Adaptive coordinated control of multi-mobile manipulator systems

Abdelkrim Brahmi*, Maarouf Saad and Guy Gauthier

Electrical Engineering Department,
Université du Québec, École de technologie supérieure,
1100 Notre-Dame Street West, Montreal,
Quebec H3C 1K3, Canada
Email: abdelkrim.brahmi.1@ens.etsmtl.ca
Email: maarouf.saad@etsmtl.ca
Email: Guy.Gauthier@etsmtl.ca
*Corresponding author

Wen-Hong Zhu

Space Exploration,
Canadian Space Agency; 6767 route de l’Aéroport,
Saint-Hubert, J3Y 8Y9, Canada
Email: WenHong.zhu@canada.ca

Jawhar Ghommam

Ecole National d’Ingénieurs de Sfax,
Research Unit on Mechatronics and Automation Systems,
BPW, 3038 Sfax, Tunisia
Email: jawhar.Ghommam@ieee.org

Abstract: This paper presents an adaptive coordinated control scheme for multiple mobile manipulator robots (MMR) moving a rigid object in coordination. The dynamic parameters of the object handled and of the mobile manipulators are considered unknown but constant. The control law and the adaptation of uncertain parameters are designed using the virtual decomposition (VDC) approach. This control approach was originally applied to multiple manipulator robot systems. The proposed control design ensures that the position error in the workspace converges to zero, and that the external force error is bounded. The global stability of the system using VDC is proven through the virtual stability of each subsystem. Numerical simulations and an experimental validation are carried out for two mobile manipulators transporting an object, and are compared with the results obtained using the computed torque approach in order to show the effectiveness of the proposed controller.

Keywords: adaptive control; coordinated control; virtual decomposition control; VDC; multi-mobile manipulator.

Biographical notes: Abdelkrim Brahmi received his BSc and MSc degrees in Electrical Engineering from the University of Sciences and Technologies of Oran, Algeria, in 1997 and 2009, respectively. He is currently pursuing his PhD degree in Electrical Engineering from Quebec University (École de Technologie Supérieure), Montreal, QC, Canada. His current research interests include nonlinear control, adaptive control applied to coordinated robotic systems.

Maarouf Saad received his Bachelor and Master degrees in Electrical Engineering from École Polytechnique of Montreal respectively in 1982 and 1984. In 1988, he received his PhD from McGill University in Electrical Engineering. He joined Ecole de technologie superieure in 1987 where he is teaching control theory. His research is mainly in nonlinear control and optimisation applied to robotics and power systems.

Guy Gauthier received his BS degree in Electrical Engineering from Université Laval, Quebec City, in 1984. Then, he received his BTech degree in Automated Manufacturing from Ecole de technologie superieure, Montreal, in 1987. He received his MS degree in Electrical Engineering from Ecole polytechnique de Montreal, in 1997. Finally he received his PhD degree in 2008 from McGill University, Montreal. He is currently a Professor at École de technologie superieure (Montreal, Canada) in the Department of Automated Manufacturing Engineering. His research area is mainly on robotic, iterative learning control and fuzzy control.

Wen-Hong Zhu received his Bachelor and MS degrees from Northwestern Polytechnical University in 1984 and 1987, respectively, and PhD degree from Xi’an Jiao Tong University in 1991. He was a Postdoctoral Fellow at Shanghai Jiao Tong University during 1991–1993, Korea Advanced Institute of Science and Technology (KAIST) in 1995, Katholieke Universiteit Leuven during 1996-1997, and University of British Columbia during 1997–1999. He worked at University of British Columbia as a Scientific Engineer from 1999 to 2001 before joining Canadian Space Agency (CSA). He is an IEEE Senior member and a holder of NSERC Discovery Grant.

Jawhar Ghommam received his BSc degree from the Institut Nationale des Sciences Appliquées et de Technologies (INSAT), Tunisia, in 2003; MSc degree in Control Engineering from the Laboratoire d’Informatique, de Robotique et de Microelectronique (LIRMM), Montpellier, France, in 2004; and PhD in Control Engineering and Industrial Computers, in 2008. He is currently an Associate Professor of Control Engineering at the INSAT, Tunisia. His research interests include nonlinear control of underactuated mechanical systems, adaptive control, guidance and control of underactuated ships, and cooperative motion of non-holonomic vehicles.
1 Introduction

The need for robots capable of locomotion and manipulation has led to the design of mobile manipulator robot (MMR) platforms. Typical examples of MMR include satellite arms, underwater robots in seabed exploration, and vehicles used in extra-planetary exploration. However, the most popular mobile manipulators are semi-automated cranes mounted on trucks. Some operations requiring the handling of heavy objects are very difficult for single mobile manipulators, and require the use and coordination of multiple mobile manipulators, which significantly complicates the robotic system, and greatly increases its control design complexity. The problem with controlling a mechanical system forming a closed kinematic chain mechanism is that it imposes a set of kinematic constraints on the coordination of the position and velocity of the mobile manipulator, thus leading to a reduction in the degrees of freedom of the entire system. Although the object internal forces produced by all mobile manipulators must be controlled, few works have been proposed to solve this control problem for the robotic systems, which have high degrees of freedom and are tightly interconnected because all manipulators are in contact with the object. Most research works in this field have thus far focused on the three main coordination mechanisms involved: decentralised control, the leader-follower control approach and motion planning. In Kume et al. (2007), a motion coordination control not involving the use of a torque/force sensor is proposed and applied to a multi-holonomic mobile manipulator. To reduce the effect of the sensor noise at the end-effector, a control scheme using constraints between the contact points and the point representing the handling object was proposed in Kosugo and Oosumi (1996), Hirata et al. (1999) and Kosugo et al. (1999). The authors then extended and applied this approach on multiple omnidirectional mobile robots manipulating a rigid object in coordination. A further extension of the method applied to manipulators with bases attached to a holonomic mobile manipulator, engaged in novel decentralised cooperation tasks, was proposed in Khatib et al. (1996) and Park and Khatib (2008).

