Modelling of aerodynamic interference of three-DOF GyroWheel rotor

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Abstract: GyroWheel is an integrated mechanism which provides both attitude control torques and measurements of attitude angular rates for tiny spacecrafts. However, aerodynamic interference is inevitable in the process of ground test which hinders the improvement of the system performance. In this paper, the modelling issue of aerodynamic interference with respect to the three-DOF GyroWheel rotor is investigated. Based on the analysis of force distributions associated with each degree of freedom, the dynamical model of the GyroWheel rotor is established with Euler’s method. According to the analysis of medium flow field, the mathematical model of the aerodynamic interference is formulated by utilising the boundary layer theory and Navier-Stokes (N-S) equations, which is illustrated by some numerical simulations in the environment of FLUENT.

Keywords: GyroWheel; aerodynamic interference; boundary layer theory; N-S equations.


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1 Introduction

Miniaturisation is one of the main development trends for spacecrafts and satellites, which leads to the reduction of cost while maintaining high reliability, integration level and aggressive performance capabilities (Gao et al., 2009; Kramer and Cracknell, 2008). From a practical point of view, attitude control system (ACS) is one of the most expensive and weighty subsystems of spacecrafts and satellites. Hence, the reduction of the volume, mass and cost of the ACS is a significant scheme for the lightweight of tiny spacecrafts, as well as improving the payloads.

GyroWheel is an innovative attitude control device that provides both an angular momentum bias and control torques about three axes while at the same time measuring the spacecraft angular rates about two axes (Tye et al., 2001; Liu et al., 2015), i.e., it integrates the actuator and rate sensing capabilities. The conception of GyroWheel is inspired from dynamical tuned gyroscope (DTG), however, it substantially departs from the classical DTG in system and structural parameters, as well as in operating principle. The structure of GyroWheel is a form of double gimbaled momentum wheel, as shown in Figure 1. It utilises a spinning rotor that is attached to the drive shaft through a novel spinning flex-gimbal suspension system which functions as a universal joint and allows the rotor to tilt in two dimensions, i.e., the rotor of GyroWheel has three degrees of freedom (DOF). A brushless DC motor is fixed on the bottom of GyroWheel case as driving device. The magnets are mounted to the surfaces of an annular-shaped gap on the underside of GyroWheel rotor while the torque coils are fixed on the stationary case, which can create the magnetic dipole moments to tilt the GyroWheel rotor. In order to be used as a three-axis actuator, the GyroWheel rotor is never really tuned since its spin speed is time-varying within ±20% of the tuned speed compared with DTG. Furthermore, it must be operated at relatively large tilt angles, but a DTG is only working at a zero tilt case. All of these introduce unique design challenges, from both of the perspective of control implementation and measurement of attitude angular rates.

In the case of ground test, a GyroWheel rotor is operated inevitably in the environment of gravity field and air flow field of the Earth, which reasonably influence the properties of the GyroWheel rotor to a great extent compared to outer space environment. Referring to other inertial components, the influence of Earth’s gravity can be reduced by sufficient ground calibration experiments, such as multi-position and rate testing (Fu et al., 2011; Bekkeng, 2009; Syed et al., 2007). However, the aerodynamic interference of the GyroWheel rotor is one of the most egregious factors which cause the deterioration of the performance, especially when it is used as an angular rates sensor. Hence, the problem of modelling and simulation of aerodynamic interference should get more attention and studies. It should be noted that the spinning rotor of GyroWheel is a three-DOF wheel so that the interference should be analysed in three-dimension space.

