A two-stage stochastic programming optimisation for sugar-ethanol-electricity production from sugarcane: a case study of Mauritius

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Abstract: Sugarcane is an economically essential plant from which juice is extracted to produce raw sugar and molasses. The raw sugar can further be refined while the molasses can be fermented to manufacture ethanol. Other than cane juice, bagasse is another by-product of sugarcane which can be used to generate electricity and as a raw material to manufacture paper and chemicals. In this paper, we present a two-stage stochastic programming model to quantify the optimal amount of sugar, ethanol and electricity to be made from sugarcane so as to minimise total production cost. The model developed is based on variation of prices of sugar, ethanol and electricity and energy required to produce them. We establish the boundedness, convexity and existence of solution of the model. The latter is then applied to data available for Mauritius and solved using a genetic algorithm approach. We estimate the optimal amount of sugar, ethanol and electricity to be produced up to the year 2027. Finally, we analyse three different scenarios, namely decrease in land under sugarcane, increase/decrease in price of sugar and increase/decrease in price of ethanol.

Keywords: modelling; optimisation; genetic algorithm; sugarcane; stochastic differential equation; Mauritius.
A two-stage stochastic programming optimisation


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1 Introduction

Mauritius is a small island located within the latitude and longitude of 20°17 south and 57°33 east. The latitude and longitude of Mauritius include 2,040 km² of total land area. It is situated in the southern Africa and is basically a group of islands found in the Indian Ocean. Hydroelectric, wind and bagasse are renewable sources of energy and they are mostly used to generate electricity for Mauritius to a level of 24% today. Bagasse accounts for about 20% alone (CSO 2014). Examples of non-renewable energy sources are gasoline, diesel oil, dual purpose kerosene, fuel oil, liquefied petroleum gas (LPG) and coal. They are used for transportation, manufacturing processes, agriculture and making of electricity. Another by-product extracted from sugarcane is molasses which is distilled to produce ethanol. Since ethanol is made from renewable source, it is a fundamental tool to replace fossil fuels for transport and hence, helps to decrease air pollution from vehicles. Brazil is the first country to have considered production of ethanol from sugarcane in view of sustaining natural resources and environment (*The Economist*, 2008).

In recent years, several works have been conducted on optimisation of sugar/ethanol from sugarcane. In particular, Silva et al. (2013) have developed a multi-choice mixed integer goal programming (MCMIGP) model and a weighted goal programming (WGP) model to address real problems in a Brazilian sugar and ethanol mill. They have shown that MCMIGP provides better solution as compared to WGP. The MCMIGP model,
based on the production system of sugar, alcohol, molasses and derivatives, is solved using the modelling language GAMs 23.6.2 and the optimisation solver CPLEX 12.2.1 and has helped to make better decisions on production planning, distribution, and energy cogeneration for the mill. A mixed integer programming model is also proposed in Paiva and Morabito (2009) so as to help decision makers to plan industrial processes of sugar, ethanol and molasses production in a Brazilian mill. In Kawamura et al. (2006), the authors have applied a multi-period linear programming (LP) model to reduce total transportation and storage costs in Sugarcane and Ethanol Producers’ Cooperative, which consists of 34 sugar mills, in Brazil. The model helps to obtain the optimal amount of each product that will be made in each mill and hence, increase overall profit. Kostin et al. (2012) have introduced a multi scenario mixed integer LP approach to optimise sugar and bio-ethanol in the Argentinean sugar cane industry, under uncertainty in demand. The programming problem is solved using the sample average approximation algorithm. The problem of the optimal design of the sugar/ethanol in the Argentinean sugarcane industry is also discussed in Durand et al. (2012), under parametric uncertainty, based on the net present value and customer satisfaction. In the latter paper, Monte Carlo simulation and genetic algorithm (GA) are applied to obtain solution of the model. Yoshizaki et al. (1996) have developed a transhipment model which estimated the logistics costs to assess the distribution of ethanol and other fossil fuels in sugarcane mills of the region of Southeastern Brazil. Other works include Buddadee et al. (2008) in which two major scenarios and a multi-objective optimisation model are used to decide about the proper utilisation scheme of excess bagasse produced in sugarcane industry whilst Illukpitiya et al. (2013) have optimised sugar and biomass feedstock production from sugarcane using a LP model which consists of seven decision variables and 23 constraints. The model maximises producers’ profit.

