Developing a multi-commodity multi-period mathematical model based on the travelling salesman problem for solving bike sharing rebalancing problem

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Abstract: The rebalance problem in the bike sharing system includes operational decisions to respond to demand fluctuations in the kidneys Bicycle stations, so that by redistributing bikes among the stations with a balancing fleet, users can be satisfied. In recent years, considering the cities growth and development and also traffic challenges it has been one of the major needs of the urban transportation network. In particular, the use of bicycles on intra-city trips In addition to reducing the heavy traffic volume, it also leads to positive ecological and environmental impacts. On the other hand, combining this balancing process with minimal cost can have positive economic effects for communities, and encourage users to use these systems more frequently. In this research, the development of a mathematical model based on travelling salesman problem (TSP) has been devised that this is multi-objective model, with considering excessive constraints and the lack of permissible and different types of bikes at these stations, and in multi-period mode, is able to minimise slacks and surpluses in stations, in addition to minimising stations balancing costs. Also, to validate the model and demonstrate its efficiency and performance, comprehensive examples have been solved using an exact solution approach.

Keywords: bike sharing system; bike sharing rebalancing problem; multi objective mathematical model; vehicle routing problem.

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1 Introduction

One of the pillars of sustainable development, especially in developed countries, is to reduce the level of dependency on personal vehicles Interurban transportation to public transportation such as rail, bus, cycling and walking. The use of a personal vehicle (car) has increased fuel consumption and, consequently, its price increase, traffic pollution, environmental pollution such as air pollution and noise pollution, etc., which is in contrast with the principles of sustainable development. In recent years, limiting the availability of personal automobiles to the core of the city, especially in metropolitan areas, is resulted in the use of motorcycles, and since motorcycles are more contaminated than cars, this has exacerbated air pollution in metropolitan areas (DeMaio et al., 2009). Recently, there are many cities around the world that encourages it's citizens to the usage of bicycles as a sustainable transport option in the environment and socially equality, and as a Supplement for other shipping systems (Raviv et al., 2013).

Bike sharing system is a common problem of distributing bikes among citizens in a simple and inexpensive process. The idea is to set up stations at the city, in which a registered user can easily borrow a bike from its special compartment and then return it to any other defined station (which has free space) (Di Gaspero et al., 2014). Usually a rental station does include a terminal and some bike stands. Terminal is defined as a device which is capable of communicating with electronic lockers, which they are connected to the bike stands. Any time a user wants to rent a bike, a signal is sent to the locker that has been left. Returning a bike to the station would be possible just when a vacant locker is available. A central control tool receives all rental and transactions and does record and report them in a real time. Consequently, each station, when there are some available bikes and some available vacant lockers then the operator would be informed about. On the other hand, bike sharing system's operators also make the info available to users.

Scientific research about the bike sharing system issue on the basis of strategy and operation subjects could be critical. These issues are respectively important in connection with system's sustainable economic performance. Solving all these challenges does require to develop different and proper optimisation approaches (Kadri et al., 2016)

A critical factor in the success of this system is the ability to meet the fluctuating demand for bicycle at each station, which is achieved by 'position change' operations, in which a number of bikes from some stations are taken to some other stations that are transported by a designated transport vehicle. One of the main complaints heard from users of these systems is the lack of access to the bike and, worse, there is no free space for delivery, and repetition of this situation will be the result of the customer's distrust and in the end, they will not leave the system (Raviv et al., 2013). Therefore, in order to increase system capacity and user satisfaction, it is necessary that bicycles be properly redistributed between stations (Caggiani et al., 2012).

Redistribution, usually by using a fleet of vehicles Limited capacity is based on a central depot that stores bicycles from stations. The issue of optimising the decision on how vehicles should rotate between stations to redistribute at least cost is known as a bike sharing rebalancing problem, which has recently attracted the attention of a great deal of researchers.

The redistribution operation can occur during the final hours of the night, when the relocation for stations can be ignored (static model), or during the day, when the distribution of bikes between stations is heavily in line with the high demand level of the (dynamic model) (Caggiani et al., 2013). Capacity planning, number and optimal route of this transport fleet are important issues, that without paying attention to its various dimensions, the cost of this operation (rebalance) will increase.