In another approach, a single or a group of MMR is designated as a leader capable of moving a desired trajectory, while the other group members follow this leader. Many papers, including Xin and Yangmin (2006), Hirata et al. (2004), Chao et al. (2009) and Fujii et al. (2007) have covered this approach. Finally, the third approach deals with the motion planning strategy, which is another fundamental problem in robotics, especially in multi-robot systems. This approach has been covered in a limited number of research works in the case of multiple MMRs, where several robots execute the task of transporting an object in coordination, in a known/unknown environment. These studies include those presented in Desai and Kumar (1997), Yamamoto and Fukuda (2002), Xiaooyan and Dunwei (2004), LaValle (2006), Latombe (1991), Furuno et al. (2003), and Zhu and Yang (2003). In Desai and Kumar (1997), an optimal trajectory was proposed for two mobile manipulators pushing a common object to a desired location; the authors in Yamamoto and Fukuda (2002) proposed a control method for multiple mobile manipulators holding a common object. Here, the measures of kinematic and dynamic manipulability were given, taking into account collision avoidance, but the dynamics of the object was however ignored. In Xiaooyan and Dunwei (2004) and LaValle (2006), a planning approach based on genetic algorithms was proposed.

Over the past few years, increased attention has been paid to the adaptive control of robotic systems with high degrees of freedom, with many research works developed
based on the approach, including those in Chen (2015), Zhao et al. (2016), Liu et al. (2016), Andaluz et al. (2012), Xing-Gang et al. (2014) and Karray and Feki (2014). This is due to the fact that this type of robotic system can be implemented in complex applications with unknown parameters. The kinematics and dynamics of these systems are characterised by uncertainty, high nonlinearity, and tight coupling, which in turn renders the control problem very complicated and difficult to solve using the classical approaches developed and explained above. One of the categories of complex robotic systems involves multiple-mobile manipulator systems holding an object. The constraints imposed on a system forming a closed kinematic chain will often cause the degrees of freedom of motion to be less than the number of actuators. In this case, not only the motions, but also the internal forces, need to be controlled. To overcome these problems, a novel adaptive control based on the virtual decomposition approach is proposed in this work.

All previous studies based on Lagrangian or Newton/Euler approaches require knowledge of the exact parameters of the system. In practice, this is difficult, and using them, the model obtained is usually uncertain. To overcome the problem of dynamic modelling and control, some researchers have proposed an adaptive control based on neural network control (Liu et al., 2014, 2013; Liu and Zhang, 2013) and fuzzy logic approaches (Mai and Wang, 2014; Yoshimura, 2015; Faten Baklouti et al., 2016). For instance, non-model-based techniques have been developed for a different type of MMR with dynamic parameter uncertainties. Another problem with existing approaches is that with them, the dynamics of the whole system are complicated. Any change in the structure of the group requires a new dynamic modelling (removal of a faulty robot or addition of a new robot to the system). Finally, for these types of tightly coupled systems with a high degree of freedom, adapting the parameters using methods based on full dynamics is very complicated due to the huge number of parameters involved.

Based on the preceding observations, in this paper, we intend to extend the work proposed in Brahmi et al. (2016) by using the adaptive decentralised control of a single MMR based on virtual decomposition control (VDC) (Zhu et al., 1997; Al-Shuka et al., 2014; Zhu, 2010; Luna et al., 2015) originally designed for fixed-base robotic systems with high degrees of freedom. Furthermore, diverging from what is seen in the available works in the literature, we propose an adaptive coordinated control based on the VDC approach.

The main contributions of this paper are summarised as follows.

1. Most approaches cited previously are fundamentally based on the Lagrangian formulation in calculating the dynamic model of robotic systems in close form. It is known that the complexity of the dynamic expression obtained is proportional to the fourth power of the number of degrees of freedom of the robotic systems (Hollerbach, 1980; Craig, 2005). This fact challenges both the numerical simulation and real-time control of robots with high degrees of freedom. To overcome this difficulty, an adaptive decentralised approach based on an extension of the VDC is proposed in this paper.

2. To overcome the problem of adaptation and modelling of systems using classical approaches, a VDC approach is implemented, which makes the control system more flexible in meeting changes to its configuration. In this case, adding a new robot or removing a faulty one from the system does not require a recalculation of the full dynamics of the system.
Using the VDC approach means that changing the dynamics of a subsystem only affects the respective local equations associated with that subsystem, while the equations associated with the rest of the system remain unchanged.

The global stability of the system’s VDC is proven through the virtual stability of each subsystem. Contrary to the original VDC stability, in this paper, all parameters are considered completely unknown, in addition to there being no known limit for the estimated parameters.

