# Modified multi-verse optimiser used for global optimisation

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**Abstract:** In this paper, modified version of multi-verse optimiser (MVO) was suggested and tested on numerical optimisation problems. MVO is an innovative optimisation approach which stimulated from the concepts of cosmology; they are named as white hole, black hole and wormhole. Mathematical modelling of this concept has been carried out to acquire exploitation, exploration and local search. Modification in MVO has been made by introducing concept of dynamic variation in population size (universe). Modified multi-verse optimiser (MMVO) was tested on 16 benchmark functions having different complexity. Statistical comparisons of other algorithms outcomes is depicted that MMVO performs better than other algorithms.

**Keywords:** optimisation; particle swarm optimisation; PSO; genetic algorithm; benchmark functions.

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## 1   Introduction

Nature is the main source of the inspiration for optimisation. These techniques are generally concerning the population-based techniques. Since a long period of time in spite of success is obtaining better result for difficult problems in global world. Many algorithms of nature are categorised under the stochastic/probabilistic process. Generally, set of random solution is created for starting any optimisation process. These solutions are combined or evolved over the course of interactions, this serves as the base of frame work for most of the population-based algorithms. Only the method of evolution or combination makes the difference between the algorithms. The process of exploration and exploitation is another concept which is common between populations-based. Researchers always try to discover search space globally by which they take the solution of the best global optimum proper balance between exploration and exploitation.

The origin of this world is considered according to big-bang theory. This is the root cause of every natural events occurring in this world. The next theory recently considered is multi-verse theory.

Optimisation techniques generally population-based techniques have one major inspiration, i.e., nature. Detail classification of algorithm on the basis of collective behaviour is discussed (Ibrahim et al., 2020). Detailed literature review is examined to study the various variants of multi-verse optimiser (MVO) algorithm as well as its strength (Dubey et al., 2018). Some of the algorithms which are stimulated from the nature and acquire optimisation as: genetic algorithm (GA) (Goldberg, 1989), which mimics the biological evolution such as mutation, selection and crossover to carry out the optimisation. Particle swarm optimisation (PSO) (Kennedy and Eberhart, 1995), which is inspired by the social behaviour of birds, insects and fish. Artificial bee colony (ABC) optimisation (Karaboga, 2005), which imitates the foraging behaviour of honey bees. Gravitational search algorithm (GSA) (Rashedi et al., 2009), uses Newtonian laws for the optimisation. Cuckoo search algorithm (CSA) (Yang and Deb, 2009), follows the brood parasitism of cuckoo birds for the optimisation. Flower pollination algorithm (FPA) (Yang, 2012), gets inspiration from the pollination of flowering plants. Grey wolf optimisation (GWO) (Mirjalili et al., 2014), which imitates the hunting behaviour of grey wolves for the optimisation. Improved multi-verse optimisation (IMVO) (Mishra et al., 2020), which gets improvement of the dynamic variation of population size in MVO. A set of random solution is created to start any optimisation process. Then these solutions are combined or evolved over the course of iterations, this is the basic framework for most of the population-based algorithms.

In this paper, a nature-inspired population-based algorithm known as modified multi-verse optimiser (MMVO) is suggested. It gets stimulation from the multi-verse theory in physics. To develop the MMVO algorithm, the concept of multi-verse theory has been considered as mathematically modelled, i.e., white hole, black hole and wormhole. The remaining part of the paper is arranged as follows: Section 2 described with the suggested algorithm (MVO) and its modification, results and discussion are depicted in Section 3 and conclusion is given in Section 4.

## 2    Multi-verse optimisation

The existence of this world depends upon a big bang theory (Khoury et al., 2002). This theory is assumed as the source of everything happening in the world. Now, there is another theory known as multi-verse theory (Tegmark, 2004), which says that one universe is produced by one big bang. We are living in a universe now, but according to multi-verse theory, in this world, there are more than one universe exists, these universes may have interaction, collisions and different laws of physics.

