Interactive intuitionistic fuzzy technique in multi-objective optimisation

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Abstract: In this paper, an interactive approach in multi objective optimisation problem under intuitionistic fuzzy environment is presented. This intuitionistic fuzzy problem is first converted into a single objective crisp optimisation problem. In proposed approach, in each step, decision maker (DM) is requested to update the reference acceptance level and reference rejection level of referred objective only. Membership and non-membership grades of all other objectives are automatically updated from the list of trade off rates in each stage. It is repeated until DM is satisfied with the optimal solution. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method and conclusions are drawn.

Keywords: interactive technique; fuzzy set; fuzzy optimisation; interactive fuzzy optimisation; intuitionistic fuzzy sets; decision-making under uncertainty; intuitionistic fuzzy optimisation; multi objective optimisation; trade off rates; Pareto optimal solution.

Reference to this paper should be made as follows: Garai, A., Mandal, P. and Roy, T.K. (2016) 'Interactive intuitionistic fuzzy technique in multi-objective optimisation', *Int. J. Fuzzy Computation and Modelling*, Vol. 2, No. 1, pp.14–26.

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This paper is a revised and expanded version of a paper entitled 'Multi objective linear programming problem with fuzzy goal' presented at the 2nd International Conference on Recent Trends in Mathematics and Its Applications (ICRTMA 2015), Vidyasagar University, India, 18–19 March 2015.

1 Introduction

Modelling of most of the real life problems involves multi objective optimisation processes that in general have conflicting objectives. And hence solutions of such problems are in general compromise solutions which satisfy each objective function to a degree of satisfaction and concept of belongingness and non-belongingness arises.

Zimmermann (1978) used the fuzzy set (IF) that was introduced by Zadeh (1965) for solving the fuzzy multi objective mathematical programming problem. Optimisation in fuzzy environment was further studied and was applied in various areas by many researchers such as Sakawa et al. (2013), etc. Fuzzy optimisation problem are more flexible and allow us to find solutions in more realistic ways than the analogous crisp optimisation technique.

Consequently IF theory has been widely developed and various modifications and generalisations have appeared. One of them is the concept of intuitionistic IFs, introduced by Atanassov (1986). It considers not only the degree of membership but also the degree of non-membership as well.

A fuzzy interactive system was introduced by Sakawa et al. (1987) which supports a decision maker (DM) in solving programming models with fuzzy constraints and fuzzy goals. Since the DM is involved in solution procedure, the interactive approaches play an important role in deriving the best preferred compromise solution. According to Ji et al. (2009) the main advantage of interactive approaches is that the DM controls the search direction during the solution procedure and, as a result, the efficient solution achieves his/her preferences.

The paper is arranged as follows. Initially few definitions are given. Then fuzzy multi objective programming problem is discussed in short. Next, interactive fuzzy multi objective optimisation technique is discussed. Then the proposed method of interactive multi objective optimisation technique under intuitionistic fuzzy environment is introduced. This method and algorithm are discussed in detail. One numerical problem is considered and solved using both interactive fuzzy and interactive intuitionistic fuzzy multi objective optimisation technique and conclusions are reached.

2 Definitions

It is well known that every set is a subset of universal set. In fact, fuzzy set theory was introduced by Zadeh and Klaua in 1965 as a paradigm shift of the classical notion of set theory. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set in the real unit interval [0, 1]. Fuzzy subset of universal set is defined as follows:

Definition 2.1: a fuzzy subset $\tilde{A}$ of universal set S is defined by its membership function $\mu_{\tilde{A}}: S \rightarrow [0, 1]$ that assigns to every $x \in S$, a real number $\mu_{\tilde{A}}(x)$ in the closed unit interval [0, 1]. Here the value of $\mu_{\tilde{A}}(x)$ at x represents the grade of membership of x in $\tilde{A}$. So nearer the value of $\mu_{\tilde{A}}(x)$ is unity, the grade of membership of x is higher in $\tilde{A}$.