Finally, the main advantages of the control schemes developed in this paper are:

a. the whole dynamics of the system can easily be found based on the individual dynamics of each subsystem (rigid object and open chains)

b. the schemes render the system control design very flexible and greatly facilitate the calculation of the dynamic system, with respect to changes in the system configuration

c. they greatly simply the adaptation of the physical parameters, which they make systematic.

The rest of the paper is organised as follows. Section 2 presents the modelling of the system, while Section 3 presents the problem control statement. Section 4 explains the control design. The simulation results are given in Section 5. Section 6 presents an experimental validation of the developed approach. Finally, a conclusion is given in Section 7.

2 Modelling system and description

Figure 1 shows the \( N \) (MMR) handling a common rigid object, with \( P_w \) being the position/orientation vector of the \( i^{th} \) MMR effector and the position/orientation vector of the object. Before presenting the developed adaptive control law based on the virtual decomposition approach, we will briefly formulate the kinematic and dynamic modelling of the \( i^{th} \) MMR and the handled object.

\textbf{Figure 1} Multiple MMR handling a rigid object (see online version for colours)
The VDC approach consists in breaking down the robotic system into a graph comprised of several objects and open chains. An object is a rigid body and an open chain consists of a series of rigid links connected one-by-one by a hinge, and having a certain degree of freedom. The dynamic coupling between the subsystems can be represented by the flow of virtual power (FVP) at the cutting point; this is the principle of virtual work (Zhu et al., 1997; Al-Shuka et al., 2014; Zhu, 2010). The decomposition is illustrated in Figure 2 as follows:

Figure 2  Virtual decomposition of the robotic system (see online version for colours)

2.1 Kinematics and dynamics of the object

2.1.1 Kinematics model of the object

Since the frames \( \{ o \} \) and \( T_i \) for all \( i \in \{ 1, N \} \) are rigidly attached, it follows that:

\[
\Gamma_{cc}^{oTT} = \left[ \begin{array}{c} V_c \\ \vdots \\ V_{Nc} \end{array} \right] = \Gamma_{ii}^{oT} \Gamma^T \quad \Gamma_{ii} \in \mathbb{R}^{6N}
\]

where \( \Gamma_{ii}^{oT} \) is the transformation matrix of force/moment, and the linear/angular velocity vectors from frame \( B \) to frame \( A \) are defined by:

\[
\Gamma = \begin{bmatrix} I_6 \\ \vdots \\ I_6 \end{bmatrix} \in \mathbb{R}^{6 N}, \quad \Gamma_{ii} = \begin{bmatrix} I_6 \\ \vdots \\ I_6 \end{bmatrix} \in \mathbb{R}^{6 N},
\]

where \( I_6 \) is the \( 6 \times 6 \) identity matrix, and the transformation matrix of force/moment, and the linear/angular velocity vectors from frame \( B \) to frame \( A \) are defined by:

\[
\Gamma_{ii}^{oT} = \begin{bmatrix} a \nu_c & \omega \nu_c \\ \vdots \\ a \nu_{Nc} & \omega \nu_{Nc} \end{bmatrix} \in \mathbb{R}^{6 N}, \quad \Gamma_{ii} = \begin{bmatrix} I_6 \\ \vdots \\ I_6 \end{bmatrix} \in \mathbb{R}^{6 N},
\]

where \( \nu_c \) is the linear velocity of the centre of gravity of the object, \( \omega \) is the angular velocity of the object, and \( a \) is a constant. The velocities at the contact points between the end-effectors and the object \( T_i, i \in \{ 1, N \}, \) and \( J_o \) is the Jacobian matrix given as follows:

\[
J_o^{oT} = \Gamma_{ii}^{oT} \Gamma^T \in \mathbb{R}^{6 N \times 6 N}, \quad \Gamma = \begin{bmatrix} I_6 \\ \vdots \\ I_6 \end{bmatrix} \in \mathbb{R}^{6 N},
\]

where \( I_6 \) is the \( 6 \times 6 \) identity matrix, and the transformation matrix of force/moment, and the linear/angular velocity vectors from frame \( B \) to frame \( A \) are defined by:

\[
\Gamma_{ii}^{oT} = \begin{bmatrix} a \nu_c & \omega \nu_c \\ \vdots \\ a \nu_{Nc} & \omega \nu_{Nc} \end{bmatrix} \in \mathbb{R}^{6 N}, \quad \Gamma_{ii} = \begin{bmatrix} I_6 \\ \vdots \\ I_6 \end{bmatrix} \in \mathbb{R}^{6 N},
\]
where $^A R_B \in \mathbb{R}^{3 \times 3}$ is the rotation matrix between frames $A$ and $B$, and $S(^A r_{AB}) \in \mathbb{R}^{3 \times 3}$ is a skew symmetric matrix built from the vector $^A r_{AB} \in \mathbb{R}^{3 \times 3}$ linking the origins of frames $A$ and $B$, expressed in the coordinates of frame $A$.

