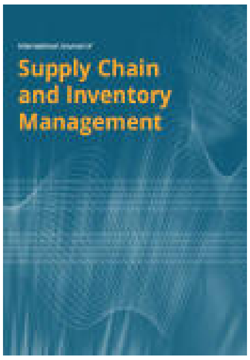

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A knapsack approach to solving the capacitated newsvendor problem

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Abstract: The capacitated newsvendor problem continues to receive considerable attention. This is attributed to its wide range of applications. In the past few decades, researchers have proposed many solution methods. Nevertheless, these methods often require users to have knowledge in certain mathematical programming techniques or complex computer programs which are not easy to apply. To complement the existing literature in this area, this paper, which is a sister to the one that has been published in Springer Nature Operations Research Forum (Abdel-Malek et al., 2020) where a constructive approach utilising the known minimum cost network, is developed. In this work, we add to the previous methods by introducing a model that utilises the known knapsack problem to optimise its solution. Numerical experiments are conducted to compare the performance of some existing methods including those yielded by its sister paper to the proposed one. Managerial insights are also provided.

Keywords: inventory control; newsvendor problem; knapsack problem; supply chain management; stochastic optimisation.

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1 Introduction

In practice, many system optimisation problems can be mapped into a newsvendor scenario. This has attributed to the publication of a plethora of papers that have appeared addressing its solution methods and applications. In addition to solving the standard inventory optimisation problem for finding the optimal lot size of products, the model is applied to a wide gambit of situations. Here we cite some of its wider applications in other fields that spans, as stated before, from inventory control, hotel room and airline seats reservation to portfolio management. For example, Aloi et al. (2012) have treated radio transmission services as virtual goods with a short life cycle. Then, they apply the newsvendor model to optimise the overall benefits when balancing service quality and facilities costs. Also, Fan and Li (2010) utilise the newsvendor model to develop an iterative process to find the optimal profit margins within a specific boundary to maximise the return on investment. Additionally, Hillier and Lieberman (2015) report a case about flight overbooking where the model is utilised to optimise the number of reservations to be double-booked. As can be seen in the cases before, in addition to being applied in the retail industries, the newsvendor model is also widely used in the service industries and the financial sectors, see Choi (2012).

Earlier methods usually utilise dynamic programming, Lagrangian approach, and iterative techniques to solve the problem. Nevertheless, the one introduced by Hadley and Whitin (1964) where dynamic programming is employed only can handle rather small size cases.

The shortcomings of their method have led many researchers to the development of more efficient methods to deal with larger-scale situations. However, these developed methods require advanced mathematical knowledge as well as the use of iterative techniques that make them cumbersome to implement. As an addition to the solution methods to its sister paper that utilises one of the known techniques to solve the problem (maximum flow minimum cost method), the introduced method uses the knapsack model to obtain its solution.

To conclude this introduction, as will be seen in the literature review section, many publications have appeared addressing solution methods to the newsvendor problem. In spite of the fact of its numerous applications and because of the difficulty of the existing solution techniques, textbooks in related fields seldom report on the capacitated newsvendor solution approaches. Therefore, our motivation here is two-fold; first is providing these two sister methods that utilise known techniques (maximum flow minimum cost method and knapsack model) for students and decision makers in related fields, and second is to demonstrate these two approximate solution methods yield good results with considerable accuracy.

The rest of this paper is structured as follows: in Section 2, the literature on the solution methods to the newsvendor problem and various models of the knapsack problem are reviewed. Section 3 presents the classical model of the capacitated newsvendor problem and provides the rationale of the proposed method. Section 4 gives numerical comparisons between the proposed method and some of the existing ones. Section 5 presents an application example of the proposed method to show its steps. Finally, in Section 6, we conclude by a summary and provide some managerial insights.

2 Literature review

In this section, we review some of the relevant literature in two veins:

1 the solution methods of the newsvendor problem

2 the knapsack problem.

