Inventory model for non-instantaneous deterioration and price-sensitive trended demand with learning effects

Nita H. Shah* and Monika K. Naik

Department of Mathematics,
Gujarat University,
Ahmedabad-380009, Gujarat, India
Email: nitahshah@gmail.com
Email: monikaknaik@gmail.com
*Corresponding author

Abstract: This paper addresses a working model for non-instantaneous deteriorating items including the learning effect on various costs and the preservation technology investment to reduce the deterioration rate. The system includes time and price sensitive demand. The objective is to maximise the total profit per unit time by finding optimal joint selling price, replenishment cycle time, the preservation technology investment per unit time, ordering quantity and the shortage period. A numerical example is presented to validate the policy investigated in this paper. Further, the sensitivity analysis about the key parameters is conducted to obtain the managerial insights.

Keywords: non-instantaneous deterioration of items; time-price dependent demand; learning effect; preservation technology.

Reference to this paper should be made as follows: Shah, N.H. and Naik, M.K. (2018) 'Inventory model for non-instantaneous deterioration and price-sensitive trended demand with learning effects', *Int. J. Inventory Research*, Vol. 5, No. 1, pp.60–77.

Biographical notes: Nita H. Shah is a Professor in the Department of Mathematics, Gujarat University, Ahmedabad. She has 25 years of research experience in inventory management, forecasting and information technology and information systems. She has published 375+ articles in international journals, including *APJOR* (Singapore), *International Journal of Production Economics*, *OMEGA*, *CCERO* (Belgium), *ECPE* (Romania), *Measuring, Control & Simulation* (France), *JIOS* (India), *IJOMS* (India), *Industrial Engineering* (India), *European Journal of Operational Research*, *IJIR*, *IJOR*, *IJMOR*, *IJBPSCM*, etc.

Monika K. Naik is a Research Scholar in the Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India. Her research interest is inventory modelling and dynamical systems of inventory parameters. She has two publications in her credit.

1 Introduction

Earlier literature mostly consists of development of the models on the assumption of either time and/or stock and/or cost dependent demand but rarely dependent on selling price as well as an unrealistic assumption were made about infinite life cycle of goods, as such most of the goods by dropping their initial value undergoes deterioration over time. Deterioration reduces the quality and physical quantity of inventory and so, simultaneously system is burdened by an additional cost. As such selling price of a product is a very crucial factor in uplifting customer's demand of a product which is directly influenced by customer's satisfaction level. Now a days, recent studies includes selling price dependent demand rate and much emphasis on deterioration is drawn to highlight the shorter life cycles of goods. Therefore, accurate inventory control of deterioration of items is considered to be an important issue to elaborate.

Gupta and Vrat (1986) developed model for stock dependent consumption rate. An EOQ model was developed by Padmanabhan and Vrat (1990) with stock dependent consumption rate and exponential decay. Datta and Pal (2001) investigated an inventory system with stock dependent and price sensitive demand. Goyal and Giri (2001) developed modelling of deteriorating inventory by reviewing studies of deteriorating items in recent trends which includes wide range of Inventory control systems with various other properties. A model for perishable products with price dependent demand was proposed by Singh (2006). Wu et al. (2006) describing the suitable deterioration pattern of various products. An optimal replenishment policies for non-instantaneous deteriorating items with stock dependent demand was introduced by Chang et al. (2010). Under the effect of learning and imprecise demand rate for an optimisation policy of inventory model was developed by Yadav et al. (2011). For price sensitive stock dependent demand under progressive payment scheme for obtaining an optimal ordering and pricing policy read (Shah et al., 2011).

Recently, in the area of operation research and revenue management, much emphasis is drawn on work comprising of joint pricing and inventory control of deteriorating items. Hou et al. (2011) proposed an inventory model for deteriorating items under partial backlogging under inflation and with stock dependent selling rate. Two warehouses inventory model for non-instantaneous deteriorating items were introduced by Singh et al. (2011) for stock-dependent demand. Initially, a two period life time product having demand of each period as a random price dependent function was investigated by Jia and Hu (2011). The similar situation was handled by Chen and Sapra (2013), in two different inventory setups: first-in-first-out (FIFO) and last-in-first-out (LIFO).

An EOQ model with preservation technology investment under two level of trade credit with demand depends on selling price and credit period by Singh et al. (2013). Two level storage and partial backlogging under inflation were studied by Kumar et al. (2013) for an inventory model with learning effect. Ahmed et al. (2013) obtained inventory models with ramp type demand rate, partial backlogging and general deterioration rate. Amutha et al. (2013) established an inventory model for deteriorating items with three parameter Weibull deterioration and price dependent demand. Guria et al. (2013) proposed and inventory model for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Singh and Rathore (2014) proposed an inventory model for deteriorating item with reliability consideration and trade credit. Parekh and Patel (2014) developed an inventory model for deteriorating items with linear demand, time varying holding cost under inflation for two warehouses. A model dealing with cooperative advertising in a closed-loop supply chain to encourage customers to return their used products was proposed by Geranmayeh et al. (2017).

Optimal price and inventory policies with permissible delay in payments for non-instantaneous deteriorating items were studied by Soni and Patel (2012). Then elaborating the earlier model, Soni and Patel (2013) included inaccurate deterioration free time and credibility constraints. An inventory model describing non-instantaneous deteriorating items was proposed by Shah et al. (2013) which consists of demand as a function of selling price and the frequency of advertisement. Further, the concept of non-instantaneous deterioration was highlighted by many other authors like Panda et al. (2013), Zhang et al. (2015), Ghoreishi et al. (2014) and Maihami and Nakhai (2012a, 2012b).