2.1.2 Dynamics model of the object

The object handled by $N$ mobile manipulators is rigid. The equation of motion of the effort based on the linear parameterisation form is given by the following equation:

$$^o F = M_o ^o \dot{V} + C_o ^o \dot{V} + G_o = Y_o \theta_o$$

(3)

where $v_o \in \mathbb{R}^3$ and $w_o \in \mathbb{R}^3$ are respectively the linear and angular velocities of the object. $^o F \in \mathbb{R}^6$ is the vector of forces applied on the object, $M_o \in \mathbb{R}^{6 \times 6}$ is the mass matrix, $C_o \in \mathbb{R}^{6 \times 6}$ represents the centrifugal and Coriolis matrix and $G_o \in \mathbb{R}^6$ is the vector of gravity, $Y_o \in \mathbb{R}^{6 \times 13}$ is the dynamic regressor matrix, and $\theta_o \in \mathbb{R}^{13}$ is known parameter vector, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu, W-H. (2010).

The net force/moment vector is given by

$$^o F^* = \sum_{i=1}^{N} ^o U_{T_i} ^T \tau_c F$$

(4)

$$= \Gamma ^o U_{T_i} ^T F$$

where $^T F = [^T F_1, ..., ^T F_N]^T$ denotes the force/moment vectors in frame $T_i$ at the contact (cutting) point for $i \in \{1, N\}$. By introducing the internal force vector $F_{int} \in \mathbb{R}^{6 \times (N-1)}$, the force/moment vectors at the contact point $T_c$ can be computed from (4) as:

$$^T F = ^T U_o \begin{bmatrix} \Phi_m & \Phi_f \end{bmatrix} \begin{bmatrix} ^o F^* \\ F_{int} \end{bmatrix}$$

(5)

where $^T U_o = ^o U_{T_i} ^{-1}$ and the matrices $\Phi_m \in \mathbb{R}^{6 \times 6}$ and $\Phi_f \in \mathbb{R}^{6 \times (6N-6)}$ are governed by:

$$\begin{bmatrix} \Gamma \Phi_m = I_6 \\ \Gamma \Phi_f = 0 \end{bmatrix}$$

(6)

Note that the matrix $[\Phi_m \Phi_f]$ is of full rank. A matrix $\Omega$ exists, which verifies:

$$\begin{bmatrix} \Gamma \\ \Omega \end{bmatrix} = [\Phi_m \Phi_f]^{-1}$$

(7)

Therefore, the internal force coordinates can be calculated from (5) based on the force/moment at the $N$ end-effectors as follows:

$$F_{int} = \Omega ^o U_{T_i} ^T F$$

(8)
2.2 Kinematics and dynamics of the $i^{th}$ mobile manipulator

Figure 3 shows the $i^{th}$ holonomic manipulator arm mounted on a non-holonomic mobile platform where the manipulator has $p$-DOF, the mobile platform has $m$-DOF, and the full robotic system has $n = m + p$-DOF.

**Figure 3** Virtual decomposition of the $i^{th}$ MMR (see online version for colours)

2.2.1 Kinematics of the $i^{th}$ mobile manipulator

The augmented linear/angular velocity vector of each frame $B_{ij}$ is defined as:

$$V_{B_{ij}} = [\dot{q}_i, V_{n_i}, V_{a_{iR}}, V_{a_{iL}}, V_{a_{im}}, \ldots, V_{a_{im}}]_T,$$

where $\dot{q}_i = [\dot{q}_{inR}, \dot{q}_{inL}, \dot{q}_0, \ldots, \dot{q}_{im}]$ are right/left wheel velocities and the $j^{th}$ joint velocities of the manipulator arm, $V_{B_{ij}} \in \mathbb{R}^6$ is the linear and angular velocity vector of the corresponding frame $B_{ij}$, and $V_{n_i} \in \mathbb{R}^6$ is the linear/angular velocity vector of the mobile platform of the $i^{th}$ MMR. In general, we can write the system in matrix form by using the Jacobian matrix:

$$V_{iB} = J_{in}\dot{q}_i \quad V_{ic} = J_{in}\dot{q}_i$$

where $V_{ic}$ is the velocity at the contact points attached to the object and the Jacobian, $J_{in}$, $J_{iq}$ are the Jacobian matrices.

2.2.2 Dynamics of the $i^{th}$ mobile manipulator

The dynamics of the $j^{th}$ rigid body of the $i^{th}$ manipulator arm based on the linear parameterisation form is given by the following equation:
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\[ F_{\beta_j}^u = M_{\beta_j} \dot{V}_{\beta_j} + C_{\beta_j} V_{\beta_j} + G_{\beta_j} = Y_{\beta_j} \theta_{\beta_j} \]  

(10)

where \( M_{\beta_j} \in \mathbb{R}^{6 \times 6} \) is the matrix of inertial term, \( C_{\beta_j} \in \mathbb{R}^{6 \times 6} \) is the matrix of centrifugal/Coriolis term, \( G_{\beta_j} \in \mathbb{R}^6 \) is the vector related to gravity, \( Y_{\beta_j} \in \mathbb{R}^{6 \times 13} \) is the dynamic regressor matrix, \( \theta_{\beta_j} \in \mathbb{R}^{13} \) is the known parameter vector, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010), and \( j \) represent the right/left wheels and the \( m \) joints of the arm manipulator.

The dynamics of the mobile platform (object) based on the linear parameterisation form is given by the following equation:

\[ F_{iv}^u = M_{iv} \dot{V}_{iv} + C_{iv} V_{iv} + G_{iv} = Y_{iv} \theta_{iv} \]  

(11)

where \( M_{iv} \in \mathbb{R}^{6 \times 6} \) is the matrix of inertial term, \( C_{iv} \in \mathbb{R}^{6 \times 6} \) is the matrix of centrifugal/Coriolis term, \( G_{iv} \in \mathbb{R}^6 \) is the vector related to gravity, \( Y_{iv} \in \mathbb{R}^{6 \times 13} \) is the dynamic regressor matrix, and \( \theta_{iv} \in \mathbb{R}^{13} \) is the known parameter vector, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010).

