Reliability block diagram methods for system reliability analysis of spatial structures

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Abstract: The safety of complex spatial structures in operation depends highly on the reliability of the whole system and its structural components. In order to determine the relationship between system reliability and reliability of its components, reliability block diagram (RBD) method, which emphasises depicting the connection mode among all the components of a system, is employed for system reliability analysis of spatial structures. With the assumption of geometric stability, a new approach, aiming at finding out key elements where damage could lead to overall collapse of the structure, is adopted here for the logic relationship identification among components. The
logic relationship, which is considered as the core component of the reliability block diagram, is utilised for reliability analysis of the system and the predetermined components. Finally, a large-span latticed grid spatial structure is investigated to validate the applicability of the RBD method for analysing the system reliability of spatial structures.

**Keywords:** reliability block diagram; RBD; system reliability; structural damage; component reliability; key elements; series-parallel system.


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### 1 Introduction

For modern civil engineering structures with structural complexity and diversity to meet the growing demands, the increasing aggressive operation environments and service life could cause serious economic losses and safety hazards once the structure fails. Due to the urgent needs for managing extensive aged civil structures for the longer service lifetime, research on the structural reliability and safety has attracted more and more attention. Therefore, structural reliability evaluation not only can reduce the occurrence of structural accidents, but also can provide knowledge basis for the repair and
maintenance strategy in order to extend the service lifetime of structure under the condition of structural safety.

The existing reliability analysis of structures can be undertaken at two different levels, namely at component level and system level (Zhao, 2000). The ultimate limit state method used in current structural design is typically based on the reliability of structural components, instead of the system, which may be inadequate in the safety evaluation of the structural system.

In the analysis of structural reliability, the structure with only one failure mode can be defined as ‘component’, while the structure with several failure modes is defined as ‘system’. The ‘system’ is considered here to be a structure that consists of a large number of components to fulfil a specified function. It is much simpler to calculate the reliability of components, comparing to the efforts required for evaluating system reliability. However, with the advance of high-strength materials and the complexity of structural design, it becomes increasingly important to analyse the reliability of the structural system with a great number of components, particularly for complex structures such as high-rise buildings (Bao and Fang, 1990). In general, the structural system that is not affected by a single failed component will be unstable only when the failed components reach a sufficient number. Therefore, there is a need to analyse the system reliability of large-scale structures such as complex framed structures.

A large number of investigations on component reliability analysis have been undertaken (Murthy et al., 2008, 2009; Smith and John Clarkson, 2005), but very limited studies on the analysis of the system’s reliability of structures, in particular spatial structures, are available. Therefore, research on how to calculate the system reliability from the component reliability will be necessary in structural reliability analysis. According to the recursive relationship from global level to local level, a complex system can be considered as a combination of several simple subsystems, where connection modes among components can be obtained by numerical analyses such as the finite element method (Frangopol et al., 2001; Gu et al., 2004).

A reliability block diagram (RBD) is a graphical analysis technique involving its distinctive components by using graphical representation. This can be used to analyse the influence of components on the system’s functional properties of failure components. The blocks and lines within the block diagram represent the groups of components and functional logic relationship, respectively. Obviously, the order of blocks linked, depending on their effects on the system, can be variable. Each subsystem can be composed of one or more elements, which cannot be further divided. Thus, the modelling of RBD can be considered as a process of element expansion.

The practical application of RBDS can be mainly found in electrical, electronic, aerospace, communication and machinery fields (Sahinoglu et al., 2004; Wang et al., 2004a; Abd-Allah, 1997; Zhang and Gao, 2008). Due to the numerous structural components in spatial structures, the challenge of the application of this method is how to find out the location in RBD, thus it has not been widely utilised in structural engineering before. By taking advantage of the division of subsystems, the position of each component in RBD system can be gradually located (Guo and Yang, 2007; Kim, 2011; Lin et al., 2010).