An investigation by Soni (2013) was undergone including optimal replenishment policies for deteriorating items with price and stock sensitive demand under permissible delay in payments. The demand was also including to the inventory level by Lu et al. (2016). Tayal et al. (2015) established an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate. Pal and Maity (2015) derived an inventory model for deteriorating items with price dependent demand and delay in payments under stochastic inflation rate. Mishra (2016) proposed an inventory model of deteriorating items with revenue sharing on preservation technology investment under price sensitive stock dependent demand. Mandal and Islam (2016) studied a fuzzy inventory model for deteriorating items, with time dependent demand, shortages, and fully backlogging. Singh et al. (2016) developed an EOQ model for deteriorating products having stock dependent demand with trade credit period and preservation technology. An EOQ model for deteriorating items with selling price-dependent demand rate with learning effect was proposed by Goyal and Chauhan (2016). Shah et al. (2016) proposed an integrated production-inventory model with preservation technology for time-varying deteriorating item under time and price sensitive demand.

The highlighting point in this article which makes it a unique one, is time and price dependent market demand rate for non-instantaneous deteriorating items with learning effect, shortages and preservation technology while many researchers like; Goyal and Chauhan (2016), Pal et al. (2015) and Mishra (2016) includes price dependent demand rate only. Singh et al. (2016) utilises stock dependent demand rate with preservation technology for deteriorating products.

This paper addresses an operating system for non-instantaneous deteriorating items including the learning effect on various costs on the total average of the system and the preservation technology investment to reduce the deterioration rate. The system includes time and price sensitive demand. The objective is to maximise the total profit per unit time by finding optimal joint selling price, replenishment cycle time, the preservation technology investment per unit time, ordering quantity and the shortage period. A numerical example is presented to validate the policy investigated in this paper. Further, the sensitivity analysis about the key parameters is conducted to obtain the managerial insights.

2 Notations and assumptions

2.1 Notations

$I_1(t)$	the inventory level at time $t(0 \leq t \leq T_d)$ in units
$I_2(t)$	the inventory level at time $t(T_d \leq t \leq T_1)$ in units
$I_3(t)$	the inventory level at time $t(T_1 \leq t \leq T)$ in units
p	selling price per unit in dollars
$Dt(t, p)$	demand rate as a function of the selling price p and time t
u	preservation technology investment per unit time to reduce deterioration rate $u \geq 0$
T	length of replenishment cycle
n	number of replenishment cycle
T_1	the time at which inventory level becomes zero
$\theta(u)$	the deterioration rate coefficient under preservation technology investment u, with, $\theta(0) = \theta_0$ the deterioration coefficient under natural conditions
T_d	the time after which deterioration occurs
Q_1	initial inventory level at the beginning of each cycle
Q_2	backordered quantity during shortages
Q	total ordering quantity
C	the purchasing cost per order in dollars
B	the rate of backlogging
A_1	the constant part per order of the ordering cost
$\frac{A_2}{n^{\alpha_1}}$	the variable part of ordering cost which decreases in each cycle due to the effect of learning
h_1	the constant part per order of holding cost
$\frac{h_2}{n^{\alpha_2}}$	the variable part of holding cost which decreases in each cycle due to the effect of learning
ϕ	shortage cost per unit
a	constant demand rate co-efficient; $a > 0$

b	elasticity of inventory level at time t, $b > 0$
c	selling price dependent demand rate co-efficient, $c > 0$
$TP(T, p, u, T_1)$	the total profit per unit time of the inventory system in dollars.

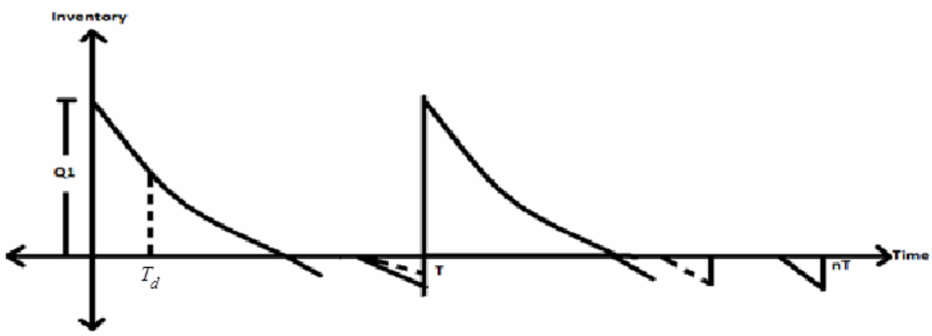
2.2 Assumptions

1. Single deteriorating item is involved in inventory system.
2. The deterioration starts after a fix time, i.e., the deterioration in the products is non-instantaneous in nature.
3. Let the demand rate for the item is $Dt(t, p) = a(1 + bt - ct^2)p^{-\eta}$, a function of time; with p as the selling price per unit, where, $a > 0$ is a scale demand, $0 \le b < 1$ denotes the linear rate of change of demand with respect to time, $0 \le c < 1$ denotes the quadratic rate of change of demand and η are mark up for selling price, where, $\eta > 1$.
4. An infinite planning horizon is considered.
5. There is no replacement or repair of deteriorating items.
6. Shortages are allowed and the occurring shortages are partially backlogged.
7. Holding cost and ordering cost per order is partly constant and partly decreases in each cycle due to the effect of learning.

3 Mathematical modelling

A mathematical model of inventory problem for non-instantaneous deteriorating items including learning effect on several costs having selling price and time dependent demand rate is proposed here. The replenishment cycle is set a T time units. The inventory level is maximum at time $t = 0$. During the time interval $[0, T_d]$ the inventory decreases due to demand only, since the products are non-instantaneous in nature and until $t = T_d$, there is no deterioration in the products. After $t = T_d$, the inventory depletes due to combined effect of demand and deterioration in the products.

Figure 1 Inventory time graph for retailer



At $t = T_1$ the inventory level becomes zero and after that shortage occurs. The occurring shortages during $[T_1, T]$ are partially backlogged. The inventory graph for the retailer is demonstrated in Figure 1.

The differential equations governing the transition of the system are given as follows:

$$\dot{I}_1(t) = -Dt(t, p); 0 \le t \le T_d \tag{1}$$

$$\dot{I}_2(t) = -\theta(u)I_2(t) - Dt(t, p); T_d \le t \le T_1 \tag{2}$$

$$\dot{I}_3(t) = -Dt(t, p); T_1 \le t \le T \tag{3}$$

With boundary conditions:

$$I_3(T_1) = 0, I_2(T_1) = 0, I_1(T_d) = I_2(T_d) \tag{4}$$

On solving the equations using boundary conditions we get the following inventory levels.

