A model of duopolistic patent contest with private provisions of industry collective goods

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Abstract: This paper studies a duopolistic patent race model where the contestants are competing in a patent race and at the same time, they are both providing an industry collective good for enhancement of the prize of the patent race. The model finds out that innovation effort peaks when the contestants have equal research and development cost efficiency. Furthermore, greater returns to scale in innovation increase innovation efforts when contestants are about equal in their innovation cost efficiency and decrease innovation efforts when contestants are very unequal in their innovation cost efficiency. Finally, the paper finds out that an increase in the supply of the industry collective good by the other contestant induces a contestant to put in a higher innovation effort. The paper suggests that public policy should aim to promote equal innovation cost efficiency between leading research and development entities to maximise innovation efforts.

Keywords: duopoly; innovation; patent race; returns to scale; rent seeking; public intermediate input; industry collective goods; asymmetry in efficiency; innovation cost; two-dimensional effort.

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1 Introduction

This paper investigates a duopolistic model of patent race with two dimension interactions. The two contestants engage in a research and development innovation contest for winning the patent. At the same time, each of them provides for an industry collective good which enhances the value of the prize of the patent race.

What are the reasons for such a two-dimensional effort model? First, in today's world, research and development is done within innovation systems instead of being done by any company in isolation. The different innovators in these systems are mutually supportive. Moreover, an innovation ecosystem requires a core business or a group of core businesses with resources and scale to provide technological leadership and direction. A large core company invests in key technologies in an innovation ecosystem and creates a stable platform that enhances the innovation environment for other companies. Therefore, the large core company both engages in innovation race against other large companies and at the same time, provides the industry collective good that enhances the innovation environment for all companies.

The industry collective good could also be a better set of legal rules for the industry. Better legal rules benefit all firms in the industry by lowering transaction costs and uncertainty. Legal rules could also benefit the industry by capturing more rents for the industry at the expense of the public. Legal effort could affect the prize value by extending the period or region where the patent rights are valid and thereby increases the winner's prospective returns.

The industry collective good could be public intermediate input for the community of contestants engaging in the patent and research race. Examples of such industry collective goods include setting of standards and adoption of common specifications, better public awareness and acceptance of the industry products or services and efforts to secure better supply of common input and shared technology.

The reason for a duopolistic model is that in today's world, large corporations and research entities dominate the landscape of research and development. Scale is a plus for innovation. For instance, Hernandez-Villafuerte et al. (2017) surveyed the literature on economies of scale and scope in health related research and development. They found that at the level of universities or research institutes, studies often pointed to positive economies of scale in biomedical and health research.

There are further reasons for a duopolistic model. In the contemporary globalised world, a company that is considered domestically large could be considered globally small. Companies need the resources to compete with foreign companies to reap the results of innovation. In addition, many of these foreign firms could be supported by the state. Empirical evidence showed that big companies in the US spent much more on research and development per worker in the post-WWII era than small firms on average (Mandel, 2011). Big firms also contributed the most important innovations. Nobel Prize distribution is a good indicator. Nobel Prize winners in the Post-WWII era were usually from the largest universities or the largest firms.

Finally, in the contemporary world, large-scale integrated systems provide many important services. These services include healthcare, education and energy. The reforms of these large-scale integrated systems are gigantic tasks and challenges to companies and societies. These systems have constituencies that are deeply rooted. Many of these constituencies resist change. Great efforts are needed to change and improve technological innovations. If a company wants to innovate in these industries, then it is usually necessary to simultaneously change multiple parts of the system. Small businesses and even small and weak states do not have the resources to innovate on such a large scale. Only a powerful state or a big business has the ability to do that.

The current innovation literature has a theory that argues that there is an inverted U relationship between competition and innovation. That is to say, at a low level of competition, rising competition increases innovation but, at a high level of competition, rising competition decreases innovation. Furthermore, the theory argues that between monopoly and perfect competition lies the ideal degree of rivalry. Kamien and Schwartz (1976) were among the earliest proponents of the inverted-U relationship between competition and innovation. The theory has empirical supports. Aghion et al. (2005), for example, confirmed the relationship with British data.

Given the dominance and advantages that large corporates and research entities have in innovation race over the smaller ones in today's world, the relevant question is not whether monopoly or atomistic competition (which includes perfect competition and monopolistic competition) is more conducive to innovation. The relevant question is whether monopoly or oligopoly is more conducive to innovation.

The history of patent race and innovation in the light bulb industry in US is a good illustration. The market structure of the industry evolved from competition to oligopoly to duopoly and finally, monopoly. Bright (1949) and Shaver (2012) gave detailed treatments of the history of the patent race of the light bulb industry. The race to invent a commercially viable light bulb started well before Edison's work began. However, before the entry of Edison to the race, the investment in pursuit of the – incandescent light bulb was quite limited. In US, the leading contestant was the Sawyer-Man team. The team had a research budget of $4000 between March and June 1878. The participation of Edison greatly increased the fervour and the scale of the race. Edison began to work on the light bulb in 1878. He led a team of engineers. Edison brought forth a far larger budget. He put about $130,000 into the research of electric lighting between 1878 and 1881. The entry of Edison attracted significant new investments to his own team. The entry also spurred increased investments to his rivals. In response to the entry of Edison, the Sawyer-Man team established a second workshop.