The vector of resulting forces/moments acting on the rigid body is computed by an iterative process as follows.

\[ F_{\alpha u} = F_{\alpha u}^* + \beta_{\alpha} U_{\alpha} \tau_{\alpha} F \]  
\[ F_{\alpha u-1} = F_{\alpha u-1}^* + \beta_{\alpha-1} U_{\alpha-1} F_{\alpha u}^* \]  
\[ \vdots \]  
\[ F_{iv} = F_{iv}^* = \beta_{iv} U_{iv} F_{iv}^* F_{iv} \]  
\[ F_{\alpha u} = F_{\alpha u}^* + \beta_{\alpha} U_{\alpha} F_{\alpha u}^* \]  
\[ F_{\alpha u-1} = F_{\alpha u-1}^* + \beta_{\alpha-1} U_{\alpha-1} F_{\alpha u}^* \]  
\[ \vdots \]  

(12)

The dynamics of the \( j \)th joint actuator of the manipulator arm and that of the right/left driving motors of the platform are expressed based on the linear parameterisation form by the following equation:

\[ \tau_{\alpha_j} = J_{\alpha_j} \dot{q}_j + \zeta_j(t) = Y_{\alpha_j} \theta_{\alpha_j} \]  

(13)

where \( J_{\alpha_j} \) denotes the moment of inertia of the \( j \)th joint motor, \( \zeta_j(t) \in \mathbb{R} \) denotes the friction force/torque, \( j = \text{wR, wL, …, m} \) is defined in (10) and \( Y_{\alpha_j} \in \mathbb{R}^{1 \times 4} \) is the dynamic regressor matrix, and \( \theta_{\alpha_j} \in \mathbb{R}^{4} \) is the known parameter vector, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010).

Finally, from (10) and (13), the dynamics of the \( i \)th MMR can be written as follows.

\[ \tau_{\alpha_i} = \tau_{\alpha_i}^* + z^T F_{\beta_j} \]  

(14)

with \( z = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T \) for the revolute joints and \( z = [0\ 0\ 1\ 0\ 0\ 0]^T \) for the prismatic joints.
3 Control problem statement

To simplify the control formulation, the following assumptions are made:

Assumption 3.1: The desired object trajectory is assumed to be smooth, and there exist $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ such that:

$$\frac{dX^d}{dt} \leq \varepsilon_1, \quad \frac{d^2X^d}{dt^2} \leq \varepsilon_2, \quad \text{and} \quad \frac{d^3X^d}{dt^3} \leq \varepsilon_3.$$ 

Assumption 3.2: The object is rigid, and all end-effectors are attached rigidly to it. As a result, there is no relative motion between the end-effector and the object.

Assumption 3.3: The parameters of the object and the mobile manipulators are unknown, but constant.

Assumption 3.4: All the joint velocities of the MMRs are available for feedback as well as for the measurement of external forces.

The control objective is to generate a set of torque inputs such that the position tracking error of the transported object in the workspace converges asymptotically to zero. Formally speaking, the control problem is to design the control input:

$$U = f\left(V_c, \dot{V}_c, X_o, \overset{o}{V}\right)$$

such that the following limits hold:

- $\lim_{t \to \infty} \|X_o - X^d\| = 0, \lim_{t \to \infty} \|\overset{o}{V} - \overset{o}{V}^d\| = 0$
- $\lim_{t \to \infty} \|F^d_{int} - F_{int}\| = \text{bounded.}$

where $X^d_o \in \mathbb{R}^6$, $\overset{o}{V}^d \in \mathbb{R}^6$ are the desired position and velocity of the object generated in the workspace, and $F^d_{int} \in \mathbb{R}^{6N}$ and $F_{int} \in \mathbb{R}^{6N}$ are the desired and measured internal forces/moment coordinates, respectively.

4 Control design

4.1 Methodology

The overall control system is designed using the following steps:

- The required velocities of the object $\overset{o}{V} \in \mathbb{R}^6$ as well as the velocities of the end-effector $V^e \in \mathbb{R}^6$ are first computed, and then the required velocity $V^f \in \mathbb{R}^{6n}$ of the $n$ fixed body frames $B_0$ illustrated in Figure 2 is calculated.
• The VDC approach is used to simplify the problem of adaptation of the parameters of the complete systems, with this problem converted into a problem of estimation of the parameters of each subsystem. From the velocities computed in the first step, the estimated parameters are calculated.

• The control law of each mobile manipulator is finally designed.

4.2 Design

Step 1 The required velocity \( {^o}V^r \in \mathbb{R}^6 \) of the object is calculated based on the desired object velocity \( {^o}V^d \in \mathbb{R}^6 \):

\[
{^o}V^r = {^o}V^d + K_j e_o
\]  

(15)

where \( e_o = X^d_o - X^o \) is the position/orientation error vector and \( K_j \) is a scalar constant.