2.1 Solution methods of the newsvendor problem

In the past few decades, many scholars have proposed many solution methods to solve the constrained newsvendor problem. Hadley and Whitin (1964) develop a dynamic programming solution procedure to solve the problem. However, their approach becomes intractable when the number of products is not small. Nahmias and Schmidt (1984) develop a heuristic solution procedure for the multi-product newsvendor problem subject to a single constraint of a specific form. Lau and Lau (1996) employ an adaptation of the 'active set methods' described in Luenberger (Paris, 1985) and implement a 'Ranking Heuristic' to make the process of reaching the optimum faster. Abdel-Malek et al. (2004) and Abdel-Malek and Montanari (2005) present a generic iterative method to solve the capacitated newsvendor problem. Abdel-Malek and Areeratchakul (2007) develop a quadratic programming procedure for the multi-constraint newsvendor. Niederhoff (2007) develops a linear approximation method (LAM) to solve the newsvendor problem. In this solution, she approximates the objective function by interpolating several break points to linearise its convex curve and use separable programming to solve the problem. However, the number of break points and the locations of interpolation affect the convergence speed and the accuracy of the results, which requires users to have more experience and judgment. Zhang et al. (2009) develop a binary solution method (BSM) which utilises a bisection algorithm to search the optimal result and it needs less insights in the judgment steps. However, it requires more iterations to achieve the optimum. In Table 1, the studies mentioned in this section as well as others are summarised and arranged in chronological order. For more details of the newsvendor model and its solutions, we refer the interested readers to the comprehensive review by Qin et al. (2011) and the handbook by Choi (2012).

As can be seen from aforementioned review, these solution methods are not easily implemented and require in depth knowledge in advanced techniques. This is what motivate our work, the development of an easy-to-use method.

Table 1 Summary of literatures on the constrained newsvendor problem

Year	*Authors*	*Model*	*Number of constraints*	*Solution*	*Optimality*
1964	Hadley and Whitin	The capacitated newsvendor model	Single	Dynamic programming	Optimum
1984	Nahmias and Schmidt	The capacitated newsvendor model	Single	Lagrangian heuristic	Optimum/near optimum
1996	Lau and Lau	The capacitated newsvendor model	Single/multiple	Ranking heuristic	Optimum
2000	Moon and Silver	Newsvendor model with fixed ordering cost	Single	Dynamic programming, marginal allocation heuristic and two stages heuristic	Optimum/near optimum
2005	Abdel-Malek and Montanari	The capacitated newsvendor model	Single/two	Generic iterative method (GIM)	Optimum/near optimum
2007	Abdel-Malek and Areeratchakul	The capacitated newsvendor model	Single/multiple	Triangle approximation and quadratic programming	Optimum/near optimum
2007	Niederhoff	The constrained newsvendor model	Multiple	LAM	Optimum
2009	Zhang et al.	The capacitated newsvendor model	Single	BSM	Optimum/near optimum
2010	Zhang	Newsvendor model with supplier quantity discount	Single/multiple	Lagrangian heuristic	Optimum
2010	Chen and Chen	Newsvendor model with reservation	Single	Lagrangian heuristic	Optimum
2015	Zhou et al.	Newsvendor model with loss constraint	Two	Loss-based marginal utility deleting method and linearization of the loss constraint	Optimum/near optimum
2015	Dash and Sahoo	Fuzzy newsvendor model	Single	Buckley's concept, fuzzy programming method and LINGO	Optimum
2015	Wang et al.	Uncertain random newsvendor model	Single	MATLAB	Optimum
2016	Wang and Qin	Uncertain random newsvendor model	Single	Hybrid simulation and genetic algorithm	Optimum
2016	Moon et al.	Newsvendor model with discounts and upgrades	Single	Distribution-free approach and BSM	Optimum
2016	Sahoo and Dash (2016)	Fuzzy newsvendor model	Single	Weighted sum method, ranking function method, chance constraint and LINGO	Optimum/near optimum

2.2 The knapsack problem

In this section, some of the relevant literature in knapsack problem is reviewed. Bitran and Hax (1981) present a recursive procedure to solve disaggregate and resource allocation problems by using convex knapsack problem and it determines the optimal value of at least one variable at each iteration. Billionnet and Calmels (1996) propose a novel method based on the elaboration of the heuristic methods proposed by Chaillou et al. (1989) and by Gallo et al. (1980). Marchand and Wolsey (1999) study a mixed 0–1 knapsack problem and derive a separation heuristic for mixed knapsack sets. Quadri et al. (2009) use pre-procedure techniques to reduce the problem size of a particular integer quadratic multi-knapsack problem and develop a branch-and-bound algorithm achieve the optimum. Wang et al. (2012) use standard branch-and-cut optimisers in CPLEX to compare quadratic and linear representations of quadratic knapsack problem. Fampa et al. (2020) relax the quadratic knapsack problem by perturbing the objective function and present a cutting plane algorithm to solve the problem.

The aforementioned synopsis review of the knapsack problem exhibits the complex nature of its solution methods. Nevertheless, and as will be seen in this work, because of the particulars of our model, its application is rather straight-forward.