For

$$\left(0 \le t \le T_d\right)$$

$$I_1(t) = \left(ap^{-\eta}\right)\begin{pmatrix} (T_d - t) + \frac{b}{2}\left(T_d^2 - t^2\right) - \frac{c}{3}\left(T_d^3 - t^3\right) \\ + \frac{\left(b\left(T_1 - T_d\right) + c\left(T_d^2 - T_1^2\right) + \left(1 + bT_d - cT_d^2\right)\left(1 - e^{(-\theta u(T_1 - T_d))}\right)\right)}{\theta u} \\ + \frac{\left((2c)\left(T_1 - T_d\right) + \left(b - 2cT_d\right)\left(\left(e^{-\theta u(T_1 - T_d)}\right) - 1\right)\right)}{\theta u^2} \\ + \frac{2c\left(1 - e^{-\theta u(T_1 - T_d)}\right)}{\theta u^3} \end{pmatrix} \tag{5}$$

For

$$\left(T_d \le t \le T_1\right)$$

$$I_2(t) = \left(ap^{-\eta}\right)\begin{pmatrix} \frac{1}{\theta u}\begin{pmatrix} b\left(T_1 - t\right) + c\left(t^2 - T_1^2\right) + \left(1 + bT_d - cT_d^2\right)\left(e^{-\theta u(t - T_d)}\right) \\ -\left(e^{-\theta u(T_1 - T_d)}\right) \end{pmatrix} \\ + \frac{\left((2c)\left(T_1 - t\right) + \left(b - 2cT_d\right)\left(\left(e^{-\theta u(T_1 - T_d)}\right) - \left(e^{-\theta u(t - T_d)}\right)\right)\right)}{\theta u^2} \\ + \frac{2c\left(e^{-\theta u(t - T_d)} - e^{-\theta u(T_1 - T_d)}\right)}{\theta u^3} \end{pmatrix} \tag{6}$$

For

$(T_1 \le t \le T)$

$$I_3(t) = ap^{-\eta}\left((t - T_1) + \frac{b}{2}(t^2 - T_1^2) - \frac{c}{3}(t^3 - T_1^3)\right) \tag{7}$$

Using the initial condition, $I_1(0) = Q_1$ we get,

$$Q_1 = (ap^{-\eta})\begin{pmatrix} (T_d) + \frac{b}{2}(T_d^2) - \frac{c}{3}(T_d^3) \\ + \frac{\left(b(T_1 - T_d) + c(T_d^2 - T_1^2) + (1 + bT_d - cT_d{}^2)(1 - e^{(-\theta u(T_1 - T_d))})\right)}{\theta u} \\ + \frac{\left((2c)(T_1 - T_d) + (b - 2cT_d)\left((e^{-\theta u(T_1 - T_d)}) - 1\right)\right)}{\theta u^2} \\ + \frac{2c(1 - e^{-\theta u(T_1 - T_d)})}{\theta u^3} \end{pmatrix} \tag{8}$$

The total profit of the operating system involves the below stated components:

1 Sales revenue:

The total ordering quantity:

$$Q = Q_1 + Q_2 \tag{9}$$

$$Sales\ Revenue\ (SR) = selling\ \ price * total\ ordering\ quantity$$

$$pQ = p\begin{bmatrix} (ap^{-\eta})\begin{pmatrix} (T_d) + \frac{b}{2}(T_d^2) - \frac{c}{3}(T_d^3) \\ + \frac{\left(b(T_1 - T_d) + c(T_d^2 - T_1^2) + (1 + bT_d - cT_d^2)(1 - e^{(-\theta u(T_1 - T_d))})\right)}{\theta u} \\ + \frac{\left((2c)(T_1 - T_d) + (b - 2cT_d)\left((e^{-\theta u(T_1 - T_d)}) - 1\right)\right)}{\theta u^2} \\ + \frac{2c(1 - e^{-\theta u(T_1 - T_d)})}{\theta u^3} \end{pmatrix} \\ -\frac{1}{3}Bacp^{-\eta}(T^3 - T_1^3) + \frac{1}{2}Babp^{-\eta}(T^2 - T_1^2) + Bap^{-\eta}(T - T_1) \end{bmatrix} \tag{10}$$

2 Purchasing cost

Purchasing cost (PC) = purchasing cost per order x total ordering quantity

= $(Q_1 + Q_2)$

As, from equation (8)

$$Q_1 = \left(ap^{-\eta}\right)\left(\begin{array}{l}(T_d)+\frac{b}{2}\left(T_d^2\right)-\frac{c}{3}\left(T_d^3\right)\\ +\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1^2\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u}\\ +\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2}\\ +\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\end{array}\right)$$

And

$$Q_2 = \int_{T_1}^{T} B\left(Dt(t,\, p)\right)dt \tag{11}$$

Therefore

Purchasing cost (PC) =

$$C\left[\begin{array}{l}\left(ap^{-\eta}\right)\left(\begin{array}{l}(T_d)+\frac{b}{2}\left(T_d^2\right)-\frac{c}{3}\left(T_d^3\right)\\ +\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1^2\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u}\\ +\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2}\\ +\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\end{array}\right)\\ -\frac{1}{3}Bacp^{-\eta}\left(T^3-T_1^3\right)+\frac{1}{2}Babp^{-\eta}\left(T^2-T_1^2\right)+Bap^{-\eta}\left(T-T_1\right)\end{array}\right] \tag{12}$$

3 Ordering cost

Ordering cost (OC) = Constant part of OC

+variable part of $OC = A_1 + \frac{A_2}{n^{\alpha_1}}$ (13)