The desired velocity at the contact point of the \( N \) mobile manipulators with the object \( V^c \in \mathbb{R}^{6N} \) is calculated from the required velocity of the object \( {^o}V^r \in \mathbb{R}^6 \):

\[
V^c = \begin{bmatrix}
V^c_1 \\
\vdots \\
V^c_N
\end{bmatrix} = {^o}U^T \left( \Gamma {^o}V^r + \Omega^T K_f \left( \tilde{F}^d_{\text{int}} - \tilde{F}_{\text{int}} \right) \right)
\]

(16)

where \( \Gamma, \Omega \) are defined in (6) to (7), \( K_f \) is a diagonal positive definite matrix, and \( \tilde{F}^d_{\text{int}}, \tilde{F}_{\text{int}} \) are the filtered internal force/moment coordinates, which are obtained as:

\[
\begin{align*}
\dot{\tilde{F}}^d_{\text{int}} &= \lambda_f \left( F^d_{\text{int}} - \tilde{F}^d_{\text{int}} \right) \\
\dot{\tilde{F}}_{\text{int}} &= \lambda_f \left( F_{\text{int}} - \tilde{F}_{\text{int}} \right)
\end{align*}
\]

(17)

with \( \lambda_f \) being a diagonal positive definite matrix.

Step 2 In this step, the goal is to virtually decompose (Brahmi et al., 2016) the robotic system into several parts and open chain elements. Each part is a rigid body, and an open chain consists of a series of rigid links connected one-by-one.

Assumption 4.1: In this paper, the manipulators are operating away from any singularity.

The required velocity in any frame \( B_j \) is given by:

\[
\begin{align*}
V^c_\alpha &= J_\alpha \dot{q}_j \\
V^c_\beta &= J_\beta \dot{q}_j
\end{align*}
\]

(18)
with $V_{\text{in}} = [q_{ij}^*, V_{\text{in}}^T, V_{\text{in}}^{T1}, V_{\text{in}}^{T2}, \ldots, V_{\text{in}}^{TN}]^T$, $J_{\text{in}}$, $J_{\text{in}}$ being the Jacobian matrix, and $\dot{q}_{ij}^* = [\dot{q}_{ij}^{\text{in}}, \dot{q}_{ij}^{\text{in}1}, \dot{q}_{ij}^{\text{in}2}, \ldots, \dot{q}_{ij}^{\text{in}N}]$ being the required joint angular velocities.

The dynamics of the object (3) based on its required velocity $aV \in \mathbb{R}^6$ and their estimated parameter is expressed in linear form by the following equation:

$$aF^* = Y_o \dot{\theta}_o + K_o (aV - V)$$  \hspace{1cm} (19)

where $aF^* \in \mathbb{R}^6$ is the required object force, $\dot{\theta}_o = \rho_o Y_o s_o \in \mathbb{R}^{13}$ is the adaptation function, and is chosen to ensure system stability, $s_o = (aV - oV)$, $\rho_o, K_o$ are positive gains, and $Y_o \in \mathbb{R}^{6 \times 13}$ is the dynamic regressor matrix, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010).

The required force/moment vectors at the $N$ end-effectors are computed from (10) as:

$$\tau_F = \tau_o U_o [\Phi_m \ \Phi_f] aF^*$$  \hspace{1cm} (20)

The control equation of the $j^{th}$ rigid body of the $i^{th}$ manipulator (10), based on its required velocity and its estimated parameters, is given in linear form by the following equation:

$$F_{ij}^* = Y_{ij} \dot{\theta}_{ij} + K_{ij} (V_{ij}^* - V_{ij})$$  \hspace{1cm} (21)

where $\dot{\theta}_{ij} = \rho_{ij} Y_{ij} s_{ij} \in \mathbb{R}^{13}$ is the adaptation function, and is chosen to ensure system stability; $s_{ij} = (V_{ij}^* - V_{ij})$, $\rho_{ij}, K_{ij}$ are positive gains, and $Y_{ij} \in \mathbb{R}^{6 \times 13}$ is the dynamic regressor matrix, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010).

The vector of resulting forces/moments acting on the $j^{th}$ rigid body is given by an iterative process (Al-Shuka et al., 2014).

We begin by computing the vector of forces at the different cutting points:

$$F_{\text{in}}^* = F_{\text{in}}^* + \rho_{\text{in}} U_{\text{in}} \tau_{\text{in}} F^*$$
$$F_{\text{in}+1}^* = F_{\text{in}+1}^* + \rho_{\text{in}} U_{\text{in}} F_{\text{in}}^*$$
$$\vdots$$
$$F_{ij}^* = F_{ij}^* + \rho_{ij} U_{ij} F_{ij}^*$$
$$F_{ij+1}^* = F_{ij+1}^* + \rho_{ij} U_{ij} F_{ij}^*$$
$$\vdots$$

The control equation of the $j^{th}$ joint actuator of the manipulator arm and the mobile platform driving motor (13) are expressed by the following expression:

$$\tau_{ij}^* = Y_{ij} \dot{\theta}_{ij} + K_{ij} (\dot{q}_{ij}^* - \dot{q}_{ij})$$  \hspace{1cm} (22)

where $\dot{\theta}_{ij} = \rho_{ij} Y_{ij} \tau_{ij} s_{ij}$ is the adaptation function, and is chosen to ensure system stability; $s_{ij} = (\dot{q}_{ij}^* - \dot{q}_{ij})$, $\rho_{ij}, K_{ij}$ are positive gains, and $Y_{ij}$ is the dynamic regressor (row)
vectors, defined in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010) and \(j = wR, wL, 1, \ldots, m\).