Figure 1 Cut-away view of the GyroWheel (see online version for colours)

To the authors’ knowledge, there is hardly any research about the aerodynamic interference of the GyroWheel rotor. However, there are some studies about the gyros in fluid, such as the tuned flexure gyro and liquid floated gyro. Ling and Chen (1998) analysed the inside gas dumping torque and gas dynamic pressure torque of a tuned flexure gyro, and the method to reduce the slanting drift was proposed. The mathematical models of the disturbances caused by gas were established, but lacked further validation (Ling and Chen, 1998). Based on FLUENT software, the inner flow field of a long-life gyro floater was calculated, and the result showed that the structure of the floater had great influence on the medium flow field produced by high speed rotary motor (Lei et al., 2007). The flowing state of liquid medium for the high speed suspended rotor gyro was studied, and the closed flow field between the rotor and the stator was simulated by using the Reynolds stress model of hydromechanics (Tang et al., 2013). The numerical simulation was developed with FLUENT software and micro particle image velocimetry (MPIV) technique was used to observe the motion of flow field and to measure the speed of the flowing field. However, there is lack of the derivation process and mathematical model as an explicit expression in the above researches.

Motivated by the discussion above, the problem of modelling and simulation of aerodynamic interference for three-DOF GyroWheel rotor is investigated in this paper. It is quite difficult to obtain model of the aerodynamic interference by experimental method which is complex and expensive under the influence of other interferences such as mechanical friction. Meanwhile, it is difficult, even impossible, to establish the accurate mathematical model, considering that the viscous fluid motion equations are a set of complicated nonlinear partial differential equations. Hence, it is a feasible and effective scheme to derive the mathematical model by rational simplification which is verified using the simulation results of FLUENT. By stress analysis, the completed dynamical model of GyroWheel is obtained. Under some reasonable hypotheses, the inner flow field between the spinning rotor suspended by a flex-gimbal and the fixed part is analysed, and the mathematical models of aerodynamic torques are established according to the theory of hydromechanics. By using FLUENT software, the motion of the closed flow field of GyroWheel is analysed,
and the aerodynamic torques acting on the spinning rotor in three-dimension space are calculated and compared with the results of mathematical models.

The rest of the paper is organised as follows. In Section 2, the dynamical model of GyroWheel is established using Euler’s method, by considering forces acting on each degree of freedom, respectively. The main results are proposed in Section 3, where the mathematical models of aerodynamic interferences are derived based on boundary layer theory and N-S equations. Based on FLUENT software, some numerical simulations and comparison analysis are given in Section 4, which are used to illustrate the validity of our work. Section 5 concludes the whole paper.

2 Dynamical model of the GyroWheel

We begin this section with the definitions of reference frames with respect to the GyroWheel system, which are used throughout the paper, i.e., the case frame $F_c:O-x_cy_cz_c$, the motor frame $F_m:O-x_my_my_m$, the gimbal frame $F_g:O-x_gy_gz_g$ and the spinning rotor frame $F_r:O-x_ry_rz_r$, meanwhile, the relationship between the reference frames is shown in Figure 2. The $z_c$ axis coincides with the motor shaft while the $x_c$ axis and $y_c$ axis coincide with the axes of the torque coils fixed on the base, respectively in the frame $F_c$. The $z_m$ axis coincides with the motor shaft while the $x_m$ axis coincides with the axis of the inner torsion elements in the frame $F_m$ fixed on the motor shaft. The $z_g$ axis coincides with the polar axis of the gimbal while the $x_g$ axis and $y_g$ axis coincide with the axes of the inner and outer torsion elements, respectively in the frame $F_g$ fixed on the gimbal. The $z_r$ axis coincides with the polar axis of the GyroWheel rotor and the $y_r$ axis coincides with the axis of the outer torsion elements in the frame $F_r$ fixed on the GyroWheel rotor. In Figure 2, $\theta$ is the rotary angle of the motor shaft while $\theta_x$ and $\theta_y$ are the rotary angles of the inner and outer torsion elements.