3 The model and method

This section is divided into three parts. In the first, we describe one of the most common multi-product newsboy models. In the second, the rationale and logic of mapping it into a knapsack approach is presented. The third subsection lists the solution steps and the flowchart of the proposed method. But before presenting the method in the following, we cite the notations used in this paper:

Notation	*Description*
a_i, b_i	Bounds of uniform distribution
B	Current budget
$B_{\min}$	Budget needed to order the unconstrained optima of all products, $B_{\min} = \sum_{i=1}^{n} c_i x_i^*$
c_i	Cost per unit
D_i	Demand of product i
$E(x_i)$	Total cost function
$E^T(x_i)$	Taylor series expansion of the total cost function
$E^L(x_i)$	Linearisation of the total cost function
$\sum_{i=1}^{n} E(x_i)$	Summation of $E(x_i)$
$\Delta E\%$	Percent gap between the optimal total cost obtained by proposed method and other methods
ε	Linearisation error
$f_i(D_i)$	PDF of product i
$F_i(D_i)$	CDF of product i

h_i	Holding cost per unit
μ_i	Mean of exponential distribution
m_i, s_j	Mean and standard deviation of normal distribution, respectively
v_i	Price per unit
x_i	Order quantity of product *i*
x_i^*	Unconstrained optimum of order quantity of product *i*, $F(x_i^*) = \frac{v_i - c_i}{v_i + h_i}$
x_i^{**}	Constrained optimum of order quantity of product *i*
BSM	Binary solution method (Zhang et al., 2009)
KSM	Knapsack method
LAM	Linear approximation method (Niederhoff, 2007)
NFM	Network flow method (Abdel-Malek et al., 2020)

3.1 The model

Equations (1)–(3) show a common formulation of the multi-product newsvendor problem. As defined in the model: x_i is the order quantity of product *i*. The unit purchase cost, unit overstocking cost and unit understocking cost of product *i* are represented by c_i, h_i and v_i respectively. Additionally, the other variables are symbolised by the stochastic demand; the random variable D_i, density function *f*(.) and the cumulative density function *F*(.). The objective function consists of three terms. On the left-hand side of equation (1), *E*(*xi*) represents the expected total cost of the vendor. On the right-hand side of equation (1):

- the first term expresses the multiplication of unit cost of product *i* and its order quantity
- the second term is the multiplication of unit holding cost of product *i* and its overstocking, $\int_0^{x_i} (x_i - D_i) f_i(D_i) dD_i$
- similarly, the third term is the multiplication of unit price of product *i* and its understocking $\int_{x_i}^{\infty} (D_i - x_i) f_i(D_i) dD_i$
- equation (2) is the available budget
- equation (3) is incorporated to avoid negative order quantity of product which is not in line with reality.

Minimise:

$$\sum_{i=1}^{n} E(x_i) = \sum_{i=1}^{n} \left[c_i x_i + hi \int_0^{x_i} (x_i - D_i) f_i(D_i) dD_i + v_i \int_{x_i}^{\infty} (D_i - x_i) f_i(D_i) dD_i \right] \quad (1)$$

$$\sum_{i=1}^{n} c_i x_i \leq B \tag{2}$$

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n \tag{3}$$

In equations (4)–(6), the capacitated newsvendor problem can be expressed as:

Maximise:

$$z = \sum_{i=1}^{n} -E^T\left(x_i\right) \tag{4}$$

Subject to

$$\sum_{i=1}^{n} c_i x_i \leq B \tag{5}$$

$$0 \leq x_i \leq x_i^* \tag{6}$$

3.2 Rationale and proof

In this subsection, we show the rationale of the proposed method (3.2.1) and the near optimality of its solutions as well as the polynomial nature of the knapsack approach in the considered scenario (3.2.2):

3.2.1 The rationale of the proposed method

The first point is to prove that the error generated by linearisation rendered by Taylor series expansion is negligible. Figure 1 shows the convexity of the newsvendor's problem objective function. When the current budget is sufficient, the unconstrained optimal amount of each product can be ordered. Nevertheless, when the current budget is less than a certain value ($B_{\min}$), the optimal lot size of one or more products needs to be less than its unconstrained optimum (x_i^*) to satisfy the constraint. Further, when the budget is even tighter, one or more products need to be removed from the purchase list and hence the corresponding optimal order quantity is zero. Consequently, one has to decide the priority of ordering the products and which of them should be discarded if it is necessary in the decision-making process.

