Lubricated loaded tooth contact analysis for spur gear pair

Callum Oglieve, Gajarajan Sivayogan,
Mahdi Mohammadpour* and Homer Rahnejat

Wolfson School of Mechanical, Electrical and
Manufacturing Engineering,
Loughborough University, UK
Email: callumoglieve@gmail.com
Email: G.Sivayogan@lboro.ac.uk
Email: m.mohammad-pour@lboro.ac.uk
Email: H.Rahnejat@lboro.ac.uk
*Corresponding author

Abstract: Gears are key components to the operation of many machines and mechanisms. However, their presence often affects system efficiency and can lead to noise, vibration and harshness (NVH) issues. Analyses described in open literature study tooth contact neglecting the effect of lubrication. In reality, contact mechanics and lubrication are closely inter-linked, requiring an integrated approach. This paper outlines a combined FEA-based TCA model with a lubricated contact mechanics analysis for real gear pairs measured from coordinate measuring machine (CMM), thus improving the prediction of gear pair efficiency, NVH and durability. An initial dry gear analysis with an estimated constant coefficient of friction in the contact is carried out. The results of this initial analysis provide input data for a subsequent tribological model in order to generate improved estimates of the contact friction for a new TCA. This approach leads to the integration of TCA and lubrication in an iterative manner.

Keywords: tooth contact analysis; TCA; lubrication; tribology; spur gear pair.

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Biographical notes: Callum Oglieve graduated from Loughborough University with a Master's in Mechanical Engineering and now works at Ricardo Automotive; designing, testing and analysing internal combustion engines.

Gajarajan Sivayogan obtained a degree in Mechanical Engineering from the University of Hull in 2015, where he also obtained the Barnes Wallis Memorial Award for his performance and effort. He started his PhD in 2016, under EPSRC Centre for Doctoral Training in Embedded Intelligence (CDT-EI) in Loughborough University and in collaboration with AVL GmbH. His PhD research focuses on the tribodynamics of hypoid gear pairs with the main aim to improve techniques for identifying and predicting frictional losses, particularly where the elastohydrodynamic lubrication (EHL) regime is dominant.

Mahdi Mohammadpour is Senior Lecturer in Dynamic Research Group of the Wolfson School of Mechanical, Electrical and Manufacturing of Loughborough University. He received his PhD in Tribodynamics from Loughborough University in 2014. His research focuses mainly on dynamics, tribology, energy efficiency and the related subjects. He is the Associate Editor for *International Journal of Powertrains*. He is also on editorial board of *Shock and Vibration Journal*.

Homer Rahnejat is the Chair of Dynamics at the Wolfson School of Mechanical, Electrical and Manufacturing of Loughborough University. He received his PhD from Imperial College in 1984. He is the Editor-in-Chief of *Proceedings of Institution of Mechanical Engineers*, *Part K: Journal of Multi-body Dynamics* and *MDPI's Lubricants*. He has been the Chairman, co-Chair and keynote speaker at many research conferences.

1 Introduction

Gears are utilised in a wide variety of industries such as; automotive, industrial machines and aerospace. The main purpose of gears is to transmit power across shafts changing the torque-speed characteristics (Budynas and Nisbett, 2011).

Analysis and testing of gear pairs is crucial to the development of gear design. Gears are subjected to significant forces which act on relatively small contact areas of the teeth, which leads to high contact pressure and stresses (Lee, 2009). Gears are also subjected to millions of load cycles, so fatigue strength and surface wear are other key considerations. Conservative safety factors are often applied in the gear design process to account for these harsh conditions (Kim and Stoker, 2009). Lubrication analysis of the gear surfaces is another key task in the design of gear pairs. The lubrication film thickness is crucial in the prediction of wear rate, friction and NVH characteristics occurring at the contacts (Paouris et al., 2015; Mohammadpour et al., 2012).

In more recent years, the emphasis of gear design has shifted towards increased efficiency, specifically in the automotive industry where emissions regulations are becoming increasingly more stringent in an attempt to reduce the fuel consumption and the harmful exhaust gas emissions (Douglas et al., 2005). Transmissions in passenger vehicles determine the engine loading and are therefore crucial to the fuel consumption and emissions (Hui and Wang, 2016). The overall efficiency of the transmission can range between 90% and 99% depending on the selected gear and design (Hui and Wang, 2016). Losses arise from; gear meshing, oil churning and bearing friction.

Traditional analytical methods for gear tooth contact analysis (TCA) have utilised two models for predicting the contact stress and the bending stress at the base of the gear tooth. These models are the Hertzian contact and Lewis bending stress respectively (Kim and Stoker, 2009).