The proposed model in this research model is extended on the basis of 2 models which have been designed and studied by Arabzad et al. (2016) and Dell'Amico et al. (2014). The One-commodity model that has been proposed by Dell'Amico et al. (2014) has aimed to minimise the cost and considered just one period of time. The presented model by Arabzad et al. (2016) is considered as a multi-commodity model that has used several types of truck and bike. Also this model minimised the rebalancing cost in one period. The proposed extended model in this research is a nonlinear mathematical modelling in the state of Multi-commodity and has three purposes. In other words, taking into account several types of bikes at stations and applying the policy of Allowed slacks and surpluses in the system, a multi-period model based on the travelling salesman problem (TSP) is presented to solve a bike sharing rebalancing problem in a static mode. In this research, the redistribution of bikes between stations is carried out by vehicles that are in sufficient number and in one of the two depots of the model. In order to select the shortest route and spend the minimum cost for balancing, vehicle routing is done. Vehicles leave the depots and then return to the depots after balancing the bikes at the stations. Vehicles with limited capacity are the same.

The paper structured as follows: in Section 2 we review pervious works, in section 3 we describe the proposed mathematical model for solving bike sharing rebalancing problem. In Section 4 to demonstrate the validity of the model, presented numerical examples. In Section 5 the results and limitations of this research and suggestions for future research are presented.

2 Literature review

The literature of bike-sharing systems is almost new because these systems have become popular in recent years. Studies about bike sharing system issue is divides in to different part of strategic design and operational design. Strategic design includes network design, location and capacity of stations. Some studies about strategic design are mentioned in the following: Vogel and Mattfeld (2010) proposed a model to estimate the effect of dynamic repositioning on service levels. Their model is not useable for repositioning operations because is not detailed enough. Lin and Yang (2011) presented a paper that examined the strategic planning of public bicycle systems along with service level constraints. Taking into account the interests of users and investors, the proposed model tries to the number and location of bicycle stations, the structure of the network of bicycle routes connected between the stations, and the routes of travel for users between each pair of destinations and destinations.

Romero et al. (2012) provided a method that simultaneously provides public bikes and modelling a personal car and checking their interaction with each other. Then, to achieve in order to optimise the location of bike stations, this model has been used to economise, efficiently and sustainably transport this model with the goal of minimising costs. To design a bicycle-car model, they analyse user behaviour to help identify the paths of decision-making variables on user behaviour. Garcia-Palomares et al. (2012) presented A GIS-based approach to estimate potential demand distribution, location of stations through Employing location and allocation models, Capacity determination Stations and characteristics of station requests. The purpose of the model is to minimise all costs, taking into account similar vehicles. Martinez et al. (2012) studied the design and implementation of the public bicycle system for the city of Lasben, who designed this new service through an innovative method, including mixed integer linear programming. This model optimises the location of the stations, size of the fleet and bike movement activities for a typical day. The model is multi-product, single-period and with the goal of maximising profits. Saharidis et al. (2014) have provided a linear programming model that gives data such as the demand pattern of day-to-day demand, the popularity of bicycles among people, and the amount of available budget is considered. Taking these data into account, the location model and optimal number of docking and stations bikes are set so that the demanded response rate is maximised for bicycles and dock.

On the other side, operational design is done considering the maximum user's satisfaction and operational costs reduction. Other issue about operational design concerns different fields such as users demand prediction, network rebalancing and bicycles availability.

Sayarshad et al. (2012) presented a mathematical model for determining the minimum bicycle requirement for a bike sharing system, so that at the same time, unanswered, unused bikes and the need to move the empty bike between stations is minimised. This multi-period model Designed using integer linear programming and their solving tools are branch and bound algorithms.

Caggiani and Ottomanelli (2012) introduced a fuzzy decision support system for the distribution processes in a bike sharing system for dynamic mode, which aims to minimise bicycle distribution costs for shared bicycle companies, determine the optimal distribution flow, distribution pattern, and The time between displacement activities is to increase the satisfaction of users, and the best distribution flow and path is determined by this method.

