A model-based process for the modelling and the analysis of avionic architectures

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Abstract: To design and analyse integrated modular avionics (IMA) architectures, this paper presents a model-based process with separation of concerns going from the business view to the scheduling analysis view. A pivot meta-model dedicated to schedulability analysis has been extended to support recent and new systems architectures as well as new analysis methods. This pivot meta-model represents the front-end of a design framework filling the gap between the system architect and the schedulability analysis expert. This is presented in this paper by showing how the design framework is used to handle IMA architecture, and how simple plugins were developed and connected to the framework. The plugins propose hierarchical schedulability analysis, as well as end-to-end response-time analysis for network architecture. This research is illustrated by discussing a case study.

Keywords: real-time embedded system; model-driven engineering; MDE; integrated modular avionics; IMA; scheduling.


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1 Introduction

A real-time system is a set of functions and messages that share hardware computing and communication resources, under the arbitration of scheduling policies or protocols. A real-time system is called critical when it should meet strict temporal requirements, otherwise catastrophic incidents may occur. Schedulability analysis is checking that the hardware resources and provided policies are sufficient for executing the software within given bounded delays, called deadlines.

Critical real-time embedded systems have been widely used in different industrial areas as avionics, which is the area focused by this paper. Indeed, the integrated modular avionics (IMA) architecture (Eveleens, 2006) has been introduced in avionics domain to cut the costs, and simplify the hardware architecture to be embedded in aircraft. Its main contributions is to allow the same physical core processor input/output module (CPIOM) to handle different applications while ensuring safety, and the same physical link to carry different flows of information. As many complex systems, avionics systems involve several stakeholders, from the manufacturer to several levels of part suppliers, involving engineers from commercial engineers, to system architects, to analysts, able to provide proof material for certification, or quality metrics. In order to reduce the cost of model exchanges, and preserve semantics and properties from a stakeholder to another, model-driven engineering (MDE) is used more and more in the industry, and in particular in aeronautics industry.

In this paper, we aim to share our experience and the feedback due to our participation in an avionics project among a large industrial consortium (over 50 companies) topped by the French Council for Research in Civil Aeronautics (CORAC). The industrial consortium has agreed on a business-centric meta-model
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(MM) dedicated to specific products relying on avionics architectures. The idea behind this business-centric MM is to be used by stakeholders (industry and academia) to share common elements of the domain. In this paper, we call it the business MM. It can allow the representation of the product architecture at several levels of the design, and may include artefacts to describe concepts ranging from the position of the screws to the processor allocation of tasks. This is largely inspired from the model-based system engineering paradigm supported by the International Council on Systems Engineering (INCOSE).

One of the central problems addressed by this paper is that, as experts in schedulability analysis, we were to provide a way to analyse schedulability of a system using the business MM as input. Nevertheless, a business MM being the result of a consensus between several stakeholders, the required information in the business MM for a schedulability analysis is: present for a part, implicit for another part, and unknown for the rest. In this paper, the focus is on schedulability analysis, but the lessons learned remain applicable to other analysis domains.

1.1 Context

Embedded avionics systems often involve hard real-time constraints intended to ensure full system correctness. Avionics software development costs can be sharply impacted by wrong design choices made in the early stages of development, but often detected after implementation. In this paper, we use the term temporal analysis to refer to schedulability analysis on processors and traversal time analysis on networks. By using temporal analysis, designers can detect unfeasible real-time architectures, and prevent costly design mistakes. Consequently, our role in the consortium was

1 to help industrial partners to check the schedulability of their designs an

2 to strengthen the certification process related to the temporal analysis (often part of the performance analysis in the avionics domain).

1.2 Problem statement

Recently, MDE paradigm facilitates the use of several analysis tools with modelling languages to get a complete design cycle ranging from modelling up to verification and validation. However, the business MM agreed by the consortium is qualified as descriptive (Selic, 2012). In other words, it is a meta-model issued from the description of the avionics domain. Thus, it is more dedicated to get models for understanding and communicating than getting models close to the specification and implementation. Indeed, the business MM does not contain all necessary temporal concepts to perform temporal analysis tests. As an example, some information is implicit. Some of the missing information is described in avionics standards, like the ARINC 653 (ARINC, 2013) standard describing system partitioning and scheduling. Some of the missing information is usually assumed in the domain, like the switching time in an ARINC 664p7 (ARINC, 2002–2005) switch. The other part of the information is missing like the priorities of the tasks in a partition, or how the local tasks clocks are synchronised, or not, on the hosting partition’s activation. This level of details is important for schedulability analysis, and for the parts developers, but obviously useless
from the plane manufacturer point of view. Therefore, it is unlikely that every required property for schedulability analysis would be part of the business MM. Moreover, some information will remain implicit, but could be assumed or expressed for the sake of schedulability analysis while other information has to be explicitly given. In the case where we would decide to develop schedulability analysis tools assuming the implicit information, then these tools would be ad-hoc and only reserved for this use. Nevertheless, our choice is to capitalise on tools, and to ease the integration of new tools issued from research. As a result, in this paper, we discuss our choice to describe the missing properties explicitly in order to allow third party tools, as well as specifically developed tools, to be used for schedulability analysis of IMA systems, while these tools can also be used in other domains. In order to allow generic tools to be used, or specific tools to be used in several domains, we will show that the separation of concerns (business and analysis) is central.

On the other hand, any chosen schedulability analysis test has to be argued for certification process. In other words, the ‘push-button’ action is not enough for our partners in order to avoid optimistic analysis (which is wrong because of the criticality of avionic systems) and oversized analysis (thus, it can be costly in terms of equipment and wiring). Moreover, in case of analysis failure, designers would like to be informed about the failure reasons.

