Reliable power flow and short circuit analysis of systems with uncertain data

Shashwati Ray*
Department of Electrical Engineering,
Bhilai Institute of Technology,
Durg (C.G.), India
Email: shashwatiroy@yahoo.com
*Corresponding author

Shimpy Ralhan
Department Electrical and Electronics Engineering,
Shri Shankaracharya Group of Institutions,
Bhilai (C.G.) India
Email: shimpyys@gmail.com

Abstract: This paper addresses the problem of uncertainties in the input parameters by specifying them as compact intervals, taking into consideration the errors in modelling and measurement of transmission line parameters and also the continuous influence of load measurement errors and fluctuations in the load demand. The power flow equations are modelled as a set of nonlinear algebraic equations which are first linearised using Taylor series expansion and the solution is obtained by the Krawczyk’s method of interval arithmetic. For the short circuit analysis the prefault conditions are obtained from power flow analysis and the faulty network is then solved using Thevenin’s equivalent network as seen from the fault point. The proposed method is applied to 3 bus, 14 bus and 30 bus IEEE test systems where load currents and fault currents for each relay are obtained in bounded form and thus well-defined relay coordination pairs are available.

Keywords: interval mathematics; load flow analysis; uncertain data; Newton Raphson method; Intlab toolbox; Krawczyk’s method; short circuit studies.


Biographical notes: Shashwati Ray received her BSc (Engg.) degree in Electrical Engineering and MTech degree in Control Systems from NIT Kurukshetra, India, and PhD degree from IIT Bombay, India in 2007. For four years she has worked as Research Associate in a project in NIT Kurukshetra, sponsored by Department of Electronics, Government of India. Currently she is a Professor in the Department of Electrical Engineering at Bhilai Institute of Technology, Chhattisgarh Swami Vivekananda Technical University, India. Her research interests include optimisation, robust control, signal processing, power systems, renewable energy sources and numerical analysis using interval analysis techniques. She is author of a great deal of research studies published in national and international journals, conference proceedings as well as book chapters.

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1 Introduction

Electrical energy has become a very important factor in the life and well-being of mankind, so a very heavy responsibility rests upon electrical supply authorities to maintain the continuity of supply under all conditions. This becomes even more difficult with the advent of numerous supply undertakings and increasing size of power systems. Steady state analysis during normal operations and fault analysis together constitute an important part of analysis of an interconnected power system. Power flow and short circuit studies are the backbone of power system analysis and design. They are necessary in planning and designing the future expansion of power systems as well as in determining the best operation of the existing systems.

The optimal load flow problem has had a long history in its development for more than 25 years. A generalised formulation of the economic dispatch problem including voltage and other operating constraints was introduced and was later named the optimal power or load flow problem (OPF) (Abdel-Hady and Abdel-Aal Hassan, 2012). Protection systems are designed to protect the power system against undesirable operating conditions that result from faults in the network. Faults can occur as a result of lightning, tree flashover, deterioration of insulation, or human error, amongst others. Short circuit analysis is one of the most widely used studies especially when considering protection systems, for which this analysis can provide the values necessary for determining relay settings and for coordination of protection devices (Rodriguez et al., 2010). Modern power systems are operating under highly stressed and unpredictable conditions because of many issues like market-oriented reforms, consumer utility, measurement errors, use of renewable generation and electric vehicles, etc. The changes resulting from these uncertain factors lead to higher requirements for the reliability of power grids. In this situation, conventional methodologies cannot be applied, so robust and reliable methods become very essential.

In power system studies, the buses in the power grid are generally divided into three categories, i.e., slack bus, load bus \(PQ\), and regulated or generator bus \(PV\). A slack bus which is taken as reference is specified with both bus voltage magnitude and phase...
angle. A PQ bus which can typically be a substation or a power plant with fixed real and reactive power is specified with both active and reactive power injections. A PV bus which is usually a substation with adjustable reactive compensation devices or a power plant with reactive reserves is specified with real power injection and bus voltage magnitude. In reality, a power plant that has adequate capacity and is responsible for frequency control is often selected as a slack bus (Almeida et al., 1994).

The load flow problem in an electric power system is concerned with solving a set of static non-linear equations describing the electric network performance. The problem is formulated on the basis of Kirchhoff’s laws in terms of active and reactive power injections and voltages at each node in the system. The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flow in each line (Saadat, 2002). In other words, the voltage at each bus and currents in each line can be obtained. The short circuit analysis consists in determining the bus voltages and line currents in various types of faults using the prefault voltages obtained from the load flow studies. Among all the faults, three phase to ground fault or symmetrical fault is the most severe and provides the worst case for the calculation of the circuit breaker ratings. Fault currents are affected by:

1. Structural system characteristics, such as impedance and length of transmission lines which are well known but with uncertainties.

2. Characteristics of the power system during operation which depend on network topology during fault (generators, transformers, lines) and voltage profile of the system.

Load flow studies and short circuit studies are the basic tools for investigating the requirements of power system, viz., generation should be sufficient to meet demand and losses, bus voltages should be in specified range and reactive powers limits of generator buses should be within limits (Grainger and Stevenson, 1994), correct choice and application of protective gears should be carefully considered, etc. This information is essential for planning for future expansion such as adding new generator sites to meet increased load demand, operation of the current state of the system, exchange of power between utilities, etc. In initial stages of planning the information about the magnitudes and phase angles of load bus voltages, reactive powers and voltage phase angles at generator buses, real and reactive power flow on transmission lines along with power at reference bus is essential to analyse the effectiveness of alternative plans for future, such as, adding new generator sites, meeting increased load demand and locating new transmission sites (Kothari and Nagrath, 2011). The voltages and powers have to be kept within certain limits for economic reliable operation which are obtained as interval values taking various uncertainties into consideration.