Caggiani and Ottomanelli (2013) presented a simulation model for the bicycle distribution process between stations, which aimed to reduce the cost of bicycle distribution and increase the level of satisfaction of users by increasing the chance of a bike or a vacancy for parking it at stations at any time (Cost of customer loss). In this model, the demand is considered as a variable. Lin et al. (2013) presented a strategic design problem for public bicycle systems, taking into account the limits on bicycle inventory. This problem is formulated as a location-based inventory model. The key design decisions that are addressed in the model are: the number and location of bike stations in the system, the creation of bike paths between bike stations, the selection of routes between bases and destinations, and the level of inventory of shared bikes which are kept on bike stands.

Dell'Amico et al. (2014) using four integrated integer linear programming models, developed a model for bike sharing rebalancing problem based on the Travelling Salesman model. In the model, it is assumed that any transportation vehicle of each station will be met exactly once, as well as additional bicycles at surplus stations can be delivered to slack stations or depots and vice versa. In this model, it is necessary to visit al stations. The proposed model has been solved using the branching and cutting algorithm.

Neumann-Saavedra et al. (2015) devised a network service design for a bike sharing system, taking into account resource constraints, user demand, and service times for stations. Given the size of the fleet and the fact that the start and the end of the fleet movement from the depot without any load, the purpose of the model is to maximise the level of service. The proposed method for solving this model is an innovative and two-step method based on mixed linear integer linear programming. Di Gaspero et al. (2016) with the aim of increasing compliance with the expectation of future demand, designing an optimal route of equipment Meliorate, along with instructions for loading/unloading bikes between stations, using two routing scheduling models based on the classic vehicle routing issue and the static model that provides planning perspectives. The objective function of this model includes reducing balancing deviation from the optimal balance target and reducing the cost of balancing and travel and service costs. In this study, two algorithms for solving the metaheuristic algorithm and branching algorithms have been used.

In his paper, Alvarez et al. (2016) introduced two main parts of demand forecasting and routing in the balance of the shared bicycle system, they designed a single-period model to optimise the quality of service in these systems. In this model, each station is allowed to be met by several vehicles several times, and the fleet route begins from the depot and ends in the warehouse (without a bicycle stock). At the end, the algorithm was solved by an innovative method and the case study from Spain was also studied. Arabzad et al. (2016) presented a math model based on travelling sales problem for balancing stations, with several types of bikes and several types of vehicles (trucks) in a single-period mode. The purpose of this model is to reduce the total travel costs (distance), the fixed cost of set upping tracks and the cost of loading bikes on tracks. In this model, integer linear programming is used and Lingo software is used to solve it.

Gosh et al. (2017) presented a linear integer math model for optimising a bike sharing system. In this research, the dynamics of the location of bicycle stations reduces the slack and excess of bikes at stations, thus effectively reducing dissatisfaction and customer complaints.

With a review of the history of the activities of researchers in the field of bike sharing rebalancing problem, we find that there are few studies that simultaneously take into account multi- commodity (bicycles) and in a multi-period mode to rebalancing the system, and Consider the problem of routing – inventory for this system, so that while meeting the needs of the stations, they will reduce the cost of balancing policies and bring the model closer to the actual situation of the community.

3 Model formulation

3.1 Model assumptions

In the proposed model, the following assumptions are considered:

- The model is a multi- commodity and multi-period.
- The location of the stations is determined and fixed.
- Number of vehicles available is enough.
- Vehicles in the fleet of transport are homogeneous, with a specified capacity and with the set-up cost specified. Also, the cost of the route is considered for all fixed devices.
- Each route to start the balancing fleet must start from one of the two existing warehouses and one of them must be completed.
- In each period, all stations must be met at exactly the same time by vehicles.
- During the balance of stations, there will be no exchange of bikes with customers (static balancing).
- Some stations are allowed to have surpluses or deficits at the end of each period, with the sum of these values being specified and defined.
- The total number of loaded bikes at different stations that assigned to a device should not exceed by the capacity of that device.
- Each station should only be serving by a device.
- The number of bikes left from each warehouse in each period is less than or equal to the stock of warehouse of that type of bike at the end of the previous period.
- The values of all parameters related to the specified problem are deterministic and fixed.