1.3 Paper contribution

MDE offers the opportunity to shorten the life-cycle of software development, by bringing means to express a system and to analyse it. However, only few model-based researches have studied the difficulty that designers face while seeking appropriate analysis tests which match their designs. In this paper, we discuss the challenge that we face to enhance the temporal analysis process of the project with industrial partners. The paper tackles the design and schedulability analysis of the case where safety-critical avionics systems are deployed on IMA architectures. We briefly highlight various possible preliminary solutions to deal with this challenge and we show the strengths of the opted solution. It consists in the utilisation of a home product called modelling oriented scheduling analysis of real-time systems (MoSaRT) framework, which has been initially developed in Ouhammou et al. (2014) to help designers to choose the most suitable analysis tools referring to the system architecture to analyse. We illustrate the process-chain from the business view, using a common exchange meta-model defining shared properties of a system, to the schedulability analysis view, using a pivot analysis MM dedicated to temporal analysis (i.e. MoSaRT design language). By pivot analysis MM we mean a MM that offers modelling artefacts for systems in order to allow temporal analysis.

1.4 Paper outline

The remainder of this paper is organised as follows. In Section 2, we introduce the main specificities of IMA architectures and especially related to schedulability and network analyses. This is important, since our goal is to extend the analysis MM for IMA to include the required information for analysis, and only this information, in order to avoid unnecessary complexity of the analysis MM. This will be followed
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by a discussion about related work and a brief presentation of MoSaRT framework. Section 3 presents our approach overview by arguing the benefits of separation of concerns and justifying the choice of MoSaRT. Section 4 presents the extension of the MoSaRT design language in order to capture the required information to analyse IMA architectures. It also describes the utilisation of a decision support that leads the analysis process independently from the designer’s knowledge. A simple case study showing the significance of such model-based chain process is presented in Section 5. Finally, Section 6 drafts our conclusions and final remarks.

2 Background and related work

2.1 Temporal analysis for IMA

Since IMA architecture contains several applications (resp. flows information) which share the same physical resources, then IMA insures freedom from interference. Freedom from interference, as defined in the standard ISO 26262 (ISO, 2011), insures that the failure of a piece of software will not create the failure of another piece of software. It also insures that the interference of two software independent parts is bounded. Moreover, an application can be studied independently from another application, even if they share the same computing resources.

Processor partitioning is a mean used in IMA to insure freedom from interference between applications running on the same CPIOM, which may be of different design assurance level (DAL), but sharing the same physical resources. The analysis of partitioned scheduling is presented in Section 7.1.

The avionics full-duplex switched ethernet (AFDX) has been introduced to replace dedicated buses like ARINC 429 (ARINC, 2004) by merging several flows of information (virtual links) on the same physical links. The analysis of AFDX end-to-end response time, also called traversal time, is presented in Section 7.2.

2.2 Discussion

Several schedulability analysis tests (like response-time analysis, processor utilisation, simulation, etc.) have been proposed for different kinds of software [e.g. self-suspended tasks (Ridouard et al., 2004), task precedences (Forget et al., 2011), transactions (Rahni et al., 2009)] and hardware [e.g. multi-core (Davis and Burns, 2011), cache memories (Phavorin et al., 2015)] architectures. These tests have been implemented in several commercial and academic analysis tools (e.g. Henia et al., 2005; Pasaje et al., 2001; Singhoff et al., 2004) in order to facilitate their utilisation by system designers through a model-based process. However, the integration of these tools with design languages [e.g. AADL, 2009; UML-MARTE (OMG, 2011)], in order to get a model-based process combining the modelling and analysis, requires a deep knowledge in schedulability analysis, in order to be sure that any transformation is conservative, and that the analysis cannot give optimistic results. Indeed, every schedulability analysis test is dedicated to a very specific set of software/hardware assumptions. Thus, the test to validate a specific architecture has to be carefully chosen regarding a large set of assumptions. The more realistic the system to analyse is, the more fine-grain, and usually the more efficient, the schedulability test is, the more underlying assumptions have to be checked to make
sure the test is suitable to the system to analyse. The tiniest mistake in the, usually implicit, underlying assumptions, can make the result of the test unfit to validate the system under analysis.

2.3 MoSaRT framework in a nutshell

Avionics software development costs can be sharply impacted by wrong design choices made in the early stages of development, but often detected after implementation. By using temporal scheduling analysis, designers could detect unfeasible real-time architectures, and prevent costly design mistakes. From a system model proposed by the system architect, a real-time expert has to determine the schedulability, or timing metrics of the system. Unless the system is using a predefined set of patterns, the design may have to go back and forth between the system architect and the analyst. This is a tedious and time consuming process.

The objective behind the MoSaRT framework is to benefit from analysts skills and architects skills in order to unify their efforts, to then avoid wrong design choices at an early design phase. MoSaRT framework helps designers to cope with the scheduling analysis and to be more autonomous during the analysis phase.

Figure 1 shows the general architecture of the framework, which contains two parts: front-end and back-end as described hereafter.

**Figure 1** General structure of MoSaRT framework (see online version for colours)
2.3.1 Front-end of MoSaRT framework: the pivot analysis meta-model

The MoSaRT front-end is based on a design language (a domain specific language DSL). It provides a set of concepts to model different architectures of real-time systems characterised by temporal extra-functional properties (e.g., execution times, deadlines, periods). The MoSaRT design language is based on Ecore (Steinberg et al., 2008) and uses Object Constraint Language (OMG, 2014) (OCL, which is a textual language fitting with Ecore and enabling to express invariants) for checking consistency (see Figure 1). MoSaRT provides a graphical language for instantiating systems through a modelling editor, that provides different model views such as hardware architecture diagram, software architecture diagram and behavioural diagram.

2.3.2 Back-end part of MoSaRT framework: the analysis repository

The MoSaRT back-end is based on the instantiation of a second meta-model called analysis repository meta-model. The instantiation contains sets of rules used to identify the analysis contexts to which a MoSaRT model (i.e., the system model) belongs (see Figure 1). An analysis context is an explicit set of assumption for a temporal analysis to be relevant. For example, the context in which the classic Joseph and Pandya (1986) response-time analysis applies is defined by the OCL constraints on the model checking that there is

1. one core
2. tasks are independent or can share critical resources using a resource management protocol
3. are periodic or sporadic
4. have deadlines less or equal to their period
5. there is no preemption delay, etc.

The analysis repository also defines how an input model can be transformed into a third-party tool implementing the test, as well as the type of answer provided by the test (response-time, task allocation, priority allocation, etc.).