Finally, from (21) to (22) and (23), the control equation of the \(i\)th mobile manipulator mobile robot can be written as follows.

\[
\tau_{wj} = \tau_{wj}^* + z F_{wj}^*
\]

with \(j = wR, wL, 1, \ldots, m\) and \(z\) defined in (10). The block diagram in Figure 4 shows the different control law calculation and implementation steps.

Figure 4  Adaptive coordinated control of \(N\) MMRs (see online version for colours)

4.3 Stability analysis

Consider the \(j\)th rigid dynamics (10) to (12) and the joint actuator dynamics (13), under the control design (21) to (23). The control objective is satisfied and the error tracking states are asymptotically stable.

Remark 1: The global stability of the system using the VDC approach is proven through the virtual stability of each subsystem (Brahimi et al., 2016; Al-Shuka et al., 2014).

Proof: To prove the stability, we consider the following Lyapunov function:

\[
V = \sum_{i=1}^{N} \left( \sum_{j=1}^{N} V_{ij} + \sum_{j=1}^{N} V_{a_{ij}} + V_{iv} \right) + V_{ob} + V_{f}
\]
where $V_j$, $V_{aj}$, $V_{iv}$, $V_{ob}$ and $V_f$ are non-negative Lyapunov candidate functions related to the $j$th rigid link, the $j$th joint, the mobile platform of the $i$th MMR, the handled object and the internal force, respectively. These Lyapunov candidate functions are chosen as follows.

$$
V_j = \frac{1}{2} \left( V'_{B_j} - V_{B_j} \right)^T M_{B_j} \left( V'_{B_j} - V_{B_j} \right) + \frac{1}{2} \sum_{k=1}^{13} \frac{\left( \theta_{ak} - \dot{\theta}_{ak} \right)^2}{\rho_{ak}}
$$

$$
V_{aj} = \frac{1}{2} J_{a_j} \left( \dot{q}_{aj} - \dot{\dot{q}}_{aj} \right)^2 + \frac{1}{2} \sum_{k=1}^{13} \frac{\left( \theta_{ak} - \dot{\theta}_{ak} \right)^2}{\rho_{ak}}
$$

$$
V_{iv} = \frac{1}{2} \left( V'_{iv} - V_{iv} \right)^T M_{iv} \left( V'_{iv} - V_{iv} \right) + \frac{1}{2} \sum_{k=1}^{13} \frac{\left( \theta_{ak} - \dot{\theta}_{ak} \right)^2}{\rho_{ak}}
$$

$$
V_{ob} = \frac{1}{2} \left( \alpha_{V^r} - \alpha_{V} \right)^T M_{o} \left( \alpha_{V^r} - \alpha_{V} \right) + \frac{1}{2} \sum_{k=1}^{14} \frac{\left( \theta_{ak} - \dot{\theta}_{ak} \right)^2}{\rho_{ek}}
$$

$$
V_f = \frac{1}{2} \left( \bar{F}_{int} - \bar{F}_{int} \right)^T K_f \bar{r}_f^2 \left( \bar{F}_{int} - \bar{F}_{int} \right)
$$

The first derivative of the Lyapunov candidate function (25) is given as follows:

$$
\dot{V} = \sum_{i=1}^{N} \sum_{j=1}^{a} \dot{V}_j + \sum_{j=1}^{n} \dot{V}_{aj} + \sum_{j=1}^{n} \dot{V}_{iv} + V_{ob} + V_f
$$

(27)

By using the definition of the virtual power and the choice of the parameter function adaptation as in (20) to (21) and (23), it is straightforward to prove that $\dot{V}$ is always decreasing, and is given as follows.

$$
\dot{V} = -\sum_{i=1}^{N} \sum_{j=1}^{a} \left( V'_{B_j} - V_{B_j} \right)^T K_{B_j} \left( V'_{B_j} - V_{B_j} \right)
$$

$$
+ K_{a_j} \left( \dot{q}_{aj} - \dot{\dot{q}}_{aj} \right)^2 + \left( V'_{iv} - V_{iv} \right)^T K_{iv} \left( V'_{iv} - V_{iv} \right)
$$

$$
- \left( \alpha_{V^r} - \alpha_{V} \right)^T K_o \left( \alpha_{V^r} - \alpha_{V} \right) - \left( \bar{F}_{int} - \bar{F}_{int} \right)^T K_f \left( \bar{F}_{int} - \bar{F}_{int} \right)
$$

(28)

The stability analysis shows that $\dot{V}$ is always decreasing, and that the system is asymptotically stable in the sense of Lyapunov. Using Barbalat’s lemma (Spong et al., 2006), we prove that the error tracking states are asymptotically stable. The reader can find the detailed proof of stability in Zhu et al. (1997), Al-Shuka et al. (2014) and Zhu (2010).
5 Simulation results

The block diagram in Figure 5 shows the different control law development and simulation steps.

Figure 5  Adaptive control of \( N \) MMRs transporting a rigid object (see online version for colours)

Numerical simulations are carried out on two identical 6DoF MMRs handling a rigid object in coordination, as illustrated in Figure 6.