3.2 Parameters

In this section the parameters which used in th model has defined in Table 1.

Table 1 Model's parameters

Symbols	*Definition*
v	Set of vertices
$\bar{v}_0$	Set of vertices except the depots(Stations 0 and $n + 1$ are depots)
A	Set of arcs
n	Number of stations
k	Trucks
b	Bike types

Table 1 Model's parameters (continued)

Symbols	*Definition*
S_b	Capacity of truck of bike type *b*
Q_{jbt}	Demand for bike type *b* at vertex j in period *t*
C_{ij}	Transportation cost of the arc (i, j)
$QTOT_{bt}$	Total demand of stations for bike type *b* in period t
P	Initial cost of implementing each truck
α	The load/unload duration cost for each bike
β	The unit cost of shortage for each bike
λ	The unit cost of holding each bike at stations
d_{bt}^{-}	The allowed shortage of bike type *b* at period t
d_{bt}^{+}	The allowed surplus of bike type *b* at period t

3.3 Variables

Variables of the model has been shown in Table 2.

Table 2 Model's variables

Symbols	*Definition*
x_{ijkt}	Taking value 1 if arc (i, j) is used by truck k in period t
f_{ijkt}^{b}	Flow over arc (i, j) for bike b with truck k in period t
w_{kt}	Taking value 1 if truck k is used to handle the demand in period t
d_{jbt}^{-}	Shortage of bike b at station j in period t
d_{jbt}^{+}	Surplus of bike b at station j in period t
$\hat{M}_{bt}$	Inventory of bike b in station 0 at the end of period t
$\bar{M}_{bt}$	Inventory of bike b in station *n* + 1 at the end of period t

3.4 Mathematical model

Mathematical model is explained in two separate sections. In the first section objective function of the model is presented and the other section will discuss constraints of the model.

3.4.1 Objective function

$$Min \sum_{i \in \bar{v}} \sum_{j \in \bar{v}} \sum_{k} \sum_{t} C_{ij}.X_{ijkt} + \sum_{k} \sum_{t} P.W_{kt} + \sum_{i \in \bar{v}} \sum_{j \in \bar{v}} \sum_{k} \sum_{b} \sum_{t} \alpha.F_{ijkbt} \tag{1.1}$$

$$Min\sum_{j\in \tilde{v}}\sum_{b}\sum_{t}d^{-}_{jbt} \tag{1.2}$$

$$Min\sum_{j\in \tilde{v}}\sum_{b}\sum_{t}d^{+}_{jbt} \tag{1.3}$$

The first part of the target function consists of three sources for costs: Balance costs include the cost of moving the transportation system between different stations, the cost of setting up the vehicles in the system and the cost of loading/loading bikes during the balancing operation, based on the research conducted by Dell'Amico et al. (2014) and Arabzad et al. (2016).

The second and third parts of the objective function minimise the amount of slacks and surpluses.

3.4.2 Constraints

$$\sum_{k}\sum_{i\in \tilde{v}}X_{ijkt}=1 \quad \forall i\in \tilde{v}_0,t \tag{2}$$

$$\sum_{k}\sum_{i\in \tilde{v}}X_{jikt}=1 \quad \forall j\in \tilde{v}_0,t \tag{3}$$

$$\sum_{k}\sum_{j\in \tilde{v}_0}X_{0jkt}+\sum_{k}\sum_{j\in \tilde{v}_0}X_{n+1,jkt}=\sum_{k}\sum_{j\in \tilde{v}_0}X_{j,n+1,kt}+\sum_{k}\sum_{j\in \tilde{v}_0}X_{j0kt} \quad \forall t \tag{4}$$

$$\sum_{i\in S}\sum_{j\in S}X_{ijkt}\le |S|-1 \quad \forall |S|\subseteq \tilde{v}_0; S\ne \varnothing, k, t \tag{5}$$