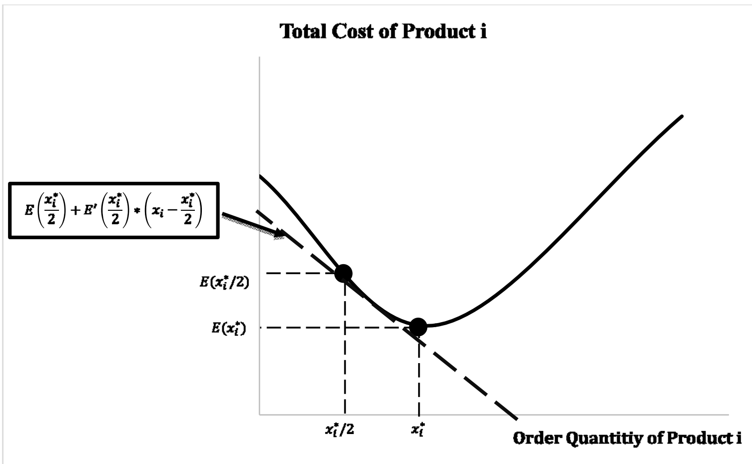
Our taxonomy to show the linearisation steps and its error is as follows:

- Using Taylor series, we expand the objective function at the midpoint of the unconstrained optimal order quantity $(x_i^*/2)$ (see Figure 1, the gradient of the broken line represents the cost reduction rate).
- The error resulting from the linearisation is calculated by using uniform and exponential demand distributions (the pattern of the normal distribution lies in between these two, see Table 1).
- Hence, the objective function of the newsvendor with uniform and exponential distributions are expressed as in equation (7) and equation (8), respectively:

$$\sum_{i=1}^{n} E_u(x_i) = \sum_{i=1}^{n} (c_i - v_i) x_i + \frac{(v_i + h_i)}{2(b_i - a_i)} x_i^2 + v_i \frac{(b_i - a_i)}{2} (UniformDist.) \tag{7}$$

$$\sum_{i=1}^{n} E_e(x_i) = \sum_{i=1}^{n} (c_i - v_i) x_i + (v_i + h_i)\left(x_i + \mu_i e^{-\frac{x_i}{n_i}}\right) + v_i \mu_i (ExponentialDist.) \tag{8}$$

Figure 1 The objective function of the newsvendor for product *i*



Subsequently, from equation (7) and equation (8), the gradient of the objective function at $x_i = x_i^*/2$ is presented in equation (9) and equation (10):

$$\sum_{i=1}^{n} E_u'(x_i) = \sum_{i=1}^{n} (c_i - v_i) + \frac{(v_i + h_i)}{(b_i - a_i)} (x_i^*/2) \tag{9}$$

$$\sum_{i=1}^{n} E_e'(x_i) = \sum_{i=1}^{n} (c_i - v_i) + (v_i + h_i)\left(1 - e - \frac{x_i^*}{2\mu_i}\right) \tag{10}$$

Applying Taylor series expansion to equation (7) and equation (8) at $x_i = x_i^*/2$ yields:

$$\begin{aligned}\sum_{i=1}^{n} E_u^T(x_i) = \sum_{i=1}^{n} &(c_i - v_i)(x_i^*/2) + \frac{(v_i + h_i)}{2(b_i - a_i)} (x_i^*/2)^2 + v_i \left(\frac{b_i - a_i}{2}\right) \\ &+ \left[(c_i - v_i) + \frac{(v_i + h_i)}{(b_i - a_i)} x_i^*/2\right] * (x_i - x_i^*/2) + \varepsilon\end{aligned} \tag{11}$$

$$\begin{aligned}\sum_{i=1}^{n} E_e^T(x_i) = \sum_{i=1}^{n} &(c_i - v_i)(x_i^*/2) + (v_i + h_i)\left(x_i^*/2 + \mu_i e^{-\frac{x_i^*}{2\mu_i}}\right) + v_i \mu_i \\ &+ \left[(c_i - v_i) + (v_i + h_i)\left(1 - e^{-\frac{x_i^*}{2\mu_i}}\right)\right] * (x_i - x_i^*/2) + \varepsilon\end{aligned} \tag{12}$$

The linearisation of equations (11)–(12) are shown in equations (13)–(14):

$$\begin{aligned}\sum_{i=1}^{n} E_u^L(x_i) = \sum_{i=1}^{n} (c_i - v_i)(x_i^*/2) + \frac{(v_i + h_i)}{2(b_i - a_i)}(x_i^*/2)^2 + v_i\left(\frac{b_i - a_i}{2}\right) \\ + \left[(c_i - v_i) + \frac{(v_i + h_i)}{(b_i - a_i)}(x_i^*/2)\right] * (x_i - x_i^*/2)\end{aligned} \tag{13}$$

$$\begin{aligned}\sum_{i=1}^{n} E_e^L(x_i) = \sum_{i=1}^{n} (c_i - v_i)(x_i^*/2) + (v_i + h_i)\left(x_i^*/2 + \mu_i e^{-\frac{x_i^*}{2\mu_i}}\right) + v_i\mu_i \\ + \left[(c_i - v_i) + (v_i + h_i)\left(1 - e^{-\frac{x_i^*}{2\mu_i}}\right)\right] * (x_i - x_i^*/2)\end{aligned} \tag{14}$$