Space structures with advantages of reasonable stress, light weight, and large stiffness are widely used in hospitals, railway stations, stadiums, and other large public buildings. Public space structures may become the main shelters when catastrophe happens (Dong et al., 2005; Lan, 2001). Therefore, the system reliability analysis for these
structures is of particular importance (Chen and Xiao, 2012, 2015). This paper describes an approach of RBD applied to structural system reliability analysis in latticed grid structures.

2 Component reliability

A structure generally operates with uncertainties during a predefined period of time under given conditions in order to avoid structural failures. Research shows that probability can be used to quantify the structural capacity accomplishing the predetermined functions, and the probability is defined here as the system’s reliability.

For any component $E_i$ of a given structure, the expected value of the failure-free time $\tau$ is represented as $MTTF$ (mean time to failure). The reliability function $R(t)$, a measure of the ability of the system to provide a required performance with no failure in the time interval $(0, t]$, can be calculated as (Birolini, 2007)

$$R(t) = \Pr \{\tau > t\}$$

$$MTTF = E[\tau] = \int_0^\infty R(t)dt$$

The failure rate $\lambda(t)$ of a component is the probability of a failure in the given interval $(t, t + \delta t]$ and did not fail in the interval $(0, t]$,

$$\lambda(t) = \lim_{\delta t \to 0} \frac{\Pr\{t < \tau \leq t + \delta t \mid \tau > t\}}{\delta t}$$

The relationship between reliability and failure rate of the component can be expressed as

$$\dot{\lambda}(t) = \frac{1}{R(t)} \frac{dR(t)}{dt}$$

Assuming reliability $R(0) = 1$ at initial state, the component reliability of the individual member of the structural system is:

$$R(t) = e^{-\int_0^t \lambda(s)ds}$$

In practices, the structures used in engineering should be designed according to structural system reliability. So the importance of structural system reliability in design process and safety evaluation is apparent.

The component failure and overall structural failure are considered as the criterion for component reliability and system reliability, respectively. The common methods focus on identifying the principal failure mode, and they are adopted for calculating system reliability (Chen et al., 2005; Feng and Dong, 1991). It is difficult to determine all failure modes, causing additional difficulty in determining the main failure mode. Therefore, the RBD method based on the overall performance of structural system is here proposed, and appropriate performance target is utilised to analyse the system reliability.
3 RBD

A RBD is an important tool for the calculation of structural system reliability, representing a direct graphical depiction of the system’s components and their connections. The diagram can be used to determine the overall system reliability if the reliabilities of its components are available. From reliability analysis, the RBD expresses the concerned system as connections of a number of components in accordance with their logical relation of reliability, and represents the effects of component failures on system performance (Guo and Yang, 2007).

From the RBD, it can be easy to identify key elements in the structural system for the fulfilment of the required functions. Here, the key elements are defined as the elements that have critical effect on the overall stability of the structure. Meanwhile, it is straightforward to obtain the logic relationship between the component reliability and system reliability. Obviously, with the different functional targets and compositions of elements, each structure has its own specific RBD. In general, the logical relationship can be classified as series, parallel or voting connections, which are the basic connections for complex connection such as bridge connections. Currently, the RBD method is mainly utilised in the electronics field (Sahinoglu et al., 2004; Wang et al., 2004b). For structural engineering, the RBD has been mainly applied in bridge structures (Estes and Frangopol, 1999).

In order to simplify the calculation by using the RBD method, the following assumptions are made (Birolini, 2007), i.e.,

1. the reliability of all components in a system is assumed to be independent of each other
2. only two states, i.e., failure state and non-failure state, are considered for all components
3. there is only one failure mode for a single component.

3.1 Series connection

For a system consisting of $n$ components, if all elements must work in order to fulfil the required functions, and any single component failure would cause the entire system failure, the relations between components are in series, and the corresponding system is defined as a series system.

Considering the series system reliability model shown in Figure 1, each block is in a random order without any effect on system model.