The analysis repository can be seen as a model-based expert system, enabling researchers and analysts to share their analysis models and tests with any user of the MoSaRT front-end. For this user, using the analysis repository consists on running an identification process (see Figure 1). This process is using OCL to look for every relevant analysis contexts for the model under analysis. Once a context is found, the appropriate scheduling analysis method is then proposed to the user. Moreover, it is possible to generate automatically the input models for a set of third-party tools by launching a transformation of the model provided by the framework to the appropriate model for the selected analysis.

2.3.3 Discussion

MoSaRT has been defined as a pivot MM between system design and temporal analyses. It has a rich semantic related to temporal analysis, based on the will to integrate a large
set of schedulability analysis state of the art models and tests. Such a framework could answer to the problem of closing the gap between the business MM of avionics and temporal analysis. Nevertheless, in its previous version, MoSaRT had a very limited support for networks. In order to be able to handle IMA applications, such concepts had to be added to MoSaRT. Moreover, even if the MoSaRT design language was already allowing to model hierarchical schedulers, which is the case for ARINC 653 compliant systems, the repository had to be populated with relevant tests for IMA schedulability analysis.

3 Overview of the contribution: from the IMA architecture to the analysis view

In this section, we first present the requirements that our deliverable has to fulfil. Secondly, we present some various directions that we have considered before opting for the solution based on the MoSaRT framework.

3.1 Business meta-model and statements

Since for industrial secrecy reason, we cannot give here the real business MM, we are using a simplified version of the business MM that contains only common knowledge about the domain. In order to illustrate such a MM, Figure 2 shows an excerpt of the hardware concepts proposed in our example business meta-model. We can notice that the only information which is useful for the temporal analysis is the number of processors (CPUNumber attribute of Equipement Class).

Some required information for temporal analysis is completely missing in the business MM. For example, the stakeholders discussing around the business MM may decide that the tasks execution time, tasks priorities, or the scheduling algorithm, do not have to be expressed. Indeed, the business MM is closely related to the engineer’s point of view.

Figure 2  Excerpt of the business meta-model: hardware part (see online version for colours)
We notice that the consensual business MM is dedicated to exchange information, understanding and communicating. Thus, it is a descriptive meta-model for designing IMA architectures. The lack of temporal properties and concepts (as other domains concepts) is done on purpose in order to avoid over complexity. Let us summarise the main requirements proposed by the consortium to integrate the temporal analysis process in the IMA design phase:

- Requirement 1: the temporal analysis process should be argued in order to justify the choice of the analysis tests.
- Requirement 2: the integration of new (academic) analysis tests that propose interesting results by taking into consideration the technology advances (software and hardware) should be eased.

Moreover, as a research lab, we would appreciate the solution to be easily applied to other domains, in order to capitalise the effort.

3.2 Investigated directions

To integrate the temporal analysis process we have explored three directions. In the following, we summarise the advantages and drawbacks of each one.

1 We have studied the possibility to enrich the business MM. We could enrich the business MM by a set of real-time concepts, and hence create a tool that directly reads inputs from a business model instance, and analyses the worst-case response times and end-to-end network latency of messages. Nevertheless, the effort would be limited to this kind of business MM (aeronautic domain), and any new analysis should be specifically developed for and connected to this business meta-model. Consequently, this direction does not help to meet requirement 2 and does not allow capitalisation.

2 We have also studied the possibility of proposing model transformations from the business MM to real-time design languages, in particular, standard ones like AADL (2009) and UML-MARTE (OMG, 2011). The drawback is that, in order to capitalise on the temporal analysis effort, two transformations have to be checked: from the business MM to AADL or UML/MARTE, then from the design language to temporal analysis tools. In the last decade, a dozen of methodologies have been proposed to reduce the utilisation difficulty. Also, each methodology is related to a special case/domain of use. Besides, although several analysis tools support AADL and MARTE, to the best of our knowledge, no one among those tools provides IMA analyses. Even if standard design languages contain enough real-time concepts enabling to meet requirement 2, but they do not propose any solution to ensure that the chosen analysis tests are the suitable ones. So, they miss requirement 1.

3 We have studied the utilisation of our MoSaRT framework (see Section 2.3). Contrary to standard design languages, the front-end of MoSaRT is only dedicated to temporal analysis domain. By using MoSaRT framework, we can meet requirement 1 thanks to its back-end (MoSaRT analysis repository), and we can also satisfy requirement 2. This latter is related to the real-time properties package contained in MoSaRT language. Finally, we can capitalise the effort, in order to take profit of the
front-end and back-end in other domains than the IMA. It facilitates the integration of tools and is also easily extensible, as we see in the sequel of the paper. Therefore, MoSaRT has to be extended to capture the scheduling analysis concepts presented in Section 7.1.1 and Section 7.2.1.

3.3 Discussion: separation of concerns (business and analysis views)

We believe that an intermediate pivot meta-model dedicated to the analysis domain is necessary to improve the validation process. There are many advantages in coupling analysis tools through the analysis meta-model (here MoSaRT design language) and not directly to the business meta-model:

- The business meta-model and the analysis meta-model do not share the same vocabulary and concepts.
- The information in the business meta-model, which is shared by several stakeholders from different scientific cultures, may not be complete regarding the needs for schedulability analysis.
- In this paper, we provide specific analysis tools for analysis of IMA systems, but these tools can be reused in another business category.
- Depending on the stakeholders effort and investment in the definition of their business meta-model, they may decide to keep it private (which is the case in our context), blocking the possibility for academia or stakeholders not part of the business meta-model definition group from offering new analysis techniques directly usable on the business meta-model. Using an intermediate open pivot meta-model (like MoSaRT design language) is allowing these stakeholders, as well as the providers of analysis methods, to communicate around an open meta-model.
- The pivot meta-model is already connected to several off-the-shelf tools, and is meant to be enriched, by our research group, and hopefully in the future by other temporal analysis research groups. The advantage is, after the effort of connecting the tool to MoSaRT, that the tool is or will be connected to several design languages and business meta-models. This also allows an easy comparison between several tools handling the same analysis problem.
- In the context of this paper, we focus on schedulability analysis by response-time and end-to-end network analysis, but we could also provide dimensioning and optimisation techniques.