MVO algorithm basically depends on three concepts, which are taken from the multi-verse theory and these concepts are: white holes, black holes and wormholes. Cyclic model of multi-verse theory (Paul and Neil, 2002), says that concoction of white holes is done by the collision of parallel universes. Black holes are in contrast with white holes. They try to attract everything also beams of light with a very high gravitational force (Michael and Kip, 1988). Wormholes are supposed to associate all other parts of cosmoses with each other. It associates another parts of cosmos, therefore it is applied a tunnel by the objects to move from one region to another region in the cosmos (Alan, 2007). Also, it gives path for the objects to travel from one cosmos to another cosmos. Models of these three components are shown in Figure 1.

**Figure 1**    (a) White hole (b) Black hole (c) Wormhole (see online version for colours)



|        |        |        |
|:------:|:------:|:------:|
|  (a)   |  (b)   |  (c)   |
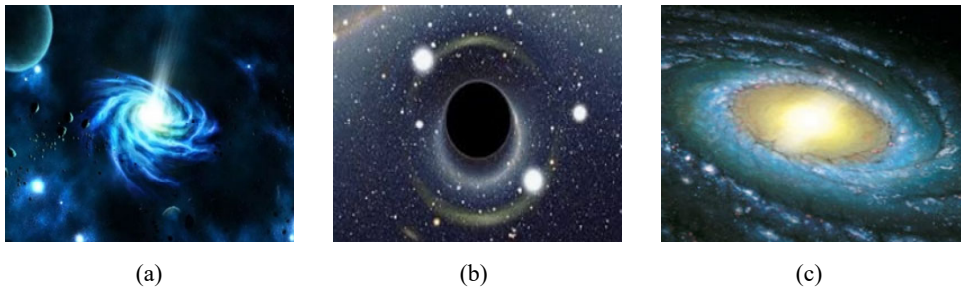
Every cosmos (universe) has an inflation rate which is generated by the expansion of cosmos. Inflation rate is a very important part for every cosmos because structure of stars, asteroids, planets, suitability for life, physical laws, white, black and wormholes based on inflation/expansion rate. White, black and wormholes are way for all cosmoses (universes) to connect with each other to obtain a fixed situation this concept gives the stimulation of MVO algorithm.

### 2.1    MVO algorithm

As we know that, exploration and exploitation are considered as search processes in case of population-based techniques. White holes and black holes are applied for the exploration of search space in MVO algorithm and wormholes are used for the exploitation of search space.

Some assumptions has been made for the optimisation such that each homogeneous solution is obtained to universe and each variable in that solution which is like as object in the cosmos (universe), as well as the supply inflation rate to every solution which is directly proportional to fitness function value of solution. Also, here we have considered

time in place of iteration because this term is common in cosmology and multi-verse theory.

    Some rules are framed to cosmoses (universes) for optimisation using MVO algorithm which are as follows (Mirjalili et al., 2016):

a    Inflation rate is directly proportional to the probability of having white holes.

b    Inflation rate is inversely proportional to the probability of having black holes.

c    Cosmoses (universes) having greater inflation rate try to send objects via white holes.

d    Cosmoses (universes) having smaller inflation rate try to receive objects via black holes.

e    Objects of all cosmoses (universes) move randomly towards best cosmos (universe) through wormholes without considering the inflation rate.

Roulette wheel selection is used to select of white or black holes and exchange of cosmos's objects. Each and every time, cosmoses (universes) based upon their rate of inflation are classified wherein one out of them gets selected using roulette wheel selection to possess a white hole. The iterative method of MVO is described as below.

*Nomenclature*

$X_q$          $q^{th}$ parameter of the best universe

*WEP*        wormhole existence probability

$r_2, r_3, r_4$   random number

$m$            number of universes

$x_p^q$          $q^{th}$ parameter of $p^{th}$ universe

$lb_q, ub_q$   lower and upper bound of $q^{th}$ variable

*TDR*         travelling distance rate

$n$             number of objects.