$$\sum_{k}X_{ijkt}\le 1 \quad \forall i,j\in \tilde{v},t \tag{6}$$

$$X_{ijkt}+\sum_{k}X_{jikt}+\sum_{i'\in \tilde{v}}\sum_{k'}X_{ji'k't}\le 1 \quad \forall i,j\in \tilde{v}_0,i,k,t,i'\ne i \text{ and } j'\ne j \tag{7}$$

$$\sum_{j\in \tilde{v}}X_{ijkt}\le 1 \quad \forall i\in \tilde{v},k,t \tag{8}$$

$$\sum_{j\in \tilde{v}_0}X_{0jkt}+\sum_{j\in \tilde{v}_0}X_{j0kt}+\sum_{j\in \tilde{v}_0}X_{n+1,jkt}+\sum_{j\in \tilde{v}_0}X_{j,n+1,kt}\le 2 \quad \forall k,t \tag{9}$$

$$\sum_{i\in \tilde{v}}\sum_{j\in \tilde{v}}X_{ijkt}\le (n+1).w_{kt} \quad \forall k,t \tag{10}$$

$$\sum_{k}\sum_{i\in \tilde{v}}\left(F_{jikbt}-F_{ijkbt}\right)-d^{-}_{jbt}+d^{+}_{jbt}=Q_{jbt} \quad \forall j\in \tilde{v}_0,t,b \tag{11}$$

$$\sum_{k}\sum_{j\in \tilde{v}_0}(F_{0jkbt}+F_{n+1,jbkt})\ge \text{Max}\{0,-QTOT_{bt}-d^{-}_{bt}\} \quad \forall t,b \tag{12}$$

$$\sum_k \sum_{j \in \tilde{v}_0} (F_{j,n+1,kbt} + F_{j0kbt}) \geq \mathrm{Max}\{0, QTOT_{bt} - d_{bt}^{+}\} \quad \forall t,b \tag{13}$$

$$\max\left\{0, Q_{ibt} + d_{ibt}^{-} - d_{ibt}^{+}, -Q_{jbt} + d_{jbt}^{-} - d_{jbt}^{+}\right\}.X_{ijkt} \leq F_{ijkbt} \leq \mathrm{Min}\left\{S_b, S_b + Q_{ibt} - d_{ibt}^{-} + d_{ibt}^{+}, S_b - Q_{jbt} - d_{jbt}^{-} + d_{jt}^{+}\right\}.X_{ijkt} \quad \forall i,j \in \tilde{v}, k,t,b \tag{14}$$

$$\sum_{j \in \tilde{v}} d_{jbt}^{-} \leq d_{bt}^{-} \quad \forall t,b \tag{15}$$

$$\sum_{j \in \tilde{v}} d_{jbt}^{+} \leq d_{bt}^{+} \quad \forall t,b \tag{16}$$

$$\hat{M}_{bt} = \hat{M}_{b,t-1} + \sum_k \sum_{j \in \tilde{v}_0} F_{j0kbt} - \sum_k \sum_{j \in \tilde{v}_0} F_{0jkbt} \quad \forall t,b \tag{17}$$

$$\bar{M}_{bt} = \bar{M}_{b,t-1} + \sum_k \sum_{j \in \tilde{v}_0} F_{j,n+1,kbt} - \sum_k \sum_{j \in \tilde{v}_0} F_{n+1,jkbt} \quad \forall t,b \tag{18}$$

$$\sum_k \sum_{j \in \tilde{v}_0} F_{0jkbt} \leq \hat{M}_{b,t-1} \quad \forall t,b \tag{19}$$

$$\sum_k \sum_{j \in \tilde{v}_0} F_{n+1,jkbt} \leq \bar{M}_{b,t-1} \quad \forall t,b \tag{20}$$

$$x_{ijkt}, w_{kt} \in \{0,1\} \tag{21}$$

$$U_{it} - U_{jt} + n.\sum_k X_{ijkt} \leq n-1 \quad \forall i,j \in \tilde{v}_0, t \tag{22}$$

$$QTOT_{bt} = \sum_{j \in \tilde{v}_0} Q_{jbt} \quad \forall t,b \tag{23}$$

In the above equations, the limitation of 2 and 3 implies that in each period all stations (except for warehouses) can be viewed exactly once and only by a vehicle. (Limit 4 states that in each period, the number of vehicles leaving the warehouse with the number the vehicle is the same input, and this limitation indicates that the start and end of the movement of the trucks to the warehouse). Limit 5 is used to use the sub-tour (loop) to ease the coding of the constraint 22 is used instead, which both represent a single paper.

