881	2.95838	3.31962	3.65593	3.97002	4.26410	4.53996	4.79908	5.04267	5.27172	5.48707
2	5.48707	5.49094	5.49457	5.49796	5.50112	5.50404	5.50673	5.50919	5.51142	5.51343	5.51521
3	5.51521	5.51528	5.51533	5.51536	5.51537	5.51536	5.51533	5.51529	5.51522	5.51514	5.51503
4	5.51537	5.51541	5.51546	5.51550	5.51554	5.51558	5.51562	5.51565	5.51568	5.51572	5.51574
5	5.51574	5.51573	5.51571	5.51569	5.51566	5.51562	5.51558	5.51553	5.51548	5.51542	5.51536
The highly nonlinear utility (normalised)											
1	4.33274	5.15477	5.48378	5.71048	5.88569	6.02840	6.14798	6.24976	6.33694	6.41150	6.47448
2	6.47448	6.48049	6.48595	6.49089	6.49533	6.49929	6.50278	6.50582	6.50841	6.51057	6.51230
3	6.51230	6.51355	6.51456	6.51533	6.51585	6.51612	6.51616	6.51595	6.51549	6.51480	6.51358
4	6.51616	6.51665	6.51713	6.51760	6.51805	6.51849	6.51891	6.51933	6.51972	6.52011	6.52047
5	6.52047	6.52048	6.52049	6.52050	6.52051	6.52040	6.52049	6.52047	6.52045	6.52042	6.52039
6	6.52051	6.52052	6.52053	6.52054	6.52055	6.52055	6.52056	6.62057	6.52057	6.52057	6.52058
7	6.52058	5.52059	6.52060	6.52061	6.52060	6.52059	6.52058	6.52056	6.52054	6.52051	6.52048
8	6.52061	6.52060	6.52060	6.52060	6.52060	6.52059	6.52059	6.52059	6.52058	6.52058	6.52058

Notes: The optimal almost-linear utility = 5.18815.
The optimal ordinary nonlinear utility = 5.51574.
The optimal highly nonlinear utility = 6.52061.
The underlined value of utility is the next, revised point.
The value of the utility in a rectangle is the optimal value.

In practice, the DM is also supposed to determine the amount (t) of movement along the best direction just obtained, which maximises his total utility. Instead, we have used our assessed utility function (24) to compute the total utility for the 0.1 intervals of (t). The total utility at $t = 0.0$ is the total utility at the initial operating point and the total utility for $t = 1.0$ is the total utility corresponding to the solution of the LP direction-finding problem (8). Thus, by presenting the total utilities for these step-size intervals in a table, we can easily determine the value of t for which the total utility is the highest, i.e., the best step size, as in (3) of Section 1.3. The corresponding total utility becomes our revised operating point and another cycle or iteration can be performed. Further, cycles may follow until the total utility remains unchanged or starts to decline and cannot be improved any more by more cycles. At this point, the maximum total utility is reached.

The cycles, the step-sizes, and the normalised total utilities according to the method of Geoffrion et al. (1972), are presented in Tables 1, 2, and 3 corresponding to V1, V2, and V3 versions of the model, respectively. Each of these tables consists of three main blocks; each block is for one type of utility function: almost-linear, ordinary nonlinear, and highly nonlinear. In each block of each table, the values of the normalised total utility are displayed for each step size (each interval of t) and for each one of the cycles or iterations needed to be performed to reach to the optimal total utility. The solution that produces the highest total utility will naturally produce the solution vector X_{ik} (as in Section 3) that shows the level undertaken from the selected projects in each year of the plan period

6 Conclusions

The purpose of this work is to develop a multi-objective LP model, which can be used as a tool for decision-making in development planning. The basic units of the model are public investment projects rather than sectors. Each project has a chance of being selected in one of three versions and a chance of starting at any year during the planning period. This provides flexibility in planning procedures. The selection of projects includes the timing of these projects. This work investigates the applicability of one of the most important and mathematically efficient methods of multi-objective decision making to developmental planning. That is the method of Geoffrion et al. (1972), an assumed hypothetical additively separable utility function has been generated and utilised to provide simulated DM responses needed by the method. In fact, three forms of utility function, differing in their degree of nonlinearity, have been used.

The data of the model are hypothetical; however, this may be viewed as an advantage since, had we obtained data from an actual less developed country setting the attention would often have been drawn to the implied relationships within the economy, instead of the model as a systematic facilitator of choice. The fictitious data, which are guided by published development planning data, have been distributed randomly. In addition, the paper demonstrates the following conclusions:

- Any development planning model should be based on planning by projects rather than by sectors.
- Mathematical programming models are needed in formulating development plans, regardless of what is usually said about the poor quality of data in less developed countries. As Ackoff (1957), has put it; "The fact that an allocation (of resources)

must be made one way or another using what data or judgment is available no matter how bad it is. To deny this is to argue that a better product is not produced from poor material by good workmanship. It is perhaps closer to the truth to lean over in the opposite direction and argue that good workmanship is most urgently needed where the materials are poor. The generation of good data in the future depends on our ability to specify today what data we will need in the future. These specifications cannot be made effectively without developing the best models and procedures of which we are capable".

To us, the essence of improving the planning process is the incorporation of multiple objective methods; planning models are better off if the issue of multiplicity of objectives is adhered to and if policy makers (as DMs) are given an active role to play in the solution process of a multiple-objective method used in planning.

One of the most important assumptions underlying multiple-objective methods [of which the method of Geffrion et al. (1972) is a good representative] is that the DMs will cooperate with the analyst, who applies the method, by accepting responsibility for the assignment of values to the relative weights of various objectives. But, it must be recognised that DMs in a development planning context are usually politicians who are often reluctant to learn the methodology of policy analysis or to spend the time answering what they might consider 'academic' questions that eventually lead to a set of well-defined relative weights. However, the fact that these methods actively involve the DM in the solution process by allowing him to interact with the analyst may facilitate implementation of the method and the acceptance of the solution obtained by the method.

In order for these methods to have the opportunity of being applied and utilised, politicians have to cooperate with the analyst.

The application of the multiple-objective methods to real- world problems with actual data will surely be beneficial for the development and refinement of these methods.

7 Suggestions for future research

Based on the findings of this article, several paths for further research can be suggested. The most important of these are:

1. The consideration of multiple or group decision making. Often, systems which are best modelled by multiple objective functions are also characterised by more than one DM.
2. The DM, if an individual, as opposed to a group, seldom makes a decision in vacuum. He or she is influenced by others; and in many instances groups, rather than individuals, make decisions. Therefore, what is needed is the development of a methodology for approaching the group decision making problem.
3. Apply multi-objective methods to decentralised planning with conflicting, multiple objectives. We believe that the introduction of multiple objective decision making, with its man-machine interactive approach, into the decentralisation procedures of multi-level planning will facilitate the implementation and hence the applicability of these procedures. This belief has to be substantiated by more research and investigation. In a way, this direction of research could be related to the one

mentioned above, since multi-level planning with multiple objectives involves various DMs searching for a compromise of their individual objectives.

4 In the development of our model we have assumed the DM aggregate utility function to be additively separable. More research is needed to investigate different forms of utility function like multiplicative, quasi-additive, etc.

5 We have also assumed that the single utility functions related to the objectives to be logarithmic. Further research can investigate the use of other forms like, for example, exponential, quadratic, etc.

It is hoped that the DM can, and will make educated compromises and judgements based on insights generated by multiple objective methods.

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