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership both. It has been introduced by Atanassov (1986) as an extension of Zadeh's notion of IF. The theory of intuitionistic IFs further extends both concepts by allowing the assessment of the elements by two functions: one for membership and another for non-membership, which belongs to the real unit interval [0, 1] and whose sum belongs to the same interval, as well. Hence intuitionistic fuzzy subset of universal set is defined as follows:

Definition 2.2: an intuitionistic fuzzy subset of universal set S is defined as

$$\tilde{A}^i = \left\{ < x, \mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x) > \middle| x \in S \right\},$$

where $\mu_{\tilde{A}^i}(x): S \rightarrow [0, 1]$ and $v_{\tilde{A}^i}(x): S \rightarrow [0, 1]$ such that $0 \le \mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x) \le 1$, here $\mu_{A^i}(x)$ and $\mu_{\tilde{A}^i}(x)$ denote the degree of membership and degree of non-membership of x in $\tilde{A}^i$ respectively.

Definition 2.3: essentially, Pareto optimality describes a state of affairs in which resources are distributed such that it is not possible to improve a single individual without also causing at least one other individual to become worse off than before the change.

A point x^* in X is said to be an Pareto optimal solution, sometimes referred to as M-Pareto optimal solution, to a generalised multi objective linear programming problem if and only if there does not exist another $x1$ in X such that $\mu_i(z_i(x_1)) \geq \mu_i(z_i(x^*))$ $\forall i$ = 1,2,…k and $\mu_j(z_j(x_1)) \neq \mu_j(z_j(x^*))$ for at least one.

Definition 2.4: a trade-off is a situation that involves losing one quality or aspect of something in return for gaining another quality or aspect. Trade studies are essentially decision-making exercises.

3 Fuzzy and intuitionistic fuzzy multi objective programming

In 1978 Zimmermann had extended fuzzy linear programming approach to following multi objective linear programming problem, with k linear objective functions $z_i(x) = c_i x$, $i = 1, 2\ldots k$ as

$$\min\ z(x) = \left(z_1(x), z_2(x), \ldots, z_k(x)\right)^T$$
subject to the constraints $Ax \le b,\ x \ge 0,$

Here $c_i = (c_{i1}, c_{i2}, \ldots, c_{in})$, $I = 1,2,\ldots k$ and $x = (x_1, x_2, \ldots, x_n)^T$ and $b = (b_1, b_2, \ldots, b_m)^T$ and A is an $m \times n$ matrix.

For each of the objective functions $z_i(x) = c_i x$, with $i = 1, 2\ldots k$, it is assumed that the DM has a fuzzy goal. Taking one objective function at a time, individual minimum and maximum values are obtained subject to given constraints for each objective function. Then following the fuzzy decision by Bellman and Zadeh (1970), the original multi objective linear programming problem may be interpreted as

$$\max \min_{i=1,2,\ldots,k} \mu_i\left(z_i(x)\right)$$
subject to the constraints $Ax \le b,\ x \ge 0,$

Here $\mu_i(z_i(x))$ is the membership function of i^{th} objective function for $i = 1, 2\ldots k$. If an auxiliary variable λ, with $\lambda = \min_{i=1,2,\ldots,k} \mu_i(z_i(x))$ is introduced, the conventional linear programming problem is obtained as

$$\max \quad \lambda$$
subject to $\lambda \le \mu_i\left(z_i(x)\right),\ i = 1,2,\ldots,k$

$$Ax \le b,\ x \ge 0$$

For this linear membership function, as suggested by Zimmermann (1978), it can be easily shown that if the optimal solution of the above problem is unique, it is also a Pareto optimal solution of the multi objective linear programming problem.

On the other hand, intuitionistic fuzzy optimisation problem was formulated and solved by intuitionistic IFs over the objectives and constraints by Angelov in his historic paper in 1997.

4 Interactive fuzzy multi objective optimisation technique

In the approach (as proposed by Zimmermann) on multi objective linear programming problem under fuzzy environment, it was assumed that the fuzzy decision of Bellman and Zadeh (1970) is the proper representation of the fuzzy preference levels of the DM. But according to Sakawa and Yano (1990), these approaches are preferable only when the DM feels that the fuzzy decision is appropriate for combining the fuzzy goals and/or constraints; and such situations seem to occur rarely in reality.