The limitation category 6 states that, in each period, the path between the two stations should not be exceeded by more than one vehicle. The limitation item 7 indicates that if a vehicle is to be visited from a station, it is necessary to leave the station and continue the route (to reach the warehouse) with the same vehicle. The restriction group 8 refers to this issue that, in each period, a specified vehicle from each station has the maximum permission to go to another station (it cannot go from multiple destinations to a destination).

Restriction category 9 states that in each period of each vehicle, after each balance operation, one must go to one of the two depots in the problem. They must also be started from the warehouses. The limitation category 10 states that, in each period, the number of paths a vehicle runs in should not exceed (Number of stations + 1). The limitation group 11 states that for each period, the demand for each node (station) is equal to the difference between the flow of the output bike and the entrance of that station, taking into account the amount considered for deficit and surplus permitted. In category 12, taking into account the slack and authorised surplus, at least the bicycle outlet from the warehouses is determined by vehicles at the beginning of each period. Restricted category 13 with considering conditions of the slack and excess allowance specifies at least the bicycle Entrance to the depots by vehicles at the end of each period. Restricted category 14, considering the capacity of vehicles, specifies the permissible limit of the bicycle displaced between each station in each period. Limitations 15 and 16 indicate that the sum slacks/surplus of bicycle types at all stations should not exceed the amount of slacks/surpluses defined at the beginning of the period. The category of limitation 17 establishes the relationship between inventories 0 of the type of bicycle at the end of each period with the previous period. The category of limitation 18 establishes the relationship between the inventory of the $n + 1$ and the type of bicycle at the end of each period with the previous period. Restrictions 19 and 20 ensure that the number of outbound bicycles from each depot in each period should not exceed the stock of that depot at the end of the previous period. Restricted category 21 shows variables of 0 and 1. The limitation group 23 states that the total demand in each period is equal to the total demand from different stations, which includes different types of bikes from that period.

4 Numerical example and results

4.1 Numerical example

In this section, in order to show the applicability of the proposed model a numerical example is described in detail. Suppose there is a city with 8 bike stations and the goal is to rebalance the determined demand for two period. There are two kinds of bike comprised of VIP bike (type 1) and common bike (type 2) and the demands for each of them are separated. Also, there are 4 trucks to rebalance the flow between stations in same capacities and initial implementing costs. Tables 3–6 show the parameters value for numerical example.

Table 3 Capacity of each truck

	Capacity (Q_k^b)	
Truck (*k*)	S_1	S_2
1	20	20
2	20	20
3	20	20
4	20	20

Table 4 Travel cost data between two points (*Cij.Xijkt*)

Origin(i) \ *Destination (j)*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1	0	2800	2100	1700	3800	1700	2000	600
2	3000	0	1200	1200	1400	1700	1600	2500
3	1800	1400	0	800	2400	1000	400	1300
4	1900	1200	600	0	2200	600	1000	1400
5	4200	1500	2700	2300	0	2800	1000	3700
5	1600	1700	1000	600	2700	0	900	1200
7	2500	1000	700	1100	2400	1200	0	2000
8	600	2600	1800	1400	3500	1500	1700	0

Table 5 Demand of each station for each bike type in period t

	Demand (q_{jbt})			
Origin (i)	q_{j11}	q_{j12}	q_{j21}	q_{j22}
1	0	0	0	0
2	–3	–3	1	4
3	–1	2	3	1
4	–3	–1	–4	–5
5	–5	1	2	1
6	–1	–2	3	5
7	5	–3	–1	–3
8	0	0	0	0

Table 6 Another example data model validation

The load/unload duration cost for each bike(α)	1
The unit cost of shortage for each bike(β)	1
The unit cost of holding each bike at stations(λ)	1
Max surplus of bike *b* at station *j* in period *t* (d^{-}_{max})	30
Max surplus of bike *b* at station *j* in period *t*(d^{-}_{min})	30

4.2 Computational results solving models

The proposed mathematical modelling model was solved by LINGO 9.0 software in a Dual-Core computer system with CPU 2.6 GHz and 6GB RAM.