This paper introduces a novel two stage programming model for the strategic planning of production of sugar, ethanol and electricity in sugar mills in Mauritius. In general, Mauritius produces a relatively small amount of sugar compared to global consumption. Other products that can be extracted from sugarcane, such as bagasse and molasses, can therefore be considered without perturbing global market demand of sugar. We take into account variation of prices of sugar, ethanol and electricity to measure the total production cost (TPC). The prices are represented by stochastic differential equations (SDEs). The model is then solved using GA which is an optimisation and stochastic search technique based on biological evolution process (Holland, 1992). The difference between GA and other traditional optimisation techniques is that it searches a population of points rather than a single point (Goldberg, 1989; Ozturk et al., 2005). GA also preserves a population of solutions while looking for better ones and is also simple to hybridise. This is the first work carried in Mauritius that shows optimisation of each product to be made in mills and will help the country in increasing total profit. The paper is organised as follows. In Section 2, the problem under study is formally stated and the equation to calculate total cost is described in Section 3. The optimisation problem to be solved is then presented in Section 4 and in Section 5 data available for Mauritius and GA are applied to obtain the numerical solution of the problem. The results are discussed in Section 6.
2 Sugar/ethanol/electricity production model for Mauritius

Figure 1 is a schematic representation of the model of sugar/ethanol/electricity production from sugarcane for Mauritius, which we term as the TPC model. The sugar cane is harvested and transported to sugar cane mills. It is then milled, cane juice and bagasse, a residue of sugar cane are obtained. The sugar cane juice is processed to produce sugar. Molasses, a by-product of sugar cane juice are also obtained. The molasses are further processed to produce ethanol. The bagasse is burnt in boilers to produce steam which is then fed in condensing extraction steam turbine coupled with an alternator. Electricity and process heat are produced. The heat is used in the processing of: cane juice to raw sugar; molasses to anhydrous ethanol; and raw sugar to refined sugar. The ethanol produced will be either exported or blended with gasoline up to a proportion of 10% to be used as fuel in motor vehicles. Vinasse, a process residue obtained during the process of converting molasses to ethanol, could be then converted to fertilisers through a concentrated molasses solids (CMS) fertiliser blending plant. The fertilisers will be used for the cultivation of sugar cane.

Figure 1 Model of sugar/ethanol/electricity production from sugarcane for Mauritius

3 Formulation of the TPC model for producing sugar, ethanol and electricity

The TPC is composed of a production cost $C$ and energy required for production $E$. We denote the amount of sugar, ethanol and electricity produced by $X_1$, $X_2$ and $X_3$ respectively. We distribute the cost $C$ amongst sugar, ethanol and electricity as follows.
Cost for producing sugar = \( \frac{\alpha_1 X_1 C}{\alpha_1 + \alpha_2 + \alpha_3} \)

Cost for producing ethanol = \( \frac{\alpha_2 X_2 C}{\alpha_1 + \alpha_2 + \alpha_3} \)

and

Cost for producing electricity = \( \frac{\alpha_3 X_3 C}{\alpha_1 + \alpha_2 + \alpha_3} \)

where \( \alpha_i, i = 1, 2, 3 \), are time dependent coefficients and \( C \) is a known cost coefficient.

The energy required for producing sugar, ethanol and electricity are given by \( \beta_1 X_1, \beta_2 X_2 \) and \( \beta_3 X_3 \) respectively, where \( \beta_i, i = 1, 2, 3 \), are known coefficients, such that \( \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 = E \).

In this paper, we assume \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) to be prices of sugar, ethanol and electricity respectively. The TPC for producing sugar, ethanol and electricity at time \( t \) is therefore described by the following equation:

\[
\frac{dP_t}{P_t} = \left( \frac{P_1(t)X_1 + P_2(t)X_2 + P_3(t)X_3}{P_1(t) + P_2(t) + P_3(t)} \right) C + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3, \tag{1}
\]

where \( P_1(t), P_2(t) \) and \( P_3(t) \) are the prices of sugar, ethanol and electricity at time \( t \) respectively. The variation of price of sugar can be represented by the following stochastic differential equation (Oksendal 2003):

\[
dP_t(t) = (\gamma P_t(t))dt + \sigma P_t(t)dW_t(t), \tag{2}
\]

where \( \gamma \) is the drift rate (rate at which average price of sugar changes), \( \sigma \) is the constant volatility and \( W_t(t) \) is the standard Wiener process with zero mean and unit rate of variance.