Consequently it is evident that an interaction with in optimisation process with DM is necessary. In the famous paper, Sakawa et al. (1987) developed an interactive fuzzy multi objective linear programming method mixing interactive approaches into the fuzzy approaches (assuming that the DM has a fuzzy goal for each of the objective functions). This method is useful in real life optimisation technique. Yet it has further scope of modification and improvement. The next section describes an interactive intuitionistic fuzzy optimisation technique that is more effective in real life.

5 Proposed interactive intuitionistic fuzzy multi objective optimisation technique

In interactive fuzzy multi objective optimisation technique, only degree of acceptance of objective function is considered to describe impreciseness. But It is well known from the renowned research by Atanassov in 1995 that when the degree of rejection (non-acceptance) is defined simultaneously with the degree of acceptance (membership) and when both these degrees are not complementary to each other, intuitionistic IF can be used as a more general and full tool for describing this impreciseness. Actually, as Angelov mentioned in 1997 that it is sometimes better to represent deeply existing nuances in problem formulation defining objectives and constraints (or part of them) by IF sets. Hence usage of intuitionistic IF is proposed in interactive optimisation technique.

In general, the linear programming problem is represented as the following vector-minimisation problem:

$$\text{optimize } z(x) = \left(z_1(x), z_2(x), \ldots, z_k(x)\right)^T$$
$$\text{subject to the constraints } Ax \le b,\ x \ge 0,$$

where $z_1(x)$, $z_2(x)$ … $z_k(x)$ are k conflicting linear objective functions.

1 The individual minimum and maximum of objective function $z_i(x) = c_i x$, $i = 1, 2\ldots k$ under given constraints are computed.

2 Taking into account these individual minimum and maximum of each objective function, together with the increase rate of satisfaction in terms of the membership degree and decrease rate of satisfaction in terms of non-membership degree such that sum of membership and non-membership degree does not exceed unity, the DM determines the membership functions $\mu_i(c_i x)$ for each $i = 1, 2 \ldots$k and non-membership function $v_i(c_i x)$ for each $i = 1, 2\ldots k$ in a subjective manner. It is noted that for minimisation type objective function, if level of acceptance are 0 and 1 z_i^0 at and z_i^1 respectively for i^{th} objective function $z_i(x) = c_i x$, then corresponding linear membership function may be taken as

$$\mu_i^R\left(z_i(x)\right) = \begin{cases} 0 & \text{if} \quad z_i(x) \ge z_i^0 \\ \dfrac{z_i^0 - z_i(x)}{z_i^0 - z_i^1} & \text{if} \quad z_i(x) \le z_i^0 \\ 1 & \text{if} \quad z_i(x) \le z_i^1 \end{cases}$$

and if level of rejection are 1 and 0 at $z_i^{0'}$ and z_i^1 respectively for i^{th} objective function $z_i(x) = c_i x$, then corresponding linear non-membership function may be taken as

$$v_i^L\left(z_i(x)\right) = \begin{cases} 1 & \text{if} \quad z_i(x) \ge z_i^{0'} \\ \dfrac{z_i^{0'} - z_i(x)}{z_i^{0'} - z_i^1} & \text{if} \quad z_i^{0'} \le z_i(x) \le z_i^1 \\ 0 & \text{if} \quad z_i(x) \le z_i^1 \end{cases}$$

Again for maximisation type, if level of acceptance are 0 and 1 at z_i^0 and z_i^1 respectively for i^{th} objective function $z_i(x) = c_i x$, then corresponding linear membership function may be taken as

$$\mu_i^L\left(z_i(x)\right) = \begin{cases} 1 & \text{if} \quad z_i(x) \geq z_i^1 \\ \dfrac{z_i(x) - z_i^0}{z_i^1 - z_i^0} & \text{if} \quad z_i^0 \leq z_i(x) \leq z_i^1 \\ 0 & \text{if} \quad z_i(x) \leq z_i^0 \end{cases}$$

and if level of rejection are 1 and 0 at $z_i^{0'}$ and z_i^1 respectively for i^{th} objective function $z_i(x) = c_i x$, then corresponding linear non-membership function may be taken as

$$v_i^R\left(z_i(x)\right) = \begin{cases} 1 & \text{if} \quad z_i(x) \leq z_i^{0'} \\ \dfrac{z_i^1 - z_i(x)}{z_i^1 - z_i^{0'}} & \text{if} \quad z_i^{0'} \leq z_i(x) \leq z_i^1 \\ 0 & \text{if} \quad z_i(x) \geq z_i^1 \end{cases}$$

For equality type objective function, both left spade and right spade of that membership function are to be taken. Here '*L*' and '*R*' denote left and right spade of function respectively.