3.4 Running example

We consider models conform to business MM as inputs to our temporal analysis framework (i.e. MoSaRT). In order to illustrate the process, we consider a small example of an IMA application (see Figure 3), composed of three processors (CPIOM for core processor input/output module in the context of IMA), executing some applications – which are not visible on the figure – sharing data over an AFDX network composed of two switches. The data is passed over the network using three virtual links.
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The notion of virtual link is presented in Section 7.2: for example, the virtual link VL_1 has a source located on CPIOM1, and as destinations CPIOM2 and CPIOM3.

Figure 3 A simple IMA system conforms to the business meta-model (see online version for colours)

Figure 4 From the business model to its corresponding temporal analysis view (see online version for colours)

Figure 4 sketches the global analysis process starting from a model conforming to the business meta-model to the analysis tools (based on mathematical background detailed in Sections 7.1 and 7.2). The gap between business meta-model and analysis tools needs a flexible way to allow an engineer, knowledgeable enough in real-time systems, to feed this missing information in real-time specific model. The business view and analysis tools will be then bridged by MoSaRT framework thanks to its analysis meta-model
and its analysis repository that orient engineers to choose the relevant temporal analysis. Note that on Figure 4, the real-time analysis domain view is independent from the business model, and can be used in other domains.

4 Extension of MoSaRT framework

Because the objective of MoSaRT framework is to be as cross-domain as possible, in order to allow the re-usability on different types of systems or distributed systems, we propose an extension of the MoSaRT design language considering very abstract networks in general. The idea is to model the concept of a network instead of having a specific meta-model for every protocol and version of protocol. Indeed, on an analysis point of view, forward analysis (detailed in Section 7) could be applied, e.g., to asynchronous transfer mode (ATM) networks, which share, on a temporal analysis point of view, many common points with AFDX.

4.1 Extending MoSaRT design language

We could be tempted to propose an extension of MoSaRT by adding an AFDX network meta-model such as the business network meta-model. Nevertheless, the analysis does not require the same kind of information as the engineer. For example, the analysis does not need information about the physical topology of the network, or which cable is connected to which port. A logical view of the topology is enough. In the analysis, we consider a succession of flow controls (admission control in the nodes, called end-systems), flows (throughput and technological delays), and arbitration policies (FIFO queue in the switches). In forward analysis, as well as in the network calculus, such flow is followed from the source of a message to its destination in order to compute its end-to-end latency. For each encountered switch output, the maximal concurrent outgoing traffic is computed and the worst-case delay of the switch is added to the jitter of the message (the jitter characterises the possible deviation between the best case ready time at the switch exit, and its worst-case). In AFDX, the arbitration points are switches, while in a CAN (Controller Area Network) networks, the arbitration point is the bus itself and can be seen as a non-preemptive shared processor among the nodes, with a caveat that the bit transmission time has to be taken into account when computing the emitting time. The objective behind the extension of the MoSaRT meta-model is to be able to abstract a network as much as possible. Thus, by using the same modelling artefacts we can be able to represent several kinds of existing (or future) wired networks.

To represent a network in general (see Figure 5), we aggregate in a network node (HpNetworkNode class) the concepts of admission control, represented as a traffic shaping property, technological delays (like the 16 microseconds delay of packet switching from switch input to switch output, or electrical transmission delay) and throughput as a service curve, and the arbitration policy. A communication channel (HpCommunicationChannel class) is connecting two processing units (end-systems in AFDX) through several network nodes of FlowCarrier type (HpFlowCarrier class). A processing unit (HpProcessingUnit class) is distinguished from a flow carrier by the fact that it is either a source or a final destination network node, while a carrier is representing any routing device. Note that the same physical device may be at the
same time routing device for one communication channel and a (source or destination) processing unit in another communication channel.

We see that the central most important class is the network node which is defined by:

- an unique origin, which is a processing unit
- a set of destinations, which can be either final (if they are processing units) or intermediate if they are flow carriers
- a traffic shaping, representing potential included limitations in the end-systems or switches (for the AFDX, this is where the BAG will be represented)
- a service curve, representing the technological delays and throughput of the physical link
- an arbitration policy, e.g. FIFO for AFDX switches output ports, or non-preemptive fixed priority for CAN networks.

**Figure 5** Excerpt the MoSaRT meta-model extension (see online version for colours)

The meta-model extension has been enriched by a set of structural rules helping to check the consistency of instances referring to MoSaRT design language. These rules insure a coherent system design. The structural rules are implemented with OCL. Examples of new added rules:

- if a processing unit is one of the destinations of a flow carrier, the flow carrier shall not have more than one processing unit as a destination
• a communication channel cannot contain cycles.

We can note that partition scheduling did not require any modification in the previous version of MoSaRT model which was allowing hierarchical scheduling. Hierarchical scheduling allows a scheduler to contain several schedulers, and the IMA assumes at least two hierarchical levels: static scheduling of the partition, within which a host real-time operating systems have their own scheduler.

4.2 Instantiating the MoSaRT analysis repository

One of the most interesting features of MoSaRT is its analysis repository (see Section 2.3), which includes a set of analysis contexts. An analysis context is defined by a set of OCL rules, which are, if met by a system model, implying that the system enters in a category of systems that can be analysed.

New detection rules have been added to the analysis repository in order to detect tasks systems that can be analysed with the test presented in Section 7.1.2. Based on the network extension, new detection rules also have been added to detect models analysable using the test presented in Section 7.2.2. These detection rules allow MoSaRT to offer these two analyses when a model fits with an IMA model. We can create a new repository or enrich an existing one. In this paper, we chose the first way, because using an existing repository would require to explain its content, which relays extensively on schedulability analysis state of the art, which is out of the scope of this paper. Therefore, the new created repository is based on the context of the tests presented in Section 2, which is defined by a set of characteristics, such as:

• the architecture of each CPIOM is uniprocessor
• tasks are independent (in fact, inter-task communications are asynchronous, meaning there is no precedence constraint between the tasks, and critical sections are short and their duration is included in the tasks’ worst-case execution times)
• the network is a set of communication channels, which can be mono-source and multi-destination
• fixed-task priority (FTP) scheduling policy is used in the partitions
• etc.