By applying the Ito formula (Oksendal, 2003) and the Ito process \( Y(t) = g(t, P_t(t)) = e^{\gamma t} P_t(t) \) to equation (2), the following solution is obtained.

\[
R(t) = e^{\gamma t} P(t-1) + \int_{t-1}^{t} e^{\gamma (u-1)} \sigma R(u) dW(u). \tag{3}
\]

We approximate \( \int_{t-1}^{t} e^{\gamma (u-1)} \sigma R(u) dW(u) \) of equation (3) trapezium rule as follows (Oksendal, 2003; Kloeden et al., 1995):

\[
\int_{t-1}^{t} e^{\gamma (u-1)} \sigma R(u) dW(u) = \frac{1}{2} (\sigma R(t) + \sigma R(t-1))(W(t) - W(t-1)).
\]

The trend function of price is therefore given by

\[
R(t) = e^{\gamma t} \frac{1 + 0.5 \sigma R(t-1)}{1 - 0.5 \sigma R(t-1)} R(t-1). \tag{4}
\]

Similarly,
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\[ P_z(t) = e^{\sigma_2} \frac{1 + 0.5\sigma_2 (W_z(t) - W_z(t-1))}{1 - 0.5\sigma_2 (W_z(t) - W_z(t-1))} P_z(t-1), \tag{5} \]

and

\[ P_z(t) = e^{\sigma_3} \frac{1 + 0.5\sigma_3 (W_z(t) - W_z(t-1))}{1 - 0.5\sigma_3 (W_z(t) - W_z(t-1))} P_z(t-1), \tag{6} \]

where \( Y_2 \) is the rate at which average price of ethanol changes, \( Y_3 \) is the rate at which average price of electricity changes, \( \sigma_2 \) and \( \sigma_3 \) are constant volatilities and \( W_2 \) and \( W_3 \) are standard Wiener processes.

4 Two stage stochastic programming problem

Let \( X = [X_1, X_2, X_3]' \) and \( Y = [Y_1, Y_2, Y_3]' \). Our objective is to minimise the TPC given by equation (1). That is, we minimise

\[ F(t) = \sum_{i=1}^{3} \left[ \left( \frac{P_i(t)}{\bar{P}_i(t) + P_z(t) + P_3(t)} \right) C + \beta_i \right] X_i \tag{7} \]

subject to

\[ 0.1X_1 + 0.005955 X_2 + 0.0086 X_3 = 0.5 \times \text{amount of land under cane} \]

\[ \Pi = \{ X \in \mathbb{R}^3 : X_1 \geq 50,000, X_2 \geq 27 \text{ and } X_3 \geq 7,740 \}, \tag{8} \]

where \( P_i(t), i = 1, 2, 3 \), are given by equations (4), (5) and (6) respectively.

\( Y_i, i = 1, 2, 3 \) are the optimal values of the second stage problem given by

Minimise

\[ f_i = \frac{1}{m} \sum_{j=1}^{m} \left[ P_{i,\text{estimated}}(j) - P_{i,\text{exact}}(j) \right], i = 1, 2, 3 \tag{9} \]

\[ \Pi_i = \{ Y \in \mathbb{R}^3 \}, \]

where \( P_{i,\text{estimated}}(j) = e^{\sigma_i} \frac{1 + 0.5\sigma_i (W_i(t) - W_i(t-1))}{1 - 0.5\sigma_i (W_i(t) - W_i(t-1))} P_{i,\text{exact}}(j-1) \) and \( m \) is the number of observations.

The constraint given by (8) is obtained from Figure 1. From 1 tonne of cane, 50% are used to obtain 0.1 tonne of sugar, 7.5 litres (L) of ethanol and 100 kwh of electricity. The amount of ethanol is converted to tonnes as follows: The density of ethanol is 0.794 kg/L and hence, 1 L of ethanol is equivalent to 0.000794 tonnes. The amount of electricity is converted to tonne of oil equivalent (toe). 1 kwh is 0.000086 toe. Hence,

\[ 0.1X_1 + 0.005955 X_2 + 0.0086 X_3 = 0.5 \times \text{amount of land under cane}. \]
4.1 Analysis of the TPC model

Result 1: The sets \( \Pi_i, i = 1, 2, 3 \), are convex.

Proof: For any two elements \( x_1, x_2 \in \mathbb{R}^3 \), \( \alpha x_1 + (1 - 2) x_2 \in \mathbb{R}^3 \). Hence, the sets \( \Pi_i \) is convex by definition of a convex set (Boyd and Vandenberghe, 2004). \( \square \)

Result 2: Let \( \Pi_i, i = 1, 2, 3 \), be convex. Then, the functions \( f_1, f_2 \) and \( f_3 \), given by equation (9), are convex.