In the mathematical formulation of the load flow problem and short circuit studies, the input data are considered to be fixed for all time at all system conditions. However, in reality the models used in power flow analysis are only approximations. The network parameters are usually approximated even though the uncertainties may arise from variances in the model parameters of transmission system elements, such as resistance, reactance and/or capacitance values due to environmental conditions and measurement errors. Also, the specified variables, like real power at PV buses, are inaccurate as they may have measurement errors (Wang and Alvarado, 1994). Further, the demands can vary in a fast and disordered way. Moreover, the uncertainty in the input data can be
enlarged due to both rounding and truncating processes that occur in numerical computations (Barboza et al., 2004). Therefore, the final results obtained by conventional methods are wrongly implemented in the system. So, to establish a real time solution, the probabilistic uncertainties should be considered which allows the analysts to incorporate both the estimate in data and solution tolerance, i.e., the uncertainty in input parameters and the effect of propagation of data inaccuracies, thus obtaining a range of values for each output quantity (Vaccaro et al., 2009; Vaccaro et al., 2013). The incorporation of randomness in fault level calculations has considerable benefits in adoption of more realistic safety factors in power system (Vicente et al., 2012).

Researchers and power engineers have recognised the importance of these uncertainties. Wang and Alvarado (1994) are the pioneers in this field where the authors have solved the interval non-linear equations in power flow problem using the Newton operator and the Gauss Seidel method. Later, in Barboza et al. (2004) load uncertainty has been dealt by solving the non-linear equations using Krawczyk’s method of Moore (1966). Recently, many authors like Dimitrovski and Tomsovic (2004) and Vaccaro et al. (2009, 2010, 2013) have used optimisation techniques to address the existence of uncertainty in power flow problem. Vicente et al. (2012) have implemented interval arithmetic and fuzzy sets theory to three phase short circuit solution of radial distribution network.

In order to overcome the aforesaid limitations and also control these numerical errors, we propose to apply the technique of interval arithmetic in the Newton-Raphson approach of power flow analysis, where the non-linear system is first linearised by the Newton Raphson method and then Krawczyk’s method of interval mathematics is applied to solve the system of linearised equations. For the short circuit analysis the prefault conditions are obtained from power flow analysis and the faulted network is then solved using Thevenin’s equivalent network as seen from the fault point. Prior to the occurrence of fault, the system is assumed to be in a balanced steady state and hence, per phase network model is used for calculation of fault current and bus voltages during fault.

In this paper, we develop a methodology by taking into account the various uncertainties. Here, all the input parameters are represented as intervals taking into consideration

- the errors in modelling and measurement of transmission line parameters
- influence of the load measurement errors and fluctuation in the demand.

The implementation is performed in MATLAB environment, using the Intlab toolbox developed by Rump (1999). The proposed methodology is tested on 3 bus, 14 bus and 30 bus IEEE test systems.

The rest of the paper is organised as follows: Section 2 reviews some basic concepts related to interval arithmetic and interval systems. In Section 3, we give the main characteristics of load flow problem. In Section 4, we describe the interval arithmetic applied to the load flow problem with the proposed approach. In Section 5, we explain the interval short circuit analysis and in Section 6, we give the results on the test problems followed by conclusion section.
Interval arithmetic

Interval arithmetic, interval mathematics, interval analysis, or interval computation, is a method developed by mathematicians since the 1950s and 1960s as an approach to putting bounds on rounding errors and measurement errors in mathematical computation, thus developing numerical methods that yield reliable results. It is an arithmetic developed by Moore (1966) that is defined on sets of intervals instead on sets of real numbers. It combines interval arithmetic with analytic estimation techniques to compute the sharpest possible interval solution set which completely contains the true solution set. The power of interval arithmetic lies in its implementation on computers. It solves problems which are unsolvable by non-interval methods and has been used recently for global optimisation, solving ordinary differential equations, linear systems, optimisation, etc. Interval arithmetic is a logical extension of standard arithmetic that uses operators defined over real intervals.

Following the notations given by Moore (1966) and Ralhan and Ray (2013), let $x = [a, b]$ be a real interval which is a bounded set of real numbers

$$\{x : a \leq x \leq b\}$$

where $a$ is the infimum (lower endpoint) and $b$ is the supremum (upper endpoint) of $x$. The width of interval is defined as $w(x) = b - a$. The midpoint of the interval is defined as $m(x) = (a + b)/2$. For a $n$ dimensional interval vector $x^* = \{x_1, x_2, ..., x_n\}$ the midpoint of interval vector $x^*$ is given by $m(x^*) = \{m(x_1), m(x_2), ..., m(x_n)\}$. The width of interval vector is $w(x^*) = \{w(x_1), w(x_2), ..., w(x_n)\}$. A degenerate interval has both its lower and upper endpoints same.