Assume that the angular velocity of the GyroWheel system with respect to the inertial space is $\omega_{b}$, thus the angular velocity of the motor shaft in the frame $F_m$ can be obtained as

$$\omega_m = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_z \end{bmatrix} + C_m^w \omega_b = \begin{bmatrix} \omega_{x_m} \\ \omega_{y_m} \\ \omega_{z_m} \end{bmatrix}$$  \hspace{1cm} (1)

where $C_m^w$ denotes the transformation matrix from the frame $F_m$ to $F_w$. Analogously, the angular velocities of the gimbal and the GyroWheel rotor in the frames $F_g$ and $F_r$ can be obtained respectively as follows.

$$\omega_g = \begin{bmatrix} \dot{\theta}_x \\ 0 \\ C_g^w \omega_m \end{bmatrix} = \begin{bmatrix} \omega_{x_g} \\ \omega_{y_g} \\ \omega_{z_g} \end{bmatrix}$$  \hspace{1cm} (2)

$$\omega_r = \begin{bmatrix} \dot{\theta}_y \\ 0 \\ C_r^w \omega_m \end{bmatrix} = \begin{bmatrix} \omega_{x_r} \\ \omega_{y_r} \\ \omega_{z_r} \end{bmatrix}$$  \hspace{1cm} (3)

where $C_g^w$ and $C_r^w$ denote the transformation matrices from the frame $F_m$ to $F_g$ and from the frame $F_g$ to $F_r$, respectively.

Figure 3  The stress analysis of motor shaft

Figure 4  The stress analysis of gimbal
The stress analyses of the motor shaft, the gimbal and the GyroWheel rotor are given in Figures 3, 4 and 5, respectively, where it is considered that the moving components of the GyroWheel listed above are all rigid bodies. As a result, the Euler dynamical equations of the motor shaft can be obtained as follows.

\[
\begin{align*}
2(K_x\dot{\theta}_x + C_x\dot{\theta}_x) &= 0 \\
2G_{ax}R_t &= 0 \\
2G_{mx}R_t + T_{mc} &= I_{mc}\omega_{mc}
\end{align*}
\]

where \(G_{ax}\) and \(G_{mx}\) denote the interaction forces between the motor shaft and gimbal acting on the motor shaft. \(R_t\) is the arm of \(G_{ax}\) and \(G_{mx}\). \(I_{mc}\) is the moment of inertia of the motor shaft along \(z_m\) axis, and \(T_{mc}\) denotes the control torque acting on the motor shaft along the \(z_m\) axis. \(K_x\) and \(C_x\) denote the torsional rigidity and the damping coefficient of the outer torsion elements.

The Euler dynamical equations of the gimbal are derived as

\[
\begin{align*}
T_{gx} - 2F_{gx}R_2 - 2(K_x\dot{\theta}_x + C_x\dot{\theta}_x) &= I_{gx}\omega_{gx} + (I_{gx} - I_{gy})\omega_{gy}\omega_{gy} \\
2G_{gx}R_t + 2(K_x\dot{\theta}_x + C_x\dot{\theta}_x) &= I_{gy}\omega_{gy} + (I_{gy} - I_{gx})\omega_{gx}\omega_{gy} \\
-2G_{gy}R_t - 2F_{gy}R_2 &= I_{gy}\omega_{gy} + (I_{gy} - I_{gx})\omega_{gx}\omega_{gy}
\end{align*}
\]

where \(T_{gx}\) and \(F_{gy}\) denote the interaction forces between the gimbal and GyroWheel rotor acting on the gimbal. \(R_2\) is the arm of \(F_{gy}\) and \(F_{gy}\). \(G_{gx}\) and \(G_{gy}\) denote the interaction forces between the motor shaft and gimbal acting on the gimbal. \(I_{gx}\) and \(I_{gy}\) are the moments of inertia of the gimbal in the frame \(F_g\), and \(T_{gx}\) represents the control torque acting on the gimbal along the \(x_g\) axis. \(K_c\) and \(C_c\) denote the torsional rigidity and the damping coefficient of the outer torsion elements.