The Edison team produced one of the first practical systems of interior lighting. They however faced very intense competition in the laboratory and the business world. There were many inventors and prior patents. The patent race carried on after the entry of

Edison. Many parties filed important patents in US and other countries. In September 1882, Edison began making large-scale electric light service in the city of New York. At that point, several other firms also mass-produced electric light bulbs and wielded their own patent portfolios. The main competitor of Edison in the legal and innovation fronts was the team of William Edward Sawyer and Albon Man. By 1890, Edison's market share had fallen to less than half.

Edison won major litigations involving his portfolio of patents on electric light technology and built on his patent portfolio and courtroom successes to obtain mergers or market sharing agreements with most of his rivals. Finally, the two strongest competitors, the Edison and Westinghouse companies, faced off each other. The confrontation was the litigation that reached the Supreme Court in the Incandescent Lamp Patent Case. The market structure changed dramatically when a court in Pennsylvania gave Edison's patent a new interpretation. The interpretation was so broad that it rendered virtually every incandescent light bulb infringing upon the patent. The court did so when invalidating a rival patent held by Sawyer and Man. After the 1895 decision of the Supreme Court, all US light bulb producers either purchased licences from General Electric or merged with it.

General Electric was the company that acquired the light bulb related patents of Edison. By 1897, General Electric became the obvious leader in the industry. Westinghouse was its only serious rival. The two companies finally reached a truce through patent cross-licensing and product pricing agreements. General Electric organised the Incandescent Lamp Manufacturers Association and invited smaller competitors to join. Through measures such as incorporation, patent litigation, licensing deals, and purchases of stock, G.E. achieved a 97% control of the market share by 1910. In sum, after the court rulings that gave General Electric overwhelming patent rights, the competitive market structure of the light bulb industry collapsed first into a duopoly and finally a monopoly. Consequently, industry innovations slowed down.

2 Literature review

Schumpeter (1942) and Arrow (1962) initiated the research on the market structure and innovation relationship. Schumpeter (1942) proposed that monopoly is more conducive to innovation than competition since monopoly profit provides incentive for investment and risk taking in research and development. On the other hand, Arrow (1962) argued that competition spurs innovation. To Arrow (1962), a monopoly has fewer incentives to innovate as innovations will cause existing products to be replaced.

After Schumpeter (1942), economists developed the "Schumpeterian Hypothesis". The hypothesis says that larger firms have more incentives to innovate than smaller firms as larger firms have larger sales. Furthermore, the hypothesis argues that firms with more market power are more willing to invest in innovation.

Several patterns were suggested for the competition-innovation relationship. Kamien and Schwartz (1976) proposed that the optimal degree of rivalry lies between the extremes of monopoly and perfect competition. Their investigation found that stronger competition in an industry will first elicit a higher research and development investment by a firm. However, it will eventually cause the intensity of innovative activity to decline. Using UK data, Aghion et al. (2005) showed that there is an inverted-U relationship between competition and innovation on the product market.

Loury (1979) investigated if there is an optimal degree of competition that maximises R&D performance. Loury (1979) discovered that increased competition reduces individual investments of firms as competition diminishes the probability of success and therefore the expected benefits of investments. However, even though the investment of an individual firm decreases, the aggregate investments of all firms increase due to the larger number of firms existing in the market. The outcome is faster innovation. Therefore, Loury (1979) argued that competition increases innovative activity.

Futia (1980) discovered that the total innovation investment of an industry might be negatively related to the number of firms when there is fast and full imitation. Spence (1984) discovered that when there is knowledge spillovers, aggregate investment will be inversely related to the number of firms.

Reinganum (1984) found that research and development investments decrease with competition for the case of fixed cost but increase with competition in a setting with variable cost. Furthermore, an increase in aggregate investment will reduce individual investment in the case of a fixed cost but increase individual investment for the flow cost setting. Reinganum (1984) pointed out that there is an inverse relationship between the magnitude of an innovation and the likelihood that it is invented by a current industry leader, independent of the type of cost, fixed or flow. An industry leader with a current advantage over the other firms has an incentive to protect his current profits, but not to innovate. An innovation with technological uncertainty might lower the expected future profits of the leader.

Harris and Vickers (1985) studied an asymmetric model of patent race. Asymmetry might exist if the two firms value the patent differently. First, as the incumbent wants to keep her monopoly position, she is more likely to value the patent higher than a challenger. Second, the incumbent has more knowledge due to her market experience. Finally, firms could have different R&D efficiency. Harris and Vickers (1985) found that the incumbent enjoys significant asymmetric advantages, and she will strive to keep her advantages.