Let $x = [a, b]$ and $y = [c, d]$ be two intervals. Let $+, -, \ast$ and $/$ denote the operation of addition, subtraction, multiplication and division, respectively. If $\otimes$ denotes any of these operations for the arithmetic of real numbers $x$ and $y$, then the corresponding operation for arithmetic of interval numbers $x$ and $y$ is

$$x \otimes y = \{x \otimes y : x \in x, y \in y\}$$

The above definition is equivalent to the following rules:

$$x + y = [a + c, b + d]$$
$$x - y = [a - d, b - c]$$
$$x \ast y = [\min(ac, bc, ad, bd), \max(ac, bc, ad, bd)]$$
$$x / y = x \ast [1/d, 1/c], 0 \notin y$$

Let $f$ be a real valued function of $n$ real variables $x_1, x_2, ..., x_n$ and $F$ be an interval function of interval variables $x_1, x_2, ..., x_n$, then $F$ is said to be an interval extension of $f$ if

$$f(x_1, x_2, ..., x_n) \in F(x_1, x_2, ..., x_n)$$

Natural interval extension of $f$ can be obtained by replacing the real variables by corresponding interval variables and the real arithmetic operations by the corresponding interval arithmetic operations. Also, $F(x_1, x_2, ..., x_n)$ contains the range of $f(x_1, x_2, ..., x_n)$
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whenever $x_i \in \mathbf{x}_i$ for all $i = 1, 2, ..., n$. An interval function $F$ is said to be inclusion monotonic if $x_i \subseteq y_i$, $i = 1, 2, ..., n$, implies

$$F(x_1, x_2, ..., x_n) \subseteq F(y_1, y_2, ..., y_n)$$

(2)

Interval functions $F(x)$ can be constructed in any programming language in which interval arithmetic is implemented, viz., C/C++ and Fortran 90/95, Maple, MATLAB, etc. However, the computations are slow and costly. Moore (1966) and Hansen (1992) stated that computing an interval bound carries a cost of 2 to 4 times as much effort as evaluating $f(x)$. Intlab (Rump, 1999) implemented with MATLAB enables basic interval operations to be performed conveniently on real and complex interval scalars, vectors and matrices.

Figure 1 A typical bus of power system

3 Load flow review

The power flow equations (Saadat, 2002; Chen et al., 2008) to describe an $n$ bus system shown in Figure 1, are formulated using the bus admittance matrix $Y$ as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

(3)

where $I_i$ is the current entering in bus $i$ and $Y_{ij}$ is the admittance between the $i^{th}$ and $j^{th}$ buses. These equations are established using the bus analysis which results in node voltages as independent variables. In polar form (3) is expressed as

$$I_i = \sum_{j=1}^{n} |Y_{ij}| \angle \theta_{ij} |V_j| \angle \delta_j$$

(4)

where $|Y_{ij}|$ is the magnitude and $\angle \theta_{ij}$ is the angle of the admittance $Y_{ij}$ and $|V_j|$ is the magnitude and $\angle \delta_j$ is the phase angle of the voltage $V_j$. The $n \times n$ bus admittance matrix $Y$ is formed by using all the elements of $Y_{ij}$. The complex power at bus $i$ is

$$P_i - jQ_i = V_i^* I_i$$

(5)
where \( P_i \) is the net real power injection and \( Q_i \) is the net reactive power injection at bus \( i \). Again, from (4) and (5)

\[
P_i - jQ_i = |V_i| \angle - \delta - \sum_{j=1}^{n} |Y_{ij}| |V_j| \angle (\theta_j + \delta_j)
\]

Thus,

\[
P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| |V_j| \cos (\theta_j - \delta - \delta_j) \quad (7)
\]

\[
Q_i = -|V_i| \sum_{j=1}^{n} |Y_{ij}| |V_j| \sin (\theta_j - \delta - \delta_j) \quad (8)
\]

The above two equations represent 2\( n \) power flow equations at \( n \) buses of a power system. The general practice in power flow studies is to identify the three types of buses in the network. At each bus \( i \), out of the four quantities \( \delta_i, |V_i|, P_i, \) and \( Q_i \), two are specified and the remaining two are determined. Specified quantities are chosen according to the criteria explained in the following subsections.

3.1 Load buses

At each non-generator bus \( i \), called a load bus or a \( PQ \) bus, both \( P_{gi} \) and \( Q_{gi} \) are zero and \( P_{di} \) and \( Q_{di} \) drawn from the system by the load are known from historical record, load forecast, or measurement, where the suffixes \( g \) and \( d \) denote generator and demand. Therefore, the two unknown quantities those to be determined are \( \delta_i \) and \( |V_i| \). Initially \( \delta_i \) is taken as 0.0.

3.2 Voltage controlled buses

Any bus of the system at which the voltage magnitude is kept at a constant is said to be a voltage-controlled bus. A generator bus is usually called a voltage-controlled or a \( PV \) bus. A prime mover of any generator can control the amount of generated megawatts (MW), whereas generator voltage magnitude can be controlled by generators excitation system. Therefore, at each generator bus \( i \) both \( |V_i| \) and \( P_{gi} \) may be properly specified. The two unknown quantities that must be determined are \( \delta_i \) and \( Q_i \). Initially \( \delta_i \) is taken as 0.0.

3.3 Slack bus

The voltage angle of a slack bus serves as a reference for the angles of all other bus voltages. Thus, the usual practice is to set \( \delta \) to zero degree. Voltage magnitude of the slack bus \( |V_i| \) is also specified. Therefore, the two unknown quantities \( P_i \) and \( Q_i \) must be determined during the load flow analysis.
3.4 Conventional load flow problem formulation

The set of equations given by (7) and (8) constitute a set of non-linear algebraic equations as a function of voltage magnitude in per unit and phase angle in radians. Since the load flow equations are non-linear, they have to be solved through iterative numerical techniques. For planning studies load flow solutions have to be carried out repeatedly, so fast solutions are required which is possible if we linearise the load flow equations (Kothari and Nagrath, 2011). Generalising

\[
P(V, \delta) = P,
\]

\[
Q(V, \delta) = Q,
\]