The Lewis bending equation is derived from a simplified model of a gear tooth by assuming that the gear tooth is a cantilever beam with a constant rectangular area (Budynas and Nisbett, 2011). The model assumes that the load is not being shared between teeth, which is not applicable for the majority of gear pairs (Budynas and Nisbett, 2011). Most gear pair designs include contact ratios above unity as they tend to be quieter and have reduced bending stresses (Herscovici, 2006). Another assumption in the model is that the greatest load will occur on the tip of the tooth. This is not entirely true, there will usually be an additional pair of teeth in contact and so the maximum load often occurs at the centre of the meshing cycle (Budynas and Nisbett, 2011). The American Gear Manufacturers Association (AGMA) has modified the Lewis bending equation so that the load is applied at the pitch radius, but this still does not account for the loads dependency on the gear mesh angle (Colbourne, 2012).

The Hertzian line contact model is used in the prediction of the maximum contact pressure between two cylinders (Hertz, 1895). The model can be utilised for gear TCA by using the relevant gear tooth geometry to replicate an equivalent two-cylinder contact situation (Budynas and Nisbett, 2011; Kim and Stoker, 2009). The model can be adapted for use with more complex gear pairs such as helical and hypoid by using an equivalent elliptic point contact situation (Hertz, 1895). The main assumptions in the model are; the gear teeth can be modelled as equivalent geometries and that there is no deflection in the gear tooth, which are not entirely valid in practice (Kim and Stoker, 2009).

Mohammadpour et al. (2009) used an energy method in order to model the real tooth geometry, as well as taking in to account the load sharing as further development of the simplified Lewis method. They also included the localised deformation of the tooth by employing the Hertzian contact.

For some gear types such as hypoid and bevel gears, ease-off topology and shell theory has been utilised in order to develop a computationally efficient TCA method (Kolivand and Kahraman, 2009; Shih, 2010).

More recently, FEA has been used to perform various analyses of gear pairs (Litvin et al., 1996). The advantage of using such software is the ability to model complex geometries and loading conditions (Kim and Stoker, 2009). Mao (2007) utilised nonlinear FEA to simulate gear teeth contact. The resulting model was used to investigate various changes to the gear teeth geometry which led to a reduction in the surface fatigue and wear. Gurumani and Shanmugam (2011) carried out crown radius studies with a FEA model, the results showed a reduction in transmission error for the spur gear pair. Transmission error contributes significantly to gear whine noise which is associated with poor NVH.

Fatourehchi et al. (2016) combined TCA with an elastohydrodynamic lubrication analysis to predict the power loss and sub-surface stresses in high performance racing gear pairs. The subsequent model was used to study the effects of parabolic tip relief and crown radius modifications on the gear pair efficiency. The results of the study showed that the tip relief increased the film thickness in the initial stages of the meshing but would lead to a reduction in contact efficiency in the rest of the meshing cycle. The crown radius modifications would lead to increased power losses but would reduce pressures at the edge of the gear flank (Fatourehchi et al., 2016). The study also showed that sub-surface stresses increased with increasing tip relief, this would lower durability as a result of increased cyclic shear stresses. Xu et al. (2007) presented a combination of TCA and tribological model for the efficiency calculations of the parallel axis gears. The presented method comprises explicit simulations of the tribological model for different range of working conditions.

The FEA models described in the literature do not account for the lubrication occurring at the contact. Therefore the presented model which considers both contributions would be desirable for improving the prediction of the; efficiency, durability and NVH characteristics of gear pairs. Additionally, commercially available software cannot account for the differences which occur from manufacturing tolerances. Hence, the presented method here enabled to generate geometry from CMM measurement which would offer a greater representation of real world gear pair interactions. Finally, the presented model provides a generic approach covering complex geometries such as hypoid gears and spiral bevel gears as well as novel applications such as beveloids.

2 Model description

2.1 Geometry

The simulations were carried out on a single gear set from a high performance racing car transmission. The reason for this choice is due to the increased importance of durability prediction of racing transmissions over consumer-based products. This is mainly due to physical testing restrictions and the constant need for pushing the limits of safety factors and margins in order to maintain competitiveness. Also, for the purpose of this study, it was preferred to have as simple as possible example of spur gears to be able to demonstrate the capabilities of the model without additional complexities.

Due to the repeating nature of the gear teeth, a simplification of the geometry can be made to reduce the size of the computational domain. The simplification is to carry out the analyses with three gear teeth on the pinion and wheel, as this is deemed sufficient to model the variance that occurs throughout the contact.