Suppose that,

$$u = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^n \\ \vdots & & \ddots & \vdots \\ x_m^1 & x_m^2 & \cdots & x_m^n \end{bmatrix} \tag{1}$$

where $n$ is the no. of variables (parameters) and $m$ is the number of solutions (universes) (Michael and Kip, 1988).

$$x_p^q = \begin{cases} x_s^q & r_1 < NI(u_p) \\ x_p^q & r_1 \geq NI(u_p) \end{cases} \tag{2}$$

where $x_p^q$ denotes the $q^{th}$ parameter of $p^{th}$ universe, $u_p$ denotes the $^p$th universe, $NI(u_p)$ is normalised rate of inflation of $p^{th}$ universe, $r_1$ is a random no. lies in [0,1], and $x_s^q$ denotes the $q^{th}$ parameter of $s^{th}$ universe selected by a roulette wheel mechanism.

According to equation (1), roulette wheel is applied to choose and obtain white holes, which depends upon normalised rate of inflation. For lower rate of inflation probability will be more when objects send via white or black hole tunnels. This mechanism gives surety for the exploration, because cosmoses (universes) are required to exchange objects and undergo sudden change in search space for the exploration. In order to carry out exploitation, we suppose that every cosmos (universe) has wormholes to move the objects via space randomly.

To update the position in optimal universe, following calculation need to be performed:

$$x_p^q = \begin{cases} \begin{cases} X_q + TDR*\left(\left(ub_q - lb_q\right)*r_4 + lb_q\right) & r_3 < 0.5 \\ X_q - TDR*\left(\left(ub_q - lb_q\right)*r_4 + lb_q\right) & r_3 \geq 0.5 \end{cases} & r_2 < WEP \\ x_p^q & r_2 \geq WEP \end{cases} \tag{3}$$

where $X_q$ denotes the $q^{th}$ parameter of best universe found so far, $TDR$ is coefficient, $WEP$ is another coefficient, $lb_q$ denotes lower bound of $q^{th}$ variable, $ub_q$ denotes upper bound of qth variable, $x_p^q$ denotes the $q^{th}$ parameter of $p^{th}$ universe, and $r_2$, $r_3$, $r_4$ are random numbers lie in [0, 1].

Wormhole existence probability (WEP) and travelling distance rate (TDR) are supposed to increase over the course of iterations to emphasise the exploitation. These coefficient can be formulated as follows:

$$WEP = \min + i*\left(\frac{\max - \min}{I}\right) \tag{4}$$

where min is minimum [0.2], max is maximum (Ibrahim et al., 2020), $i$ denotes current iteration and $I$ denotes maximum iteration.

$$TDR = 1 - \frac{i^{1}/k}{I^{1}/k} \tag{5}$$

where $k = 6$, defining accuracy of exploitation over the course of iterations.

In MVO algorithm, optimisation starts by producing a set of random universes. In every iteration, objects lying in the cosmos (universe) having high rate of inflation try to travel towards the universe having low rate of inflation through white or black holes. Meanwhile, every cosmos (universe) undergoes sudden teleportation in its objects via wormholes towards best cosmos (universe). This process will be continued whenever the require criterion achieve. The computational complexity of MVO algorithm is given as:

$$O(IMVO) = O\left(I\left(O(quick\ sort) + m*n*\left(O(roulette\ wheel)\right)\right)\right) \tag{6}$$

$$O(IMVO) = O\left(I\left(m^2 + m*n*\log m\right)\right) \tag{7}$$

where $m$ is the no. of cosmoses (universes), $I$ is the maximum number of iterations, and $n$ is the number of objects.