3 For initial reference membership levels $\hat{\mu} = (\hat{\mu}_1, \ldots, \hat{\mu}_k)^T$, usually unity and reference non-membership levels $\hat{v} = (\hat{v}_1, \hat{v}_2 \ldots, \hat{v}_k)^T$, usually zero, the corresponding Pareto optimal solution, if it exists, is obtained by solving the following problem:

$$\text{minimize} \max_{i=1,2,\ldots,k} \left\{\hat{\mu}_i - \mu_i\left(z_i(x)\right)\right\}$$
$$\text{minimize} \max_{i=1,2,\ldots,k} \left\{v_i\left(z_i(x)\right) - \hat{v}_i\right\}$$
$$\text{subject to } Ax \leq b, x \geq 0$$

i.e., equivalently

$$\begin{aligned} &\text{minimize } v \\ &\text{minimize } w \\ &\text{subject to } \left\{\hat{\mu}_i - \mu_i\left(z_i(x)\right)\right\} \leq v, i = 1, 2, \ldots, k \\ &\qquad\qquad \left\{v_i\left(z_i(x)\right) - \hat{v}_i\right\} \leq w, i = 1, 2, \ldots, k \\ &\qquad\qquad Ax \leq b, x \geq 0, v \geq 0, w \geq 0 \end{aligned}$$

with $v = \max_{i=1,2,\ldots,k} \{\hat{u}_i - \mu_i(z_i(x))\}$ and $w = \max_{i=1,2,\ldots,k} \{v_i(z_i(x)) - \hat{v}_i\}$. Here all membership functions $\mu_i(z_i(x))$, $i = 1, 2\ldots k$ and non-membership functions $v_i(z_i(x))$, $i = 1, 2\ldots k$ are linear, the above problem is linear programming problem.

This two-objective linear optimisation problem is then converted into following single objective optimisation problem as follows:

$$\begin{aligned} &\min \quad v + w \\ &\text{subject to } \left\{\hat{\mu}_i - \mu_i\left(z_i(x)\right)\right\} \le v, i = 1,2,\ldots,k \\ &\left\{v_i\left(z_i(x)\right) - \hat{v}_i\right\} \le w, i = 1,2,\ldots,k \\ &Ax \le b, x \ge 0, v \ge 0, w \ge 0 \end{aligned}$$

4 In order to help the DM express a level of preference for membership and non-membership functions at next level of interaction, DM is first requested to choose an appropriate standing objective function, say $z_i(x)$, among all objectives $z_j(x)$, $j = 1, 2\ldots k$ and corresponding reference membership level and reference non-membership level. For membership function, trade-off information between this standing membership function $\mu_i(z_i(x))$ and each of the other membership functions is analytically computed and similarly for non-membership function, trade-off information between standing non-membership function $v_i(z_i(x))$, corresponding to same objective function $z_i(x)$, and each of the other non-membership functions is analytically computed. The reference membership levels and reference non-membership levels for other objective functions are obtained from that trade off rates.

   These trades off rates, for both membership and non-membership functions, together with that Pareto optimal solution are then supplied back to DM, as was done by Sakawa et al. in 1998.

5 If the DM is satisfied with the values of objective function together with current membership and non-membership levels in that Pareto optimal solution, stop. In that case, the current Pareto optimal solution is the satisfying solution of the DM.

   Otherwise, the DM is requested to update the current reference membership and non-membership level of only standing objective function $z_i(x)$, by considering the last optimal values of the membership functions and non-membership function together with both trade-off rates and return to step 3. It should be stressed to the DM that any improvement of one membership function can be achieved only at the expense of at least one of the other membership functions.