Grossman and Shapiro (1987) studied changes in the innovation effort of firms during the process of a patent race. They discovered that competition is most intense when both firms are even and each has completed the initial phase of the research project. Moreover, when a firm behind closes up with a leading competitor, both firms intensify their innovation efforts. Besides, when the two firms are at different stages in the innovation process, the one that is ahead has a greater incentive to invest in research and development than the one that is behind.

Weeds (2002) observed that in a patent system, a patent holder receives the exclusive rights for the patented product for a specified period of time. Therefore, firms invest in research and development and file patents to secure expected future benefits. When several firms contest the same invention, the innovation race is then effectively a tournament or rent seeking contest. Baye and Hoppe (2003) established the strategic equivalence of rent seeking and patent race in single dimensional effort framework. Faria et al. (2014) studied a patent race with both research and legal efforts and analysed the symmetrical equilibrium. Faria et al. (2014) established the existence of a strategic equivalence between rent seeking and patent race in the two-dimensional effort context.

Tullock (1980) and Hirshleifer (1988, 1989, 1991, 1995) pioneered the formal modelling of rent seeking contest. In their models, the ratio of efforts between the contestants decides the probability of victory between them or their respective share of contested prize captured and secured. In the rent seeking literature, Clark and Konrad

(2007) began the examination of the influence of two or more types of effort on the prize values. They extended the single dimension contests framework into multiple dimension contests. Epstein et al. (2008) studied a rent seeking contest whereby both the rent seekers and the recipients of rent seeking efforts engage in rent seeking contests. They concluded that existing theories and empirical works underestimate the extent of rent dissipation.

This paper studies firms simultaneously engaging in a patent race and providing for an industry collective good (or public good). Samuelson (1954) first proposed the concept of public goods. The two key characteristics of a public good are non-rivalry and non-excludability in consumption. Olson (1965) applied the concept of public goods to explain how groups undertake or fail to undertake collective action for their group collective interest. Olson (1982) explained the economic performance of nations using the concept of public goods, collective interests and rent seeking. Economic performance suffers when interest groups rent seeking undermine the collective interests of the whole nation. Tamai (2018) studied dynamic private supply of public goods under certainty and found that uncertainty increases donations in the short run but decreases donations in the long run.

After Olson (1982), the two strands of literature, public goods and rent seeking, converged. Ursprung (1990) investigated rent seeking for a public good in a model of political competition. He found that there is substantial under dissipation of the public-good rent that the groups contest. Gradstein (1993) argued that while the government can overcome the free-rider problem in public goods provision by compelling payment, the government could also be lobbied by rent seeking efforts. These rent seeking efforts waste resources and do not result in first-best provision. Baik (1993) studied rent seeking for a public good with groups composed of people with different valuations of a public good and finds out that there is low rent dissipation. Hillman and Long (2019) investigated the theoretical issues of measuring social cost when there are rent seeking for contestable benefits. Dickson et al. (2018) found that rent seeking efforts could either decrease or increase when the stake dwindles.

3 The model

This paper builds on the above strands of literature to analyse how the returns to scale in research and development interacts with asymmetry in innovation cost efficiency between contestants to affect innovation efforts in a patent race game which the contestants also contribute efforts towards the collective good of the industry.

In the model of this paper, the variation in innovation cost efficiency between the two contestants decides how close the market structure is to monopoly or duopoly (which is a special form of oligopoly). When the two competitors have equal innovation cost efficiency, the market structure is a symmetrical duopoly. As one of the competitor becomes more cost-efficient in innovation than the other, the duopoly becomes more asymmetrical and becomes more like monopoly. In the extreme case when one competitor is infinitely more cost-efficient in innovation than the other, the market structure becomes a monopoly.

The two players in the model are Firm 1 and Firm 2. The firms are competing to innovate to produce a new kind of private good for the market. An example is a new kind of medicine for treating some kind of disease. The industry, however, also shares an industry collective good which increases the value of the innovation contest. For instance, the collective good could be a public education campaign to provide awareness about the disease and its treatment and thereby increase demand for the new medicine.

The expected profit functions of Firm 1 and 2 are

$$\pi_1(X_1, X_2, L_1, L_2) = v(L_1, L_2)\, p(X_1, X_2) - c_1(X_1, L_1) \tag{1}$$

$$\pi_2(X_1, X_2, L_1, L_2) = v(L_1, L_2)\big(1 - p(X_1, X_2)\big) - c_2(X_2, L_2) \tag{2}$$

$v(L_1, L_2)$ is the prize of the contest. Firm 1 and 2 could raise the value of the prize by supplying the industry collective good. L_1 is the industry collective good provision of Firm 1 and L_2 is the industry collective good contribution of Firm 2. The prize relates to the provision effort of the two firms linearly:

$$v(L_1, L_2) = w(s + L_1 + L_2) \tag{3}$$

w is the increase in the value of the prize given a unit increase in the provision of the industry collective good. s is the supply of the industry collective good by parties other than the two firms competing in the patent race. The industry collective good is non-rivalrous and non-excludable in consumption and produces benefits for all parties in the industry.