**Figure 2**    Flowchart for multi-verse optimisation (see online version for colours)
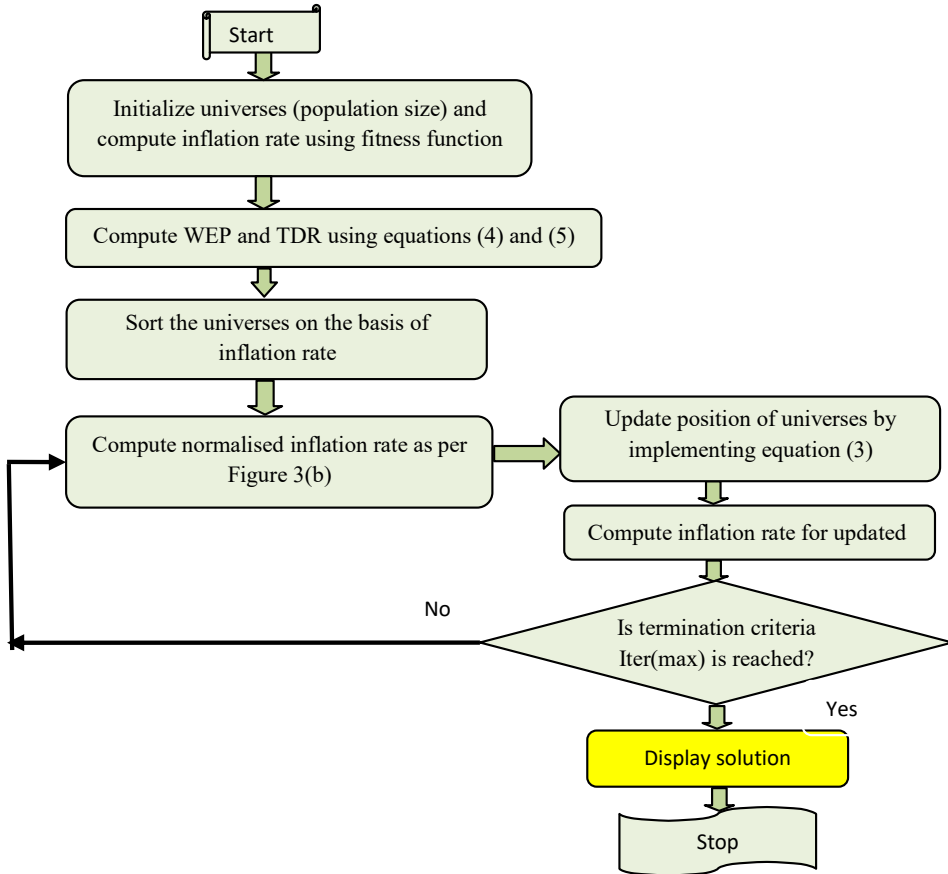


## 2.2    Modified MVO algorithm

Modification in MVO is made in two ways:

a    cosmos (universe) numbers are varied iteratively

b    introduce more randomness in algorithm that helps to minimise stagnation of standard MVO.

These steps are depicted in Figure 3(a) and Figure 3(b).

**Figure 3** Pseudo code of modification phase of MVO

```
x = Iter/Iter_max;
    if (x ≤ 0.5)
        Universes_no = 150
    end
    if (x > 0.5)
        Universes_no = 50
    end
```

(a)

```
for each Universe i
    for each Universe j
      if r2 < WEP
      r2 ∈ random(0, 1)
       r3 ∈ random(0, 1)
      if r3 < 0.7
       Universes(i, j) = Best_universe(1, j) + TDR*((ub – lb)*rand + lb);
       end
      if r3 > 0.7
       Universes(i, j) = Best_universe(1, j) – TDR*((ub – lb)*rand + lb);
      end
       end
    end
end
```

(b)

**Table 1**    Uni-modal benchmark functions

| Function | Dimension | Range | Fmin |
|---|---|---|---|
| $F_1(x) = \sum_{i=1}^{n} x_i^2$ | 50 | [–100, 100] | 0 |
| $F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$ | 50 | [–10, 10] | 0 |
| $F_3(x) = \sum_{i=1}^{n} \left( \sum_{j-1}^{i} x_j \right)^2$ | 50 | [–100, 100] | 0 |
| $F_4(x) = \max_i \{ \lvert x_i \rvert, 1 \leq n \leq i \}$ | 50 | [–100, 100] | 0 |
| $F_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ | 50 | [–30, 30] | 0 |
| $F_6(x) = \sum_{i=1}^{n} (x_i + 0.5)^2$ | 50 | [–100, 100] | 0 |
| $F_7(x) = \sum_{i=1}^{n} ix_i^4 + random(0, 1)$ | 50 | [–1.28, 1.28] | 0 |

## 3    Numerical benchmarks and simulation results

In this paper, 16 benchmark functions are selected, dimension of these test functions is considered as 50 and these test functions are given in Table 1, Table 2 and Table 3. The employed benchmark functions are determined into three groups: uni-modal benchmark functions, multi-modal benchmark functions and composite benchmark functions. In uni-modal benchmark functions, is only one global optimum, therefore, it is useful to test the exploitation of algorithms. Multi-modal benchmark functions having only one global optimum and multiple local optima. Whereas, composite benchmark functions are providing the balance of exploitation and exploration which presented in Table 3.