If \((V_k, \delta_k)\) is an initial estimate and \(\Delta V\) and \(\Delta \delta\) are small deviations in voltage magnitudes and angles respectively, except the slack bus then

\[
P(V_k + \Delta V, \delta_k + \Delta \delta) = P,
\]

\[
Q(V_k + \Delta V, \delta_k + \Delta \delta) = Q.
\]

Expanding (10) in Taylor series about an initial estimate \((V_k, \delta_k)\) and neglecting higher order terms, we obtain

\[
P_{V_k, \delta_k} + \Delta V \frac{\partial P_{V_k, \delta_k}}{\partial V} + \Delta \delta \frac{\partial P_{V_k, \delta_k}}{\partial \delta} = P_k
\]

\[
Q_{V_k, \delta_k} + \Delta V \frac{\partial Q_{V_k, \delta_k}}{\partial V} + \Delta \delta \frac{\partial Q_{V_k, \delta_k}}{\partial \delta} = Q_k
\]

Hence,

\[
\begin{bmatrix}
P_k - P_{V_k, \delta_k} \\
Q_k - Q_{V_k, \delta_k}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P_{V_k, \delta_k}}{\partial V} & \frac{\partial P_{V_k, \delta_k}}{\partial \delta} \\
\frac{\partial Q_{V_k, \delta_k}}{\partial V} & \frac{\partial Q_{V_k, \delta_k}}{\partial \delta}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

(12)

Elements of the Jacobian matrix in (12) are the partial derivatives of (7) and (8) for \(i = 2, \ldots, n\), evaluated at \((V_k, \delta_k)\) and bus 1 is assumed to be the slack bus. Using the initial estimates given in Sections 3.1, 3.2 and 3.3, \(P_{V_k, \delta_k}\) and \(Q_{V_k, \delta_k}\) are calculated using (7) and (8) respectively for each bus. Writing in compact form

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_1 J_2 \\
J_3 J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

(13)

Expressing (13) as,

\[
Jx = b
\]

(14)

where,

\[
x =
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

(15)
\[
J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}
\]  \tag{16}

and

\[
b = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}
\]  \tag{17}

Each \( J_1, J_2, J_3, J_4 \) are \((n-1) \times (n-1)\) matrices. The diagonal and off-diagonal elements of \( J_1 \) are

\[
\frac{\partial P_i}{\partial \delta_j} = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \sin(\theta_j - \delta_i + \delta_j)
\]

\[
\frac{\partial P_i}{\partial \delta_j} = -|V_i| |V_j| |Y_{ij}| \sin(\theta_j - \delta_i + \delta_j)(j \neq i)
\]  \tag{18}

The diagonal and off-diagonal elements of \( J_2 \) are

\[
\frac{\partial P_i}{\partial |V_j|} = 2 |V_i| |Y_{ij}| \cos(\theta_j - \delta_i + \delta_j)
\]

\[
\frac{\partial P_i}{\partial |V_j|} = -|V_i| |V_j| |Y_{ij}| \cos(\theta_j - \delta_i + \delta_j)(j \neq i)
\]  \tag{19}

The diagonal and off-diagonal elements of \( J_3 \) are

\[
\frac{\partial Q_i}{\partial \delta_j} = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \cos(\theta_j - \delta_i + \delta_j)
\]

\[
\frac{\partial Q_i}{\partial \delta_j} = -|V_i| |V_j| |Y_{ij}| \cos(\theta_j - \delta_i + \delta_j)(j \neq i)
\]  \tag{20}

The diagonal and off-diagonal elements of \( J_4 \) are

\[
\frac{\partial Q_i}{\partial |V_j|} = -2 |V_i| |Y_{ij}| \sin(\theta_j - \delta_i + \delta_j)
\]

\[
\frac{\partial Q_i}{\partial |V_j|} = -|V_i| |V_j| |Y_{ij}| \sin(\theta_j - \delta_i + \delta_j)(j \neq i)
\]  \tag{21}

The above equations have inherent assumptions which include the factors, such as the three phase system is in balanced, steady-state condition so that the frequency and voltage are constant, the real and reactive power demands are known and specified precisely, the real power injection and voltage magnitude of generators are fixed, and the network topology and impedances are known precisely. But these are not valid, at least in true sense. The uncertainties may be induced by modelling errors due to the approximations in the values of the resistances, reactances and shunts in the models.
which are used to represent transmission lines and transformers. The practical power systems are very large with tens of thousands of buses. As a consequence, the probability of data errors increases dramatically with system size. Therefore, we take the data uncertainty into consideration by characterising the load and transmission line parameters as a range of real values or a real interval instead of crisp values.

4 Load flow solution using interval method

As the uncertainties in the data could affect the deterministic power flow solution to a considerable extent, reliable solution algorithms that incorporate the effect of data uncertainties into the load flow analysis are therefore required. In this way, the uncertainty propagation effect is explicitly considered and the bounded values for power flow studies can be assessed. Thus, the voltages, angles and powers are obtained as intervals that include all computational errors and all the possible results obtained by computations with real numbers. Known real and reactive power injections $P_i, Q_i$, as well as other input line parameters are modelled as intervals that are estimated in the beginning of the process.