Analogously, the Euler dynamical equations of the GyroWheel rotor can be obtained as follows:

\[
\begin{align*}
2F_{rx}R_2 - T_{dx} &= I_{rx}\omega_{rx} + (I_{rx} - I_{ry})\omega_{ry}\omega_{ry} \\
T_{ry} - T_{dy} - 2(K_y\dot{\theta}_y + C_y\dot{\theta}_y) &= I_{ry}\omega_{ry} + (I_{rx} - I_{ry})\omega_{rx}\omega_{ry} \\
2F_{ry}R_2 - T_{dz} &= I_{ry}\omega_{ry} + (I_{rx} - I_{ry})\omega_{rx}\omega_{ry}
\end{align*}
\]

where \(F_{rx}\) and \(F_{ry}\) denote the interaction forces between the gimbal and GyroWheel rotor acting on the GyroWheel rotor. \(I_{rx}\), \(I_{ry}\) and \(I_{rz}\) are the moments of inertia of the GyroWheel rotor in the frame \(F_r\). \(T_{dx}\), \(T_{dy}\) and \(T_{dz}\) denote the disturbance torques caused by air flow along the \(x_r\), \(y_r\) and \(z_r\) axes, respectively.

Without loss of generality, assume that the base of the GyroWheel is fixed, thus the dynamical model of the three-DOF system can be obtained as

\[
M\ddot{q} + C\dot{q} + Kq = T_c - DT_d + F_{nl}(q, \dot{q})
\]

where \(\dot{q} = [\theta_x, \theta_y, \theta_z]^T\) denotes the state vector. The inertia matrix, damping and stiffness matrix are given as follows. \(T_c\) and \(T_d\) denote the control torque offered by the torque coils and the disturbance torque introduced by air action, respectively, and \(F_{nl}\) denotes the nonlinear terms.

\[\begin{bmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{bmatrix}, \begin{bmatrix}
C_{xx} & C_{xy} & C_{xz} \\
C_{yx} & C_{yy} & C_{yz} \\
C_{zx} & C_{zy} & C_{zz}
\end{bmatrix}, \begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{bmatrix}\]

\[\begin{bmatrix}
T_{dx} \\
T_{dy} \\
T_{dz}
\end{bmatrix}, \begin{bmatrix}
f_{nlx} \\
f_{nly} \\
f_{nlz}
\end{bmatrix}, \begin{bmatrix}
\cos \theta_x & 0 & \sin \theta_x \\
0 & -1 & 0 \\
-\cos \theta_x & \sin \theta_x & 0 & \cos \theta_y
\end{bmatrix}\]

Remark 1: It should be noted that \(T_{gx}\) and \(T_{gy}\) cannot be used directly since they are different from the outputs of the torque coils \(T_{cx}\) and \(T_{cy}\), where the transformation is given in the sequel. Analogously, \(\theta_x\) and \(\theta_y\) cannot be measured directly according to the configuration of the tilt sensors, and the relationship between \(\theta_x\), \(\theta_y\) and \(\phi_x\), \(\phi_y\) is also provided here, where the latter can be conveniently measured in the case frame.

\[
\begin{bmatrix}
T_{gx} \\
T_{gy}
\end{bmatrix} = \begin{bmatrix}
\cos \theta_x & \sin \theta_x & 0 \\
-\cos \theta_x & \sin \theta_x & 0 & \cos \theta_y
\end{bmatrix} \begin{bmatrix}
T_{cx} \\
T_{cy}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\theta_x \\
\theta_y
\end{bmatrix} = \begin{bmatrix}
\cos \theta_x & \sin \theta_x \\
-\sin \theta_x & \cos \theta_y
\end{bmatrix} \begin{bmatrix}
\phi_x \\
\phi_y
\end{bmatrix}
\]

According to the equation (7), the disturbance torque \(T_{dz}\) which is caused by the aerodynamic interference, has direct effect on the dynamical model of GyroWheel. This effect would interfere with the measurement of attitude angular
rates and the control torques about three axes in the case of ground test. Hence, it is significant and necessary to investigate the model issue of the aerodynamic interference as presented in next Section, which can contribute to improve the measurement and control accuracy of the GyroWheel system.

3 Mathematical modelling

In this section, the medium flow field around the three-DOF GyroWheel rotor is analysed under reasonable hypotheses with the systematic and environmental parameters listed in Table 1, and all of the analysis and calculation for the aerodynamic interference are developed in the spinning rotor frame $F_t$.