4 Holding cost

$$\text{Holding cost } (HC) = \left(h_1 + \frac{h_2}{n^{\alpha_2}}\right)\left(\int_0^{T_d} I_1 dt + \int_{T_d}^{T_1} I_2 dt\right) \tag{14}$$

$$
\begin{aligned}
&=\left(h_1+\frac{h_2}{n^{\alpha_2}}\right)\left(\int_0^{T_d}\left(ap^{-\eta}\right)\left(\left(T_d-t\right)+\frac{b}{2}\left(T_d^2-t^2\right)-\frac{c}{3}\left(T_d^3-t^3\right)\right.\right.\\
&+\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u}\\
&+\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2}\\
&\left.+\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\right)dt+\int_{T_d}^{T_1}\left(ap^{-\eta}\right)\left(\frac{1}{\theta u}\left(b\left(T_1-t\right)+c\left(t^2-T_1^2\right)\right.\right.\\
&+\left(1+bT_d-cT_d^2\right)\left(e^{-\theta u(t-T_d)}\right)-\left(e^{-\theta u(T_1-T_d)}\right)\Big)\\
&+\frac{\left((2c)\left(T_1-t\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-\left(e^{-\theta u(t-T_d)}\right)\right)\right)}{\theta u^2}\\
&\left.\left.+\frac{2c\left(e^{-\theta u(t-T_d)}-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\right)dt\right)
\end{aligned}
\tag{15}
$$

5 Shortage cost

Shortage cost = shortage cost per unit * market demand

$$SC=\phi\left(\int_{T_1}^{T} Dt\left(t,\, p\right)dt\right)=\phi\left(\int_{T_1}^{T} a\left(1+bt-ct^2\right)p^{-\eta}dt\right) \tag{16}$$

6 Lost sale cost

$$\text{Lost sale cost}=LSC=\int_{T_1}^{T}(1-B)\left(Dt(t,\, p)\right)dt \tag{17}$$

where, B represents the rate of backlogging.

7 Preservation technology investment

$$\text{Preservation technology investment}=PTI=uT \tag{18}$$

The total profit per unit time is calculated by

$$\text{Total profit per unit time}=\frac{1}{T}\left(\text{Sales Revenue-}\left(\begin{array}{l}\text{Purchasing Cost + Ordering Cost}\\ \text{+ Holding Cost + Shortage Cost}\\ \text{+Lost Sell Cost+Preservation}\\ \text{Technology Investment}\end{array}\right)\right)$$

$$TP\left(p,T,u,T_1\right)=\frac{1}{T}\left(SR-\left(PC+OC+HC+SC+LSC+PTI\right)\right)$$

$$
\begin{aligned}
&TP(p,T,u,T_1)= \\
&\frac{1}{T}\Bigg(p\bigg[\left(ap^{-\eta}\right)\bigg(\left(T_d\right)+\frac{b}{2}\left(T_d^2\right)-\frac{c}{3}\left(T_d^3\right) \\
&+\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1^2\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u} \\
&+\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2}+\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\bigg) \\
&-\frac{1}{3}Bacp^{-\eta}\left(T^3-T_1^3\right)+\frac{1}{2}Babp^{-\eta}\left(T^2-T_1^2\right)+Bap^{-\eta}\left(T-T_1\right)\bigg] \\
&-C\bigg[\left(ap^{-\eta}\right)\bigg(\left(T_d\right)+\frac{b}{2}\left(T_d^2\right)-\frac{c}{3}\left(T_d^3\right) \\
&+\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1^2\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u} \\
&+\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2}+\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\bigg) \\
&-\frac{1}{3}Bacp^{-\eta}\left(T^3-T_1^3\right)+\frac{1}{2}Babp^{-\eta}\left(T^2-T_1^2\right)+Bap^{-\eta}\left(T-T_1\right)\bigg]-\left(A_1+\frac{A_2}{n^{\alpha_1}}\right) \qquad (19) \\
&-\left(h_1+\frac{h_2}{n^{\alpha_2}}\right)\Bigg(\int_0^{T_d}\left(ap^{-\eta}\right)\bigg(\left(T_d-t\right)+\frac{b}{2}\left(T_d^2-t^2\right)-\frac{c}{3}\left(T_d^3-t^3\right) \\
&+\frac{\left(b\left(T_1-T_d\right)+c\left(T_d^2-T_1^2\right)+\left(1+bT_d-cT_d^2\right)\left(1-e^{(-\theta u(T_1-T_d))}\right)\right)}{\theta u} \\
&+\frac{\left((2c)\left(T_1-T_d\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-1\right)\right)}{\theta u^2} \\
&+\frac{2c\left(1-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\bigg)dt+\int_{T_d}^{T_1}\left(ap^{-\eta}\right)\bigg(\frac{1}{\theta u}\Big(b\left(T_1-t\right)+c\left(t^2-T_1^2\right) \\
&+\left(1+bT_d-cT_d^2\right)\left(e^{-\theta u(t-T_d)}\right)-\left(e^{-\theta u(T_1-T_d)}\right)\Big) \\
&+\frac{\left((2c)\left(T_1-t\right)+\left(b-2cT_d\right)\left(\left(e^{-\theta u(T_1-T_d)}\right)-\left(e^{-\theta u(t-T_d)}\right)\right)\right)}{\theta u^2} \\
&+\frac{2c\left(e^{-\theta u(t-T_d)}-e^{-\theta u(T_1-T_d)}\right)}{\theta u^3}\bigg)dt\Bigg)-\phi\left(\int_{T_1}^{T}a\left(1+bt-ct^2\right)p^{-\eta}dt\right) \\
&-\int_{T_1}^{T}(1-B)\left(R(t,\,p)\right)dt-uT\Bigg)
\end{aligned}
$$

In order to increase the total profit of the system and cash flow to acquire optimal scenario we maximise the total profit by applying the following necessary and sufficient condition:

$$\frac{\partial TP(p,T,u,T_1)}{\partial p}=0,\ \frac{\partial TP(p,T,u,T_1)}{\partial T}=0,\ \frac{\partial TP(p,T,u,T_1)}{\partial u}=0,\ \frac{\partial TP(p,T,u,T_1)}{\partial T_1}=0 \tag{20}$$

Adopting below stated algorithm for the solution:

Step 1 Allocate hypothetical values to the inventory parameters.