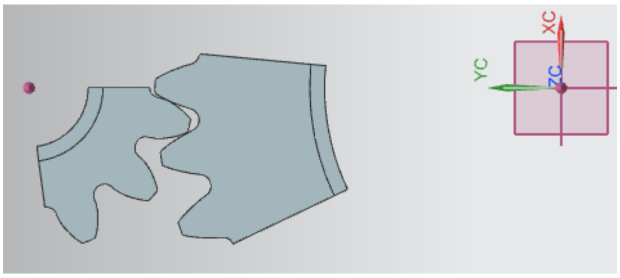
To ensure accurate representation of the complex gear tooth profile a coordinate measuring machine (CMM) is used instead of the ideal CAD data. CMM outputs the x, y, z coordinates along the profile with high precision (±1.5 μm). The raw data output from the machine can be used as an input into the CAD software (NX 8.5) and a spline tool can be used to interpolate between measurement points. The profiles can then be extruded to form the solid gears sections. Further gear specifications are shown in Table 1.

Table 1 Gear pair specification

Gear type	Spur
Pinion no. teeth	13
Wheel no. teeth	35
Gear module	3.8
Centre distance	90 mm
Gear width	13.3 mm

The individual gear components can be assembled so that they are positioned on the input and output shafts with the defined centre distance. The gears are aligned to a nominal position at the start of the meshing cycle shown in Figure 1.

Figure 1 Gear geometry assembly (see online version for colours)



2.2 Material

The material is defined as linear elastic with a constant Young's modulus. The material of the gears is steel FND 15NiMoCr10 with a density of 7,800 kg/m^3, Young's modulus of 206 GPa and Poisson's ratio of 0.3 (Aubert and Duval, 2016).

2.3 Analysis solver settings

The FEA solution is to be analysed as a quasi-static type simulation, where separate static analyses are carried out at discrete points in the meshing cycle (Lee, 2009). The benefit to this type of simulation is the reduced computational time over dynamic simulations. This is especially important for this analysis due to the high mesh densities and the additional lubrication model. The major downside of the static analysis is that the dynamic effects will be neglected. These include the inertial contributions (Lee, 2009).

The high speed rotation of the gears means the inertia will lead to increased contact forces and deflection. This effect is expected to be small in comparison to the high torque loading and the relatively low inertia gear pairs. If the results show great disparity with the expected values then a dynamic analysis will be required.

Torsional vibration output from engine into transmissions can lead to dynamic phenomena such as gear rattle. During this condition gears can lose contact and in extreme cases contact can be made on the reverse face of the gear. Gear rattle is most prominent at low rotational speeds and low engine loads as the decreased loading on the output gear wheel can lead to angular accelerations greater than the drag torque (Seaman et al., 1984). The effects of gear rattle are expected to be insignificant for the gears in this simulation as the engine speed and load is very high.

2.4 Contact modelling

Abaqus FEA software is utilised for the simulations detailed in this report. The main driving force behind this choice is the flexibility and its advanced contact models. The general contact interaction is applied to both of the gear teeth profile surfaces with the pinion and wheel defined as master and slave surfaces respectively. The normal and tangential contact interactions are enabled with the 'hard contact pressure-overclosure relationship' and 'friction penalty' options applied respectively. The hard contact over-closure option is utilised to ensure that the gear surfaces do not penetrate each other whilst in contact from the high loading conditions expected, but will still allow deformation of both bodies at the contact (Howard et al., 2001). A coefficient of friction is required for the friction penalty option of the tangential contact which is considered to be 0.05 initially, but is to be calculated by the tribological model for later iterations.

2.5 Constraints and loading

To model each of the gears on the shafts, the inside surface of the hub is kinematically coupled in all degrees of freedom to a fixed node at the centres of the gears (Howard et al., 2001). The coupling function means the rotation exerted on the node will be transferred to all of the gears nodes (Abaqus Analysis User Guide 6.14, 2014).

The node at the centre of the pinion is constrained in all directions except for the rotation about the *z* axis. The torque can then be applied in this rotation axis to initiate contact. The node at the centre of the output wheel is fixed in all directions and rotations to replicate the loading.

2.6 Mesh generation

The mesh type and size must be defined to ensure accurate representation of the geometry and the solution (Benham et al., 1996). The mesh however should not be too fine, as this can lead to an over defined mesh, in this situation the solution will converge to the same answer as a coarser mesh, but have an unnecessary increase in computational time (Benham et al., 1996). A mesh sensitivity study can be used to identify convergence of a particular output result from a simulation with respect to the number of nodes or size of mesh.