Firm 1's probability of winning the patent contest is $p(X_1, X_2)$ and Firm 2's probability of winning is $\big(1 - p(X_1, X_2)\big)$. $p(X_1, X_2)$ is specified as:

$$p(X_1, X_2) = \frac{X_1^{\,m}}{X_1^{\,m} + X_2^{\,m}} \tag{4}$$

The specification uses the Tullock-Hirshleifer ratio form contest function. X_1 is the research and development expenditures of Firm 1 and X_2 is the research and development expenditures of Firm 2. m is the returns to scale in innovation or research and development. A greater returns to scale in innovation increases the probability of winning of the contestant with greater effort.

The cost function of Firm 1 is $c_1(X_1, L_1)$ and the cost function of Firm 2 is $c_2(X_2, L_2)$:

$$c_1(X_1, L_1) = a_1 X_1 + \frac{L_1^2}{2} \tag{5}$$

$$c_2(X_2, L_2) = a_2 X_2 + \frac{L_2^2}{2} \tag{6}$$

a_1 is the innovation cost efficiency parameter of Firm 1 and a_2 is the innovation cost efficiency parameter of Firm 2. $\frac{L_1^2}{2}$ is the spending on the industry collective good by Firm 1 and $\frac{L_2^2}{2}$ is the spending on the industry collective good by Firm 2.

Substituting the above equations into the expected profit functions, Firm 1 solves

$$\max_{X_1, L_1} \pi_1(X_1, X_2, L_1, L_2) = w(s + L_1 + L_2)\left(\frac{X_1^{\,m}}{X_1^{\,m} + X_2^{\,m}}\right) - a_1 X_1 - \frac{L_1^2}{2} \tag{7}$$

Similarly, Firm 2 optimises

$$\max_{X_2, L_2} \pi_2\left(X_1, X_2, L_1, L_2\right) = w\left(s + L_1 + L_2\right)\left(\frac{X_2^{\ m}}{X_1^{\ m} + X_2^{\ m}}\right) - a_2 X_2 - \frac{L_2^2}{2} \quad (8)$$

For notational simplicity, this paper writes $p\left(X_1, X_2\right)$ as p.

$$\frac{\partial \pi_1}{\partial X_1} = w\left(s + L_1 + L_2\right) mp(1-p) X_1^{-1} - a_1 = 0 \quad (9)$$

$$\frac{\partial \pi_1}{\partial L_1} = wp - L_1 = 0 \quad (10)$$

Equation 9 and 10 are the first order conditions of Firm 1's optimisation.

$$\frac{\partial \pi_2}{\partial X_2} = w\left(s + L_1 + L_2\right) mp(1-p) X_2^{-1} - a_2 = 0 \quad (11)$$

$$\frac{\partial \pi_2}{\partial L_2} = w\left(1 - p\right) - L_2 = 0 \quad (12)$$

Equations (11) and (12) are the first order conditions of Firm 2's maximisation.

Firm 1 and Firm 2 equate the marginal returns from investing in research and development to win the patent race with the marginal costs of engaging in research and development. Firm 1 and Firm 2 also equate the marginal returns of provisions of the industry collective good with the marginal costs of the provision.

$$\frac{\partial^2 \pi_1}{\partial X_1^2} = \left[-1 + m\left(1 - 2p\right)\right] w\left(s + L_1 + L_2\right) mp(1-p) X_1^{-2} < 0 \quad (13)$$

$$\frac{\partial^2 \pi_1}{\partial L_1^2} = -1 < 0 \quad (14)$$

$$\frac{\partial^2 \pi_1}{\partial X_1 \partial L_1} = \frac{\partial^2 \pi_1}{\partial L_1 \partial X_1} = wmp(1-p) X_1^{-1} > 0 \quad (15)$$

$$\frac{\partial^2 \pi_1}{\partial X_1^2} \frac{\partial^2 \pi_1}{\partial L_1^2} - \frac{\partial^2 \pi_1}{\partial X_1 \partial L_1} \frac{\partial^2 \pi_1}{\partial L_1 \partial X_1} > 0 \quad (16)$$

Equations (13)–(16) are the second order conditions for the maximisation problem of Firm 1.

$$\frac{\partial^2 \pi_1}{\partial X_2^2} = \left[-1 + m\left(1 - 2p\right)\right] w\left(s + L_1 + L_2\right) mp(1-p) X_2^{-2} < 0 \quad (17)$$

$$\frac{\partial^2 \pi_2}{\partial L_2^2} = -1 < 0 \quad (18)$$

$$\frac{\partial^2 \pi_2}{\partial X_2 \partial L_2} = \frac{\partial^2 \pi_2}{\partial L_2 \partial X_2} = wmp(1-p)X_2^{-1} > 0 \tag{19}$$

$$\frac{\partial^2 \pi_2}{\partial X_2^2}\frac{\partial^2 \pi_2}{\partial L_2^2} - \frac{\partial^2 \pi_2}{\partial X_2 \partial L_2}\frac{\partial^2 \pi_2}{\partial L_2 \partial X_2} > 0 \tag{20}$$

Equations (17)–(20) are the second order conditions of Firm 2's optimisation.