6 Numerical example of interactive intuitionistic fuzzy optimisation technique

Let us consider the following fuzzy linear programming problem:

$$\begin{aligned} &\text{fuzzy max} \quad z_1 = 5x_1 + 5x_2, \\ &\text{fuzzy min} \quad z_2 = 5x_1 + x_2, \\ &\text{fuzzy max} \quad z_3 = 3x_1 - 8x_2, \\ &\text{subject to} \quad 5x_1 + 7x_2 \le 12, 9x_1 + x_2 \le 10, -5x_1 + 3x_2 \le 3, x_1 \ge 0, x_2 \ge 0, \end{aligned}$$

The problem is solved in steps as follows:

1 The individual minimums and maximums of each objective function $z_i(x) = c_i x$, $i = 1, 2, 3$ under the given constraints are computed and obtained as in Table 1.

2 Assume that the DM specifies the corresponding sub intervals as given in Table 2. Then linear membership and linear non-membership functions are taken as follows:

$$\mu_1^L(z_1(x)) = \begin{cases} 1 & \text{if} \quad z_1(x) \ge 10 \\ \dfrac{z_1(x)-0}{10} & \text{if} \quad z_1(x) \le 10, \\ 0 & \text{if} \quad z_1(x) \le 0 \end{cases}$$

$$v_1^R(z_1(x)) = \begin{cases} 1 & \text{if} \quad z_1(x) \le -2 \\ \dfrac{10-z_1(x)}{12} & \text{if} \quad -2 \le z_1(x) \le 10 \\ 0 & \text{if} \quad z_1(x) \ge 10 \end{cases}$$

$$\mu_2^R(z_2(x)) = \begin{cases} 0 & \text{if} \quad z_2(x) \ge 6 \\ \dfrac{6 - z_2(x)}{5} & \text{if} \quad 1 \le z_2(x) \le 6, \\ 1 & \text{if} \quad z_2(x) \le 1 \end{cases}$$

$$v_2^L(z_2(x)) = \begin{cases} 0 & \text{if} \quad z_2(x) \le 1 \\ \dfrac{z_2(x)-1}{7} & \text{if} \quad 1 \le z_2(x) \le 8 \\ 1 & \text{if} \quad z_2(x) \ge 8 \end{cases}$$

$$\mu_3^L(z_3(x)) = \begin{cases} 1 & \text{if} \quad z_3(x) \ge 0 \\ \dfrac{z_3(x)+10}{10} & \text{if} \quad 0 \le z_3(x) \le 10, \\ 0 & \text{if} \quad z_3(x) \le -10 \end{cases}$$

$$v_3^R(z_3(x)) = \begin{cases} 1 & \text{if} \quad z_3(x) \le -12 \\ \dfrac{0-z_3(x)}{12} & \text{if} \quad -12 \le z_3(x) \le 0 \\ 0 & \text{if} \quad z_3(x) \ge 0 \end{cases}$$

3 Suppose that, DM takes unity as initial reference membership levels for z_i i.e., $\hat{\mu}_i = 1, \forall i$ and zero as initial reference non-membership levels for z_i i.e., $\hat{v}_i = 0, \forall i$. Then the corresponding minimax problem is obtained as

$$\min\{\max(1-\mu_1(z_1), 1-\mu_2(z_2), 1-\mu_3(z_3))\}$$

$$\max\{\min(v_1(z_1)-0, v_2(z_2)-0, v_3(z_3)-0)\}$$

subject to $5x_1 + 7x_2 \le 12, 9x_1 + x_2 \le 10, -5x_1 + 3x_2 \le 3, x_1 \ge 0, x_2 \ge 0,$

Let $v = \max(1 - \mu_1(z_1), 1 - \mu_2(z_2), 1 - \mu_3(z_3))$ and $w = \max(v_1(z_1) - 0, v_2(z_2) - 0, v_3(z_3 - 0)$, then the problem is formulated to be a single objective optimisation problem as

$$\min \quad v+w$$

subject to the constraints

$$\left\{\hat{\mu}_1-\mu_1\left(z_1(x)\right)\right\}\le v,\left\{\hat{\mu}_2-\mu_2\left(z_2(x)\right)\right\}\le v,\left\{\hat{\mu}_3-\mu_3\left(z_3(x)\right)\right\}\le v,$$

$$\left\{v_1\left(z_1(x)\right)-\hat{v}_1\right\}\le w,\left\{v_2\left(z_2(x)\right)-\hat{v}_2\right\}\le w,\left\{v_3\left(z_3(x)\right)-\hat{v}_3\right\}\le w,$$

$$5x_1+7x_2\le 12, 9x_1+x_2\le 10, -5x_1+3x_2\le 3, x_1\ge 0, x_2\ge 0, v\ge 0, w\ge 0,$$