The wedge type mesh was utilised in the model as it produces elements with a triangular prism shape. This means the triangular faces can be placed on the profile and side surfaces of the gear to form consistent layers throughout the face width of the gear. The advantage of triangular faces in the wedge elements is the ability to accurately represent complex geometry with reduced element count. This is important to ensure the tooth profile is sufficiently modelled (Figure 2). The quadrilateral faces of the wedge elements are used across the face width which maintains a uniform surface (Figure 3). Another benefit is that the number of elements is lower compared to the equivalent tetrahedral element type. The mesh was further refined by utilising 2nd order elements which have additional nodes at the mid-points of the element sides, this leads to a better approximation of the surface geometry and interpolation of results (Abaqus Analysis User Guide 6.14, 2014). The downside is an increase in the number of nodes in each element and therefore increased computation effort.

A base size mesh of 1mm was applied to both gears and the element size is reduced on the geometry that is deemed important. The mesh size on the tooth profile edge was refined so that the element size is 0.1 mm. This would ensure accurate representation of the profile and additionally provide sufficient number of nodes at the contact patch (Kim and Stoker, 2009).

Figure 2 Gear assembly meshed (front face) (see online version for colours)

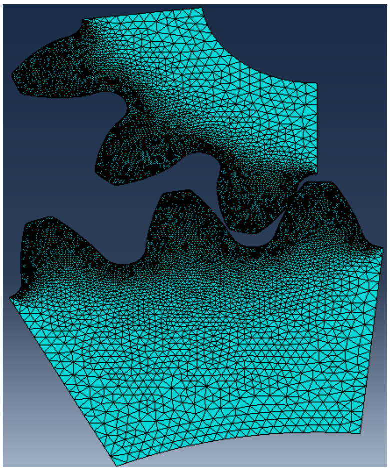
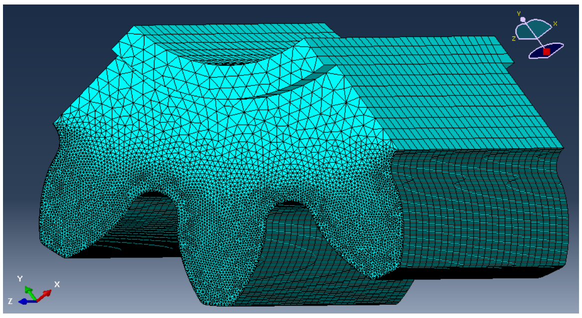


Figure 3 Pinion mesh (isometric) (see online version for colours)



2.7 FEA output results

The results of importance in the lubricated contact analysis software are the contact forces, pressure and the nodes in contact. The latter is used in combination with the node positions to calculate the contact kinematics and radii of curvatures. These values are calculated at each node in the defined contact surface.

2.8 Automated model generation

The use of the quasi-static modelling method means a model is required at each rotational step in the meshing cycle. The creation of individual models is timely as the geometry requires alterations and a new FEA model is required. To reduce the user interaction, the model generation should be automated.

The method for automation is for the user to generate one complete FEA model at the start of the meshing cycle. A Matlab script is then used to modify the mesh for each rotational step in the meshing cycle. The node coordinates can be read and modified by rotation around the required axis of the gear. The output gear wheel is centred about the origin so the x and y coordinates can be manipulated by using the rotation matrix in equation (1). The pinion is not centred about the origin, so it must be translated to the origin, rotated, and then translated back to its original centre.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

2.9 Lubricated contact analysis

The effect of the lubrication within the contact is considered by initially estimating the coefficient of friction in a so called 'dry' analysis. The subsequent contact results will then be used in the lubrication model to improve the coefficient of friction values for use in the next 'wet' iteration of the FEA model.

The full analysis process for the initial conception design of the LLTCA simulation is shown in the flowchart in Appendix 1. The method chosen for validating the results from the first FEA 'dry' analysis in this report is to compare with the contact results obtained from commercially available gear analysis software. This step will not be necessary once the FEA methodology has shown to obtain consistent conformity with other analysis data.

The high contact forces experienced within highly loaded gear pairs will result in an elastohydrodynamic lubrication regime. The film thickness for such conditions is found using the analytical equation defined by Chittenden et al. (1986).

$$h_{c0}^* = 4.31 U_e^{0.68} G_e^{0.49} W_e^{-0.073} \left[1 - \exp\left(-1.23 \left(\frac{r_y}{r_{xy}} \right)^{2/3} \right) \right] \tag{2}$$

$$W_e = \frac{\pi W}{2 E_r r_{xy}^2}, U_e = \frac{\pi \eta_0 u}{4 E_r r_{xy}}, G_e = \frac{2 E_r \alpha}{\pi}, h_{c0}^* = \frac{h_{c0}}{r_{xy}} \tag{3}$$

For the considered gear pair and loading, the contact is operating in the Eyring traction regime (Fatourehchi et al., 1996). The coefficient of friction occurring at the EHL contact is found from Evans and Johnson (1986).