Therefore, the percent difference between the value of the total cost function and its linearisation when the constrained optimal order quantities of products are substituted into them can be calculated by equation (15):

$$\Delta E\% = \frac{\sum_{i=1}^{n} E(x_i^{**}) - \sum_{i=1}^{n} E^L(x_i^{**})}{\sum_{i=1}^{n} E(x_i^{**})} \le \varepsilon \tag{15}$$

In order to find the differences between the optimal total costs obtained by the introduced method and some of existing ones, we conduct a series of numerical experiments by considering different profit margins (v_i / c_i) of products under situations with selected existing budgets in a reasonable range ($B/B_{\min}$). As shown in Table 2, the percentage of differences generated by Taylor series expansion are acceptable in most scenarios. It should be emphasised though that in our experiments, the ratio v_i / c_i is chosen to demonstrate the worst-case scenario stemming from the approximation. This means that the proposed linearisation of the objective function of the newsvendor dominates the cost curve over the interval $[0, x_i^*]$ (ε is negligible in equations (11)–(15).

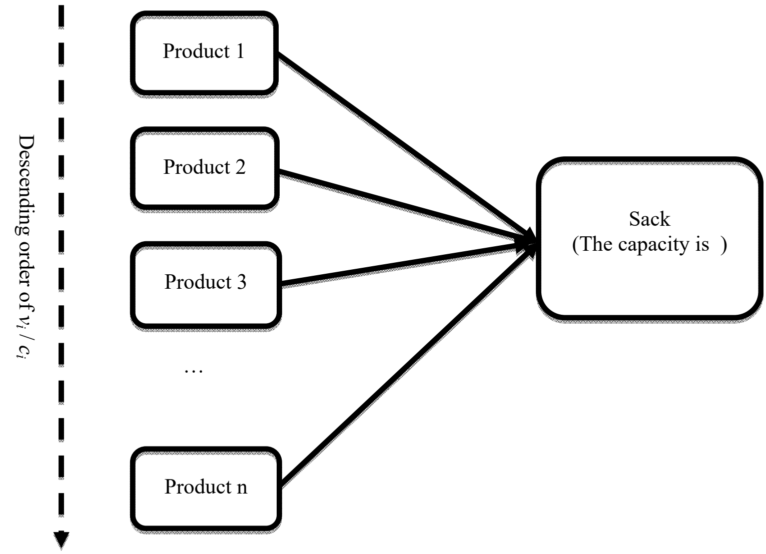
Table 2 The error generated by linearisation

$\Delta E\%$	$B/B_{min} = 0.3$		$B/B_{min} = 0.5$		$B/B_{min} = 0.7$		$B/B_{min} = 0.9$	
	UNI	*EXP*	*UNI*	*EXP*	*UNI*	*EXP*	*UNI*	*EXP*
$v_i / c_i = 1.5$	2.11%	1.18%	2.45%	1.35%	1.93%	0.97%	0.93%	0.38%
$v_i / c_i = 1.75$	3.66%	2.18%	4.33%	2.48%	3.48%	1.77%	1.68%	0.89%
$v_i / c_i = 2$	5.22%	3.27%	6.28%	3.73%	5.16%	2.65%	2.49%	1.02%

3.2.2 Mapping to the knapsack approach

In order to show the polynomial nature of the knapsack approach, we consider equations (4)–(6) and Figure 2. The solution methods of the general knapsack problem are based on branch and bound and/or dynamic programming techniques. Both of them are known to be usually NP. They require taken into consideration the earlier decisions in the search of the optimality. In our case, we do not need to go backward, the assignments are made in descending order of products' profitability (v_i / c_i). Hence, an increase of the number of products is linear in nature of complexity (polynomial in time). Therefore, in the solution steps, the products are sorted in descending order of ratio (v_i / c_i) as priority to put them in the sack. (Therefore, the polynomial nature of the introduced method is shown, one must note that as shown in Section 2, a necessary optimality condition for the capacitated newsvendor problem is the full use of its available resource which is equivalent to fill up the capacity of the knapsack as much as possible).

Figure 2 Knapsack representation of the capacitated newsvendor problem



3.3 Solution steps and the flowchart

In the following, we describe the steps of the proposed method as it relates to the knapsack problem (Figure 3 shows its flowchart):

Step 0 Solve the unconstrained optimum for each product by $F(x_i^*) = \frac{v_i - c_i}{v_i + h_i}$, $i = 1, 2, \ldots n$.

Step 1 Insert the unconstrained optimum of each product in the constraint, $\sum_{i=1}^{n} c_i x_i \leq B$, if $\sum_{i=1}^{n} c_i x_i^* \leq B$, let $x_i^{**} = x_i^*$ and go to Step 5, otherwise.