**Figure 1** RBD of structure in series

The probability of component $E_i$ working without failure in the interval $(0, t]$ is:

$$R_i(t) = Pr\{\tau_i > t\} = Pr\{E_i\}$$ (6)
Based on the definition of the series system, the system does not fail in the interval \((0, t]\) if and only if all elements work in non-failure condition at the same time. Thus, according to the theory of full probability and the failure mode of each component \(\lambda_i(x)\) \((i = 1, 2, \ldots, n)\), the probability of the whole system working without failure is:

\[
R_{S0}(t) = \Pr \{ E_1 \cap \ldots \cap E_n \} = \prod_{i=1}^{n} R_i(t)
\]

\[
e^{-\int_{0}^{t} \sum_{i=1}^{n} \lambda_i(x) dx}
\]

where \(S0\) stands for the whole system starting point of \(t = 0\).

### 3.2 Parallel connection

For a system consisting of \(n\) elements, the required function is fulfilled if at least one of the \(n\) elements work without failure. This connection mode is in parallel, and the corresponding system is defined as the parallel system. The system model for parallel system is shown in Figure 2.

**Figure 2** RBD of structure in parallel

The reliability of any element \(E_i\) working without failure in the interval \((0, t]\) is written as:

\[
R_i(t) = \Pr \{ \tau_i > t \} = \Pr \{ E_i \}
\]

With the definition of parallel system and the theory of full probability, in the case with only two elements, the probability of system working in non-failure condition is as follows:

\[
R_{S0}(t) = \Pr \{ E_1 \cup E_2 \} = \Pr \{ E_1 \} + \Pr \{ E_2 \} - \Pr \{ E_1 \cap E_2 \}
\]

\[
R_{S0}(t) = R_1(t) + R_2(t) - R_1(t)R_2(t)
\]

\[
e^{-\int_{0}^{t} \lambda_1(x) dx} + e^{-\int_{0}^{t} \lambda_2(x) dx} - e^{-\int_{0}^{t} [\lambda_1(x) + \lambda_2(x)] dx}
\]

In the cases with general situations, the system reliability for a given system with \(n\) elements is:

\[
R_{S0} = 1 - \prod_{i=1}^{n} (1 - R_i)
\]
3.3 Voting connection

A system with $n$ components, where $k$ elements are necessary to perform the required function and the remaining $n-k$ elements are in reserve, is designated as $k$-out-of-$n$ redundancy system or voting system. It is obvious that the voting system is a series system when $k = n$ and is parallel system when $k = 1$.

From the theory of full probability, the system reliability for $k$-out-of-$n$ redundancy system is:

$$R_{SO}(t) = \sum_{i=k}^{n} \binom{n}{i} R^i(t)(1-R(t))^{n-i}$$

(12)

3.4 Complex connection

The system reliability can be calculated from above equations for relatively simple engineering systems. However, the structural systems in civil engineering often have complex connections, such as series parallel connection, parallel series connection and other combined connections, as shown in Figure 3. In the process of calculations, it is effective to simplify the complex system into a combination of several basic connections step by step.

**Figure 3** Simplification of RBD

In the case for the system shown in Figure 3, from the reliability for each individual components, the system reliability can be obtained from:

$$R_7(t) = R_1(t)R_2(t)R_3(t)$$

(13)

$$R_6(t) = R_4(t)R_5(t)$$

(14)

$$R_5(t) = R_7(t) + R_6(t) - R_7(t)R_6(t)$$

(15)

And the system reliability is calculated from:

$$R_{SO}(t) = R_6(t)R_8(t)$$

$$= [R_6(t) + R_7(t) - R_7(t)R_6(t)]R_8(t)$$

$$= [R(t)R_2(t)R_3(t) + R_4(t)R_5(t) - R(t)R_2(t)R_3(t)R_4(t)R_5(t)]R_8(t)$$

(16)
4 Key elements

In general, as component reliability increases, the system reliability of the existing structures increases. However, the increase in component reliability may not necessarily improve the system reliability, causing unnecessary waste of resources. Structures in operation usually require for regular maintenance, thus it may not be possible to test and repair all components in practical operation. Therefore, there is a need to determine the relatively important elements for undertaking system reliability analysis and scheduling maintenance during the system’s service life. For the complex civil engineering structures in practice, the representation of the key elements in RBD can become difficult and time-consuming. Therefore, these important elements should be determined in the reliability analysis for the structural systems.