$$\mu = 0.87 \alpha \tau_0 + \frac{1.74 \tau_0}{P_{avg}} \ln \left[\frac{1.2}{\tau_0 h_{c0}} \left(\frac{2 K_l \eta_0}{1 + 9.6 \xi} \right)^{1/2} \right] \tag{4}$$

$$\xi = \frac{4 K_l}{\pi h_{c0}^*} \left(\frac{P_{avg}}{E_r r_{xy} K_g \rho_g c_g u} \right)^{1/2} \tag{5}$$

The radius of curvature of the teeth profiles can be calculated from the x and y node coordinates using equation (6) (Gray, 1997). The equivalent radius (r_{xy}) is calculated from the instantaneous radius of curvature at the contact point of the unloaded pinion and wheel geometry [equation (7)].

$$r_p = \left| \frac{\left(1 + y_{p^2}'\right)^{1.5}}{y_p''} \right|, \quad r_w = \left| \frac{\left(1 + y_{w^2}'\right)^{1.5}}{y_w''} \right| \tag{6}$$

$$\frac{1}{r_{xy}} = \frac{1}{r_p} + \frac{1}{r_w} \tag{7}$$

The speed of entraining motion of the lubricant is calculated from the rolling velocities (Merritt, 1971).

$$v_p = r_p \omega_p \left[\sin \varphi + \frac{L}{r_{pp}} \right], \quad v_w = r_w \omega_w \left[\sin \varphi - \frac{L}{r_{pw}} \right] \tag{8}$$

$$u = \frac{v_p + v_w}{2} (m/s) \tag{9}$$

The contact results from the FEA model are used to calculate the total contact force and subsequently the average contact pressure using equations (10) and (11) respectively.

$$W = \sum_{i=1}^{n} P_i A_i \quad (N) \tag{10}$$

$$P_{avg} = \frac{W}{\sum_{i=1}^{n} A_i} \quad (Pa) \tag{11}$$

3 Results and discussion

3.1 Loaded tooth contact analysis (LTCA)

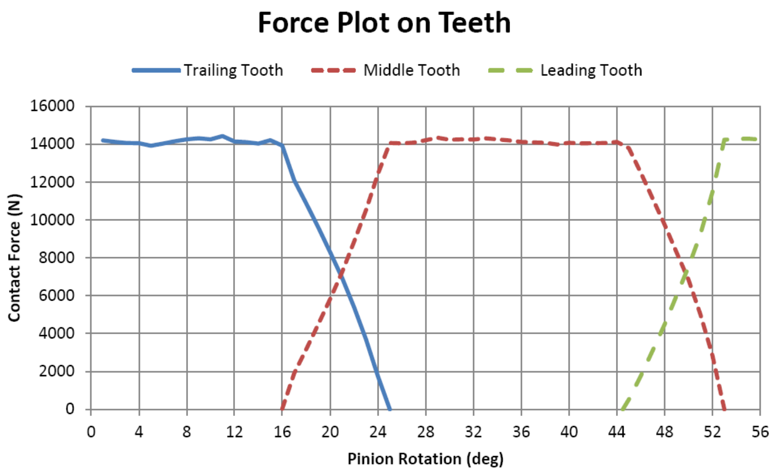
The initial gear analysis is carried out with a constant coefficient of friction which is estimated to be 0.05 for a lubricated steel to steel contact (Budynas and Nisbett, 2011). The simulation is carried out for two complete meshing cycles (56°) to ensure repeatability. A modest pinion rotational step size of 1° is selected as the analysis is quasi-static. The loading on the pinion is 312 Nm and the rotational speed is 11,758 rpm.

The Matlab script created an analysis file for each rotational step using the original analysis input file at the initial point in the meshing cycle (0°). A batch script is created to run each simulation one-by-one with the number of processors defined as the maximum of 4, this will help reduce computation time. Each simulation takes approximately 50 minutes leading to a total simulation time of around 48 hours. The computational time could be reduced by analysing a single gear mesh cycle as well as a coarser time step.

The FEA software outputs the contact results for every surface node, so a Matlab script is used to remove nodes results which are not in contact. The nodes in contact can be identified by a non-zero contact pressure value. The script is also used to associate the contact results with the nodal coordinates.

The contact forces acting on the pinion tooth are calculated throughout the meshing cycle by using equation (10). The contact node positions are used to identify whether multiple tooth contact is occurring, if multiple contact is detected, the script will separate and attribute the contact to the relevant teeth. The contact load variation results are shown in Figure 4.