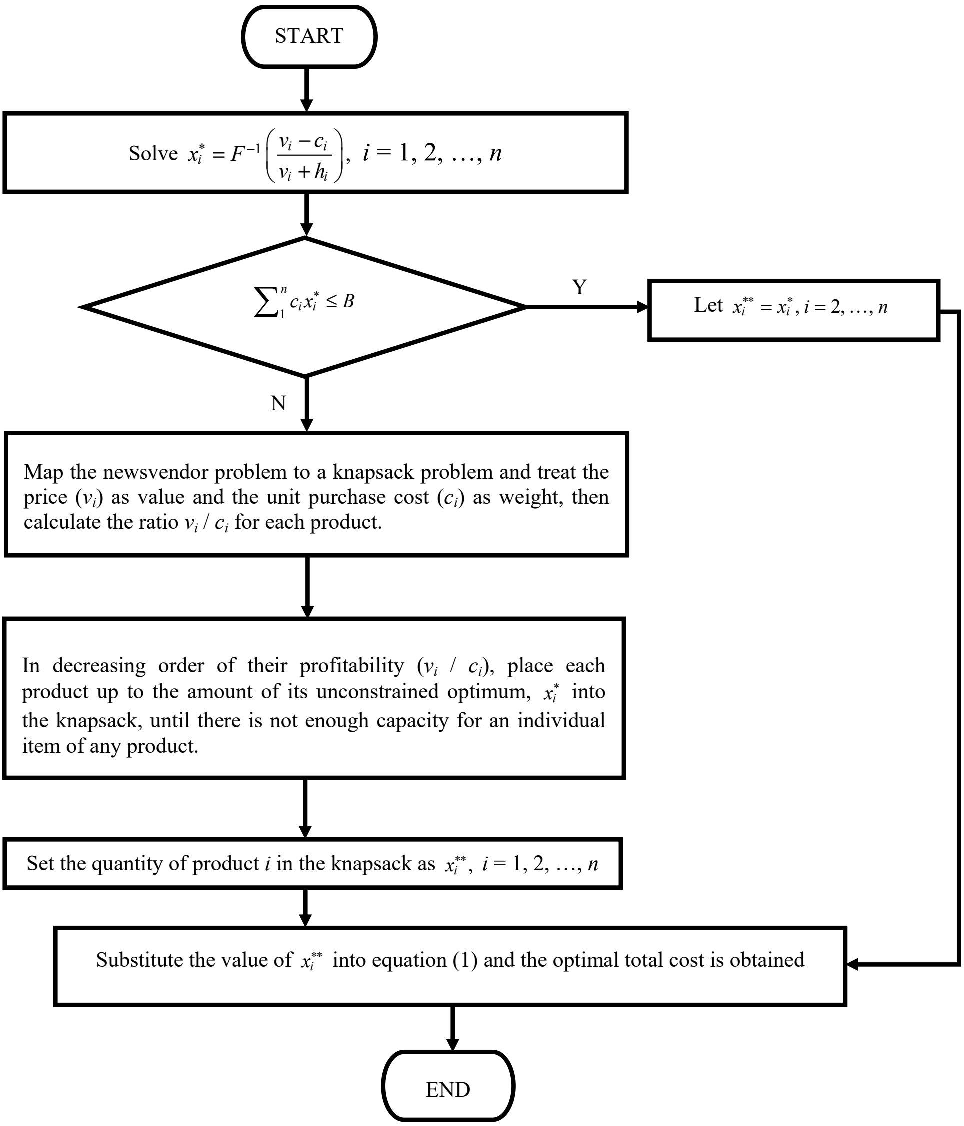
Step 2 Treat the price (v_i) as value and the unit purchase cost (c_i) as weight, then calculate the ratio v_i / c_i for each product.

Step 3 Rearrange the products according to the ratio (v_i / c_i) in descending order.

Step 4 In decreasing order of their profitability (v_i / c_i), place each product up to the amount of its unconstrained optimum, x_i^* into the knapsack, until there is not enough capacity for an individual item of any product and let the quantity of items of each product in the knapsack be x_i^{**}, $i = 1, 2, \ldots, n$.

Step 5 Substitute the value of x_i^{**} into equation (1) and the optimal total cost is obtained.