**Table 2**    Multi-modal benchmark functions

| Function | Dimension | Range | Fmin |
|---|---|---|---|
| $F_8(x) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{\|x_i\|}\right)$ | 50 | [−500, 500] | 0 |
| $F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$ | 50 | [−5.12, 5.12] | 0 |
| $F_{10}(x) = -20\exp\left(-0.2\sqrt{\dfrac{1}{n}\sum_{i=1}^{n} x_i^2}\right)$ $-\exp\left(\dfrac{1}{n}\sum_{i=1}^{n} \cos(2\pi x_i)\right) + 20 + e$ | 50 | [−32, 32] | 0 |
| $F_{11}(x) = \dfrac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\dfrac{x_i}{\sqrt{i}}\right) + 1$ | 50 | [−600, 600] | 0 |
| $F_{12}(x) = \dfrac{\pi}{n}\Big\{10\sin(\pi y_1)$ $+\sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10(\sin(\pi y_{i+1}))^2] + (y_n - 1)^2\Big\}$ $+\sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = \left(1 + \dfrac{x_i + 1}{4}\right)$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$ | 50 | [−50, 50] | 0 |
| $F_{13}(x) = 0.1\Big\{\sin^2(3\pi x_1)$ $+\sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + \sin^2(2\pi x_n)]\Big\}$ $+\sum_{i=1}^{n} u(x_i, 5, 100, 4)$ | 50 | [−50, 50] | 0 |

For the simulation analysis the number of cosmoses (universes) is considered as 150 for have of maximum iteration and 50 for the remaining iteration. The maximum number of iteration which is nothing but the stopping criterion considered as 100.

**Table 3** Composite benchmark functions

| | | | |
|---|---|---|---|
| $F_{14}(CF1)$ | 50 | [–5, 5] | 0 |
| $f_1, f_2, f_3, \ldots\ldots\ldots\ldots\ldots, f_{10}$ = *sphere function* | | | |
| $[\sigma_1, \sigma_2, \sigma_3, \ldots\ldots\ldots\ldots\ldots, \sigma_{10}] = [1, 1, 1, \ldots\ldots, 1]$ | | | |
| $[\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots\ldots\ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots\ldots, 5/100]$ | | | |
| $F_{15}(CF2)$ | 50 | [–5, 5] | 0 |
| $f_1, f_2, f_3, \ldots\ldots\ldots\ldots\ldots, f_{10}$ = *Griewank's function* | | | |
| $[\sigma_1, \sigma_2, \sigma_3, \ldots\ldots\ldots\ldots\ldots, \sigma_{10}] = [1, 1, 1, \ldots\ldots, 1]$ | | | |
| $[\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots\ldots\ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots\ldots, 5/100]$ | | | |
| $F_{16}(CF3)$ | 50 | [–5, 5] | 0 |
| $f_1, f_2, f_3, \ldots\ldots\ldots\ldots\ldots, f_{10}$ = *Griewank's function* | | | |
| $[\sigma_1, \sigma_2, \sigma_3, \ldots\ldots\ldots\ldots\ldots, \sigma_{10}] = [1, 1, 1, \ldots\ldots, 1]$ | | | |
| $[\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots\ldots\ldots, \lambda_{10}] = [1, 1, 1, \ldots\ldots\ldots, 1]$ | | | |

