Thermally radiative flow of a viscoelastic nanofluid with Newtonian heating

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Abstract: This research paper studies the impact of thermally radiative 3D viscoelastic nanofluid flow upon a stretchy paper with Newtonian heating. Appropriate similarity variables are used to remodel the governing non-linear PDEs to ODEs and they are analytically solved by adopting the homotopy analysis method (HAM). The disparity of fluid velocities, temperature, nanoparticle volume fraction, skin friction coefficients and local Nusselt number of various parameters is pointed out. It is noticed that, enhancing the Newtonian heating parameter increases the fluid temperature. Also found that the nanoparticle volume fraction enlarges with growing the thermophoresis parameter.

Keywords: viscoelastic; nanofluid; Brownian motion; radiation; Newtonian heating and thermophoresis effect; heat transfer; stretching sheet.
1 Introduction

The heat transfer mechanism plays a significant role in many industrial and engineering processes. Especially heating and cooling processes are occurring in such fields. The ordinary fluids, such as water, glycol, ethylene, oil, toluene generally have poor thermal conductivity. So they have poor heat transfer characteristics. The nanosized (1–100 nm) metallic particles, like iron, copper, titanium and associated oxides are suspended in the ordinary fluids to enhance the fluid thermal conductivity. These fluids are mainly used in computer chips while enlarging the storage capacity with small size. Ahmed et al. [1] addressed the energy transfer phenomenon of a nanofluid flow over a stretching tube with heat sink/source. Their results show that the suction/injection parameter leads to suppress the local Nusselt number. The stretched flow of a nanofluid was described by Khan and Pop [2]. They predicted that the reduced Sherwood number enhances for large values of
Brownian motion parameter. Many researchers are interested to study the nanofluids in different aspects [3–7]. The thermal radiation occurs when the disparity of surface and ambient temperatures is high. In numerous engineering processes, the energy boundary layer thickness can be changed by use of thermal radiation. For example, gas turbines, power plants, drying the food products, missiles technology, disposal of nuclear waste, satellites, etc. Thermal radiation of a viscoelastic fluid flow with convective heating is analytically investigated by Srinivas and Muthuraj [8]. They noted that the fluid temperature suppresses with enhancing the viscoelastic parameter. Some important investigations highlighting the thermal radiation can be found in Srinivas et al. [9], Eswaramoorthi et al. [10], Karthikeyan et al. [11], Niranjan et al. [12], Lesnic et al. [13] and Ramzan and Yousaf [14]. The heat transfer from the surface is proportional to the local surface temperature is called Newtonian heating and has many potential applications, such as thermal energy storage, design of heat exchangers, nuclear turbines, etc. Moreover, some recent attempts, such as Hayat et al. [15], Merkin [16] and Makinde [17,18] focused on the Newtonian heating.

Motivated by the above studies, we aim to analyse the impact of thermally radiative flow of a viscoelastic nanofluid with Newtonian heating. The governing problem is solved analytically using HAM, (see Bhuvaneswari et al. [19] and Loganathan et al. [20]).

2 Mathematical formulation

We consider the 3D radiative flow of a viscoelastic nanofluid over a stretchy paper. The nanofluid having uniform size, shape and single phase. The paper is place at $Z > 0$. Let $T_W$ and $C_W$ be the fluid temperature and nanoparticle volume fraction and they are assumed as invariant at the paper which is larger than the ambient fluid temperature $T_\infty$ and nanoparticle volume fraction $C_\infty$. In our model, we incorporate the Brownian motion and the thermophoresis properties. The Newtonian heating is implemented in the heat transfer process. Under the above inferences, the governing equations are developed as follows,