Figure 3 Flowchart of the knapsack method



4 Numerical comparisons

In this section, we conduct numerical experiments to compare the performance between knapsack method (KSM), network flow method (NFM) and other existing ones such as BSM and LAM (These two latter methods give exact optimal or near it). Table 3 and Table 4 list a reasonable range of the governing parameters of each product (The products in Table 3 share the same type of demand PDF while products in Table 4 have a mix). For cases listed in Table 3 and Table 4, Table 5 exhibits the optimal values of the objective functions obtained by KSM, NFM, BSM, and LAM as well as the percent differences amongst them. For these percentages of budget constraint tightness, 30%, 50%, 70% and 90% (that is compared to $B_{\min}$, the amount needed to purchase the unconstrained optimal quantity for all products), the optimal results are obtained. It can be shown from the table that the differences among the optimal costs are most of the time negligible (0.19% to 3.48%). Therefore, one can apply the proposed method with confidence. Additionally, the comparisons support the observation which is made in Section 3 regarding the leading effect of the ratio, v_i / c_i on the procurement strategy. Further managerial insights are presented in the conclusion section.

Table 3 Parameters of products with the same demand distribution functions

Product	*Parameters*			*UNI*		*EXP*	*NOR*	
	v_i	h_i	c_i	a_i	b_i	μ_i	m_i	s_i
1	35	4	22	0	150	55	166	35
2	27	3	16	0	200	78	193	64
3	20	2	12	0	180	105	200	67
4	19	1	10	0	125	110	168	56
5	33	5	25	0	149	150	135	45
6	40	7	15	0	250	63	250	83
7	17	1	9	0	165	179	150	50
8	22	5	10	0	177	89	170	57
9	39	6	21	0	208	98	210	70
10	25	3	15	0	155	123	120	40

Table 4 Parameters of products with different demand distribution functions

Product	*Parameters*				
	v_i	h_i	c_i	a_i	b_i
1	35	4	22	0	150
2	27	3	16	0	200
3	20	2	12	0	180
				μ_i	
4	19	1	10	110	
5	33	5	25	150	
6	40	7	15	63	
				m_i	s_i
7	17	1	9	150	50
8	22	5	10	170	57
9	39	6	21	210	70

Table 5 Numerical comparisons between the knapsack method and some existing methods

	UNI $B_{\min} = 10{,}424$															
Budget	*9,400*				*7,300*				*5,200*				*3,100*			
Method	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*
Total cost	20,449	20,449	20,330	20,371	21,119	21,119	20,648	20,699	21,759	21,759	21,294	21,307	22,774	22,774	22,319	22,401
ΔE%			0.58%	0.38%			2.28%	2.03%			2.18%	2.12%			2.04%	1.67%
	EXP $B_{\min} = 7{,}228$															
Budget	*6,500*				*5,060*				*3,600*				*2,200*			
Method	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*
Total cost	24,935	24,935	24,865	24,888	25,352	25,352	25,032	25,049	25,753	25,753	25,387	25,399	26,301	26,301	25,947	25,969
ΔE%			0.28%	0.19%			1.28%	1.21%			1.44%	1.39%			1.36%	1.28%
	NOR $B_{\min} = 24{,}634$															
Budget	*22,000*				*17,200*				*12,300*				*7,400*			
Method	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*
Total cost	34,654	34,654	34,331	34,430	37,090	37,090	35,842	35,887	39,540	39,540	38,542	38,565	42,666	42,666	41,816	41,845
ΔE%			0.94%	0.65%			3.48%	3.35%			2.59%	2.53%			2.03%	1.96%
	MIX $B_{\min} = 12{,}265$															
Budget	*11,000*				*8,600*				*6,100*				*3,700*			
Method	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*	*KSM*	*NFM*	*BSM*	*LAM*
Total cost	24,581	24,581	24,472	24,504	25,363	25,363	24,960	24,970	26,588	26,588	26,032	26,058	28,347	28,347	28,169	27,666
ΔE%			0.45%	0.32%			1.62%	1.58%			2.13%	2.03%			0.63%	2.46%

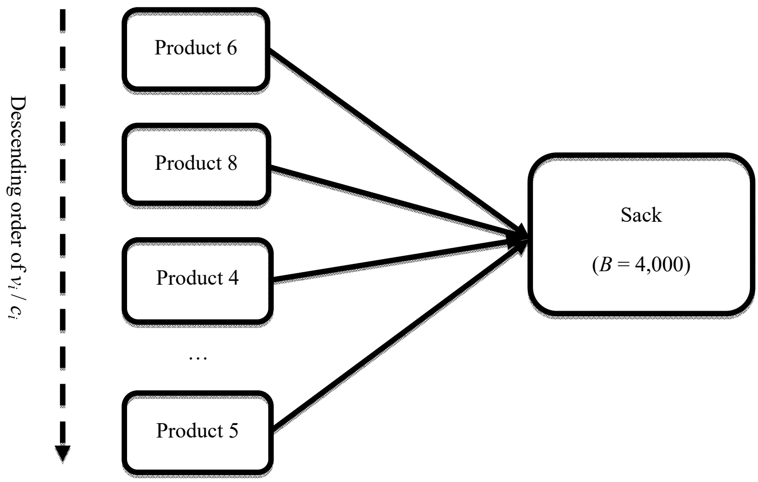
Table 6 The optimal results of the application