Step 2 Solve equations in (20) simultaneously by mathematical software Maple XVIII.

Step 3 Check second order (sufficient conditions).

$$\frac{\partial^2 TP(p,T,u,T_1)}{\partial p^2}\le 0,\ \frac{\partial^2 TP(p,T,u,T_1)}{\partial T^2}\le 0,\ \frac{\partial^2 TP(p,T,u,T_1)}{\partial u^2}\le 0,\ \frac{\partial^2 TP(p,T,u,T_1)}{\partial T_1^2}\le 0$$

Step 4 Compute profit per unit time by using (19), ordering quantity from equation (9).

Optimum selling price from equation (20) and optimum replenishment cycle length by using equation (20).

The objective is to make total profit per unit time maximum with respect to selling price (p), cycle time (T), preservation technology investment (u) and time at which inventory becomes zero (T_1). Now, we examine the working of the model with numerical values for the inventory parameters.

4 Numerical example and sensitivity analysis

In this section, we provide a numerical example to illustrate the above mathematical model.

Example 1: Assume the following inventory parametric values to validate the model.

$$a=20{,}000\ units,\ b=62\%,\ c=75\%,\ \eta=1.2,$$
$$\phi=\$50/unit,\ T_d=1.5\ days,\ h_1=\$0.1/unit$$
$$h_2=\$0.1/unit,\ A_1=\$500/unit,\ A_2=\$150/unit,$$
$$C=\$30/unit,\ \alpha_1=0.2,\ \alpha_2=0.2,$$
$$B=0.1,\ \theta_0=0.0999,\ \xi=0.9999,\ n=4$$

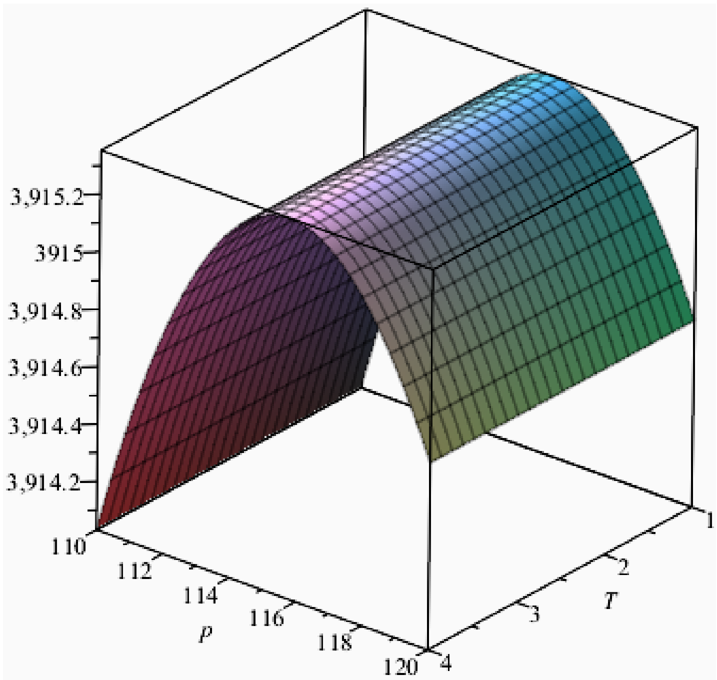
Solution:

By following the above algorithm, we obtain the below stated optimal values:

- the optimal cycle time is $T = 2.2497$ *days*
- the optimal retail price is $p = \$115.6841$ / *unit*
- the preservation technology investment is $u = \$49.7880$ / *unit*
- the time at which inventory becomes zero is $T_1 = 1.9297$ *days*
- the corresponding total profit is TP = \$3,915.3526
- the order quantity $Q = 98.8993$ *units*.

The concavity of total profit is shown from Figure 2.

Figure 2 Concavity of total profit (see online version for colours)



Now, for Example 1, we are examining the effects of various inventory parameters on total profit, decision variables selling price, cycle time, preservation technology investment, time at which inventory becomes zero by varying them as –20%, –10%, 10% and 20% as shown in Figure (3) to Figure (7) and Table 1 shows the impact on decision variables on varying various inventory parameters.

Figure 3 Replenishment cycle verses variation of various inventory parameters (see online version for colours)

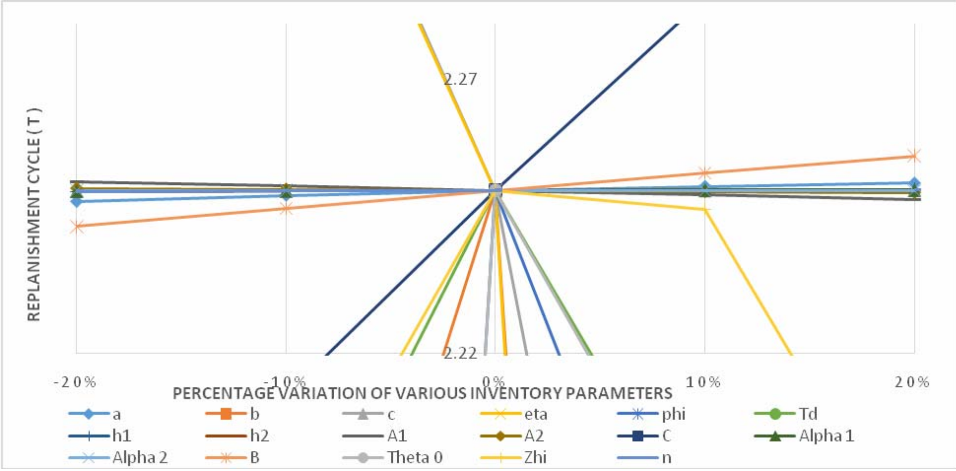


Figure 4 Selling price verses variation of various inventory parameters (see online version for colours)