All elements in a system can be divided into key elements and non-key elements. Based on the overall structural failure criterion, a key element or an important element is defined as the element that can lead to the failure of further structural elements and thus cause the collapse of large structural section or the entire structure, after its failure. Moreover, the structural overall performance evaluation criterion may depend on operation situations, leading to the different key elements. Thus, the key elements of a structure are undetermined, largely relying on the given performance criteria.

As stated in the definition of key elements, it should be more effective to adjust the geometry and material properties of key elements in order to improve the system reliability. Thus, during modelling for the RBD, the determination of corresponding key elements under the pre-set overall performance criterion could be a crucial step.

For the determination of key elements, different methods have been adopted in many investigations (Torres-Toledano and Sucar, 1998; Bennetts, 1982; He et al., 2009; Zhan, 2014). A method is proposed by He et al. (2009) to determine whether the damaged element is a key element, based on the fact whether the local damage can cause the strength degradation of other elements. To overcome the disadvantages, Zhan (2014) proposed a more comprehensive and reasonable approach, by using the geometric stability and nodal displacements. In the case with finite displacement generated and the self-equilibrium in new formed structure, the further damage, including cable slackening and internal force exceeding the material strength, should be considered to emphasise the overall performance of structure. Obviously, the key elements are connected in series, while elements that can fail with no damage on structural integrity are connected in parallel. After the key elements are identified and the RBD is constructed, the system reliability of complex structures can be obtained from reliability analysis (Chen and Alani, 2012, 2013).

5 Case study

The reliability analysis from component to system is carried out to take the predetermined overall performance into account, by integrating the RBD method with the key element identification method. Geometric stability and finite nodal displacement are considered as the criterion for performance assessment in the case study. Obviously, all elements of statically determinate structures are in series connection, while the connection mode of statically indeterminate structures depends on the specific situations.
5.1 Truss structure

The structure shown in Figure 4 is a statically indeterminate structure with one redundant element.

Figure 4 A statically indeterminate plane truss structure

It should be noted that the new formed structure after removing any one of components may become unstable. From structural analysis, only one of the elements with numbering from three to eight can be removed to become a statically determinate structure, which forms a 5-out-of-6 voting system. The RBD with parallel connection modes is shown in Figure 5.

Figure 5 RBD of the statically indeterminate structure

Assume that all elements have the same reliability with $R$, the system reliability is calculated from

$$R_{50}(t) = R^t \sum_{i=5}^{k} \binom{n}{i} R^i (1-R)^{n-i}$$

(17)

5.2 Latticed grid structure

A latticed grid structure with a design reference period of 50 years, simply supported at four corners, is employed for further investigations, as shown in Figure 6. The structure has overall plane dimensions of $9 \times 9$ m and height of 1.5 m. The numerical model for the structure has 25 nodes and 72 elements, including 24 upper chord elements, 12 lower chord elements and 36 diagonal elements. Every member has the same specification, $\phi 140 \times 4$. The structure is simply supported at four corners where their displacements in all three directions are constrained.
The space grid structure system can be divided into three parts, i.e., upper chords, lower chords and diagonal members. These three parts are analysed below in detail. On the basis of geometric topology relationship between elements and connections, a structure can be classified as geometrically stable system and geometrically unstable system. The first-order natural frequency of a value of zero is a sufficient condition for validating if the structure becomes a mechanism, since the structure has a rigid body motion. Comparing with other indices, the first-order natural frequency is easier to be obtained from the finite element analysis, such as by using software package ANSYS. Therefore, the first-order frequency is adopted here to check the stability of the structures concerned.