The power flow equations (11), (12), and (13) have been obtained after linearisation using Taylor series expansion method. To know bounds for the remainder term of the Taylor formula, such bounds can be obtained by interval bounding of the $(n+1)^{st}$ derivative, which can be obtained with polynomial algebra. The non-linear function is represented by a Taylor polynomial and the sharp remainder intervals can computed by the method shown in Berz and Hoffstätter (1998). In our work we have ignored the remainder term assuming that it would be small. Since we have taken the uncertainty in the transmission line parameters, i.e., resistances and reactances, the Jacobian $J$ is an interval matrix. Also, the variations in power lead to interval vectors (which are actually non-linear in nature but are linearised). Magnitude and angle of voltages are unknown intervals to be determined for all the buses except the slack bus. In order to reach to the solution, we use the Krawczyk method which is an iterative method to solve interval linear system of equations (Moore et al., 2008; Hansen, 1992) as explained in the subsequent paragraphs.

Consider a system of finite system of linear equations represented as

$$Jx = b$$

(22)

i.e., $x = J^{-1}b$

(23)

Let $Y$ be an approximate inverse of $J$, i.e.,

$$Y = J^{-1}$$

(24)

Multiplying both sides of (22) by $Y$. We have

$$YJx = Yb$$

(25)

From (23) and (25), we get

$$[I - YJ]x = J^{-1}b - Yb$$

(26)

Let us consider

$$E = I - YJ$$

(27)
Therefore, equation (26) can be written as
\[ Ex = x - Yb \] (28)
or
\[ x = Yb + Ex \] (29)
The norm of an interval matrix say \( A \) is given by
\[ \| A \| = \max_j \sum_i |A_{ij}| \] (30)
Since \( E \) is an interval matrix \( \| E \| = \max_i \sum_j |E_{ij}| \). So if \( \| E \| \leq 1 \) using (30), then the sequence
\[ x^{(k+1)} = Yb + Ex^{(k)} \cap x^{(k)}; k = 0,1,2,\ldots \]
\[ x^{(0)} = [-1,1] | Yb \| / (1- \| E \|) \]; \( i = 0,1,\ldots, n \) (31)
is a nested sequence of interval vectors containing the unique solution to (22) for every interval matrix \( J \) and every interval vector \( b \). From (31) the new estimates for the bus voltage angle and magnitude are respectively obtained as
\[ \delta^{(k+1)} = \delta^{(k)} + \Delta \delta^{(k)} \] (32)
\[ |V^{(k+1)}| = |V^{(k)}| + \Delta |V^{(k)}| \] (33)
These values are then used to update the values of \( \Delta P \) and \( \Delta Q \) at every iteration. Since non-linearities in the power system can be said to be encompassed by intervals in the linearised system (Kearfott, 1991; Hansen, 1992), (32) and (33) give us a bounded and converging solution for the non-linear power flow equations.

5 Interval short circuit analysis

The variation in the magnitude of the short-circuit current is due to two main components, viz., the equivalent system impedance at the fault point which produces a decaying DC component, and the performance of the synchronous generator which result in a decaying AC component. The rate of decay depends on the instantaneous value of the voltage at the time of the fault and also on the power factor of the system at the fault point (Kamdar et al., 2013; Kersting and Greg, 2012).

Prior to the occurrence of fault, the system is assumed to be in a balanced steady state and hence, per phase network model is used for calculation of fault current and bus voltages during fault. The generators are represented by a constant voltage source behind a suitable reactance which may be sub-transient, transient or normal d axis reactance. Since the short circuit analysis is used for relay setting and coordination, we have used the transient reactance for our calculations. The transmission lines are represented by their models with all impedances referred to a common base.

A prefault load flow solution provides the information about the prefault bus voltages \( V(0) \) (Terzija and Dobrijevic, 2007). A typical bus of an \( n \) bus power system network with a balanced three phase fault, through fault impedance \( Z_f \) at the \( k^{th} \) bus is shown in
Figure 2. The fault at the \( k^{th} \) bus through fault impedance \( Z_f \) will cause a change in the voltage of all the buses by \( \Delta V \). This change can be calculated using Thevenin’s circuit by applying a voltage \( V_k(0) \) at \( k^{th} \) bus, short circuiting all other voltage sources and replacing the sources and loads by their equivalent impedances. Applying superposition theorem, the bus voltages during a fault \( V(\text{F}) \) can be obtained as the sum of prefault bus voltages \( V(0) \) and the change \( \Delta V \) in bus voltages due to fault. So,

\[
V(\text{F}) = V(0) + \Delta V \tag{34}
\]

Fault current \( I(\text{F}) \) is given as

\[
I(\text{F}) = Y \Delta V \tag{35}
\]

Figure 2  Fault at \( k \)-th bus of a power system

From (34) and (35)

\[
V(\text{F}) = V(0) = Z I(\text{F}) \tag{36}
\]

where \( Z = Y^{-1} \). Current entering every bus is zero except at the faulted bus. This is taken as negative current entering the bus as the current at the faulted bus is leaving the bus. Therefore, the fault current is

\[
I(\text{F}) = [0 \cdots 0 \cdots I_k(\text{F}) \cdots 0]^T \tag{37}
\]

So, from (36) and (37) during fault the \( k^{th} \) bus voltage is

\[
V_k(\text{F}) = V_k(0) - Z_{kk} I_k(\text{F}) \tag{38}
\]

where \( Z_{kk} \) are the diagonal elements of \( Z \).

From Thevenin’s circuit, during the fault conditions, the voltage \( V_k(\text{F}) \) at \( k^{th} \) bus is given as the product of \( Z_f \) and fault current \( I_k(\text{F}) \)

\[
V_k(\text{F}) = Z_f I_k(\text{F}) \tag{39}
\]

Substituting for \( V_k(\text{F}) \) from (39) into (38), short circuit current or fault current at the \( k^{th} \) bus is given by

\[
I_k(\text{F}) = \frac{V_k(0)}{Z_{kk} + Z_f} \tag{40}
\]
Here we obtain the elements of $Z$ matrix not by inverting $Y$, but by the using the line impedances.