**Table 4** Results of uni-modal benchmark functions

| Function | MMVO | | MVO (Mirjalili et al., 2016) | | PSO (Mirjalili et al., 2016) | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | Mean | SD |
| $F_1$ | 0.5010 | 0.1002 | 2.08583 | 0.648651 | 3.552364 | 2.853733 |
| $F_2$ | 4.4556 | 10.2025 | 15.92479 | 44.7459 | 8.716272 | 4.929157 |
| $F_3$ | 310.0885 | 70.3062 | 453.2002 | 177.0973 | 2380.963 | 1183.351 |
| $F_4$ | 2.2314 | 0.8809 | 3.123005 | 1.582907 | 21.5169 | 6.71628 |
| $F_5$ | 310.7155 | 462.0721 | 1,272.13 | 1,479.477 | 1,132.486 | 1,357.967 |
| $F_6$ | 0.4550 | 0.0664 | 2.29495 | 0.630813 | 86.62074 | 147.3067 |
| $F_7$ | 0.0311 | 0.0102 | 0.051991 | 0.029606 | 0.577434 | 0.318544 |

**Table 5** Results of multimodal benchmark functions

| Function | MMVO | | MVO (Mirjalili et al., 2016) | | PSO (Mirjalili et al., 2016) | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | Mean | SD |
| $F_8$ | –12,413.021 | 931.0624 | –11,720.2 | 937.1975 | –6,727.59 | 1,352.882 |
| $F_9$ | 214.4299 | 40.8172 | 118.046 | 39.34364 | 99.83202 | 24.62872 |
| $F_{10}$ | 1.3191 | 0.5642 | 4.074904 | 5.501546 | 4.295044 | 1.308386 |
| $F_{11}$ | 0.5600 | 0.0622 | 0.938733 | 0.059535 | 624.3092 | 105.3874 |
| $F_{12}$ | 2.017 | 1.0112 | 2.459953 | 0.791886 | 13.38384 | 8.969122 |
| $F_{13}$ | 0.1134 | 0.0834 | 0.222672 | 0.086407 | 21.11298 | 12.83179 |

**Figure 4**  Plot of benchmark function and their convergence curve (see online version for colours)
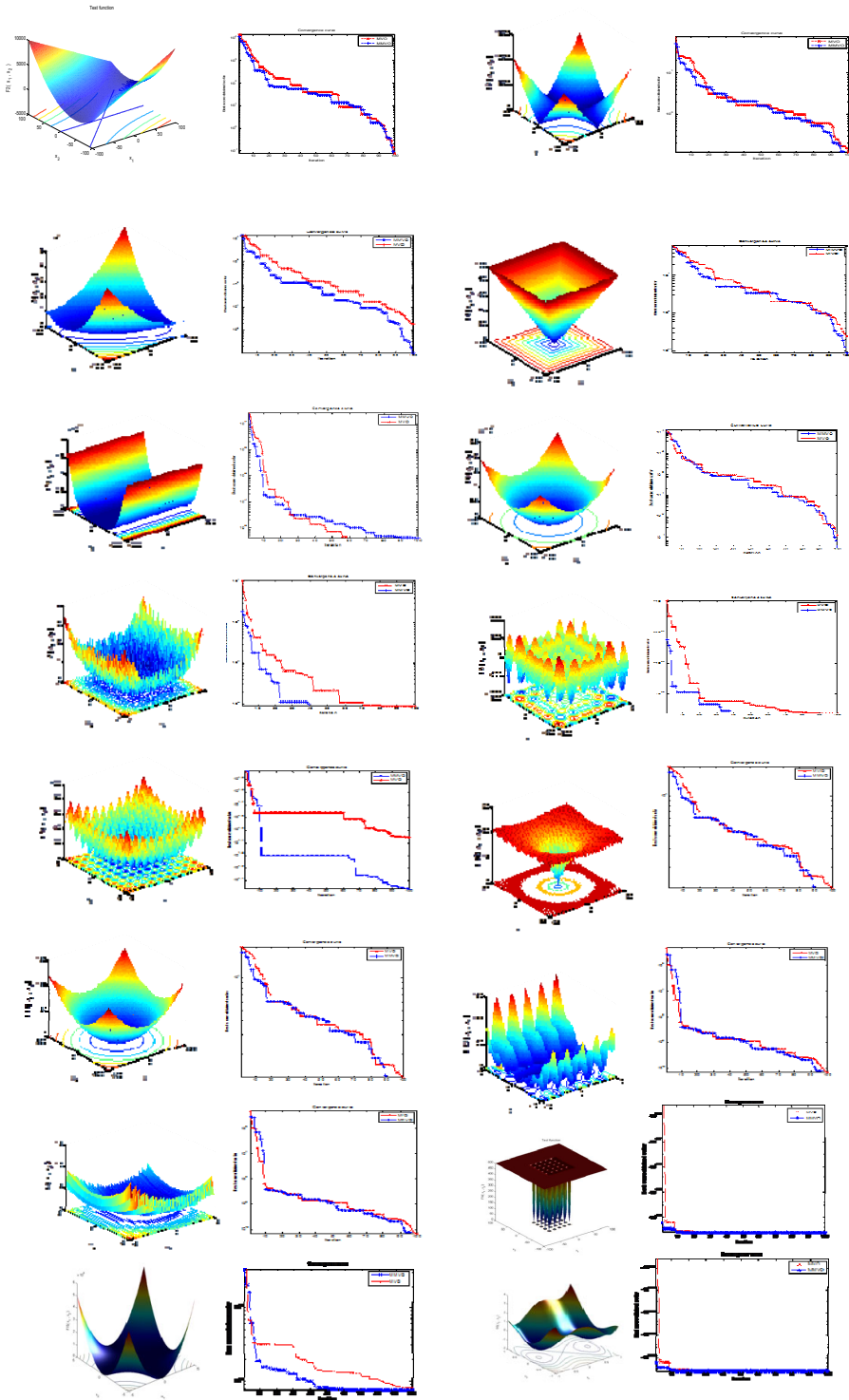
The benchmark functions were tested by performing consecutive 30 runs and the obtained statistical results of test functions are tabulated in Table 4, Table 5 and Table 6. The obtained results are also compared with other methods as MVO (Mirjalili et al., 2016) and PSO (Mirjalili et al., 2016) from the tabulated results, it is clearly observed that MMVO performs better in terms of mean value and the standard deviation (SD) while comparison to other reported methods.