\begin{align}
U_{1y} + U_{2z} + U_{3x} &= 0 \\
U_{1y}U_{1y} + U_{1y}U_{2z} + U_{1y}U_{3x} &= k_i \left[ U_{1y}U_{1y} + U_{1y}U_{2z} - U_{1y}U_{3x} - 2U_{1y}U_{1y} - 2U_{1y}U_{3x} \right] \\
U_{2z} + U_{2z}U_{2z} + U_{2z}U_{3x} &= k_i \left[ U_{2z}U_{2z} + U_{2z}U_{2z} - U_{2z}U_{2z} - 2U_{2z}U_{2z} - 2U_{2z}U_{2z} \right] \\
U_{3x} + U_{3x}U_{3x} + U_{3x}U_{3x} &= k_i \left[ U_{3x}U_{3x} + U_{3x}U_{3x} - U_{3x}U_{3x} - 2U_{3x}U_{3x} - 2U_{3x}U_{3x} \right] \\
U_{1y}T_{1y} + U_{2z}T_{1y} + U_{3x}T_{1y} &= \alpha_x T_{1y} + \tau \left[ D_T C_{1y} T_{1y} + \frac{D_T}{T_\infty} T_{1y} \right] - \frac{1}{\rho C_p} \frac{\partial q}{\partial x} \\
U_{1y}C_{1y} + U_{2z}C_{1y} + U_{3x}C_{1y} &= D_T C_{1y} + \frac{D_T}{T_\infty} T_{1y}
\end{align}
where \((U_1, U_2, U_3)\) are velocity components in \((x_1, x_2, x_3)\) directions, \(\nu\) is the kinematic viscosity, \(k\) is the fluid material parameter, \(D_b\) is the Brownian motion coefficient, \(D_t\) is the thermophoretic diffusion coefficient, \(\alpha_n\) is the thermal diffusivity, \(\tau\) is the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid, \(\rho\) is the density of the fluid, \(C_p\) the specific heat, \(q_r = -\frac{\bar{\sigma}T^2}{\bar{m}}\) is the Rosseland radiative heat flux with the Boltzmann constant \(\bar{\sigma}\) and mean absorption coefficient \(\bar{m}\).

The boundary conditions are,
\[
U_1 = U_1(x_1) = ax_1, U_2 = U_2(x_2) = bx_2, U_3 = 0, T = -hT, C = C_v \quad \text{at} \quad x_3 = 0 \quad (6)
\]
\[
U_1 \to 0, U_2 \to 0, U_1(x_1) \to 0, U_2(x_2) \to 0, T \to T_\infty, C \to C_v \quad (7)
\]

The similarity variables are defined as follows,
\[
\eta = \frac{a}{\nu} x_1, \quad U_1 = ax_1 F'(\eta), \quad U_2 = ax_2 G'(\eta), \quad U_3 = -\sqrt{\nu} \left[ F(\eta) + G(\eta) \right]
\]
\[
\theta(\eta) = \frac{T - T_\infty}{T_\infty}, \quad \phi(\eta) = \frac{c - c_v}{c_v} \quad (8)
\]

Substituting equation (8) in equations (2)–(4), we have
\[
\frac{d^2 F}{d\eta^2} - \frac{\partial F}{\partial\eta} + (F + G) \frac{d^2 F}{d\eta^2} + 2 \left( \frac{dF}{d\eta} + \frac{dG}{d\eta} \right) \frac{d^2 F}{d\eta^2} = 0 \quad (9)
\]
\[
\frac{d^2 G}{d\eta^2} - \frac{\partial G}{\partial\eta} + (F + G) \frac{d^2 G}{d\eta^2} + 2 \left( \frac{dF}{d\eta} + \frac{dG}{d\eta} \right) \frac{d^2 G}{d\eta^2} = 0 \quad (10)
\]
\[
\left(1 + \frac{4}{3} R \right) \frac{d^2 \theta}{d\eta^2} + \Pr (F + G) \frac{d\theta}{d\eta} + Nb \frac{d\phi}{d\eta} + Nt \left( \frac{d\theta}{d\eta} \right)^2 = 0 \quad (11)
\]
\[
\frac{d^2 \phi}{d\eta^2} + Pr Le (F + G) \frac{d\phi}{d\eta} + Nb \frac{d^2 \theta}{d\eta^2} = 0 \quad (12)
\]

The corresponding boundary conditions becomes,
\[
F(0) = 0, G(0) = 0, \frac{dF(0)}{d\eta} = \frac{dG(0)}{d\eta} = 1, \frac{dF(\infty)}{d\eta} = c, \frac{dG(\infty)}{d\eta} = 0,
\]
\[
\frac{dG(\infty)}{d\eta} = 0, \frac{d^2 F(\infty)}{d\eta^2} = 0, \frac{d^2 G(\infty)}{d\eta^2} = 0
\]
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\[
\frac{d\theta(0)}{d\eta} = -N[1 + \theta(0)], \theta(\infty) = 0, N\frac{d\phi(0)}{d\eta} + N\frac{d\theta(0)}{d\eta} = 0, \phi(\infty) = 0
\]

where \( K = k_1 a / \nu \) – viscoelastic parameter, \( Pr = v / \alpha \) – Prandtl number, \( Nb = \tau D_0 (C_1 - C_2) / v \) – Brownian motion parameter, \( Nt = \tau D_0 (T - T_\infty) / v T_\infty \) – thermophoresis parameter, \( Le = \alpha / De \) – Lewis number, \( c = b / a \) – stretching ratio, \( R = 4\sigma^* T_\infty^4 / km^* \) – thermal radiation parameter and \( N = h\sqrt{\nu / a} \) – Newtonian heating parameter.