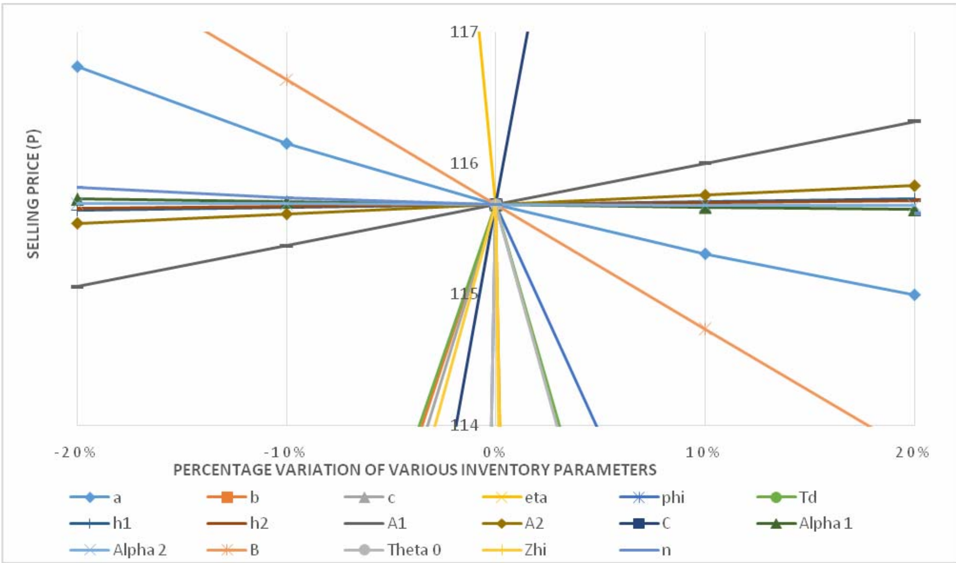


Figure 5 Preservation technology investment verses variation of various inventory parameters (see online version for colours)

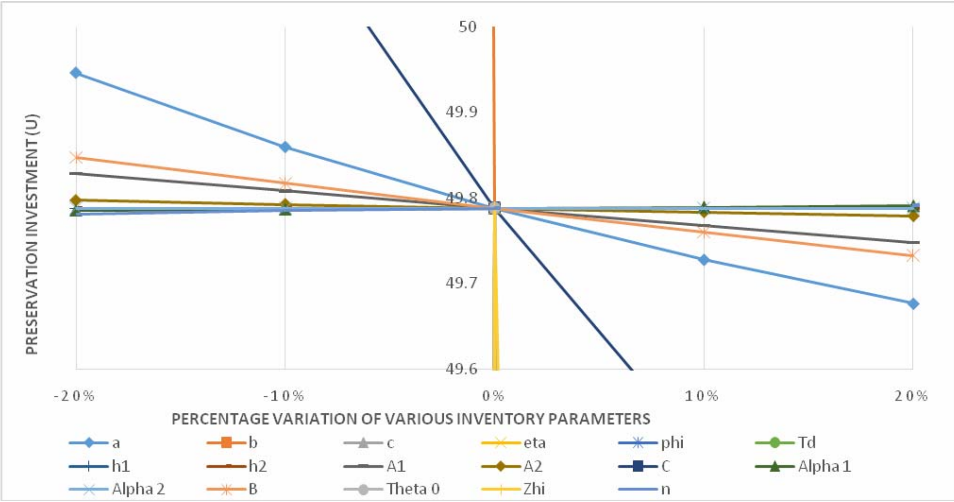


Figure 6 Time at which inventory becomes zero verses variation of various inventory parameters (see online version for colours)

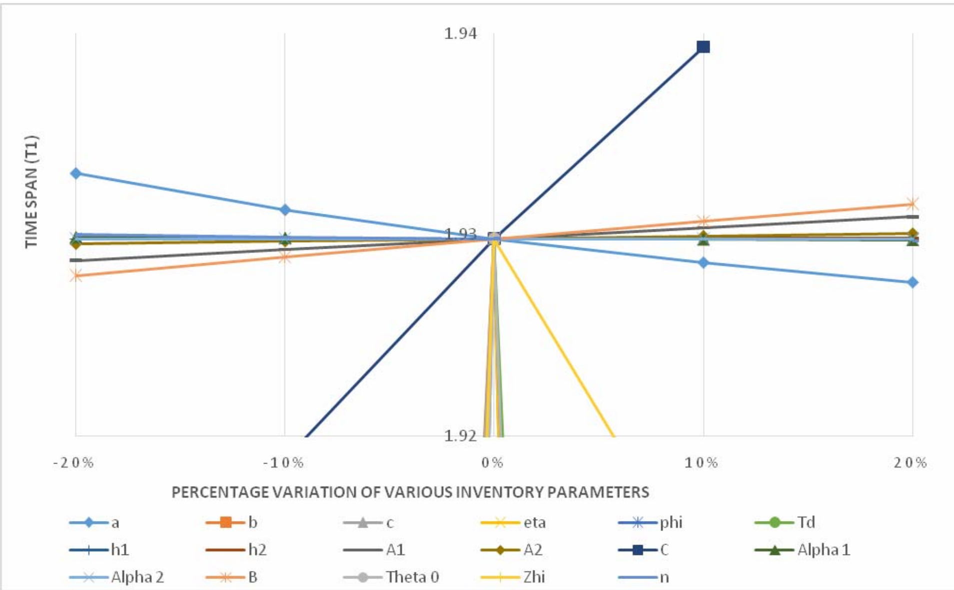
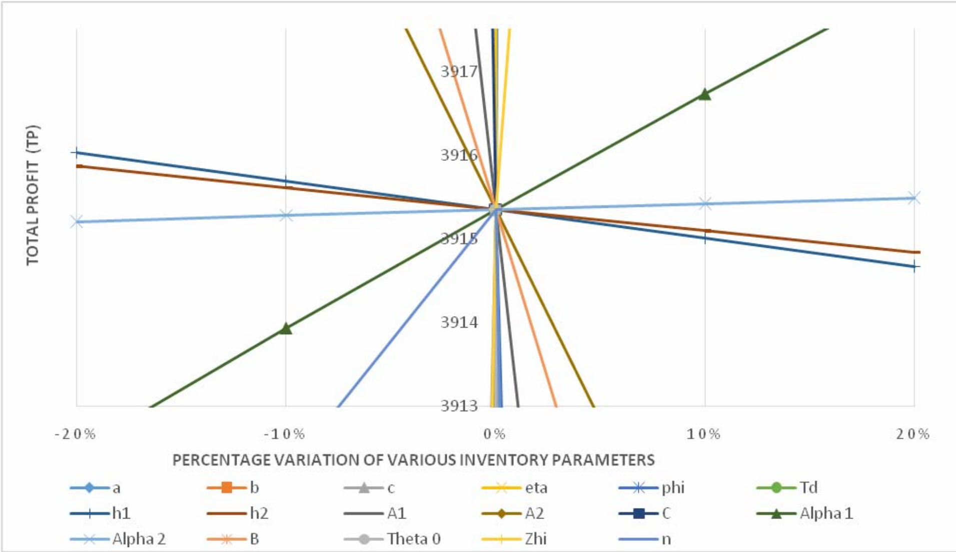