After simplification, it becomes

$$\min \quad v+w$$

subject to the constraint

$$1-\left((5x_1+5x_2-0)/10\right)\le v, 1-\left((6-5x_1-x_2)/5\right)\le v,$$

$$1-\left((3x_1-8x_2+10)/10\right)\le v,\left((10-5x_1-5x_2)/12\right)-0\le w,$$

$$\left((5x_1+x_2-1)/7\right)-0\le w,(0-3x_1+8x_2)/12-0\le w,$$

$$5x_1+7x_2\le 12, 9x_1+x_2\le 10, -5x_1+3x_2\le 3, x_1\ge 0, x_2\ge 0, v\ge 0, w\ge 0,$$

The solution using Lingo (14.0.1) is obtained as given in Table 3.

4 In order to help the DM to express level of reference for membership and non-membership functions at next level of interaction, DM is requested to select an appropriate standing objective from family the objectives $z_i(x)$, $i = 1, 2, 3$. Suppose DM selects z_3 as appropriate standing objective function. Then for membership level, trade-off information between standing membership function $\mu_3(z_3(x))$ and each of the other membership functions is analytically computed and obtained as [using chain rule of partial differentiation]

$$-\frac{\partial\mu_3}{\partial\mu_1}=-\frac{\partial\mu_3}{\partial z_3}\frac{\partial z_3}{\partial z_1}\frac{\partial z_1}{\partial\mu_1}=-\frac{\partial\mu_3}{\partial z_3}\frac{\partial z_3}{\partial z_1}\left(\frac{\partial\mu_1}{\partial z_1}\right)^{-1}=43/20$$

$$-\frac{\partial\mu_3}{\partial\mu_2}=-\frac{\partial\mu_3}{\partial z_3}\frac{\partial z_3}{\partial z_2}\frac{\partial z_2}{\partial\mu_2}=-\frac{\partial\mu_3}{\partial z_3}\frac{\partial z_3}{\partial z_2}\left(\frac{\partial\mu_2}{\partial z_2}\right)^{-1}=11/8$$

And for non-membership level, trade-off information between standing non-membership function $v_3(z_3(x))$, corresponding to same objective function $z_3(x)$, and each of the other non-membership functions is analytically computed and obtained as

$$-\frac{\partial v_3}{\partial v_1}=-\frac{\partial v_3}{\partial z_3}\frac{\partial z_3}{\partial z_1}\frac{\partial z_1}{\partial v_1}=-\frac{\partial v_3}{\partial z_3}\frac{\partial z_3}{\partial z_1}\left(\frac{\partial v_1}{\partial z_1}\right)^{-1}=43/20$$

$$-\frac{\partial v_3}{\partial v_2}=-\frac{\partial v_3}{\partial z_3}\frac{\partial z_3}{\partial z_2}\frac{\partial z_2}{\partial v_2}=-\frac{\partial v_3}{\partial z_3}\frac{\partial z_3}{\partial z_2}\left(\frac{\partial v_2}{\partial z_2}\right)^{-1}=77/48$$

These trades off rates, for both membership and non-membership functions, together with that Pareto optimal solution are sent to DM.

5 Suppose the DM is satisfied; then stop. Otherwise on the basis of supplied information, DM updates the reference levels of membership and non-membership function of standing objective function z_3 as $\mu_3(z_3) = 0.6$ and $\nu_3(z_3) = 0.3$ respectively. The other values of reference membership and non-membership function are obtained from trade off rates as in Table 4.