5.2.1 Upper chords analysis

Figure 7 shows the upper chords as a part of the whole structure. According to the locations of elements and symmetry, the upper chords can be further divided into three parts, i.e., middle (Subsystems 1 and 3), edge (Subsystem 2) and corner (Subsystem 4). The details of these subsystems are listed in Table 1.
Figure 7  Subsystems of upper chords (a) Subsystem 1 (b) Subsystem 2 (c) Subsystem 3 (d) Subsystem 4

Table 1  Upper chords subsystem

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Element number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10, 11, 13, 17</td>
</tr>
<tr>
<td>2</td>
<td>3, 9, 14, 23</td>
</tr>
<tr>
<td>3</td>
<td>4, 6, 8, 12, 15, 19, 18, 20</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 5, 7, 16, 21, 22, 24</td>
</tr>
</tbody>
</table>

To simplify calculation, the reliability of individual elements are assumed to be independent of each other. Reliability analysis can then be undertaken by assuming that each component follows exponential distribution with constant failure rate $\lambda = 1.026 \times 10^{-3}$/year. The failure rate is estimated from the probability of failure of the typical structural members subject to steel corrosion due to aggressive environment (Xiao et al., 2014). For all elements during the service life ($t = 50$ years), the component reliability is written as:

$$ R_i(t) = e^{-\lambda t} = e^{-\left(1.026 \times 10^{-3}\times 50\right)} \approx 0.95 $$

(18)

For the four series subsystems, their reliability values are obtained, respectively, as:

$$ R_{Sub1}(t) = \Pr \{ E_{i10} \cap E_{i11} \cap E_{i13} \cap E_{i17} \} = \prod_{i=1}^{4} R_i(t) \approx 0.8145 $$

(19)

$$ R_{Sub2}(t) = \Pr \{ E_3 \cap E_6 \cap E_{14} \cap E_{23} \} = \prod_{i=1}^{4} R_i(t) \approx 0.8145 $$

(20)
From the analysis of natural frequency for various structures after removing different subsystems in upper chords, the results show that the first-order natural frequency of the structure becomes zero, when Subsystem 3 or Subsystem 4 is removed from the structural system. This indicates that the modified structure will collapse due to its instability. On the other hand, the removal of Subsystem 1 or Subsystem 2 will not lead to structural collapse, and the maximum nodal displacement in the modified structure subjected to normal loads is almost the same as that of the original structure. Therefore, the Subsystem 3 and Subsystem 4 can be considered as series relationships in the whole system, and further analysis is needed for Subsystems 1 and 2.

5.2.2 Lower chords analysis

Figure 8 shows the lower chords of the structural system with two subsystems, i.e., Subsystem 5 and Subsystem 6.

Figure 8 Subsystems of lower chords

Similarly, according to structural symmetry, the lower chords can be divided into inner part (Subsystem 5) and peripheral part (Subsystem 6). The details of these two subsystems are listed in Table 2.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Element number</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>28, 30, 32, 33</td>
</tr>
<tr>
<td>6</td>
<td>25, 26, 27, 29, 31, 34, 35, 36</td>
</tr>
</tbody>
</table>

The reliability values of these two series subsystems can be calculated, respectively, from

$$R_{Sub3}(t) = \Pr\{E_4 \cap E_6 \cap E_8 \cap E_{12} \cap E_{15} \cap E_{19} \cap E_{18} \cap E_{20}\}$$

$$= \prod_{i=1}^{8} R_i(t) \approx 0.6634$$ (21)

$$R_{Sub4}(t) = \Pr\{E_1 \cap E_2 \cap E_5 \cap E_7 \cap E_{16} \cap E_{21} \cap E_{22} \cap E_{24}\}$$

$$= \prod_{i=1}^{8} R_i(t) \approx 0.6634$$ (22)
The results from the analysis of structural dynamic characteristics show that the removal of elements in Subsystem 5 will not have significant effect on the remaining structure, while removing elements in Subsystem 6 will lead to vanishing the first-order natural frequency. As a result, Subsystem 6 is considered as series part of the entire system, and further analysis is needed for Subsystem 5.