Table 1 Impact on decision variables on varying inventory parameters (see online version for colours)

Inventory parameters	*Impact on decision variables on varying inventory parameters –20% to 20%*					*Managerial insights*
	Replenishment cycle (T)	*Selling price (p)*	*Preservation technology investment (u)*	*Time at which inventory becomes zero (T_1)*	*Total profit (TP)*	
Scale demand (a)	↑	↓	↓	↓	↑	Each parameter is sensitive to a
Linear demand rate (b)	Fluctuates	Fluctuates	↓	Fluctuates	Fluctuates	Preservation cost decreases with b
Quadratic demand rate (c)	↓	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Cycle length decreases with c
Markup for selling price (η)	↓	↓	Fluctuates	Fluctuates	↓	Cycle length, selling price and Total profi decreases with η
Shortage cost per unit (ϕ)	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Each parameter fluctuates with T_d
Time at which inventory becomes zero (T_d)	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Each parameter fluctuates with T_d
Constant part of holding cost (h_1)	↑	↑	↓	↑	↓	Preservation cost and total profit decreases with h_1
Variable part of holding cost (h_2)	↑	↑	↓	↑	↓	Preservation cost and total profit decreases with h_2
Constant part of ordering Cost (A_1)	↓	↑	↓	↑	↓	Cycle length, preservation cost and total profit decreases with A_1
Variable part of ordering cost (A_2)	↓	↑	↓	↑	↓	Cycle length, preservation cost and total profit decreases with A_2
Purchasing cost (C)	↑	↑	↓	↑	↑	Preservation cost decreases with C
Markup (α_1)	↑	↓	↑	↓	↑	Selling price and inventory vanish time reduces with α_1
Markup (α_2)	↓	↓	↑	↓	↓	Only preservation cost increases with α_2
Rate of backlogging (B)	↑	↓	↓	↓	↓	Increase in cycle length with B
Deterioration co-efficient (θ_0)	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Fluctuates	Each parameter fluctuates with θ_0
Markup for preservation constant (ξ)	Fluctuates	Fluctuates	Fluctuates	Fluctuates	↑	Total profit increases with ξ
Number of shipments (n)	↑	↓	Fluctuates	↓	Fluctuates	Increase in cycle length with n

Figure 7 Total profit verses variation of various inventory parameters (see online version for colours)



5 Conclusions

In this article a system for non-instantaneous deteriorating items including the learning effect on various costs on the total average of the system and the preservation technology investment to reduce the deterioration rate is considered. The system includes time and price sensitive demand. The objective is to maximise the total profit per unit time by finding optimal joint selling price, replenishment cycle time, the preservation technology investment per unit time, ordering quantity and the shortage period. It is calculated, how a vendor can get the optimise policy and analyse the changes in the policy behaviour, as the various parameters changes.

A numerical example is presented to validate the policy investigated in this paper. Further, the sensitivity analysis about the key parameters is conducted to obtain the managerial insights. This model can be generalised for multivariate demand function and for different conditions of trade credit.

References

Ahmed, M.A., Al-Khamis, T.A. and Benkherouf, L. (2013) 'Inventory models with ramp type demand rate, partial backlogging and general deterioration rate', *Applied Mathematics and Computation*, Vol. 219, No. 9, pp.4288–4307.

Amutha, R. and Chandrasekaran, E. (2013) 'An inventory model for deteriorating items with three parameter Weibull deterioration and price dependent demand', *International Journal of Engineering Research & Technology*, Vol. 2, No. 5, pp.1931–1935.

Chang, C.T., Teng, J.T. and Goyal, S.K. (2010) 'Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand', *International Journal of Production Economics*, Vol. 123, No. 1, pp.62–68.

Chen, L.M. and Sapra, A. (2013) 'Joint inventory and pricing decisions for perishable products with two period lifetime', *Naval Research Logistics*, Vol. 60, No. 5, pp.343–366.

Datta, T.K. and Pal, A.K. (2001) 'An inventory system stock-dependent, price sensitive demand rate', *Production Planning and Control*, Vol. 12, No. 1, pp.13–20.

Geranmayeh, A.F., Rabbani, M. and Taleizadeh, A.A. (2017) 'Cooperative advertising in a closed-loop supply chain to encourage customers to return their used products', *International Journal of Inventory Research*, Vol. 4, No. 1, pp.4–22.

Ghoreishi, M., Mirzazadeh, A. and Weber, G.W. (2014) 'Optimal pricing and ordering policy for non-instantaneous deteriorating items under inflation and customer returns', *Optimization*, Vol. 63, No. 12, pp.1785–1804.

Goyal, A.K. and Chauhan, A. (2016) 'An EOQ model for deteriorating items with selling price dependent demand rate with learning effect', *Nonlinear Studies*, Vol. 23, No. 4, pp.541–550.

Goyal, S.K. and Giri, B.C. (2001) 'Recent trends in modeling of deteriorating inventory', *European Journal of Operational Research*, Vol. 134, No. 1, pp.1–16.