Table 1 Simulation parameters list

<table>
<thead>
<tr>
<th>Structure parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of GyroWheel rotor, $R$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Height of GyroWheel rotor, $h$</td>
<td>30 mm</td>
</tr>
<tr>
<td>Maximum title angle, $\theta_t$</td>
<td>$\pm 5^\circ$</td>
</tr>
<tr>
<td>Working angular velocity, $\omega_w$</td>
<td>3,500 ± 500 rpm</td>
</tr>
<tr>
<td>Temperature, $T$</td>
<td>20°C</td>
</tr>
<tr>
<td>Atmospheric pressure, $p_0$</td>
<td>101,325 Pa</td>
</tr>
<tr>
<td>Density of air, $\rho$</td>
<td>1.205 kg/m$^3$</td>
</tr>
<tr>
<td>Coefficients of viscosity of air, $\mu$</td>
<td>$1.81 \times 10^{-5}$Pa∙s</td>
</tr>
</tbody>
</table>

3.1 Analysis of the inner flow field

Compressibility of the atmosphere is one of the most significant characteristics where the density of this medium always varies sensitively with the velocity of the airflow and the temperature nearby (Dong and Zhang, 2003). Under laboratory conditions, the variations in temperature can be controlled and restricted, that is, an isothermal condition is guaranteed. Hence, the relationship of relative density change with respect to the angular velocity of the GyroWheel rotor, as shown in Figure 6, can be obtained as

$$ \frac{d\rho}{\rho} = \frac{y}{2} M^2 $$ (10)

where $\rho$ and $M$ denote the density and Mach number of the atmosphere. Obviously, the relative density change around the GyroWheel rotor is quite small so that the air can be considered as a kind of incompressible fluid in the inner flow field in the case that the spinning velocity of the GyroWheel rotor is relatively lower.

In order to describe the state of the fluid motion, the notion of Reynolds number is introduced, which is denoted by the ratio of the non-viscous force to the viscous force (Chekmarev, 2004). Generally, the state of the fluid motion is laminar flow while the Reynolds number is less than 2300, and the state of the fluid motion is turbulence flow while the Reynolds number is greater than 4,000. According to Figure 6, the state of air motion in the inner flow field is obviously turbulent flow for the GyroWheel rotor.

$$ Re = \frac{\rho R^2 \omega_w}{\mu} $$ (11)

where $\mu$ denotes the coefficients of viscosity, $R$ denotes the radius of the GyroWheel rotor, and the angular velocity of the GyroWheel rotor is denoted by $\omega_w$. 

Figure 6 Reynolds number and relative density under 20°C (see online version for colours)

3.2 Mathematical model of axial disturbance torque

Based on the relative motion theory and the viscosity of the air flow field, a turbulent boundary layer will be formed nearby the spinning GyroWheel rotor which can be divided into two parts: the inner of the boundary layer and the external of the boundary layer. For the sake of brevity, a cylindrical coordinates frame $F_r':O-r\beta z$ is founded instead of the frame $F_t$ where the origin and $z$ axis of the two frames are coincident.

The velocity gradient of air motion inside the boundary layer is large due to the no-slip condition that there is no slip of the fluid relative to the surface when the fluid is next to a solid surface, which means that the frictional resistance caused by the viscosity plays a major role which causes the disturbance torque $T_{de}$. It is convenient to obtain the frictional resistance by using the boundary layer theory proposed by Ludwig Prandtl (Kunkel and Marusic, 2006; Jiménez et al., 2010).