After solving the above problem with the proposed model of this study, the paths formed from the feasible answer in each period are as follows:

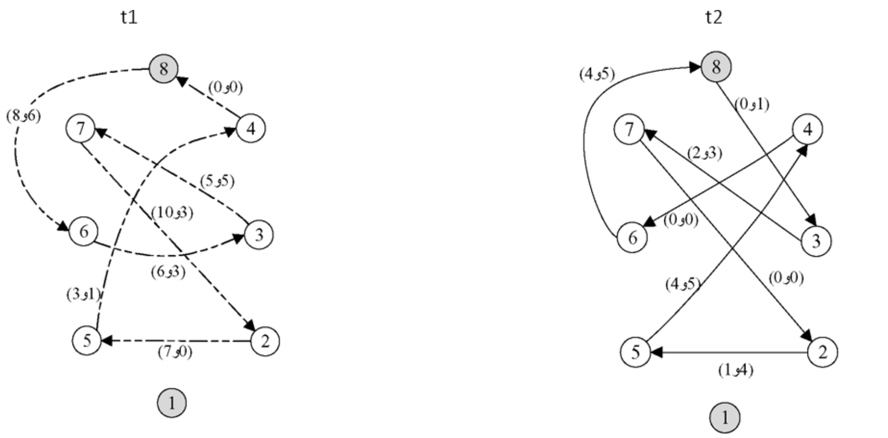
Explaining that, in Table 7, the numbers in parenthesis are the number of type 1 and type 2 bikes that have been transmitted to each station. As seen from the above outputs, each station is visited once in each period.

Table 7 routes formed in feasible condition in validation example

t	*Truck number*	*Origin (i)*	*The stations that are met by the truck*						*Destination (j)*
1	3	8	6	3	7	2	5	4	8
2	3	8	3	7	2	5	4	6	8

Also, the number of bicycles transported in different routes did not exceed the capacity of the trucks. Figure 1 the transport network obtained in the first example shows schematically in two periods, respectively.

Figure 1 Transportation network of validation example in two periods



As it is shown in Figure 1, the beginning and end of the movement of trucks started from the depots and ended up in depots. Table 8 explain flow of transmitted bike between each stations and depots. Table 9 is specified Stations with slacks and surpluses and the amount of these slacks and surpluses according to the type of bicycle.

Also, at the end of the first period inventory 7 is (87, 97) and depot 1 is (102, 100), and in the second period, inventory 1 is (102, 87) and depot number 7 is (103, 102). The results obtained from the model solving are completely consistent with the assumptions and constraints in the model.

Table 8 The flow of the bicycle transmitted in each of the periods in the validation example

t	*Truck number*	*Number of the bikes at the exit time from the depot*	*Flow over arc (i, j)*					*Number of the bikes at the entrance time to the depot*
1	3	(6,8)	(6,3)	(5,5)	(10,3)	(7,0)	(3,15)	(0,0)
2	2	(0,1)	(2,3)	(0,0)	(1,4)	(4,5)	(0,0)	(4,5)

Table 9 The amount of bike slack and surplus in the stations in each of the period in the validation example

d^-_{jb1} / d^+_{jb1}	d^-_{jb2} / d^+_{jb2}
$d^-_{721} = 1$	$d^-_{512} = 1$
$d^-_{511} = 1$	$d^-_{612} = 1$
$d^+_{611} = 1$	$d^-_{322} = 1$
$d^+_{621} = 1$	$d^+_{312} = 1$
	$d^+_{712} = 1$

In Figures 2–4, the feasible output of the schematic obtained from the feasible results obtained by solving three other numerical examples is shown:

The results of solving all the examples ensure the correctness of the output and validity of the model. So, considering the correctness and logic of the outputs, its validation can be verified. It should be noted that this issue is in the large dimension of Np-Hard.