6 Results

In this section we test the performance of the proposed method. For this we have carried out the tests on three standard systems, namely IEEE 3 bus, 14 bus and 30 bus test power systems taken from the Power System Test Case Archive (http://www.ee.washington.edu/research/pstca/), by interval load flow method and interval short circuit analysis. We specify the loads and generation in Mega Watt (MW) and Mega Volt Ampere Reactive (MVAR), respectively, bus voltages in per unit, and their angles in degrees. Loads and generations are converted into per unit quantities on the base Mega Volt Ampere (MVA) selected. Tests are performed on all the systems by introducing the uncertainties as

1. Measurement error of 2% in the transmission line parameters was considered.
2. A 10% variation in active and reactive powers of load and generator were carried out.

The comparison of results with conventional and interval load flow methods for various test systems with various test cases are given exhaustively in Ray and Ralhan (2016).

6.1 IEEE 3 bus system

The three-bus network shown in Figure 3 consists of one load bus ($PQ$ bus, bus 2), one generator bus (PV bus, bus 3) and a slack bus (reference bus, bus 1) and has three circuits whose parameters, i.e., resistance, reactance and capacitance are shown in Table 1. Table 2 shows power flow data for the test case.

Figure 3  Three-bus test case
Table 1  Line data for three-bus test case

<table>
<thead>
<tr>
<th>Bus</th>
<th>Bus</th>
<th>$R_{p.u.}$</th>
<th>$X_{p.u.}$</th>
<th>$C$ (MV$\text{ar}$)</th>
<th>Tr/TapSet.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.02</td>
<td>0.04</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.01</td>
<td>0.03</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.0125</td>
<td>0.025</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2  Bus data for three-bus test case

<table>
<thead>
<tr>
<th>Bus</th>
<th>$V$(p.u.)</th>
<th>Angle (deg)</th>
<th>$P_D$</th>
<th>$Q_D$</th>
<th>$P_G$</th>
<th>$Q_G$</th>
<th>$Q_{min}$</th>
<th>$Q_{max}$</th>
<th>MVAr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0</td>
<td>–4</td>
<td>–2.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.04</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

With the proposed load flow method we obtain the results as shown in Table 3. In the table we show the magnitude and angle of voltages at all system buses ($|V|$ is magnitude of bus voltage, $\delta$ is the angle in degrees) and $Q_{G}$ is the reactive powers of generator and slack buses. The reactive power at slack bus is [1.0008, 1.8217].

Table 3  Interval load flow solution for 3 bus system

<table>
<thead>
<tr>
<th>Bus</th>
<th>$V$(p.u.)</th>
<th>Angle (deg)</th>
<th>$P_D$</th>
<th>$Q_D$</th>
<th>$P_G$</th>
<th>$Q_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[1.05,1.05]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[1.47, 2.89]</td>
<td>[1.00, 1.82]</td>
</tr>
<tr>
<td>2</td>
<td>[0.96,0.98]</td>
<td>[–3.50,–1.89]</td>
<td>[4.00,4.00]</td>
<td>[2.50, 2.50]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>3</td>
<td>[1.04,1.04]</td>
<td>[–1.09,0.09]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[2.00,2.00]</td>
<td>[0.12,2.79]</td>
</tr>
</tbody>
</table>

With the proposed interval short circuit studies, the fault currents at all the buses with magnitude and phase angles are shown in Table 4.

Table 4  Three phase interval short circuit for 3 bus system

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Prefault Voltage $V(0)$</th>
<th>Fault current $I(F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Magnitude)</td>
<td>(Angle)</td>
</tr>
<tr>
<td>1</td>
<td>[1.0500, 1.0501]</td>
<td>[0.000,0.000]</td>
</tr>
<tr>
<td>2</td>
<td>[0.9637, 0.9797]</td>
<td>[–3.5024,–1.8880]</td>
</tr>
<tr>
<td>3</td>
<td>[1.0400, 1.0401]</td>
<td>[–1.0918, 0.0949]</td>
</tr>
</tbody>
</table>

6.2 IEEE 14 bus system

The 14 bus IEEE test case shown in Figure 4 has been analysed in the same way. Table 5 shows the results with the proposed interval load flow method, where $Q_G$ is the reactive powers of generator and slack buses. The reactive power at slack bus is [–0.2386, –0.0762].

With the proposed interval short circuit studies, the fault currents at all the buses with magnitude and phase angles are shown in Table 6.
Table 5  Interval load flow solution for 14 bus system