Figure 6  Two identical 6DoF mobile manipulators (see online version for colours)
The desired trajectory of the centre of gravity of the object is generated in the Cartesian space. The object displacement is along the X-axis, with a sinusoidal trajectory along the Y-axis, and no rotation along the Z-axis. In this case, there is no displacement along the Z-axis, and no rotation along the X-axis and the Y-axis. The starting point is \( P_0 = (x_0, y_0, z_0, \beta_0) = (2, 0.5, 1, 0) \) and the final point is \( P_f = (x_0, y_0, z_0, \beta_0) = (2, 0.5, 1, 0) \). The controls gains of the controller are chosen to be \( K_{ijB} = 25, \ K_{ija} = 15, \ K_o = 50, \ K_i = 5, \ \rho_o = .7, \ \rho_{ijB} = 0.8 \) and \( \rho_{ija} = 0.8 \). The trajectory tracking is presented in Figure 7 and Figure 8. A good position and orientation tracking can be observed. The convergence of the error to zero along the XYZ positions and the moment along the Z-axis are presented in Figure 8.

Figure 7 Desired and real trajectories of the object (see online version for colours)

Figure 8 Error in X-axis, in Y-axis, in Z-axis and in orientation (see online version for colours)
6 Experimental validation

In this section, the proposed control scheme is implemented in real time on two identical MMRs named Mob_ETS located in the GREPCI laboratory. In this experimental test, a Zigbee technology communication is used between the application program implemented in Simulink MATLAB® and the MMRs. The adaptive control developed and simulated in the previous section is implemented and compared to the computed torque approach in real time using real-time workshop (RTW) by Mathworks®. Since the external end-effector force is unavailable for measurement, we use an end-effector observer proposed in Alcocera et al. (2003) to estimate it in this section. Figure 9 shows the complete structure design of the control.

The two wheels of the $j^{th}$ MMR platform are actuated by two DC motors, HNGH12-2217Y (DC-12V-200RPM 30:1), and its angular positions are given by using encoder sensors (E4P-100-079-D-H-T-B). All the joints of the manipulator arm are actuated by a Dynamixel motor (MX-64T).

The desired trajectory of the centre of gravity of the object is generated in the Cartesian space. The object displacement is along the X-axis, with a sinusoidal trajectory along the Y-axis. The starting point is $P_0 = (x_0, y_0, z_0, \beta)$ $(0.1, -0.1, 0.42, 0)$ and the final point is $P_f = (x_0, y_0, z_0, \beta)$ $(3, 0.1, 0.47, 0)$. The control gains of the controller are chosen to be $K_{\beta_0} = 25$, $K_{\beta_0} = 15$, $K_o = 5$, $K_i = 5$, $\rho_o = 0.7$, $\rho_{\beta_0} = 0.8$ and $\rho_{\beta_0} = 0.8$. The sampling time is set to 0.015 seconds.
The trajectory tracking is presented in Figure 10 and Figure 11. A good position and orientation tracking can be observed. The convergence of the errors to zero along the XYZ positions and the moment along the Z-axis are presented in Figure 11, and the convergence of the parameters of the first mobile manipulator during the adaptive control is illustrated in Figure 12 as an example, where Figure 12(a) represents the convergence of all estimated parameters of the first manipulator robot $\hat{\theta}_{B_j}$ with $j = w_R, w_L, 1, \ldots, m$ and Figure 12(b) to Figure 12(c) shows the convergence of the parameters of only two links $\hat{\theta}_{B_1}, \hat{\theta}_{B_2}$ of this manipulator mobile robot.

Figure 10  Desired and real trajectories of the object (see online version for colours)

Figure 11  Error in X-axis, in Y-axis, in Z-axis and in orientation (see online version for colours)
To show the effectiveness of the control strategy tested above, the computed torque (Slotine and Li, 1991; Papadopoulos and Poulakakis, 2000) is used for the same mobile manipulators. Figures 13 to 14 show the experimental results for the computed torque approach using the same desired trajectories.

**Figure 13** Desired and real trajectories of the object (see online version for colours)
Figure 14  Error in X-axis, in Y-axis, in Z-axis and in orientation (see online version for colours)

For purposes of comparison, the multi-mobile manipulators handling the object are controlled by applying the computed torque method, using the same desired trajectory. The tracking of the position and orientation in the workspace is shown in Figure 13, and errors along the XYZ positions and the moment along the Z-axis are presented in Figure 14. According to the experimental results shown in Figure 15, the resulting tracking errors of the proposed control strategy (dashed line) are smaller than those found using the computed torque method (solid line). This illustrates the effectiveness of the adaptive coordinated approach developed in this paper.

Figure 15  Errors: adaptive control (dashed red line), computed torque (solid blue line)
7 Conclusions

In this paper, a coordinated control scheme for multiple MMRs transporting a rigid object in coordination has been presented. The desired trajectory of the object is generated in the workspace and the parameters of the handling object and that of the mobile manipulators are estimated online using the virtual decomposition approach. In this study, the external forces are considered available. The control law is designed based on the virtual decomposition approach, and the global stability of the system is proven through the virtual stability of each subsystem. The proposed control design ensures that the workspace position error converges to zero asymptotically. This controller is tested and is compared with the computed torque approach. The simulation and experimental results show the effectiveness of the proposed control and illustrate the validation of the theoretical development.

References


Adaptive coordinated control of multi-mobile manipulator systems