If a design does not respect these characteristics, it does not mean that the structure of the design is incorrect. However, when all characteristics are respected we can conclude that the analysis context of the design is the one presented in Section 2.

Figure 6 is our instance of the MoSaRT analysis repository. This instance contains a set of identification rules, where each rule is an OCL constraint (e.g. rule1 in Figure 6 describes the constraint stating that the nodes are uniprocessor). We also link the implemented analysis test to the context in order to check the schedulability of the design by calculating the response times of the tasks. We also compute the end-to-end delays of the message, in order for the designer to be able to check for end-to-end delays implied by the use of the network. Note that in the current version, every task is time-based, i.e. communication delays imply data aging, but not task jitters on the processors (the term used in avionics is ‘sampling’). Once the new instance of the
repository is ready to be used, we can connect it to a design expressed using MoSaRT language.

**Figure 6** Excerpt of a repository containing the analysis context related to hierarchical scheduling (see online version for colours)

5 **Case study**

Since the MoSaRT design language has been extended to support IMA architectures, we can illustrate this extension through a case study. The case study that we treat in this section is based on the example presented in Section 3.4. In the following, we presents steps leading to get the analysis results.

5.1 **Step 1: modelling using extended MoSaRT design language**

The architecture represents a communication between a set of tasks executed in different CPIOMs communicating through an avionics full duplex (AFDX) network. The networking is based on various switches and physical links.

Therefore, Figure 7 illustrates a part of the MoSaRT model related to the network part of the system shown in Figure 3. The instance illustrated in Figure 7 is expressed using MoSaRT concrete syntaxes. Two syntaxes are provided by MoSaRT. The first one enables to get a tree-structure view and the second one is graphical and allows to show the instances through different diagrams. For instance, the second part of Figure 7 represents the network diagram where VL-relationships between CPIOMs are shown. Indeed, via the extended design language we can model:

- Virtual links as instances of \textit{HpCommunicationChannel}.
- Output ports of the switches as instances of \textit{HpFlowCarrier}, which include the transmission delay of the physical transmission from the output port to the input port of the physically connected \textit{HpNetworkNode}. If this node is a \textit{HpFlowCarrier}, we also integrate the switch technological delay.
- Source CPIOM end-systems of virtual links as instances of \textit{HpProcessingUnit}, where the BAG and \(S_{\text{max}}\) (see Section 7.2) parameters are used to define the traffic shaping, and including the transmission delay on the link to the next switch, as well as the switch technological delay.
Destination CPIOM end-systems as instances of *HpProcessingUnit*.

Each partition executed on the CPIOM contains the tasks modelled as *SoSchedulableTask*.

Each partition is modelled as a child *SoSpaceProcess* of the main process.

**Figure 7** Excerpt of analysis model expressed in MoSaRT language
(see online version for colours)

Concerning processor scheduling, we focus on the first partition of the CPIOM1, whose content is given in Figure 8. The initial MoSaRT language was already allowing to model such a system. It is expressed using MoSaRT design language in Figure 9. It shows a hierarchical architecture composed of three partitions where the first partition contains three independent tasks.

Table 1 presents the timing properties related to tasks and partitions. The considered period of the partition *Partition1_1* is 30 ms and the time slices (i.e. time intervals) allocated to it are (0, 5), (10, 15) and (20, 25). Figure 9 represents a part of a MoSaRT model compliant with the software architecture of the example. The model highlights the software structure (i.e. tasks and partition) and temporal behaviour of tasks (i.e. task activities and triggers).
Figure 8  Content of the first partition of CPIOM1 (see online version for colours)

Figure 9  MoSaRT model corresponding to Figure 8 (see online version for colours)
Table 1  Timing properties of the task-set

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Priority</th>
<th>Period (ms)</th>
<th>Deadline (ms)</th>
<th>Execution time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task1</td>
<td>15</td>
<td>8</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Task2</td>
<td>10</td>
<td>14</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>Task3</td>
<td>5</td>
<td>30</td>
<td>30</td>
<td>4</td>
</tr>
</tbody>
</table>

5.2 Step 2: implementing a test prototype

To the best of our knowledge, the two tests presented in Section 7 could not be found in any freely available schedulability analysis toolsets. They were fairly easy to program, because the most time consuming part of a tool implementing these tests would be to connect them to a design framework in order to call them when necessary. Using MoSaRT framework, we show how it is easily possible to connect such a tool to a design language, as well as to check if an input model can be handled by the developed tool. Hereafter, we show how the analysis repository can be used to lead designers to this tool for analysis.

5.3 Step 3: utilisation of MoSaRT analysis repository

If one needs to be assisted to analyse models compliant with IMA architecture specifications, one has to use the analysis repository of MoSaRT. For example, if we launch the MoSaRT identification process from the design of the example treated in Section 3.4, we can check if it is compliant with the context added in the new repository. In other words, we can check if an input model fits with the underlying hypothesis of the tests: independent tasks, IMA hierarchical scheduler, sporadic constrained deadlines tasks, uni-processor CPU, no loop in the network, complete and consistent virtual links. Moreover, the identification process (a.k.a. detection process) of MoSaRT presents the result of this step. Figure 10 shows the analysis test corresponding to the characteristics of the design. The result window recapitulates the rules that are verified by the design and also the suitable analysis model. References related to tests and models are also presented in case when designers need more details. This allows the designer to check why his model can be analysed, satisfying Requirement 1 of Section 3.1.

5.4 Step 4: analysis results

Note that if the designer agrees with the proposed tests, the latter can be performed from the MoSaRT framework by doing a transformation to the input model of the analysis tools. The same process can be repeated several times with different architectures, like the network architecture shown in Figure 7. Figure 11 summarises the different results related to the case study and then calculated by the implemented prototype analysis tool. The results consider the worst-case response times and the worst case end-to-end delays.
Figure 10  Result of a MoSaRT identification process (see online version for colours)

Figure 11  Analysis result of the case study (see online version for colours)
6 Conclusions and learned lessons

We have presented a whole chain of modelling and analysis which can be applied for IMA real-time analysis. First, we presented how temporal analysis can be computed in the context of partitioned IMA systems and also for end-to-end latency on AFDX networks. As MDE is more and more used in the aeronautics industry, we can expect, in this domain, as well as in other domains, the stakeholders to define a common meta-model that can be used for exchange purpose, but also for multi-analysis purposes, while shortening the duration of the design process.