Product	v_i	h_i	c_i	x_i^*	$c_i x_i^*$	μ_i	v_i / c_i	$x_{i,KSM}^{**}$	$E(x_{i,KSM}^{**})$	$x_{i,NFM}^{**}$	$E(x_{i,NFM}^{**})$	$x_{i,BSM}^{**}$	$E(x_{i,BSM}^{**})$	$x_{i,LAM}^{**}$	$E(x_{i,LAM}^{**})$
6	40	7	15	48	717	63	2.67	48	1,997	48	1,997	11	1,823	11	1,825
8	22	5	10	52	523	89	2.20	52	1,675	52	1,675	19	1,959	17	1,972
4	19	1	10	66	658	110	1.90	66	1,823	66	1,823	25	1,959	22	1,970
7	17	1	9	105	947	179	1.89	105	2,663	105	2,663	41	1,855	47	1,841
9	39	6	21	50	1,051	98	1.86	50	3,410	50	3,410	5	4,915	0	4,950
2	27	3	16	36	570	78	1.69	6	2,043	6	2,043	37	2,019	37	2,021
3	20	2	12	47	570	105	1.67	0	2,100	0	2,100	66	2,710	76	2,689
10	25	3	15	54	815	123	1.67	0	3,,075	0	3,075	37	1,694	38	1,694
1	35	4	22	22	491	55	1.59	0	1,925	0	1,925	31	3,462	36	3,437
5	33	5	25	35	886	150	1.32	0	4,950	0	4,950	29	2,873	26	2,888
SUM					7.228				$25,661		$25,661		$25,270		$25,286
Budget					$4,000								$\Delta E\% = 1.55\%$		$\Delta E\% = 1.48\%$

5 Application

For an available budget of \$4,000 and in order to illustrate the steps of applying the proposed method, we use the products listed in Table 3 (without loss of generality, we apply it to the exponential distribution). For the ease of comparison, Table 6 shows the parameters of the products and the results obtained by the knapsack method and two other existing ones (BSM and LAM). Figure 4 visualises the solution steps of the application. It should be noted from this example as well as other numerical experiments that the total cost function of the newsvendor is relatively insensitive to the purchase volume of each product. In other words, one can achieve the same optimal cost using varying combination of order quantities of different products.

Figure 4 Knapsack representation of the application



6 Conclusions

In this work, we develop a polynomial in time solution method for the capacitated newsvendor problem. Like its sister paper in the same vein, where the known maximum flow minimum cost approach is utilised, this method adds the use of the standard knapsack problem in rendering the solutions. It should be mentioned that although both methods are approximate, they yield the same results. Further, there are insignificant differences between both of them and the more complex solution methods that are known to render improvements over the ones proposed. Because of the nature of the problem, where there are upper bounds for the order quantities, the knapsack problem which is known to be non-polynomial (NP) is reduced to the polynomial (P) category. Thus, and like its sister paper, does not require in applying it, the use of advanced mathematical programming technique, iterative methods, and/or computer programs. To evaluate the performance of the proposed method, extensive numerical experiments are made covering wide range of parameters and most common demand distribution functions (uniform, exponential and normal). Their results show that the differences between the optimal cost rendered by both knapsack approach (KSM) and the maximum flow minimum cost method (NFM) and other existing ones are narrow (0.19% to 3.48%). Further, the illustrative example presented exhibits the ease of the application of proposed method where ten products are considered. The solution is shown to be easily obtained by a handheld calculator. Among the salient features of the knapsack solution approach are two. The first one is, it can easily accommodate various types of demands PDFs for each product (uniform, exponential, normal, etc…). Second, our analysis reveals the following managerial insights:

1 The product that has higher profit/cost ratio should be ordered first, which means that the ratio (v_i / c_i) dominates the procurement decision.

2 The total cost function of the newsvendor is relatively insensitive to the order quantity of each product that is, in some cases, the same total cost can be achieved by different combinations of products' lot sizes.

3 When the available budget is greater than or equal to half of the minimum budget that can be used to order unconstrained optimal quantity for each product, the gap between the minimum cost rendered by the knapsack method and other existing methods becomes modicum (<3.48%).

Without losing generality and in addition to budget, the capacity constraint in the model can also be replaced by space, weight or others depending on the scarcity of each recourse. Additionally, the proposed method could serve as means to practitioners to decide whether if it is necessary to delve into more complicated methods with more involved iterative techniques. (This is because cost benefit generated by additional computational may be low and not warranting the effort).

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