The skin friction coefficients and local Nusselt number are defined as follows

\[
C_{F_1} = \nu U_{i_1}
\]
\[
+ K_1 \left[ U_{i_1} U_{2_{i_1}} + U_{i_2} U_{1_{i_2}} + U_{i_3} U_{1_{i_3}} + U_{i_4} U_{1_{i_4}} + 2U_{i_5} U_{i_4} - U_{i_5} U_{i_4} \right]_{i_1=0}
\]
\[
C_{F_2} = \nu U_{i_3}
\]
\[
+ K_1 \left[ U_{i_1} U_{2_{i_3}} + U_{i_2} U_{1_{i_3}} + U_{i_3} U_{1_{i_3}} + U_{i_4} U_{1_{i_4}} + 2U_{i_5} U_{i_4} - U_{i_5} U_{i_4} \right]_{i_1=0}
\]
\[
k_i \left[ \frac{\partial T}{\partial x} \right]_{x_1=0} \quad Nu = - \frac{k_i (T_n - T_\infty)}{k_i (T_n - T_\infty)}
\]

Then, the reduced form of the skin friction coefficients and local Nusselt number are derived as

\[
C_{F_1, \sqrt{Re}} = \frac{d^2 F}{d\eta^2} + k_1 \left[ 2 \frac{dF}{d\eta} \frac{dF}{d\eta} - (F + G) \frac{dF}{d\eta} + \left( \frac{dF}{d\eta} + \frac{dG}{d\eta} \right) \frac{d^2F}{d\eta^2} \right]_{g=0}
\]
\[
C_{F_2, \sqrt{Re}} = \frac{d^2 G}{d\eta^2} + k_1 \left[ 2 \frac{dG}{d\eta} \frac{dG}{d\eta} - (F + G) \frac{dG}{d\eta} + \left( \frac{dF}{d\eta} + \frac{dG}{d\eta} \right) \frac{d^2G}{d\eta^2} \right]_{g=0}
\]
\[
\frac{Nu}{\sqrt{Re}} = N \left[ 1 + \frac{4}{3} \frac{R}{3} \left( 1 + \frac{1}{3} \frac{d\theta}{d\eta} \right) \right]_{g=0}
\]

3 Methodology

The governing equations are analytically solved by using HAM. The initial approximations of HAM solutions are expressed as \( F_0 = 1 - e^{-\eta}, G_0 = c(1 - e^{-\eta}), \theta_0 = (N/1 - N)e^{-\eta} \) and \( \phi_0 = -(NtNe^{-\eta}) / (Nh(1 - N)) \). After simplifying, the \( n \)th order HAM equations, we get \( F^n(\eta) = F^n(\eta) + B_1 e^\eta + B_2 e^{-\eta}; G^n(\eta) = G^n(\eta) + B_3 e^\eta + B_4 e^{-\eta}; \theta^n(\eta) = \theta^n(\eta) + B_5 e^\eta + B_6 e^{-\eta} \) and \( \phi^n(\eta) = \phi^n(\eta) + B_7 e^\eta + B_8 e^{-\eta} \). The particular solutions \( F^n(\eta), G^n(\eta), \theta^n(\eta) \) and \( \phi^n(\eta) \) having the auxiliary
parameters $h_p, h_c, h_g$ and $h_\phi$. These parameters are important in the convergence of the HAM solutions. Figure 1(a)–(d) epitomise the range values of HAM parameters $h_p, h_c, h_g$ and $h_\phi$. It is noticed that the range values are suppressed for enhancing the viscoelastic parameter. In addition, the range values of viscoelastic fluid is smaller compared to the Newtonian fluid. We choose $h_p = h_c = h_g = h_\phi = -1$ for better approximations.