<table>
<thead>
<tr>
<th>Bus</th>
<th>V(p.u.)</th>
<th>Angle (deg)</th>
<th>P_D</th>
<th>Q_D</th>
<th>P_G</th>
<th>Q_G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[1.06,1.06]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[2.00,2.66]</td>
<td>[-0.23,-0.08]</td>
</tr>
<tr>
<td>2</td>
<td>[1.04,1.04]</td>
<td>[-5.78,-4.20]</td>
<td>[0.22, 0.22]</td>
<td>[0.13,0.13]</td>
<td>[0.40,0.40]</td>
<td>[-0.07,1.00]</td>
</tr>
<tr>
<td>3</td>
<td>[1.01,1.01]</td>
<td>[-14.38,-11.11]</td>
<td>[0.94,0.94]</td>
<td>[0.19,0.19]</td>
<td>[0.00,0.00]</td>
<td>[-0.13,0.66]</td>
</tr>
<tr>
<td>4</td>
<td>[1.01,1.02]</td>
<td>[-11.57,-8.95]</td>
<td>[0.48,0.48]</td>
<td>[-0.04,0.04]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>5</td>
<td>[1.01,1.02]</td>
<td>[-9.89,-7.64]</td>
<td>[0.08,0.08]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>6</td>
<td>[1.07,1.07]</td>
<td>[-16.15,-12.69]</td>
<td>[0.11,0.11]</td>
<td>[0.07,0.07]</td>
<td>[0.00,0.00]</td>
<td>[-0.76,1.18]</td>
</tr>
<tr>
<td>7</td>
<td>[1.05,1.05]</td>
<td>[-14.88,-11.62]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>8</td>
<td>[1.09,1.09]</td>
<td>[-14.88,-11.62]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.21,0.27]</td>
</tr>
<tr>
<td>9</td>
<td>[1.03,1.04]</td>
<td>[-16.63,-13.04]</td>
<td>[0.29,0.29]</td>
<td>[0.17,0.17]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>10</td>
<td>[1.03,1.04]</td>
<td>[-16.86,-13.22]</td>
<td>[0.09,0.09]</td>
<td>[0.06,0.06]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>11</td>
<td>[1.04,1.05]</td>
<td>[-16.64,-13.06]</td>
<td>[0.03,0.03]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>12</td>
<td>[1.05,1.06]</td>
<td>[-17.10,-13.43]</td>
<td>[0.06,0.06]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>13</td>
<td>[1.04,1.05]</td>
<td>[-17.16,-13.46]</td>
<td>[0.13,0.13]</td>
<td>[0.06,0.06]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>14</td>
<td>[1.02,1.03]</td>
<td>[-18.02,-14.11]</td>
<td>[0.15,0.15]</td>
<td>[0.05,0.05]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
</tbody>
</table>

6.3 IEEE 30 bus system

The 30 bus IEEE test case shown in Figure 5 has been analysed in the same way. Table 7 shows the results with the proposed interval load flow method. The reactive power at slack bus is [−0.2386, −0.0762].
Table 6  Three phase interval short circuit for 14 bus system

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Prefault Voltage $V(0)$</th>
<th>Fault current $I(F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Magnitude)</td>
<td>(Angle)</td>
</tr>
<tr>
<td>1</td>
<td>[1.0600, 1.0601]</td>
<td>[0.0000, 0.0000]</td>
</tr>
<tr>
<td>6</td>
<td>[1.0700, 1.0701]</td>
<td>[–16.1464, 12.6910]</td>
</tr>
<tr>
<td>12</td>
<td>[1.0517, 1.0553]</td>
<td>[–17.1040, –13.4341]</td>
</tr>
<tr>
<td>13</td>
<td>[1.0445, 1.0497]</td>
<td>[–17.1591, 13.4585]</td>
</tr>
<tr>
<td>14</td>
<td>[1.0155, 1.0272]</td>
<td>[–18.0192, –14.1115]</td>
</tr>
</tbody>
</table>

Figure 5  30 bus test system
Table 7  
Interval load flow solution for 30 bus system

<table>
<thead>
<tr>
<th>Bus</th>
<th>$V$(p.u.)</th>
<th>$\text{Angle (deg)}$</th>
<th>$P_D$</th>
<th>$Q_D$</th>
<th>$P_G$</th>
<th>$Q_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[1.06,1.06]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[2.25,2.98]</td>
<td>[–0.26,–0.08]</td>
</tr>
<tr>
<td>2</td>
<td>[1.04,1.04]</td>
<td>[–6.36,–4.65]</td>
<td>[0.22,0.22]</td>
<td>[0.13,0.13]</td>
<td>[0.40,0.40]</td>
<td>[–0.09,1.07]</td>
</tr>
<tr>
<td>3</td>
<td>[1.02,1.02]</td>
<td>[–9.04,–6.99]</td>
<td>[0.02,0.02]</td>
<td>[0.01,0.01]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>4</td>
<td>[1.01,1.01]</td>
<td>[–10.92,–8.43]</td>
<td>[0.07,0.07]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>5</td>
<td>[1.01,1.01]</td>
<td>[–16.20,–12.58]</td>
<td>[0.94,0.94]</td>
<td>[0.19,0.19]</td>
<td>[0.00,0.00]</td>
<td>[–0.21,0.92]</td>
</tr>
<tr>
<td>6</td>
<td>[1.01,1.01]</td>
<td>[–12.87,–9.96]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>7</td>
<td>[1.00,1.01]</td>
<td>[–14.83,–11.50]</td>
<td>[0.23,0.23]</td>
<td>[0.11,0.11]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>8</td>
<td>[1.01,1.01]</td>
<td>[–13.69,–10.58]</td>
<td>[0.30,0.30]</td>
<td>[0.30,0.30]</td>
<td>[0.00,0.00]</td>
<td>[–0.62,1.23]</td>
</tr>
<tr>
<td>9</td>
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<td>[–16.23,–12.67]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>10</td>
<td>[1.04,1.05]</td>
<td>[–17.99,–14.09]</td>
<td>[0.06,0.06]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>11</td>
<td>[1.08,1.08]</td>
<td>[–16.23,–12.67]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.13,0.19]</td>
</tr>
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<td>[0.11,0.11]</td>
<td>[0.07,0.07]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
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<td>[1.07,1.07]</td>
<td>[–17.19,–13.44]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.05,0.15]</td>
</tr>
<tr>
<td>14</td>
<td>[1.04,1.05]</td>
<td>[–18.19,–14.22]</td>
<td>[0.06,0.06]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
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<td>[1.03,1.04]</td>
<td>[–18.28,–14.30]</td>
<td>[0.08,0.08]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>16</td>
<td>[1.04,1.05]</td>
<td>[–17.83,–13.96]</td>
<td>[0.03,0.03]</td>
<td>[0.02,0.02]</td>
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<td>[0.00,0.00]</td>
</tr>
<tr>
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<td>[1.03,1.05]</td>
<td>[–18.18,–14.23]</td>
<td>[0.09,0.09]</td>
<td>[0.06,0.06]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
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<td>[–18.96,–14.83]</td>
<td>[0.03,0.03]</td>
<td>[0.01,0.01]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>19</td>
<td>[1.02,1.03]</td>
<td>[–19.15,–14.98]</td>
<td>[0.10,0.10]</td>
<td>[0.03,0.03]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
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<td>[0.02,0.02]</td>
<td>[0.01,0.01]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>21</td>
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<td>[–18.50,–14.46]</td>
<td>[0.17,0.17]</td>
<td>[0.11,0.11]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
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<td>[1.02,1.04]</td>
<td>[–18.48,–14.45]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>23</td>
<td>[1.02,1.04]</td>
<td>[–18.73,–14.62]</td>
<td>[0.03,0.03]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>24</td>
<td>[1.01,1.03]</td>
<td>[–18.94,–14.75]</td>
<td>[0.09,0.09]</td>
<td>[0.07,0.07]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>25</td>
<td>[1.01,1.03]</td>
<td>[–18.49,–14.39]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>26</td>
<td>[0.99,1.01]</td>
<td>[–19.01,–14.70]</td>
<td>[0.03,0.03]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>27</td>
<td>[1.02,1.03]</td>
<td>[–17.90,–13.96]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>28</td>
<td>[1.01,1.01]</td>
<td>[–13.60,–10.55]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>29</td>
<td>[1.00,1.02]</td>
<td>[–19.29,–15.01]</td>
<td>[0.02,0.02]</td>
<td>[0.01,0.01]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
<tr>
<td>30</td>
<td>[0.98,1.01]</td>
<td>[–20.28,–15.78]</td>
<td>[0.11,0.11]</td>
<td>[0.02,0.02]</td>
<td>[0.00,0.00]</td>
<td>[0.00,0.00]</td>
</tr>
</tbody>
</table>