5.2.3 Diagonal members analysis

Diagonal members with node numbering are shown in Figure 9. From inner towards outer, the diagonal elements can be categorised as six subsystems as shown in Table 3.

![Figure 9](image)

Table 3 Diagonal member subsystem

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Element number</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>53, 54, 55, 56</td>
</tr>
<tr>
<td>8</td>
<td>41, 44, 49, 50, 59, 60, 66, 67</td>
</tr>
<tr>
<td>9</td>
<td>42, 43, 51, 52, 57, 58, 65, 68</td>
</tr>
<tr>
<td>10</td>
<td>37, 48, 62, 71</td>
</tr>
<tr>
<td>11</td>
<td>38, 40, 45, 47, 61, 63, 70, 72</td>
</tr>
<tr>
<td>12</td>
<td>39, 46, 64, 69</td>
</tr>
</tbody>
</table>
The reliability values of these six series subsystems can be obtained, respectively, from

\[ R_{\text{Sub}1}(t) = \Pr \left\{ E_{33} \cap E_{34} \cap E_{35} \cap E_{36} \right\} = \prod_{i=1}^{4} R_i(t) \approx 0.8145 \]  

\[ R_{\text{Sub}2}(t) = \Pr \left\{ E_{41} \cap E_{44} \cap E_{49} \cap E_{50} \cap E_{59} \cap E_{60} \cap E_{66} \cap E_{67} \right\} = \prod_{i=1}^{8} R_i(t) \approx 0.6634 \]  

\[ R_{\text{Sub}3}(t) = \Pr \left\{ E_{42} \cap E_{43} \cap E_{51} \cap E_{52} \cap E_{57} \cap E_{58} \cap E_{65} \cap E_{68} \right\} = \prod_{i=1}^{8} R_i(t) \approx 0.6634 \]  

\[ R_{\text{Sub}4}(t) = \Pr \left\{ E_{37} \cap E_{48} \cap E_{62} \cap E_{71} \right\} = \prod_{i=1}^{4} R_i(t) \approx 0.8145 \]  

\[ R_{\text{Sub}5}(t) = \Pr \left\{ E_{38} \cap E_{40} \cap E_{45} \cap E_{47} \cap E_{61} \cap E_{63} \cap E_{70} \cap E_{72} \right\} = \prod_{i=1}^{8} R_i(t) \approx 0.6634 \]  

\[ R_{\text{Sub}6}(t) = \Pr \left\{ E_{39} \cap E_{46} \cap E_{64} \cap E_{69} \right\} = \prod_{i=1}^{4} R_i(t) \approx 0.8145 \]  

Similarly, the results from the analysis of structural dynamic characteristics show that the removals of any element in Subsystem 7, 8, 9, 10, or 11 will directly lead to the modified structure unstable. However, the modified structure keeps stable after the removal of the elements in Subsystem 12.

The Subsystems 1, 2, 5 and 12 and their relationships are further analysed for stability. From the mechanism analysis for pin-jointed spatial structures, there are three cases for stability check (Fowler and Guest, 2000), i.e.,

1. \[ W = B + C - 3J > 0 \], the space grid structure system has redundant elements, i.e., indeterminate system
2. \[ W = B + C - 3J = 0 \], the space grid structure system has no redundant elements, i.e., determinate system
3. \[ W = B + C - 3J < 0 \], the space grid system is mechanism, i.e., instable system.