Without loss of generality, assume that $dS$ is an area microelement on the GyroWheel rotor surface, where the air flows along tangential direction at any instant. Thus, the caused friction resistance and the disturbance torque can be derived as follows:

$$ dF = \frac{1}{2} C_f \rho (\omega_w r)^2 dS $$ (12)

$$ T_{de} = \iint_S r dF $$ (13)

where $r$ denotes the distance from the area element to the polar axis and $C_f$ is the frictional resistance coefficient of the turbulence flow, which is introduced in Guo (2008).
It should be noted that $d\Sigma$ can be represented as $rd\beta dr$ on the top and bottom surface of the GyroWheel rotor where $\beta$ denotes the rotational angle of the fluid microelement in frame $F_r$. Analogously, $d\Sigma$ can also be represented as $hrd\beta$ on the lateral surface of the GyroWheel rotor where $h$ denotes the height of the GyroWheel rotor.

Substituting (12) and (14) into (13) yields,

$$
T_{dz} = \left(h + \frac{2}{5} \right) R^4 \rho \omega_c^2 \frac{1.328}{\sqrt{Re}}
$$

### 3.3 Mathematical models of lateral disturbance torques

The lateral disturbance torques of $T_{dx}$ and $T_{dy}$ are mainly caused by the non-viscous forces which are caused by the uneven pressure distribution on the top and the bottom surfaces of the GyroWheel rotor. The air, away from the surface of the GyroWheel rotor, can be viewed as a non-viscous fluid since the non-viscous force is far greater than the viscous force according to the large Reynolds number.

Generally, the movement state of the fluid microelement can be described by the N-S equations and the continuity equation, where the viscous term can be neglected as

$$
\begin{align*}
    f_r &= \frac{1}{\rho} \left( \frac{\partial p}{\partial r} - u_r \frac{\partial u_r}{\partial r} \right), \\
    f_\beta &= \frac{1}{\rho r} \left( \frac{\partial p}{\partial \beta} - \frac{u_\beta}{r} \frac{\partial u_r}{\partial \beta} - u_r \frac{\partial u_\beta}{\partial \beta} \right), \quad f_z = 0,
\end{align*}
$$

where $u_r$, $u_\beta$ and $u_z$ denote the velocity of the microelement along the axis $r$, $\beta$ and $z$, respectively. Noted that $u_\beta = \omega_c r$ and $u_z = 0$. $f_r$, $f_\beta$ and $f_z$ denote unit volume forces acting on the fluid microelement where $f_r = 0$, $f_\beta = -\omega_c^2 r$ and $f_z = 0$, and $p$ denotes the pressure of the air.

The continuity equation is given as follows,

$$
\frac{\partial u_r}{\partial r} + \frac{u_r}{r} \frac{\partial u_r}{\partial \beta} + \frac{\partial u_z}{\partial z} = 0
$$

According to (16) and (17), the pressure field around the GyroWheel rotor can be obtained as

$$
\frac{\partial p}{\partial r} = 0
$$

which means that the distribution of the atmosphere pressure is symmetrical on the top and the bottom surfaces of the GyroWheel rotor, so that the lateral disturbance torques could be neglected from a theoretical point of view.

Above all, the aerodynamic interference causes two varieties of disturbance torques, the axial disturbance torque $T_{dz}$ and the lateral disturbance torques $T_{dx}$, $T_{dy}$. The axial disturbance torque is the major component of the aerodynamic interference which increases with respect to the rising of the angular velocity, meanwhile, the lateral disturbance torques are almost non-existent, as shown in Figure 7.

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**4 Numerical illustrations**

So far, the numerical simulation is playing an important role in the research area of fluid dynamics (Liu and Zhang, 2015; Rose and Jinu, 2016; Malek and Azar, 2016). Thousands of engineers throughout the world benefit from the use of FLUENT software as an integral part of the design and optimisation phases of their product development. In order to verify the validity of the mathematical models developed in Section 3, some numerical simulation examples are presented in the sequel, where the flow field and pressure field around the three-DOF GyroWheel rotor are constructed in FLUENT software.

**4.1 Modelling in FLUENT**

The geometric models of the inner flow field and the GyroWheel rotor are built by GAMBIT software, which is a pre-processing module of FLUENT for partitioning the grid, setting the boundary condition and implementing other tasks. The GyroWheel rotor is considered as a free cylindrical rotor, and the air is considered as an incompressible fluid in the turbulent state for reducing the complexity of the simulation while the calculation accuracy can be satisfied.