Figure 2 Transportation network the second example in two periods

Figure 3 Transportation network the third example in two periods

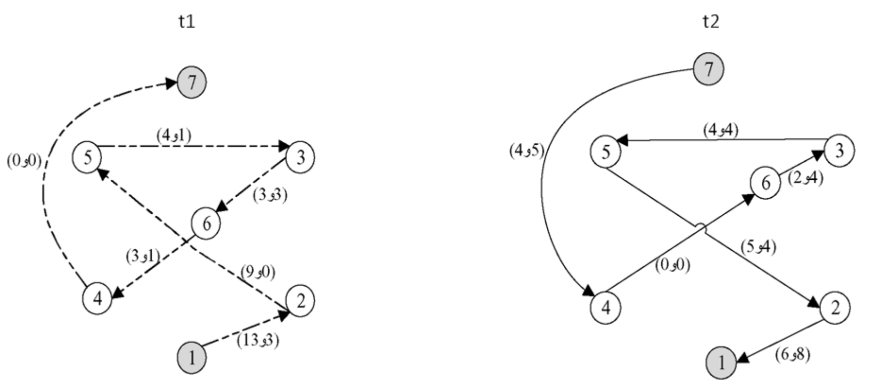
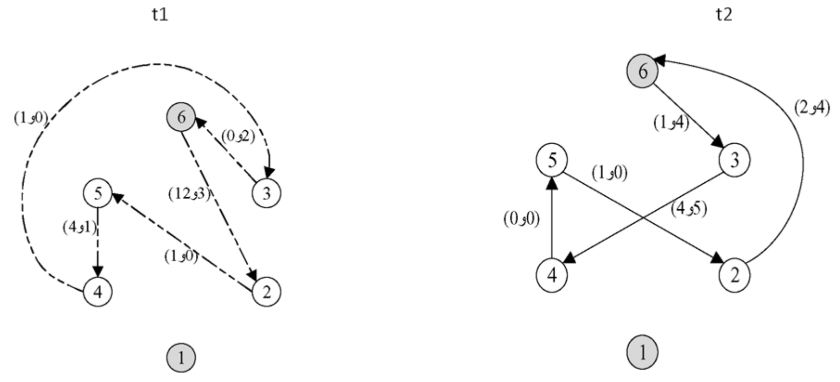


Figure 4 Transportation network the fourth example in two periods



5 Conclusion

In the present study, a multi- commodity and multi-period based on travelling sales man problem is proposed for solving the problem of rebalancing bike sharing problem in static mode. Balancing operation of stations by the homogeneous fleet of vehicles with limited capacity is sufficient in two existing depots in the model and in hours when there is no exchange of products at the stations. In order to select the shortest route and spend the least cost, the routing of the vehicle has taken place. The Proposed routing-inventory model is able to answer the questions like the number of required trucks to rebalance the stations, the sequence that each of the stations met by each of the trucks, the type and the number of bikes which are picked from or delivered to each station, the warehouses inventory at the end of the period. Also, the results of solving this model guarantees the issue that in addition to rebalance operation, applying the inventory policies such as slack and surplus permitted at some stations, this model is able to do this task with the less cost.

Therefore, the results of proposed model could have some positive function like satisfaction of users and decision makers of the system, reducing the air pollution and improving general health.

Also most of the research constraints and some suggestions for the future studies are mentioned in the following:

In this study parameters has considered as definite ones which considering the uncertainty terms can cause the model to be more closer to the real society's conditions. In each period, all the stations (with or without the demand) are met by the trucks. Changing the model in the way that just the stations which have the demand are being met, not only can reduce the costs but it also can balance the time. In the current study, the available vehicles in the balancing fleet are assumed Homogenous (the same capacity and cost) the in the future studies considering the Heterogeneous navy it can provide the term that each truck is selected which the capacity is commensurate with the route demand. It should be mentioned that this research hasn't surveyed such issues like Prioritising stations to be met by trucks, limited capacity of the Warehouse and limited distance that each truck can pass and all are the challenging subjects which could be discussed in the future studies.

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