Our contribution consists in showing how a temporal analysis-oriented pivot language, like MoSaRT design language for schedulability analysis, can be used in this context. We first discussed various investigated directions. Hence, we presented the advantages of using an open pivot language, for re-usability, potential incompleteness regarding a specific analysis domain, but also for potential privacy concerns around the business meta-model. Then, we showed how a pivot language may offer the expert modelling artefacts whose semantics fit with the schedulability analysis expert semantics, without specific business related jargon. Moreover, some implicit business assumptions are typically not expressed, and may be added when transforming a model from the business meta-model to the temporal analysis meta-model. We also discussed the modelling philosophy of such a pivot meta-model by trying not to describe concepts by their technological pendant, but by the concepts themselves. Thus, the meta-model is more prone to be extensible to new technologies. It is the case for MoSaRT design language where hierarchical scheduling concepts allows an IMA CPIOM to be represented, while it can also be used to represent a classic Unix process hierarchy. Consequently, the utilisation of MoSaRT framework allow us as a project member to meet requirements provided by other project partners. In the future, we plan to release an open source stable version of MoSaRT framework. Moreover, we will add various analysis tools to compare and discuss the output results.

We believe that the separation of concerns has many advantages as it has been shown in the paper. However, we can also enumerate several limits. The main one is the lack of traceability. After analysis and impacting the analysis model regarding the analysis, we notice the absence of a feedback to the business model (instance of business meta-model). For this, we can imagine another meta-model which can play the role of a glue between analysis models of different analysis domains and the core elements of the business meta-model. In our case, porting automatically the analysis results in the business meta-model would be an interesting feature. Another important issue is the transformation from the business domain to the analysis domain. Currently, the transformation is done automatically based on a model transformation program. Besides, the analysis model resulting from the transformation is enriched by several temporal information by engineers. The transformation program implements indeed a set of transformation rules provided by the engineer experience. Nevertheless, an open question is how to verify its conformity referring to both business domain and analysis domain.
References


7 Annex

7.1 Partitioned scheduling in IMA

The classic way to arbitrate the computing resources in a partitioned system is to use a hierarchical scheduler. In the IMA standard ARINC 653 (ARINC, 2013), a two level scheduler is used: a low level cyclic scheduler is statically assigning the computing resources to the partitions, using a pre-defined table. Then, in each partition, a real-time operating system is assigning the processor to the tasks, using its own scheduling policy. On a timing behaviour point of view, freedom from interference means that the interference of a partition on another one should be null or bounded. In most models, like it is the case in this paper, absence of interference is assumed.

The seminal research on partitioned scheduling of Deng et al. (1997) proposes two hierarchical levels. Later, Feng and Mok (2002) proposed the resource partitioning model and schedulability analysis based on the supply bound function (sbf), which is an elegant mathematical way to extend non-hierarchical schedulability analysis to hierarchical scheduling. Shin and Lee (2003) introduced the concepts of temporal interface with the periodic resource model. For applications containing FTP schedulers, Kuo and Li (1999) and Saewong et al. (2002) proposed dedicated hierarchical schedulers. In the context of server based temporal partitioning, numerous techniques have been proposed. Lipari and Bini (2005) used the abstraction of a periodic server in order to solve the problem of creating a partitioning such that the system is schedulable. Davis and Burns (2005) proposed an upper bound on the response time of the tasks when the partitions are scheduled by an FTP. An exact worst-case response-time can also be obtained (Balbastre et al., 2009).

We present hereafter a scheduling model for hierarchical scheduling fitting with the ARINC 653 standard, and techniques allowing to compute the worst-case response times of the tasks.

7.1.1 Analysis model

A (temporal) partition can be defined as a collection of time intervals defining statically the allocation of the computing resource to a partition.

Definition: a resource partition is noted \( \Pi = (\Gamma, P) \) where:

- \( \Gamma = \{(S_1, E_1), (S_2, E_2), \ldots, (S_N, E_N)\} \) is a set of time intervals, where \( S_i \) is the offset of a time interval allocated to the resource partition, related to the starting time of the period of the partition. \( E_i \) is the ending date of the time interval. The size of the \( i^{th} \) time interval is given by \( E_i - S_i \).
- The size of the period of the resource partition is \( P \).

A resource partition on a single CPU processing resource must obviously satisfy non-overlapping property: \( 0 \leq S_1 < E_1 \leq S_2 < E_2 \ldots \leq S_N < E_N \leq P \). For simplicity, we suppose that \( P \) is common to every partition of a CPU, corresponding to the MAF (MAjor Frame) in ARINC 653. A simple algebraic transformation using greatest common divisor and least common multiple could be used to transform a classic ARINC 653 partition using MIF (MIInor Frames) into a resource partition model.
An IMA application is a set of tasks, allocated to a resource partition on a CPU. In the sequel, we consider sets of sporadic independent tasks. Each application has its own FTP scheduler, which is scheduling its tasks in its resource partition.

**Definition:** an application $j$ contains $n_j$ sporadic tasks $\tau_i^j, i = 1..n_j$. A task $\tau_i^j$ is defined by $(C_i^j, D_i^j, T_i^j)$ where:

- $C_i^j$ is the task’s worst-case execution time (WCET)
- $D_i^j$ is the relative deadline of the task, i.e., the maximum allowed time interval for the task to be completed after each of its activation
- $T_i^j$ is the period of the task, i.e., the minimum interval of time between two successive releases of the task.

Since we consider that every task $\tau_i^j$ is scheduled by a FTP scheduler, we order the tasks by priority order in each application. The task $\tau_1^j$ has the highest priority while $\tau_{n_j}^j$ has the lowest priority.