**Table 6**    Results of composite benchmark functions

| Function | MMVO | | MVO (Mirjalili et al., 2016) | | PSO (Mirjalili et al., 2016) | |
|---|---|---|---|---|---|---|
| | *Mean* | *SD* | *Mean* | *SD* | *Mean* | *SD* |
| $F_{14}$ | 0.998 | 0.1792 | 10.00017 | 31.62288 | 20 | 63.2455 |
| $F_{15}$ | 0.0007204 | 0.00016235 | 30.00705 | 48.30615 | 60 | 84.3274 |
| $F_{16}$ | −1.0316 | $6.7679 \times 10^{-16}$ | 50.00061 | 52.70461 | 20 | 42.1637 |

The comparison of convergence curves of all the tested functions were plotted and presented in Figure 4. Here, it is clearly observed that convergence characteristics of MMVO are found to be better than MVO and PSO.

## 4    Conclusions

In this paper, a modified version of optimisation algorithm stimulated from the concepts of cosmology and multi-verse theory in physics is suggested. MMVO is based on the three concepts, i.e., white, black and wormholes. Sixteen benchmark functions have been considered to study MMVO. These test functions provides the information related to exploitation, exploration and convergence of MMVO.

In cosmoses (universes), white holes might be produced due to high rate of inflation, so that they can move objects to other cosmoses and help them to improve their rate of inflation. In cosmoses (universes), black holes might be produced due to low rate of inflation, so that they can receive objects from other cosmoses which in turn modify their inflation rate. White or black hole tunnels are used for the moving of objects from cosmoses (universes) having high rate of inflation to cosmoses (universes) having low rate of inflation, such that overall rate of inflation of all cosmoses (universes) can be modified over the course of iterations. Wormholes have the tendency to appear randomly in any universe without considering the rate of inflation, which maintains the diversity of cosmoses (universes) over the course of iterations. White or black hole tunnels has the requirement of immediate changes in cosmos (universe), which causes exploration of search space. Sudden changes in cosmoses (universes) leads to the resolution of problem of stagnation of local optima. Wormholes re-span variables of cosmoses (universes) around obtained best solution in a random manner, which modifies exploitation. The convergence of MMVO has been modified by accentuating local search proportional to the number of iterations.

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