Equations (9) and (11) give

$$\frac{X_1}{X_2} = \frac{a_2}{a_1} \tag{21}$$

The ratio of the innovation effort of Firm 1 to that of Firm 2 equals the ratio of innovation cost efficiency of Firm 1 to that of Firm 2.

The probability that Firm 1 will win the patent contest is

$$p = \frac{a_2{}^m}{a_1{}^m + a_2{}^m} = \frac{R^m}{1+R^m} \tag{22}$$

$R \equiv \frac{a_2}{a_1}$ is the relative innovation cost efficiency of Firm 1 and 2. The contestant which is more cost-efficient in innovation puts in greater innovation effort relative to the rival.

In the Nash equilibrium, the provisions of the industry collective good, the innovation efforts and the profit levels of the two contestants are:

$$L_1 = wp \tag{23}$$

$$L_2 = w(1-p) \tag{24}$$

$$X_1 = w(s+w)\frac{m}{a_1}p(1-p) \tag{25}$$

$$X_2 = w(s+w)\frac{m}{a_2}p(1-p) \tag{26}$$

$$\pi_1 = wp\left[(s+w)\left(1-m(1-p)\right) - \frac{wp}{2}\right] \tag{27}$$

$$\pi_2 = w(1-p)\left[(s+w)(1-mp) - \frac{w(1-p)}{2}\right] \tag{28}$$

For comparative static analysis, the Jacobian is

$$|J| = \begin{vmatrix} \frac{\partial^2 \pi_1}{\partial X_1^2} & \frac{\partial^2 \pi_1}{\partial X_1 \partial L_1} \\ \frac{\partial^2 \pi_1}{\partial L_1 \partial X_1} & \frac{\partial^2 \pi_1}{\partial L_1^2} \end{vmatrix} = \frac{\partial^2 \pi_1}{\partial X_1^2}\frac{\partial^2 \pi_1}{\partial L_1^2} - \frac{\partial^2 \pi_1}{\partial X_1 \partial L_1}\frac{\partial^2 \pi_1}{\partial L_1 \partial X_1} > 0 \tag{29}$$

To analyse the effect of the industry collective good provision of rival on a contestant's provision of industry collective good and research and development effort, the following are derived:

$$\frac{\partial^2 \pi_1}{\partial X_1 \partial L_2} = wmp(1-p)X_1^{-1} > 0 \tag{30}$$

$$\frac{\partial^2 \pi_1}{\partial L_1 \partial L_2} = 0 \tag{31}$$

An increase in the rival's provision of industry collective good increases a contestant's provision of the industry collective good.

$$\frac{\partial L_1}{\partial L_2} = \frac{\partial L_1}{\partial L_2} + \frac{\partial L_1}{\partial X_1}\frac{\partial X_1}{\partial L_2} = \frac{\begin{vmatrix} \dfrac{\partial^2 \pi_1}{\partial X_1^2} & -\dfrac{\partial^2 \pi_1}{\partial X_1 \partial L_2} \\ \dfrac{\partial^2 \pi_1}{\partial L_1 \partial X_1} & -\dfrac{\partial^2 \pi_1}{\partial L_1 \partial L_2} \end{vmatrix}}{|J|} = \frac{1}{|J|}\left(\frac{\partial^2 \pi_1}{\partial L_1 \partial X_1}\frac{\partial^2 \pi_1}{\partial X_1 \partial L_2}\right) > 0 \tag{32}$$

The first term is the direct effect and is zero. The second term is the indirect effect and is positive.

An increase in the rival's provision of industry collective good increases a contestant's innovation effort.

$$\frac{\partial X_1}{\partial L_2} = \frac{\partial X_1}{\partial L_2} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial L_2} = \frac{\begin{vmatrix} -\dfrac{\partial^2 \pi_1}{\partial X_1 \partial L_2} & \dfrac{\partial^2 \pi_1}{\partial X_1 \partial L_1} \\ -\dfrac{\partial^2 \pi_1}{\partial L_1 \partial L_2} & \dfrac{\partial^2 \pi_1}{\partial L_1^2} \end{vmatrix}}{|J|} = \frac{1}{|J|}\left(-\frac{\partial^2 \pi_1}{\partial X_1 \partial L_2}\frac{\partial^2 \pi_1}{\partial L_1^2}\right) > 0 \tag{33}$$

The first term is the direct effect and is positive. The second term is the indirect effect and is zero.