Then move to step 3. DM should consider the increase rate of satisfaction in terms of the membership degree and decrease rate of satisfaction in terms of non-membership degree such that sum of membership and non-membership degree does not exceed unity. Hence the corresponding problem from step 3 becomes

$$\min \quad v + w$$

subject to the constraints

$$0.27907 - \left((5x_1 + 5x_2 - 0)/10\right) \le v,\ 0.436364 - \left((6 - 5x_1 - x_2)/5\right) \le v,$$
$$0.6 - \left((3x_1 - 8x_2 + 10)/10\right) \le v,\ \left((10 - 5x_1 - 5x_2)/12\right) - 0.139535 \le w,$$

$$\left((5x_1 + x_2 - 1)/7\right) - 0.187013 \le w,\ \left((0 - 3x_1 + 8x_2)/12\right) - 0.3 \le w,$$
$$5x_1 + 7x_2 \le 12,\ 9x_1 + x_2 \le 10,\ \{-\}5x_1 + 3x_2 \le 3,\ x_1 \ge 0,\ x_2 \ge 0,\ v \ge 0,\ w \ge 0,$$

As before, the solution by LINGO (14.0.1.57) [LINGO is used as it provides completely integrated package that includes a powerful language for expressing optimisation models, a full featured environment for building and editing problems, and a set of fast built-in solvers] gives the solution set as in Table 5. If the DM updates the reference level of non-membership function for z_3 to be 0.2, then the corresponding problem gives the Pareto optimal solution as given in Table 6. If the DM is satisfied with the current value of membership, non-membership and hence value of objective functions of this Pareto optimal solution, stop. Then the current Pareto optimal solution is the satisfying solution for DM. Otherwise the DM updates the current reference membership level and reference non-membership level for the standing objective function $z_3(x)$ only, by considering the values of the membership functions and non-membership functions together with trade-off rates, such that sum of membership and non-membership grade does not exceed unity in any case and returns to step 3.

In this example, if further iterations are done, the optimal solutions at stage 3 are obtained as Table 7.

It should be stressed to the DM that any improvement of one membership function can be achieved only at the expense of at least one of other membership functions. If the same problem is solved under simple fuzzy environment, then the optimal solutions are obtained as in Table 8.

7 Conclusions

The concept of interactive intuitionistic fuzzy optimisation is introduced and applied here. In interactive fuzzy optimisation, for one level of acceptance 0.6 for z_3, only one optimal solution set $\{z_1 = 2.7907, z_2 = 0.727275, z_3 = -4\}$ is obtained.

But in proposed approach, for same level of acceptance 0.6 for z_3, different optimal solution sets are obtained corresponding to different rejection levels i.e., under intuitionistic fuzzy environment, more accurate conversion of imprecise linguistic information (as obtained from DM) is used, e.g., for reference non-membership level 0.3 of z_3, optimum value of z_1 is 6.31 and for that of 0.2, it is 6.23. Thus DM may select the solution that best fits his/her aspiration level in IF environment. Therefore this method is more flexible than usual fuzzy interactive method.

Moreover in proposed method, DM supplies reference membership and reference non-membership level for one objective on every step. The reference levels (both membership and non-membership) for other objective functions are obtained analytically by using trade off rates. As Monghasemi et al. (2015) has mentioned that decision-making approaches can aid decision-makers in selecting the most appropriate solution among numerous potential Pareto optimal solutions and trade off rates are effective tools in finding such information. In simple fuzzy interactive process, DM supplies the reference level for all membership functions at each stage. Thus our method is more analytical in nature. Also, it saves the precious time of DM.

Finally, it may be noted that customer (in this case, DM) is our king and his/her choice is must be given highest priority in imprecise environment. Till date, there is no standard form of optimisation problem in imprecise environment and it needs to be customised according to requirement. Effort is on to find a concrete and everywhere-accepted format of imprecise real-life optimisation problem and a standard solution procedure of it.

Acknowledgements

This research work is supported by University Grants Commission, India vide minor research project [PSW-071/13-14 (WC2-130) (S.N. 219630)]. The first author sincerely acknowledges the contributions and is very grateful to them.