Figure 4 Tooth contact force variation through meshing cycle (see online version for colours)

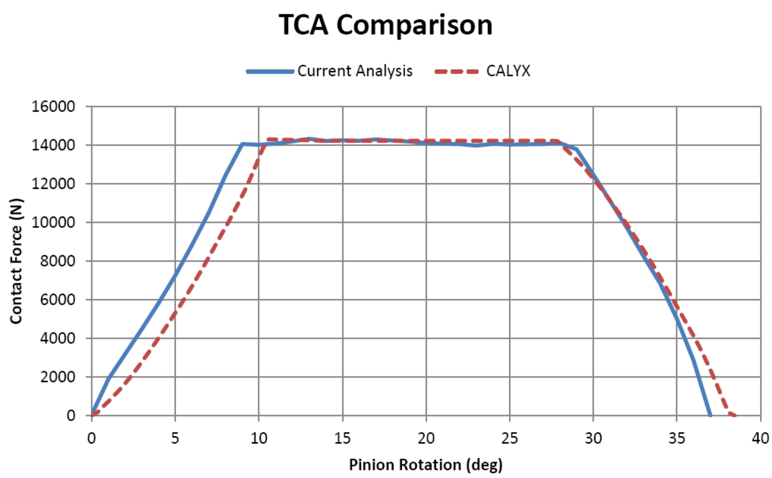


The results demonstrate the load sharing effect for gear pairs with contact ratios higher than unity, characterised by the transfer of load to the next tooth. The results show relatively stable contact forces under single contact conditions and a low variation in the loading/unloading of teeth under multiple contact conditions.

3.1.1 Results validation

The initial simulation is validated against the results of a similar simulation using commercially available TCA software CALYX (Fatourehchi et al., 2016). CALYX uses the perfectly defined geometry of the teeth from the design process rather than the real geometry which is based on the manufactured parts. The variation of the contact force acting on a single tooth throughout the meshing cycle is used as a comparison between the two models. The results are shown in Figure 5.

Figure 5 Current analysis and CALYX TCA force comparison (see online version for colours)

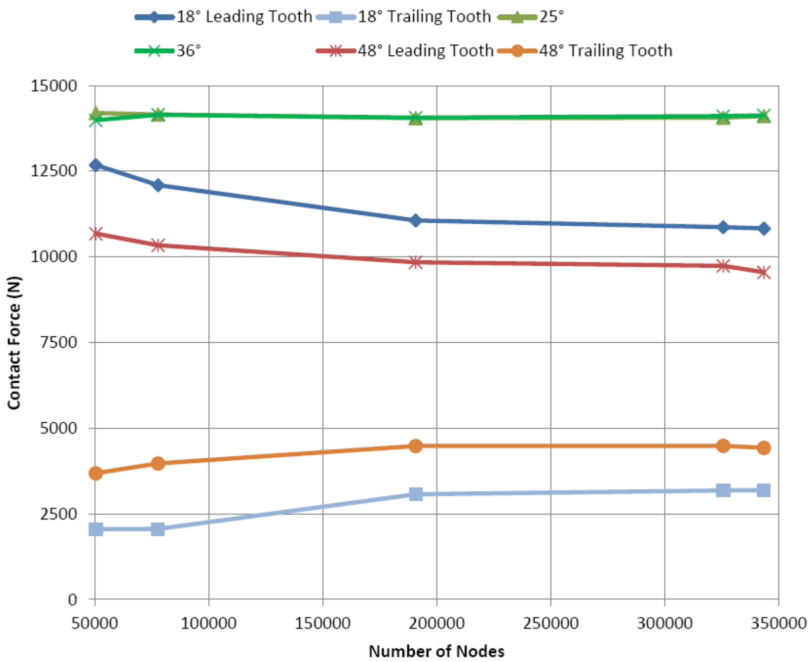


The results of the current analysis show good conformity with the TCA software. The contact force occurring in the single contact is very close with a maximum deviation of –1.8%. The key differences between the two simulation results are the duration of the; single contact and the meshing cycle, with differences of 9.93% and –3.77% respectively. This is considered to be as result of the geometrical differences. The contact ratios obtained from CALYX and the current FEA are 1.45 and 1.51 respectively.

3.1.2 Mesh sensitivity study

As discussed previously, the results show good correlation with commercial TCA software. The computational times are far greater in the FEA model than the TCA software. To address the issue of higher computational time, a mesh sensitivity study was carried out to understand the mesh densities effect on the contact forces (Figure 6). The study was carried out at four points in the meshing cycle; 18°, 25°, 36° and 48° which represent the contact closing, single contact initiation, single contact and contact opening respectively. The mesh densities were altered by reducing the mesh size on the tooth flanks from the nominal value of 0.1 mm to 0.75, 0.5, 0.25 and 0.075 mm.