The density of grid must be rationally controlled since the quality of grid has a great influence on the computation time and accuracy. In our design, the unstructured grid is used as the body grid of the space around the GyroWheel rotor while the surface grid of the GyroWheel rotor is a kind of mixture grid combined with quadrilateral grid and...
Modelling of aerodynamic interference of three-DOF GyroWheel rotor

triangular grid, as shown in Figure 8, where the total number of the body grid is up to 950504 based on the demands of high precision and rapid speed.

**Figure 8** The grids partitioning, (a) body grids around the GyroWheel rotor (b) surface grids of the GyroWheel rotor

Based on finite volume method, discretised Reynolds-averaged Navier-Stokes (N-S) equations are used as the governing equations (Xu et al., 2011). The pressure-based solver is chosen which is mainly used for the incompressible flow and the micro-compressible flow, and the turbulence is modelled using the standard k-ε model in this paper (Gavelli et al., 2008).

\[
\frac{\partial u_i}{\partial x_j} = 0
\]

\[
\rho \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_i} + \rho \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \rho g_i - \frac{\partial}{\partial x_j} \left( \rho \omega_i \omega_j \right)
\]

where \( u_i \) and \( u_j \) (\( i, j = 1, 2, 3 \)) denote the average velocity along three axes, respectively. The parameter \( \nu \) is the coefficient of kinetics viscous of the air, and \( g_i \) denotes the volume forces. Reynolds stress is described as \(-\frac{\partial}{\partial x_j} \rho \omega_i \omega_j \). The iterative computations are finished until the residuals are less than \( 10^{-3} \), as shown in Figure 9.

**4.2 Grid independence analysis**

The numerical simulation results are greatly influenced by the grid scheme. In order to illustrate the reasonableness and feasibility of the grid which is used in this paper, the grid independence should be taken into consideration. In our case, the total number of the grid is changed by choosing different maximum cell volume while the grid is refined around the whole inner flow field for comparing the influence of different grids. Considering the working range of the GyroWheel rotor, the simulation results of different grids are shown in Figure 10 where the grid numbers of the Grid1, Grid2 and Grid3 are 231336, 494959 and 950504, respectively. Without loss of generality, the case of \( T_{dx} \) is considered here due to the equivalent relationship between \( T_{dx} \) and \( T_{dy} \). Obviously, the simulation results are hardly influenced by the change of grid while the grid number is more than 494959 which means that the grid independence is achieved. Hence, the grid quality is further ensured and the simulation results are believable while the grid number is up to 950504 which is used in this paper.

**Figure 9** Residuals in FLUENT (see online version for colours)

**Figure 10** Simulation results of different grids while the tilt angle is 0\(^\circ\), (a) the lateral disturbance torques \( T_{dx} \) (b) the axial disturbance torques \( T_{dz} \) (see online version for colours)
**Figure 10** Simulation results of different grids while the tilt angle is 0°, (a) the lateral disturbance torques $T_{dx}$ (b) the axial disturbance torques $T_{dz}$ (continued) (see online version for colours)

**Figure 11** Velocity contour in the plane of $z = 0$ while the tilt angle is 0° under 3,500 rpm (see online version for colours)

**Figure 12** Simulation results while the tilt angle is 0° under 3,500 rpm, (a) pressure contour on the surface of GyroWheel rotor (b) velocity vectors near GyroWheel rotor (c) the disturbance torques (see online version for colours)

4.3 Analysis of the simulation results

Figure 11 shows the velocity contour of air in the plane of $z = 0$, and the central white part is the cross section of the GyroWheel rotor. It can be seen from the local enlarged part that the velocity of air changes rapidly in a thin region which is known as the boundary layer, where frictional resistance plays a dominant role to the aerodynamic disturbance. The boundary layer is so thin that the pressure distribution around the GyroWheel is almost identical whether inside or outside the boundary layer as proposed in the Section 3.3.