### 7.1.2 Response time analysis

Since we consider the system application per application, for the sake of simplicity, in the sequel, we will drop the exponent $j$ of the application number. Response-time analysis (RTA) in the IMA context is an extension of the classic RTA, which is based on the request bound function (rbf) (Joseph and Pandya, 1986). When considering a sporadic task $\tau_i$, released at time 0, its worst-case response time cannot be lower than its own WCET $C_i$.

**Classic non hierarchical RTA**

1. Therefore, we know that the processor will be busy at least in the time interval $[0, C_i]$. Let’s denote $rbf_1^i$ the value of the end of this interval. In this time interval, any release of a higher priority task $\tau_k, k < i$ is postponing the end of the execution of $\tau_i$. The amount of releases of higher priority tasks in the interval $[0, rbf_1^i]$ occurs if all the higher priority tasks are released at the time 0, and execute with their shortest period (they are sporadic). This is called the critical instant. Each task $\tau_k, k < i$ can postpone the end of $\tau_i$ by up to $\left\lceil \frac{rbf_1^i}{T_k} \right\rceil C_k$.

2. The processor will be busy with tasks whose priority is at least the priority of $\tau_i$ at least in the time interval $[0, rbf_2^i = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{rbf_1^k}{T_k} \right\rceil C_k]$. In this time interval, the amount of time every higher priority task can postpone the end of $\tau_i$ may be higher, and given, for each task $\tau_k, k < i$ by $\left\lceil \frac{rbf_2^i}{T_k} \right\rceil C_k$.

3. Going back to point 1, substituting $rbf_1^i$ by $rbf_2^i$, we obtain at the smallest fixed point of $rbf_2^i$ the worst-case response time of $\tau_i$ as long as it is not greater than its period (because if it was greater than its period, then $\tau_i$ may delay itself and the formula should be adapted like in Lehoczky (1990)).
A model-based process

We therefore consider the classic RTA test stating that the worst-case execution time of a constrained deadline task ($D_i \leq T_i$) is obtained as the smallest fixed-point $rbf_i^*$ of the equation:

$$rbf_i^* = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{rbf_j^*}{T_k} \right\rceil C_k, \text{ if } rbf_i^* \leq T_i$$

(1)

If $rbf_i^* > T_i$ then $\tau_i$ will miss its deadline.

**RTA for IMA**

The main difference between hierarchical and classical RTA is that, in classic RTA, the processor dedicates one time unit of its time every time unit, while in hierarchical scheduling, the processor dedicates one time unit every time unit to the elected processor partition only. This phenomenon is accounted for by using a supply function of the processor.

The supply function is depending on the beginning of the MAF. Since there is no information concerning the offset between a considered instant for a task under analysis, and the beginning of the MAF, we have to consider any offset between the tasks critical instant to study and the offset in the MAF. The lowest amount of processor supplied to a processor partition

$$\Gamma = \{(S_1, E_1), (S_2, E_2), \ldots, (S_N, E_N)\}$$

is given, in an interval of length $t$, by considering the minimal supply function assuming the critical instant of the tasks coincides with $E_1, E_2, \ldots, E_N$ (Feng and Mok, 2002). These points are called critical points.

**Definition**: The supply function of the processor in a time interval $[0, t]$, depends on the critical point $E_1, E_2, \ldots, E_N$ coinciding with the critical instant at the date 0. It is denoted $sf(E_i, t)$.

When computing the time required by the processor to compute the workload given by the $rbf$, we need to consider how long, in the worst case, is necessary for the processor partition to be supplied with enough processor time to compute the $rbf$. This is given by the reverse supply function $sf^{-1}$ [see (Feng and Mok, 2002) for the way to compute this function].

**Theorem (Feng and Mok, 2002)**: The WCET of the first job of a task $\tau_i$ of an application scheduled in a processor partition $\Gamma = \{(S_1, E_1), (S_2, E_2), \ldots, (S_N, E_N)\}$ is given by the maximal for $E_j$, $j = 1..N$ of the smallest fixed-point $rbf_i^*(E_j) = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{sf^{-1}(rbf_j^*(E_j))}{T_k} \right\rceil C_k$. If this value is not greater than the task’s period $T_i$, then it is giving the worst-case response time of the task.

**7.2 AFDX in IMA**

The AFDX is the backbone network of an IMA architecture. AFDX offers higher throughput (based on 100 Mb/s Ethernet standard), weight saving, high reliability and integrity. Traffic constraints are enforced by the network in order to guarantee worst-case end-to-end delays of avionics communications.
Several approaches have been proposed in order to analyse end-To-end (ETE) delays in the AFDX context. A simulation approach (Scharbarg and Fraboul, 2007) gives a panorama in terms of average network load but cannot be used for the certification, since rare events leading to underestimate the real upper bound can go unseen. Model checking (Adnan et al., 2012) aims to exhibit scenarios leading to an exact worst-case ETE, but faces with combinatorial explosion for configurations with more than a few links and nodes. Another kind of approach consists in computing sure upper bounds of worst-case ETE delays instead of exact values.

The first method among them, used in the AFDX for certification purpose, was Network Calculus (NC) (Grieu, 2004; Le Boudec and Thiran, 2001), based on \((\min; +)\) algebra (Cruz, 1991). It is based on a per node computation using time-cumulative curves and traffic envelopes. A stochastic extension to NC has also been proposed in Scharbarg et al. (2009). In a holistic approach (Tindell and Clark, 1994), local delays are propagated from an ingress to an egress point of distributed system. Such an approach has been used in Gutiérrez et al. (2012), focused on an AFDX End-System analysis. The trajectory approach (TA) (Martin and Minet, 2005) relies on an analogy between frames served in network nodes and non-preemptive tasks being scheduled on a uniprocessor system. It has been applied in the AFDX context where it can lead to tighter bounds than NC (Bauer et al., 2010). However, TA cannot compute bounds for some types of configurations (high load on several nodes among a given path) and it has been proved (Kemayo et al., 2014) that in some corner cases, the bounds computed with TA can be underestimated and thus cannot be used for certification purpose. The forward end-to-end delays analysis (FA) (Kemayo et al., 2014) overcomes the drawbacks of TA while keeping tighter bounds than NC. The FA method is presented hereafter.