Figure 6 Mesh sensitivity at 18°, 25°, 36° and 48° (see online version for colours)



The results of the mesh sensitivity study show that for single contact conditions (25° and 36°) there is little variation in the contact forces, suggesting an overly refined mesh. For the double contact conditions (18° and 48°) the contact forces show considerable disparity with the coarser mesh densities. This difference is substantially reduced as the mesh density is increased and is reasonably converged at 200,000 nodes. The mesh sensitivity study shows that there is scope to reduce the mesh size in order to reduce total computation time with little effect on the contact forces. However, a high mesh density will be desirable to maintain an adequate number of nodes within the contacts, as this will lead to a better definition of the radius of curvature of the gear teeth and therefore improve the lubrication analysis results.

3.2 Lubrication analysis

Equations (7) and (9) are used to calculate the gear pair equivalent radius and speed of entraining motion at each gear mesh step, the comparison of the calculated values by the current analysis and the CALYX results are shown in Figures 7 and 8.

Figure 7 Current analysis and CALYX TCA equivalent radius comparison (see online version for colours)

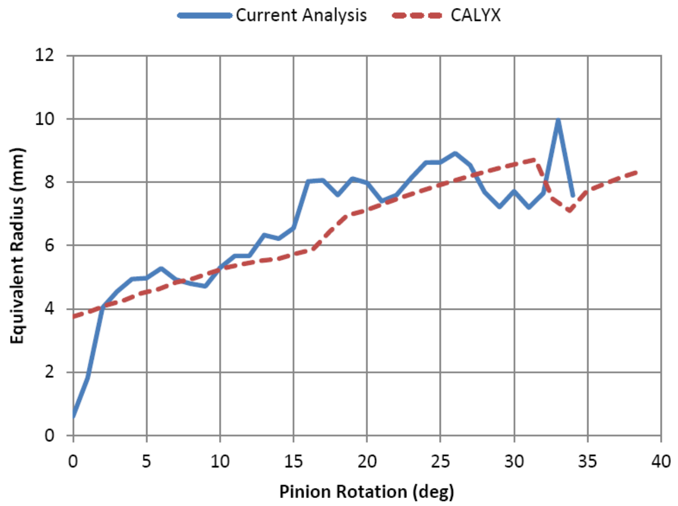
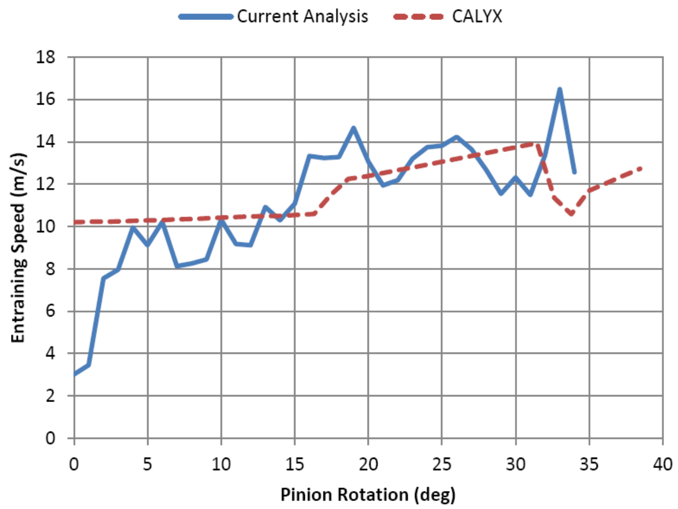
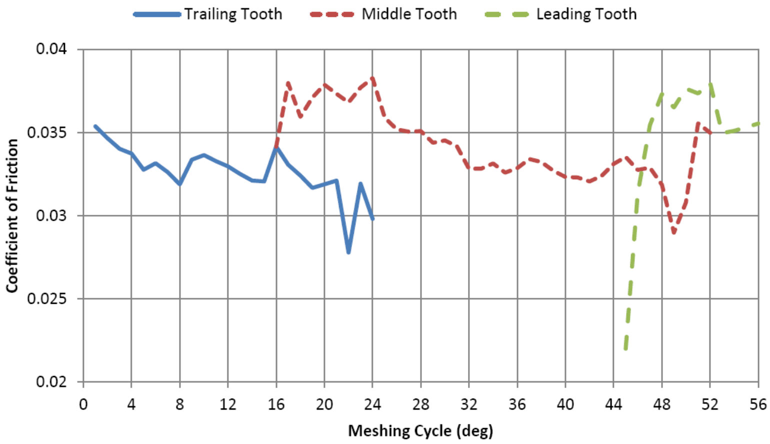


Figure 8 Current analysis and CALYX TCA entraining speed comparison (see online version for colours)



The lubrication analysis is used to calculate the coefficient of friction occurring in the contacts. The results in Figure 9 show the variation of the coefficient of friction throughout the meshing cycle. The values are considerably lower than the estimated value of 0.05 used for the 'dry' analysis.