Proof. Since the exponential function \( f(x) = e^{\alpha x} \) is convex for any \( \alpha \in \mathbb{R} \) and \( \lambda f \) is convex for any scalar \( \lambda \geq 0 \) (Boyd and Vandenberghe, 2004), it follows that \( f(Y) \), \( i = 1, 2, 3 \) being sum of convex functions and non-decreasing, we deduce that \( f_1, f_2 \) and \( f_3 \) are therefore convex by Theorem 1. \( \square \)

Result 3: Let \( \Pi_i, i = 1, 2, 3 \), be convex. Then, \( f_1, f_2 \) and \( f_3 \) have a unique minimum in \( \Pi_1, \Pi_2 \) and \( \Pi_3 \) respectively.

Proof: Since \( \lim_{t \to \infty} Y_i = 1, 2, 3 \) being sum of convex functions and non-decreasing, we deduce that \( f_1, f_2 \) and \( f_3 \) are coercive. The Proof follows from Theorems 2 and 3 which are:

Theorem 2: If \( f: \mathbb{R}^n \to \mathbb{R} \) is continuous and coercive, then \( f \) has at least one global minimiser (Boyd and Vandenberghe, 2004).

Theorem 3: Let \( f: \mathbb{R}^n \to \mathbb{R} \) be convex. If \( \Pi \in \mathbb{R}^n \) is a local minimum for \( f \), then \( x \) is a global minimum for \( f \) (Boyd and Vandenberghe, 2004). \( \square \)

Result 4: The function \( F(t) \) as defined in (7) is convex.

Proof: At the end of the second stage programming, we obtain value for \( P_i(t) \), \( i = 1, 2, 3 \). These values are substituted in equation (7) to obtain the function \( F(X_1, X_2, X_3) \) in the following form: \( F(X_1, X_2, X_3) = \sum_{i=1}^{3} \gamma_i X_i \), where \( \frac{P_i(t)}{P_1(t) + P_2(t) + P_3(t)} C + \beta_i, i = 1, 2, 3 \). Thus, \( F(X_1, X_2, X_3) \) is a linear function in \( X_1, X_2 \) and \( X_3 \) and hence, convex on \( \Pi \). \( \square \)

Result 5: Let \( F \) be convex, \( F^* \) be the optimal value of problem 7. The set \( \Pi = \{ X \in \mathbb{R}^3: X_1 \geq 50,000, X_2 \geq 27 \text{ and } X_3 \geq 7,740 \} \) is non-empty, closed and convex. If \( X^* \) is the optimal set of problem 7, then \( X \) is non-empty, compact and convex.

Proof: The set \( C = \{ X \in \Pi: 0.1X_1 + 0.005955X_2 + 0.0086X_3 = 0.5 \times \text{amount of land under cane} \} \) is a hyperplane, which is a closed set. Since \( C \subset \Pi \) and \( \Pi \) is bounded, the set \( C \) is also bounded. Hence, \( C \) is compact. Since \( F \) is continuous and convex, by Weierstrass Theorem (Boyd and Vandenberghe, 2004), the optimal value \( F^* \) is finite and its optimal set \( X^* \) is non-empty. Since \( X^* = C \cap \{ X \in \mathbb{R}^3: F(X) \leq F^* \} \), \( X^* \) is also closed and
bounded. That is, $X^*$ is compact. $C$ is a hyperplane which is convex. The set \( \{ X \in \mathbb{R}^3 \mid F(X) \leq F^* \} \) is also convex by convexity of $F$. Hence, $X^*$ being intersection of two convex sets, is convex.

5 Numerical Solution of TPC model

GA is implemented in MATLAB® using the in-built function `ga.m` and different user-specified parameters such as population size, number of generations, selection function and crossover function to estimate the parameters $Y_1$, $Y_2$, $Y_3$, $X_1$, $X_2$ and $X_3$. The second stage problems are first solved based on the price data available for the year 2000 to 2009. The selection stochastic uniform and the crossover scattered yield lowest root mean square error with the following parameters, Population Size: 300, Number of Generations: 1,500, Crossover Probability: 0.8 and Mutation Probability: 0.02. The values of $Y_1$, $Y_2$ and $Y_3$ are found to be 0.0243, 0.0132 and 0.0402 respectively. Figure 2 shows the variation of prices of sugar, ethanol and electricity up to the year 2027.