7.2.1 Analysis model

An AFDX network is composed of a set of ingress and egress points called End-Systems (ESs), interconnected via AFDX switches (SWs) with full-duplex links. Each SW output port multiplexes the traffic incoming from several SW input ports. Packets arriving on one input port can also be duplicated to several output ports. Thus, a Virtual Link (VL) is defined as a multicast flow connecting one source ES to one or multiple destination ESs. The number of ESs, SWs and VLs as well as the routes are fully static. A VL is a virtual channel with a given bandwidth reservation. At its ingress point, a VL has to comply with:

- a maximal frame length, denoted \(F_{\text{max}}\)
- a minimum delay between the transmission of two consecutive frames, called bandwidth allocation gap (BAG).

The enforcement of this contract is guaranteed by a traffic shaping unit (packet scheduler) at ES level and a traffic policing unit in each AFDX SW.

The analysis model for such a network is thus composed of a set of \(p\) multiplexing nodes \(S = \{N_1, \ldots, N_p\}\), and a set of \(n\) flows \(\Gamma = \{v_1, \ldots, v_n\}\). A multiplexing node \(N_i\) is a Store-and-Forward output port (from either an ES or a SW), characterised by a constant output rate (\(R = 100\) Mb/s in the case of AFDX), a FIFO output buffer and a technological latency (the switch fabric delay). A flow \(v_i\) is sporadic, unidirectional and unicast: multicast VLs are duplicated at each fork in the path and mapped on several
flows. A flow is thus characterised by its path (an ordered list of nodes, necessarily interconnected by a physical link), a maximum transmission time in an output port \(C_i = F_{\text{max}}/R\), and a period \(T_i\) (in a sporadic sense) equal to the BAG.

The delay incurred by a packet crossing the network (see Figure ??) is composed of:

- a waiting time, highly variable in each node, due to the amount of pending traffic in the output port, called backlog ;
- a transmission time in every node, corresponding to the parameter \(C_i = F_{\text{max}}/R\);
- a constant delay \(L\), composed of the switching delay (16 µs in this case) and the electrical propagation time (often neglected).

### 7.2.2 Forward end-to-end delays analysis

In order to present the details of an End-to-End delay computation for a flow with FA, we will focus on a frame \(f_i\) generated by a flow \(v_i\), following a path \(P_i\).

The worst-case ETE delay is computed iteratively for each node \(h\) of \(P_i\), from the source ES \((\text{first}_i)\) to the last output port \((\text{last}_i)\) directly connected to the destination ES. The worst case delay incurred by \(f_i\), from its generation time in \(\text{first}_i\) until its arrival in \(h\) is denoted \(S_{\text{max}}^h\). By definition: \(S_{\text{max}}^{\text{first}_i} = 0\). Let us denote \(h + 1\) the node after \(h\) in \(P_i\). \(S_{\text{max}}^{h+1}\) can be computed by induction, knowing \(S_{\text{max}}^h\) and the maximum delay incurred in node \(h\) (highlighted by the shaded area in Figure ??). This delay is composed of the transmission time of the pending frames in the FIFO buffer at the arrival time of \(f_i\), called the backlog and denoted \(B_{\text{klg}}^h\), plus its own transmission time (\(C_i\)). By adding the constant latency \(L\), we thus have:

\[
\begin{align*}
S_{\text{max}}^{\text{first}_i} &= 0 \\
S_{\text{max}}^{h+1} &= S_{\text{max}}^h + B_{\text{klg}}^h + C_i + L
\end{align*}
\]  

(2)

The worst-case ETE delay is then equal to the delay incurred while reaching its destination node plus the worst-case backlog in \(\text{last}_i\) (see Formula (4)) plus its own transmission time:

\[
R_i = S_{\text{max}}^{\text{last}_i} + B_{\text{klg}}^{\text{last}_i} + C_i
\]  

(3)

The main difficulty is thus the computation of \(B_{\text{klg}}^h\), the worst-case scenario generating the maximum possible amount of pending packets in the FIFO buffer at arrival of \(f_i\) in node \(h\). This maximisation will be obtained using the same request bound function concept as exposed in Section 7.1.2.

Compared with Formula (1), there are a few changes in the hypothesis though: packet scheduling is non-preemptive and uses a FIFO policy (instead of fixed priority). Let \(t\) be the arrival time of \(f_i\) in \(h\), then the \(rbf\) includes all the frames arriving in the closed time interval \([0, t]: rbf_{\text{h}}^i(t) = \sum_{v_j \in P_h} \left(1 + \left\lfloor \frac{t}{T_j} \right\rfloor \right) C_j\).

However, the \(T_j\) constraint (sporadic sense period) is only guaranteed in the source nodes, thus in a given node \(h\), the minimum arrival time between two consecutive frames of a flow can be reduced by the amount of jitter incurred in the preceding nodes. The jitter is the difference between the minimal and the maximal traversal time up to
node \( h \): \( J^h_j = S_{\text{max}}^h_j - S_{\text{min}}^h_j \), where \( S_{\text{min}}^h_j = (|h_j| - 1) \times (C_i + L) \) is the delay incurred if no traffic is encountered in every crossed node (\(|h_j|\) being the number of hops between first in \( P_j \) and \( h \)).

The backlog generated in node \( h \) during a time interval of length \( t \) is then equal to the transmission time of the frames arrived during this interval (excluding \( f_i \)) minus the time during which the output port has served the queue, which is \( t \). The worst-case backlog can finally be found by taking the maximum for any arrival time \( t \):

\[
B_{\text{klg}}^h = \max_{0 \leq t \leq B^h} \left\{ \sum_{v_j \in \Gamma_h} \left( 1 + \left\lfloor \frac{t + J^h_j}{T_j} \right\rfloor \right) C_j - C_i - t \right\}
\]

(4)

\( B^h \) is a sufficient bound on \( t \), computed with a fixed-point algorithm on the utilisation factor \( (\frac{C_i}{T_i}) \) which can be found in Kemayo et al. (2014).