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Appendix

Table 1 Individual minimum and maximum of each objective function

Objective function	*Minimum value*	*Maximum value*
Z_1	0	10
Z_2	0	6
Z_3	–11.1	3.333

Table 2 Subinterval for membership and non-membership function

Objective function	*Value of membership function*		*Value of non-membership function*	
	$\mu_i(z_i) = 0$	$\mu_i(z_i) = 1$	$v_i(z_i) = 1$	$v_i(z_i) = 0$
Z_1	0	10	–2	10
Z_2	6	1	8	1
Z_3	–10	0	–12	0

Table 3 Solution at stage 1

Stage	*Optimal values*			
1	x_1^* 0.475	$\mu_1^*(z_1)$ 0.586	$v_1^*(z_1)$ 0.345	Z_1^* 5.856
	x_2^* 0.696	$\mu_2^*(z_2)$ 0.586	$v_2^*(z_2)$ 0.295	Z_2^* 3.072
		$\mu_3^*(z_3)$ 0.586	$v_3^*(z_3)$ 0.345	Z_3^* –4.143

Table 4 Updated reference level for each objective function

Objective function	Z_1	Z_2	Z_3
Membership values	0.279	0.436	0.6
Non-membership values	0.139	0.187	0.3

Table 5 Solution at stage 2

Stage	*Updated reference level*		*Optimal values*			
2 (0.3)	$\mu_1(z_1)$ 0.279	$v_1(z_1)$ 0.139	x_1^* 0.554	$\mu_1^*(z_1)$ 0.631	$v_1^*(z_1)$ 0.307	Z_1^* 6.31
	$\mu_2(z_2)$ 0.436	$v_2(z_2)$ 0.187	x_2^* 0.708	$\mu_2^*(z_2)$ 0.503	$v_2^*(z_2)$ 0.354	Z_2^* 3.48
	$\mu_3(z_3)$ 0.6	$v_3(z_3)$ 0.3		$\mu_3^*(z_3)$ 0.6	$v_3^*(z_3)$ 0.333	Z_3^* –4

Table 6 Next solution

Stage	*Updated references*		*Optimal values*			
2 (0.2)	$\mu_1(z_1)$ 0.279	$v_1(z_1)$ 0.093	x_1^* 0.543	$\mu_1^*(z_1)$ 0.623	$v_1^*(z_1)$ 0.314	Z_1^* 6.23
	$\mu_2(z_2)$ 0.436	$v_2(z_2)$ 0.124	x_2^* 0.704	$\mu_2^*(z_2)$ 0.516	$v_2^*(z_2)$ 0.354	Z_2^* 3.42
	$\mu_3(z_3)$ 0.6	$v_3(z_3)$ 0.2		$\mu_3^*(z_3)$ 0.6	$v_3^*(z_3)$ 0.333	Z_3^* –4

Table 7 Solution at stage 3

Stage	*Updated references*		*Optimal values*			
3	$\mu_1(z_1)$ 0.325	$v_1(z_1)$ 0.093	x_1^* 0.573	$\mu_1^*(z_1)$ 0.581	$v_1^*(z_1)$ 0.349	Z_1^* 5.814
	$\mu_2(z_2)$ 0.509	$v_2(z_2)$ 0.125	x_2^* 0.589	$\mu_2^*(z_2)$ 0.509	$v_2^*(z_2)$ 0.350	Z_2^* 3.454
	$\mu_3(z_3)$ 0.7	$v_3(z_3)$ 0.2		$\mu_3^*(z_3)$ 0.7	$v_3^*(z_3)$ 0.25	Z_3^* –3

Table 8 Solution in classical fuzzy environment

Stage	*Updated references*	*Optimal values*		
1	$\mu_1(z_1)$ 1	x_1 0.475	$\mu_1(z_1)$ 0.585	Z_1 5.856
	$\mu_2(z_2)$ 1	x_2 0.696	$\mu_2(z_2)$ 0.585	Z_2 3.071
	$\mu_3(z_3)$ 1		$\mu_3(z_3)$ 0.585	Z_3 –4.143
2	$\mu_1(z_1)$ 0.279	x_1 0.042	$\mu_1(z_1)$ 0.279	Z_1 2.790
	$\mu_2(z_2)$ 0.436	x_2 0.516	$\mu_2(z_2)$ 1	Z_2 0.727
	$\mu_3(z_3)$ 0.6		$\mu_3(z_3)$ 0.6	Z_3 –4