The simulation results where the tilt angle are 0° is shown in Figure 12, where the pressure contour and velocity vectors on the surface of the GyroWheel rotor are given, respectively. The calculated disturbance torques are shown with respect to the angular velocity of the GyroWheel rotor, which coincides with the mathematical results presented in Section 3. In order to distinguish the influence of different
tilt angles, a comparison example is given where the tilt angle of the GyroWheel rotor is 5°, as shown in Figure 13. It is obvious that the pressure distribution is almost identical on the top and bottom of the GyroWheel rotor, where the errors of disturbance torques are shown in Figure 14, which in turn illustrates that the change of tilt angle will not introduce additional torques acting on the lateral channel of the GyroWheel rotor.

**Figure 13** Simulation results while the tilt angle is 5° under 3,500 rpm, (a) pressure contour on the surface of GyroWheel rotor (b) velocity vectors near GyroWheel rotor (c) the disturbance torques (see online version for colours)

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**Figure 14** Errors of disturbance torques between the tilt angle of 0° and 5° in FLUENT (see online version for colours)

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**Figure 15** Secondary flow of air near the GyroWheel rotor while the tilt angle is 5° under 3,500 rpm, (a) the axial motion of air on the x = 0 surface (b) the axial motion of air on the y = 0 surface (c) the radial motion of air on the z = 0 surface (see online version for colours)
Also, the change of tilt angle hardly influences the trend of the axial disturbance according to the simulation results, because the main formation reason of the axial disturbance is the change of angular velocity which is not influenced by the tilt angle, as shown in Figure 12 and Figure 13. Further, the influence of the motion of the GyroWheel rotor mainly loads on the part of air close to surface where the velocity vectors are much larger than other parts, which means that the effect of the far flow field is small.

According to Figure 7, Figure 12, Figure 13 and Figure 14, the simulation results are mostly coincident with the results of mathematical model established in Section 3, which verify the correctness of main results of this paper. However, there exist some errors between the mathematical models and the simulation results with $T_{dx}$, $T_{dy}$ and $T_{dz}$, which is obvious according to Figure 14, where the lateral or radial disturbance torques are non-zero although their magnitudes are definitely small. The main reason for this is that the secondary flow of air motion is ignored while the mathematical model is established for the sake of brevity. As an illustrated example shown in Figure 15, the secondary flow is analysed with an angular velocity of 3,500 rpm, which consists of axial motion as well as radial motion of air around the GyroWheel rotor, and it is distinct that the pressure distribution is uneven on the top, bottom and lateral surfaces of the GyroWheel rotor. Actually, $T_{dx}$ and $T_{dy}$ caused by the secondary flow can be ignored since they are several orders of magnitude smaller than the control torques produced by the torque coils.

5 Conclusions

In this paper, the modelling problem of aerodynamic interference is addressed in the case of ground test for GyroWheel with three DOF, which is a novel attitude control and measuring sensor for tiny spacecrafts. Based on the boundary layer theory and N-S equations, the mathematical model of the aerodynamic disturbance is established, which can be divided into the axial disturbance torque and the lateral disturbance. Then, the mathematical model of the aerodynamic disturbance is illustrated by numerical simulations with FLUENT software. According to the simulation results, the air motions around the GyroWheel rotor mainly consist of the tangential movement and secondary flow. Furthermore, the lateral disturbance, which is caused by the secondary flow, can be ignored although there exists a larger tilt angle in the operating process. However, the axial disturbance leads to the major component of the aerodynamic interference while the GyroWheel is tested in the ground, which is caused by the tangential movement of air and increases with respect to the angular velocity of the GyroWheel rotor.

In the current work, the GyroWheel rotor is simply regarded as a cylindrical rotor. Hence, we should step forward to study the influence of different complex shape of the GyroWheel rotor on aerodynamic interference. Besides, it is very valuable and necessary to research the precision servo control problem of the GyroWheel system considering the influence of the aerodynamic interference which will improve the accuracy and stability of the GyroWheel system. Future research work will focus on these topics.

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