With the proposed interval short circuit studies, the fault currents at all the buses with magnitude and phase angles are shown in Table 8.
### Table 8  Three phase interval short circuit for 30 bus system

<table>
<thead>
<tr>
<th>Bus No</th>
<th>Prefault Voltage $V(0)$</th>
<th>Fault current $I(F)$</th>
</tr>
</thead>
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7 Conclusion

In this study, methods for considering the uncertainties of the input parameters in the load flow solution and short circuit studies for power systems have been presented. Based on interval arithmetic, the proposed methodology can consider the uncertainties in both the load demand and the transmission line parameters successfully. The solutions which we have obtained from the interval arithmetic based methods encompass all the solutions obtained from conventional methods thus guaranteeing the results (Ray and Ralhan, 2016). The solutions obtained by the proposed method provide more information in qualitative terms as the values obtained also include the unknown uncertainties. Conventional solutions give only deterministic values which are approximated results.

Interval methods are often affected by overestimation, hence the computed error bounds become overly pessimistic. Even though Vaccaro et al. (2010) have stated that use of interval arithmetic in power flow analysis leads to solutions which are not useful for practical applications, alternative evaluation schemes using affine arithmetic can be applied to overcome this problem. The dependency problem and the wrapping effect are particular sources of overestimation in interval computations. Dependency problem occurs due to the failure of interval arithmetic to identify the different occurrences of the same variable. For reducing both the dependency problem and the wrapping effect, interval arithmetic has been extended with symbolic computations using Taylor models in Neher et al. (2007). The dependency problem can be also eliminated by applying a suitable extension of the Jacobian matrix as discussed in Kearfott (1991). Also, the condition imposed on the stopping criterion can be made more stringent, which has been taken to be as the width of the box in our computations. Consequently, this technique can be made computationally more robust and reliable, thus yielding better and accurate solutions, which has been taken as future work.

This analysis helps to ensure that cables, transformers, lines are sized properly to carry the variable load. From the results, it can be determined whether the system voltages remain within specified limits and the equipments are not getting overloaded. The profiles of bus voltages and angles help us to identify real and reactive power flows and minimise the transmission losses. These studies become important in planning and expansion while ensuring that each generator runs within the specified limits and demand is met without overloading the power infrastructure. The reactive power bounds provide us with information regarding the injection of reactive power into the system, in order to keep the power factor close to unity.

Also, in the initial stages of planning and design studies of power systems, the proposed technique will be useful to save time, effort, and the resources required. The results obtained by the proposed method take into account the demand error which is very significant, whence, the load uncertainty is not accounted for, the load flow solutions obtained would be a ‘snapshot’ for a single specific configuration and operating conditions of the power system. If uneven variations in active and reactive powers in different buses are considered then we can arrive at a more realistic solution. The same is being considered as a part of the future work.

The information gained from short circuit fault studies is used for proper relay setting and coordination. It is also used to obtain the rating of the prospective switchgears. With the proposed method, maximum load currents and fault currents for each relay are available and thus we can have well-defined coordination pairs.
New techniques that are computationally robust and reliable are needed for the analysis of power systems with uncertainties, especially in view of the increasing use of renewable power sources, such as wind, hydro and solar power which are highly variable. However, the requirements of interval arithmetic computation are greater than the conventional load flow methods.

References