![Figure 2: Prices of sugar, ethanol and electricity](see online version for colours)

We then use the values of $Y_1$, $Y_2$ and $Y_3$ to solve problem (7) using GA and the same user-specified parameters given above. The values of known constants are as follows: $C = 19,148$, $\beta_1 = 3,778$, $\beta_2 = 222$, $\beta_3 = 435$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.1$. In scenario 1, the amount of land available for sugarcane is assumed to be constant. In our study, we take it as 4,500,000 which is in fact the current surface area under sugarcane. Figure 3 shows the optimal values of sugar, ethanol and electricity to be produced and the total production up to the year 2027.
Figure 2 shows an increase in market price of sugar from 2015 to 2018. Our results from Figure 3 indicate that the optimal amount of sugar to be produced must increase from 2015 to 2016, decrease in 2017 and increase in 2018. The corresponding amount of ethanol decrease from 2,675 tonnes to 2,500 tonnes. The optimum production of electricity from sugarcane follows a similar trend to that of sugar. For the period of 2024 to 2027, we predict a rise in the market price of sugar. In order to minimise the TPC, the results of our experiments, as shown in Figure 3, indicate that the optimum amount of sugar produced must be increased progressively from 2024 to 2027. The optimum amount of ethanol to be produced will have to be increased to 2,775 tonnes by the end of 2025 and decreased progressively to 2,650 tonnes by the end of 2027. The optimum amount of electricity produced from cane must be decreased to 1.242 tonnes by the end of 2025, increased to 1.252 tonnes by the end of 2026 and decreased to 1.25 tonnes by December 2027. In general, we observe that whenever there is a rise in price of sugar, in most cases, it is more profitable to increase the production of sugar whilst decreasing the amount of ethanol produced. On the other hand, we see that even if the price of ethanol is high, we tend to decrease the production of ethanol so as to reduce TPC.

6 Scenarios and discussion

Figure 4 shows that from 1993 to 2012, there has been a general tendency to decrease the amount of land under sugarcane and correspondingly a decrease in the amount of cane harvested. It is therefore crucial for us to investigate how the optimum production of sugar, ethanol and electricity will vary when the amount of land under cane is constantly decreased to a certain value.
In scenario 2, we assume that allocation of land for sugarcane decreases by 2,000 annually from 2014 to 2027. Figure 5 shows the optimal values of sugar, ethanol and electricity to be produced and the total production up to the year 2027 under scenario 1 and scenario 2 super imposed. We can observe that the trend in the optimum value of each of the three by-products to be produced is the same. However, trivially, since the amount of cane harvested is constantly decreased we can observe that the corresponding optimum yearly value decreases accordingly.

In scenario 3, we assume that the price of sugar is 5% less or 5% more than the predicted one. Figure 6 shows the optimal values of sugar, ethanol and electricity to be produced and the total production up to the year 2027 under scenario 1 and scenario 3
superimposed. If the predicted price is 5% more than the predicted one, then we observe a rise in the TPC. We also see a decrease in the TPC if the predicted price is 5% less than the predicted one.

**Figure 6** Scenario 3: optimal values of sugar, ethanol and electricity and the total production when price of sugar decreases or decreases (see online version for colours)

**Figure 7** Scenario 4: optimal values of sugar, ethanol and electricity and the total production when price of ethanol decreases or increases (see online version for colours)
In scenario 4, we assume that the price of ethanol is 5% less or 5% more than the predicted one. Figure 7 shows the optimal values of sugar, ethanol and electricity to be produced and the total production up to the year 2027 under scenario 1 and scenario 4 superimposed. We observe that an increase of 5% in the prediction of price of ethanol will lead to a decrease in TPC. On the other hand, a 5% decrease in prediction of price of ethanol results in a rise in production cost.

7 Concluding remarks

In this work, we forecast the optimal amount of sugarcane by-products, namely sugar, ethanol and electricity to be produced with a minimum TPC for Mauritius for 2014–2027. We use a two-stage stochastic programming model which is based on variation of prices of sugar, ethanol and electricity and energy required to produce them. We show that the minimisation problem is convex, bounded and has a unique solution. We use ga.m in MATLAB in order to generate our results. We consider three scenarios which are the progressive decrease of land under sugarcane, increase in market price of sugar and ethanol and decrease in market price of sugar and ethanol. Our numerical experiments give the corresponding optimal amount of sugar, ethanol and electricity to be produced each year from 2014 to 2028. This model can be used as a policy decision mechanism in the economic planning of the country.

References