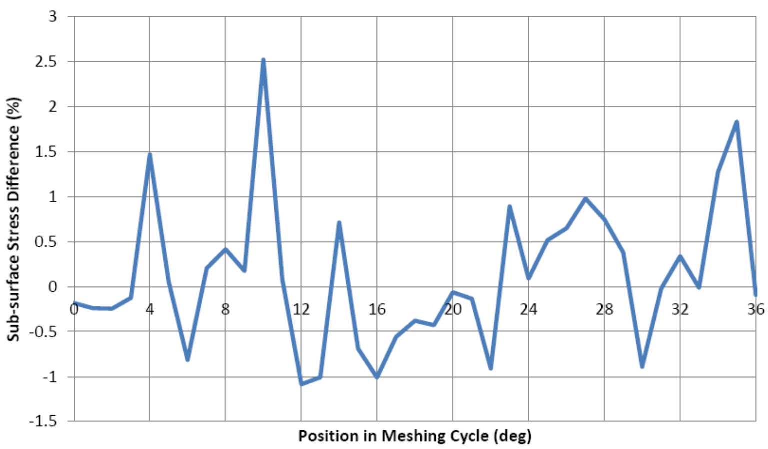
Figure 9 Coefficient of friction through meshing cycle (see online version for colours)



3.3 *Lubricated loaded tooth contact analysis (LLTCA)*

The coefficient of friction values calculated in the previous section are used in the 'LLTCA' to improve the representativeness of the contact conditions. The lubrication analysis was then reiterated with the results from the wet analysis and showed that the contact conditions converged within two iterations. The maximum percentage change to the coefficient of friction was –0.36% for the two consequent iterations.

Figure 10 Maximum sub-surface stress difference between 'LTCA' and 'LLTCA' analyses (see online version for colours)



As discussed previously, the sub-surface stresses arising at the contact play an important role in the durability of the gear pair. The improvements to the contact conditions will have an effect on the sub-surface stresses due to the change in traction force, resulting from the differing coefficient of frictions. To investigate the effect, the maximum sub-surface shear stress is compared between the dry (LTCA) and wet (LLTCA) analyses, shown in Figure 10.

The sub-surface stresses vary between a maximum and minimum difference of 2.51% and –1.09% respectively. The reasons for this variation is attributable to the change in the coefficient of friction, which reduces surface shear force and changes in the contact position due to the differing contact equilibriums.

4 Conclusions

A finite element analysis method to analyse gear tooth contact is generated from geometrical data obtained through CMM. The analysis is carried out with a quasi-static methodology where individual analyses are computed at steps through the meshing cycle. One model is generated by the user and a Matlab script is used to automate the mesh rotation to each required step. The results from the initial LTCA are validated against results obtained from commercial LTCA software and show good correlation.

The lubrication results from the initial dry model show that the coefficient of friction differs from the assumed estimated value and its value is dependent on the position within the meshing cycle. The calculated coefficient of friction values are utilised within the next iteration of the wet analysis termed the LLTCA. The improved contact definition results in a change of the sub surface stresses with a maximum difference of 2.51%. The results from the LLTCA are iterated in the lubrication analysis to determine whether the contact lubrication (coefficient of friction) has converged. The results showed a maximum difference of –0.36% within two iterations.

Overall, the TCA model described was able to accurately analyse gear contact behaviour whilst taking in to account the real geometry. The coupling of LTCA with analytical lubrication models led to improved contact conditions and the use of CMM geometrical data provides a means of improving gear interaction predictions. Due to the flexibility offered by FEA software, the model can be modified to work with various other types of complex gear systems such as; helical, bevel and hypoid.

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Nomenclature

A	Contact area	U_e	Dimensionless speed parameter
c_g	Gear specific heat capacity	v_p	Pinion rolling velocity
E_r	Reduced elastic modulus	v_w	Wheel rolling velocity
G_e	Dimensionless material parameter	W	Tooth contact force
h_{c0}	Central film thickness	W_e	Dimensionless load parameter
h_{c0}^*	Dimensionless central film thickness	α	Pressure viscosity coefficient
K_g	Gear thermal conductivity	η_0	Lubricant viscosity
K_L	Lubricant thermal conductivity	θ	Rotation angle
P	Contact pressure	μ	Coefficient of friction
P_{avg}	Average contact pressure	ξ	Asperity density per unit area
r_p	Pinion radius of curvature	ρ_g	Gear density
r_w	Wheel radius of curvature	τ_o	Eyring stress
r_{xy}	Equivalent radius	ω_p	Pinion angular rotational speed
r_y	Side leakage radius of curvature	ω_w	Wheel angular rotational speed
u	Lubricant entraining speed		

Appendix 1

Figure A1 Flowchart of simulation methodology