The direct effect could be derived using the implication function rule which gives the reaction function of the research and development effort of Firm 1 with respect to the industry collective good provision of Firm 2:

$$\frac{\partial X_1}{\partial L_2} = -\frac{\dfrac{\partial^2 \pi_1}{\partial X_1 \partial L_2}}{\dfrac{\partial^2 \pi_1}{\partial X_1^2}} = \frac{X_1}{\left[1-m(1-2p)\right](s+L_1+L_2)} > 0 \tag{34}$$

To study how an increase in the rival's innovation effort affects a contestant's provision of industry collective good and innovation effort, the following are derived:

$$\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2} = -w(s+L_1+L_2)m^2 X_1^{-1} X_2^{-1} p(1-p)(1-2p) \tag{35}$$

$$\frac{\partial^2 \pi_1}{\partial L_1 \partial X_2} = -wmp(1-p)X_2^{-1} < 0 \tag{36}$$

Equation (37) tells how an increase in the rival's innovation effort affects a contestant's innovation effort:

$$\frac{\partial X_1}{\partial X_2} = \frac{\partial X_1}{\partial X_2} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial X_2} = \frac{\begin{vmatrix} -\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2} & \frac{\partial^2 \pi_1}{\partial X_1 \partial L_1} \\ -\frac{\partial^2 \pi_1}{\partial L_1 \partial X_2} & \frac{\partial^2 \pi_1}{\partial L_1^2} \end{vmatrix}}{|J|} = \frac{-\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2}\frac{\partial^2 \pi_1}{\partial L_1^2}}{|J|} + \frac{\left(\frac{\partial^2 \pi_1}{\partial L_1 \partial X_2}\frac{\partial^2 \pi_1}{\partial X_1 \partial L_1}\right)}{|J|} \tag{37}$$

The first term is the direct effect and its sign is the same as that of $(1-2p)$. The second term is the indirect effect through the size of Firm 1's industry collective provision and it is negative.

The first term direct effect gives the reaction function $X_1(X_2)$ which could be derived through applying the implicit function theorem on the first order condition of Firm 1 with respect to X_1.

$$\frac{\partial X_1}{\partial X_2} = -\frac{\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2}}{\frac{\partial^2 \pi_1}{\partial X_1^2}} = -\frac{m(1-2p)X_1}{\left[1-m(1-2p)\right]X_2} \tag{38}$$

Note that $\frac{\partial X_1}{\partial X_2} > 0$ when $X_1 > X_2$, $\frac{\partial X_1}{\partial X_2} = 0$ when $X_1 = X_2$ and $\frac{\partial X_1}{\partial X_2} < 0$ when $X_1 < X_2$.

Similarly, the derivation of the slope of the reaction function $X_2(X_1)$ uses the implicit function theorem

$$\frac{\partial^2 \pi_2}{\partial X_2 \partial X_1} = -w(s+L_1+L_2)m^2 X_1^{-1} X_2^{-1} p(1-P)(1-2p) = 0 \tag{39}$$

$$\frac{\partial X_2}{\partial X_1} = -\frac{\frac{\partial^2 \pi_2}{\partial X_2 \partial X_1}}{\frac{\partial^2 \pi_2}{\partial X_2^2}} = -\frac{m(1-2p)X_2}{\left[1-m(1-2p)\right]X_1} \tag{40}$$

$\frac{\partial X_2}{\partial X_1} > 0$ when $X_2 > X_1$, $\frac{\partial X_2}{\partial X_1} = 0$ when $X_2 = X_1$ and $\frac{\partial X_2}{\partial X_1} < 0$ when $X_2 < X_1$.

For local strategic stability, the requirement is

$$\left|\frac{\partial X_1}{\partial X_2}\frac{\partial X_2}{\partial X_1}\right| = \frac{\left[m(1-2p)\right]^2}{\left[1-m(1-2p)\right]^2} < 1 \tag{41}$$

The above is satisfied for $m(1-2p) < \frac{1}{2}$.[1]

Equation (42) shows how the innovation effort of Firm 2 affects Firm 1's provision of the industry collective good.

$$\frac{\partial L_1}{\partial X_2} = \frac{\partial L_1}{\partial X_2} + \frac{\partial L_1}{\partial X_1}\frac{\partial X_1}{\partial X_2} = \frac{\begin{vmatrix} \frac{\partial^2 \pi_1}{\partial X_1^2} & -\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi_1}{\partial L_1 \partial X_1} & -\frac{\partial^2 \pi_1}{\partial L_1 \partial X_2} \end{vmatrix}}{|J|} \tag{42}$$

$$= \frac{1}{|J|}\left(-\frac{\partial^2 \pi_1}{\partial X_1^2}\frac{\partial^2 \pi_1}{\partial L_1 \partial X_2} + \frac{\partial^2 \pi_1}{\partial L_1 \partial X_1}\frac{\partial^2 \pi_1}{\partial X_1 \partial X_2} \right)$$

The first term is the direct effect and is negative. The second term is the indirect effect and its sign is opposite to the sign of $(1-2p)$.