**Figure 1** $h$ curves of (a) $F''(0)$; (b) $G''(0)$; (c) $\theta'(0)$ and (d) $\phi'(0)$ with different $K$ values
Figure 2  Various values of $K$ on (a) $F'(\eta)$; (b) $G'(\eta)$; (a) $\theta(\eta)$ and (d) $\phi(\eta)$.
Figure 3 Various values of $K$ on (a) $F'(\eta)$; (b) $G'(\eta)$; (a) $\theta(\eta)$ and (d) $\phi(\eta)$
4 Results

In this section, we provide the outcomes of $K$, $c$, $R$, $Nb$, $Nt$ and $N$ for $x$-direction velocity ($F'$), $y$-direction velocity ($G'$), temperature profile ($\theta$), nanoparticle volume fraction ($\phi$) and local Nusselt number ($Nu / \sqrt{Re}$) with constant values of $Pr = 1.2$ and $Le = 1$. The features of $K$ on $F'(\eta), G'(\eta), \theta(\eta)$ and $\phi(\eta)$ are described in Figure 2(a)–(d). It is perceived from these figures that the both direction velocities are abridged for larger value $K$. However the fluid temperature and nanoparticle volume fraction show opposite behaviour for strengthening the $K$ values.

Figure 3(a)–(d) epitomise the effect of $c$ on $F'(\eta), G'(\eta), \theta(\eta)$ and $\phi(\eta)$. It is witnessed that the $x$-direction velocity is an diminishing function of $c$, see Figure 3(a). On the contrary, the large values of $c$ upsurge the $y$-direction velocity. In addition, there is no $y$-direction velocity when $c = 0$, see Figure 3(b). From Figure 3(c) and (d), the fluid temperature and nanoparticle volume fraction suppresses on enhancing $c$ values. The changes of $\theta(\eta)$ for various $Nt$ values with and without radiation are sketched in Figure 4(a) and (b). It is found that the fluid temperature enhances with rising $Nt$ values for both case.

Figure 4 Variations of temperature profile (a) with radiation and (b) without radiation for $Nt$ values (see online version for colours)
Figure 5(a) and (b) illustrate the effect of $R$ and $N$ on $\theta(\eta)$. We found that, the larger amount of $R$ enhances the energy transport of the fluid and this causes the fluid temperature to rise. Physically, larger thermal conjugate parameter enhances the heat transfer coefficient and this causes the fluid temperature to rise and the thermal boundary layer thickness to increase, see Figure 5(b).

**Figure 5** Variations of temperature profile on (a) radiation and (b) Newtonian heating parameter (see online version for colours)

The effects of $N_t$ and $N_b$ values of nanoparticle volume fraction are plotted in Figure 6(a) and (b). It is found from these figures that nanoparticle volume fraction increases with raising $N_t$ and $N_b$ values.

Figure 7(a) and (b) represent the impact of $K$ and $c$ on $C_{f_x}, \sqrt{Re}$ and $C_{f_x}, \sqrt{Re}$ and show that both skin friction coefficients are suppressed on enhancing $K$ and $c$ values. From the Table 1, we conclude that 15th order is enough for both velocities, temperature and nanoparticles volume fraction profiles. Local Nusselt numbers for various values of $c$, $R$, $N$ and $N_t$ are listed in Table 2 for viscoelastic fluid and viscous fluid. It is seen
that the heat transfer gradient is an enhancing functions of \( N \) and \( Nt \) and it suppresses on raising \( c \) and \( R \) values for both fluids. In addition, we found that the heat transfer gradient of viscoelastic fluid is becomes larger compared to viscous fluid.

**Figure 6** Variations of nanoparticle volume fraction profile on (a) \( Nt \) values and (b) \( Nb \) values (see online version for colours)

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<th>(-\theta(0))</th>
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Figure 7  Variations of (a) $C_{f_1c} \sqrt{Re}$ and (b) $C_{f_2c} \sqrt{Re}$ for different values of $K$ and $c$ (see online version for colours)

Table 2  Local Nusselt number for different values of $c$, $R$, $N$ and $N_t$

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<th>$R$</th>
<th>$N$</th>
<th>$N_t$</th>
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<th>$VEF$</th>
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Table 2  Local Nusselt number for different values of \(c, R, N\) and \(Nt\) (continued)

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5 Conclusion

We studied the impact of different physical parameters on a viscoelastic fluid flow in a stretching sheet. In our investigations, we found that larger values of viscoelastic parameter suppress the auxiliary parameter \((h)\) range. Both direction velocities and its corresponding boundary layer thicknesses are reduced for higher values of viscoelastic parameter. Larger conjugate heating parameter produces an enhancement of fluid temperature. Brownian motion parameter creates a growth of nanoparticle volume fraction. Skin friction coefficients are a decreasing function of both viscoelastic parameter and stretching ratio parameter. Higher values of thermal radiation cause the heat transfer gradient to decay.

References