Here \( B \) represents the number of element, \( C \) represents the number of constrain in bearing and \( J \) represents the number of node. In order to maintain structure geometrically stable, at most \( 72 + 12 - 25 \times 3 = 9 \) elements in the structure can be removed. As shown before, there are at least four elements in each subsystem. Therefore, at most two subsystems in the whole structure can be removed to keep the remaining structure stable. From the analysis above, the elements in Subsystems 3, 4, 6, 7, 8, 9, 10 and 11 are essential, since they are in series connection in the structure. However, for the remaining four subsystems, namely Subsystems 1, 2, 5, and 12, if any two of these subsystems work in
non-failure condition, the structural system will remain stable. The results for the first-order natural frequencies of modified structures after removing the corresponding combination of subsystems are summarised in Table 4.

Table 4  First-order frequencies for various subsystem removal combinations

<table>
<thead>
<tr>
<th>Removal of combination of subsystems</th>
<th>First-order frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem 1 + Subsystem 2</td>
<td>20.782</td>
</tr>
<tr>
<td>Subsystem 1 + Subsystem 5</td>
<td>0.0000</td>
</tr>
<tr>
<td>Subsystem 1 + Subsystem 12</td>
<td>0.0000</td>
</tr>
<tr>
<td>Subsystem 2 + Subsystem 5</td>
<td>19.942</td>
</tr>
<tr>
<td>Subsystem 2 + Subsystem 12</td>
<td>15.252</td>
</tr>
<tr>
<td>Subsystem 5 + Subsystem 12</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

From Table 4, the removal of combinations including Subsystem 2 will keep the modified structures stable, i.e., Subsystem 2 is removable without critical effect on the stability of the entire structural system. However, for Subsystems 1, 5 and 12, there are three possible combinations as shown in italics in Table 4. To keep the structure stable, two of three cases of these possible combinations cannot be removed, and only one of these possible combinations can be removed, generating 2-out-of-3 voting system. Thus, the corresponding RBD is shown in Figure 10, and the reliability value is expressed as follows

\[
R_{oo}(t) = \sum_{i=2}^{3} \binom{n}{i} R^{i}(t)(1 - R(t))^{n-i} \approx 0.9928
\]

Figure 10 2-out-of-3 voting system

Finally, the whole system consists of eight series connection subsystems (Subsystems 3, 4, 6, 7, 8, 9, 10 and 11) and one 2-out-of-3 voting system (Subsystems 1, 5 and 12). Thus the reliability value of the entire structure can be obtained from

\[
R_{S}(t) = \prod_{j=3\text{if }j\neq 5}^{11} R_{\text{Sub}}(t) \cdot R_{oo}(t) \approx 0.056
\]

Two processes are here included in combination of the entire system, namely from components to subsystems and from subsystems to the entire system. During expansion of RBD, in order to figure out the logic relationship among subsystems, all components in one subsystem are removed while the performance of remaining system is investigated. From the results, the logic relationships among subsystems are clear and dependent. However, with regard to the relationships among components within a
subsystem, it is assumed here that each component in a subsystem is essential for an entire subsystem, i.e., the relationships of components within a subsystem are conservatively simplified as in series. Thus, the reliability of system is conservative as well.

6 Conclusions

On the basis of the results for the numerical examples, the following conclusions are drawn:

1. The RBD method mainly adopted in electronic circuits is an important and applicable tool for the reliability analysis of civil engineering systems.

2. Under the condition with pre-confirmed goal for overall performance, the proposed approach for determining key elements is an effective technique for modelling RBD of a given complex structure.

3. The key elements of a specific structure can be determined, and the determination of key element relies highly on the given performance criterion associated with different goals.

4. The specified performance goal can be chosen for complex spatial structures, and then the system reliability is obtained from the reliability of individual components in the system. However, the system reliability for other types of complex structures, such as cable structures and shell structures, etc., should be further investigated under different performance criterions.

References