4 Comparative statics

The analysis focuses on Firm 1.

$$\frac{\partial X_1}{\partial s} = w\frac{m}{a_1}p(1-p) > 0 \tag{43}$$

Proposition 1: *An increase in the industry collective good provision by other parties raises the innovation effort of Firm 1.*

Proof. Equations (33), (34) and (43) give the result.

Q. E. D.

An increase in the provisions of the industry collective good by other parties raises a contestant's provision of the industry collective good and a contestant's profit.

$$\frac{\partial L_1}{\partial s} = 0 \tag{44}$$

$$\frac{\partial \pi_1}{\partial s} = wp > 0 \tag{45}$$

(By Envelope Theorem)

$$\frac{\partial X_1}{\partial w} = (s+2w)\frac{m}{a_1}p(1-p) > 0 \tag{46}$$

An increase in the value of the industry collective good raises the innovation effort of Firm 1.

$$\frac{\partial L_1}{\partial w} = p > 0 \tag{47}$$

An increase in the value of the industry collective good increases the industry collective good provision of Firm 1

$$\frac{\partial \pi_1}{\partial w} = \left(s + L_1 + L_2\right) p > 0 \tag{48}$$

(By Envelope Theorem)

An increase in the value of the industry collective good increases the profit level of Firm 1

$$\frac{\partial L_1}{\partial a_1} = \frac{\partial L_1}{\partial p}\frac{\partial p}{\partial R}\frac{\partial R}{\partial a_1} < 0 \tag{49}$$

An increase in the innovation cost efficiency of Firm 1 increases the provisions of industry collective good by Firm 1.

$$\frac{\partial \pi_1}{\partial a_1} = -X_1 < 0 \tag{50}$$

(By Envelope Theorem)

An increase in the innovation cost efficiency of Firm 1 increases the profit level of Firm 1.

$$\frac{\partial X_1}{\partial a_1} = \frac{\partial X_1}{\partial a_1} + \frac{\partial X_1}{\partial p}\frac{\partial p}{\partial R}\frac{\partial R}{\partial a_1} < 0 \tag{51}$$

An increase in innovation cost efficiency has two effects on research effort. One is the direct effect which is positive. The other is the indirect effect which acts through relative innovation cost efficiency and the probability of winning the contest.

Please note that

$$\frac{\partial p}{\partial R} = mp\left(1 - p\right) R^{-1} > 0 \tag{52}$$

An increase in relative innovation cost efficiency raises the chance of winning the contest.

$$\frac{\partial L_1}{\partial R} = w\frac{\partial p}{\partial R} > 0 \tag{53}$$

Greater relative innovation cost efficiency raises the industry collective good provision of the firm.

$$\frac{\partial \pi_1}{\partial R} = w\left(s + L_1 + L_2\right)\frac{\partial p}{\partial R} > 0 \tag{54}$$

Greater relative innovation cost efficiency raises the profit level of the firm.

Proposition 2: *Innovation effort peaks when the contestants are equal in innovation cost efficiency.*

Proof.

$$\frac{\partial X_1}{\partial R} = w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial R} \tag{55}$$

Equation (55) is positive for $R<1$, zero for $R=1$, and negative for $R>1$. In other words, X_1 peaks at $p = 1/2$.

Q. E. D.

Proposition 2 echoes the finding of Aoki (1991) that innovation efforts are maximised when firms are equally effective in research and development.

$$\frac{\partial p}{\partial m} = p(1-p)\ln R \tag{56}$$

$\frac{\partial p}{\partial m} > 0$ for $R > 1$, $\frac{\partial p}{\partial m} = 0$ for $R = 0$ and $\frac{\partial p}{\partial m} < 0$ for $R < 1$. Greater returns to scale in innovation increase the winning chance of the firm more cost-efficient in innovation and decrease the winning chance of the firm less cost-efficient in innovation.

$$\frac{\partial \pi_1}{\partial m} = w(s+L_1+L_2)\frac{\partial p}{\partial m} \tag{57}$$

$$\frac{\partial L_1}{\partial m} = \frac{\partial L_1}{\partial p}\frac{\partial p}{\partial m} \tag{58}$$

Greater returns to scale in innovation raise the profits and industry collective good provision effort of the firm more cost-efficient in innovation and reduce the profits and industry collective good provision effort of the firm less cost-efficient in innovation.

Proposition 3: *Greater returns to scale in innovation increase innovation effort when contestants are closely equal in innovation cost efficiency and decrease innovation effort when contestants are greatly unequal in innovation cost efficiency.*

Proof.

$$X_1 = w(s+w)\frac{m}{a_1}p(1-p) \tag{59}$$

$$\begin{aligned}\frac{\partial X_1}{\partial m} &= \frac{\partial X_1}{\partial m} + \frac{\partial X_1}{\partial p}\frac{\partial p}{\partial m} \\ &= w(s+w)\frac{1}{a_1}p(1-p) + w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}\end{aligned} \tag{60}$$

In equation (60), $w(s+w)\frac{1}{a_1}p(1-p)$ and $w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}$ at $R=1$ and $w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}$ becomes increasingly negative as R deviates away from zero. When $R<1$, $(1-2p)>0$ and $\frac{\partial p}{\partial m}<0$, $w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}<0$; when $R=1$, $(1-2p)=0$ and $\frac{\partial p}{\partial m}=0$, $w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}=0$; when $R>1$, $(1-2p)<0$ and $\frac{\partial p}{\partial m}>0$, $w(s+w)\frac{m}{a_1}(1-2p)\frac{\partial p}{\partial m}<0$.

Q. E. D.

An increase in the returns to scale in innovation has two effects on the marginal influence of relative innovation cost efficiency on probability of victory. One is the direct effect and the other is the indirect effect. By the direct effect, a higher level of returns to scale causes both contestants to increase innovation effort as size confers greater advantage. By the indirect effect, the more cost-efficient contestant becomes more powerful and the less efficient player weaker given the greater returns to scale. According to Proposition 2, that dampens the contest and makes both players put in less innovation effort. The greater the asymmetry in innovation cost efficiency, the greater the indirect effect. The indirect effect is zero if the two contestants have equal innovation cost efficiency. Therefore, if the contestants are close in innovation cost efficiency, then greater returns to scale in innovation increase innovation efforts since the direct effect dominates the indirect effect. On the other hand, if the contestants are far apart in innovation cost efficiency, then greater returns to scale in innovation reduce innovation efforts since the indirect effect dominates the indirect effect.

5 Conclusions and policy implications

Propositions (1)–(3) are the key results of the paper. They have significant public policy implications. One key question is whether the industry collective good is a public good to the general society or is it a rent seeking activity? If it is a public good to the general society, then under Nash equilibrium, the competing firms underprovide the public good. The policy proposal is for public authority to adopt measures that increase the provision of the public good. Besides the conventional consideration of underprovision by private entities in comparison to the Samuelsonian optimal provision of public good, this paper argues that there is an additional reason to increase the provision of the public good. That reason is that with a greater provision of the industry collective good, there would be greater research and development efforts by the firms in the patent race.

If the industry collective good is a rent-seeking activity to the society at large, then the public policy suggestion is to discourage it. However, there is an additional consideration here. Discouraging the provision of the industry collective good would lower the research and development efforts of firms. The public authority therefore needs to balance the trade-off between benefits from a lower level of the rent-seeking industry collective good and the loss resulting from the ensuing lower level of research and development efforts by the firms in the patent race.

This paper observes that research and development effort peaks when the contestants are equal in innovation cost efficiency. Furthermore, if the two rivals are very close in innovation cost efficiency, then an increase in returns to scale in research and development raises the innovation efforts of the contestants. On the contrary, if the two rivals are very unequal in innovation cost efficiency, then an increase in returns to scale in research and development lowers the innovation efforts of the contestants. Policy makers should therefore try to attain equal or close innovation efficiency between patent race participants since innovation drives economic advances. Equal innovation efficiency leads to higher innovation efforts and more growth. This point is especially important since in today's world, large corporations and research entities dominate the research and development landscape.

This paper offers another perspective on the inverted-U relationship of innovation. For innovation, the optimal market structure for innovation is neither atomic competition nor monopoly, given the dominance of large corporations and universities in innovation. The optimal structure is oligopoly with the largest companies having equal or very closely equal strength in the product market and in research and development. As one of the competitors becomes more powerful against the rest, innovation declines.

The paper is against concentrating innovation capacity in a single firm or research entity. An equal distribution of innovative capacity among a few large corporates or universities intensifies competition and raises innovation. Furthermore, better knowledge about the market and product enhances innovation efficiency and hence innovation capacity is linked to market share of the product. Therefore, the policy proposal is to promote oligopolistic competition and limit monopoly power. Mergers and acquisitions to take advantage of economies of scale in production and innovation when there are many small firms in the industry should be given green light. However, mergers and acquisitions must not proceed to the extent of establishing a monopoly. The limit is a duopoly of two firms with equal strengths in the product market and in innovation.

The paper argues that there should be public concerns over the strong incentives of equally efficient and powerful oligopolies to form explicit or implicit cartels, including coordinating research and development activities. Once a cartel is formed, the industry effectively becomes a monopoly and innovation falters. As documented in Bright (1949) and Shaver (2012), the court decision that gave General Electric overwhelming patent rights in the then competitive light bulb industry created a monopoly and consequently, innovation died out. Lampe and Moser (2016) also found that empirically patent pools weakened competition and discouraged innovation during the regulatory tolerance in antitrust area under the New Deal. Therefore, public policy should discourage collusion in innovation activities.

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Note

[1]Refer to Tirole (1988, pp.323–324) for the condition of strategic stability.