Gupta, R. and Vrat, P. (1986) 'Inventory models for stock-dependent consumption rate', *Opsearch*, Vol. 23, No. 1, pp.19–24.

Guria, A., Das, B., Mondal, S. and Maiti, M. (2013) 'Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment', *Applied Mathematical Modelling*, Vol. 37, No. 1, pp.240–257.

Hou, K-L., Huang, Y-F. and Lin, L-C. (2011) 'An inventory model for deteriorating items with stock-dependent selling rate and partial backlogging under inflation', *African Journal of Business Management*, Vol. 5, No. 10, pp.3834–3843.

Jia, J. and Hu, Q. (2011) 'Dynamic ordering and pricing for a perishable goods supply chain', *Computers and Industrial Engineering*, Vol. 60, No. 2, pp.302–309.

Kumar, N., Singh, S.R. and Kumari, R. (2013) 'Learning effect on an inventory model with two level storage and partial backlogging under inflation', *International Journal of Services and Operations Management*, Vol. 16, No. 1, pp.105–122.

Lu, L., Zhang, J. and Tang, W. (2016) 'Optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand', *International Journal of Systems Science*, Vol. 47, No. 6, pp.1480–1494.

Maihami, R. and Nakhai, K.I. (2012a) 'Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging', *Mathematical and Computer Modelling*, Vol. 55, No. 5, pp.1722–1733.

Maihami, R. and Nakhai, K.I. (2012b) 'Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand', *International Journal of Production Economics*, Vol. 136, No. 1, pp.116–122.

Mandal, W.A. and Islam, S. (2016) 'Fuzzy inventory model for deteriorating items, with time depended demand, shortages, and fully backlogging', *Pakistan Journal of statistics and Operational Research*, Vol. 12, No. 1, pp.101–109.

Mishra, V.K. (2016) 'Inventory model of deteriorating items with revenue sharing on preservation technology investment under price sensitive stock dependent demand', *International Journal of Mathematical Modelling & Computations*, Vol. 21, No. 6, pp.37–48.

Padmanabhan, G. and Vrat, P. (1990) 'An EOQ model for items with stock dependent consumption rate and exponential decay', *Engineering Costs and Production Economics*, Vol. 18, No. 3, pp.241–246.

Pal, M. and Maity, H.K. (2015) 'An inventory model for deteriorating items with price dependent demand and delay in payments under stochastic inflation rate', *Thailand Statistician*, Vol. 13, No. 2, pp.127–143.

Panda, S., Saha, S. and Basu, M. (2013) 'Optimal pricing and lot-sizing for perishable inventory with price and time dependent ramp-type demand', *International Journal of Systems Science*, Vol. 44, No. 1, pp.127–138.

Parekh, R.U. and Patel, R. (2014) 'Deteriorating items inventory models for two warehouses with linear demand, time varying holding cost under inflation and permissible delay in payments', *International Journal of Mathematics and Statistics Invention*, Vol. 2, No. 5, pp.39–48.

Shah, N.H., Chaudhari, U. and Jani, M.Y. (2016) 'An integrated production-inventory model with preservation technology investment for time-varying deteriorating item under time and price sensitive demand', *International Journal of Inventory Research*, Vol. 3, No. 1, pp.81–98.

Shah, N.H., Patel, A.R. and Louc, K.R. (2011) 'Optimal ordering and pricing policy for price sensitive stock-dependent demand under progressive payment scheme', *International Journal of Industrial Engineering Computations*, Vol. 2, No. 3, pp.523–532.

Shah, N.H., Soni, H.N. and Patel, K.A. (2013) 'Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates', *Omega*, Vol. 41, No. 2, pp.421–430.

Singh, S.R. and Rathore, H. (2014) 'An inventory model for deteriorating item with reliability consideration and trade credit', *Pakistan journal of Statistics and Operational Research*, Vol. 10, No. 3, pp.349–360.

Singh, S.R., Gupta, V. and Goel, A. (2013) 'An EOQ model with preservation technology investment when demand depends on selling price and credit period under two level of trade credit', in *Proceeding of International Conference on Computational Intelligence: Modeling Techniques and Applications*, Vol. 10, pp.227–235.

Singh, S.R., Khurana, D. and Tayal, S. (2016) 'An economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology', *Uncertain Supply Chain Management*, Vol. 4, No. 1, pp.29–42.

Singh, S.R., Malik, A.K. and Gupta, S.K. (2011) 'Two warehouses inventory model for non-instantaneous deteriorating items with stock dependent demand', *International Transactions in Applied Sciences*, Vol. 3, No. 4, pp.749–760.

Singh. S.R. (2006) 'Production inventory model for perishable product with price dependent demand', *Ref. des ERA*, Vol. 1, No. 1, pp.1–10.

Soni, H.N. (2013) 'Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment', *International Journal of Production Economics*, Vol. 146, No. 1, pp.259–268.

Soni, H.N. and Patel, K.A. (2012) 'Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: fuzzy expected value model', *International Journal of Industrial Engineering Computations*, Vol. 3, No. 3, pp.281–300.

Soni, H.N. and Patel, K.A. (2013) 'Joint pricing and replenishment policies for non-instantaneous deteriorating items with imprecise deterioration free time and credibility constraint', *Computers and Industrial Engineering*, Vol. 66, No. 4, pp.944–951.

Tayal, S., Singh, S.R., Sharma, R. and Singh, A.P. (2015) 'An EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate', *International Journal of Operational Research*, Vol. 23, No. 2, pp.145–162.

Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006) 'An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging', *International Journal of Production Economics*, Vol. 101, No. 2, pp.369–384.

Yadav, D., Kumari, R. and Singh, S.R. (2011) 'Optimization policy of inventory model under the effects of learning and imprecise demand rate', *International Journal of Inventory Control & Management*, Vol. 1, No. 1, pp.39–53.

Zhang, J., Wang, Y., Lu, L. and Tang, W. (2015) 'Optimal dynamic pricing and replenishment cycle for non-instantaneous deterioration items with inventory-level-dependent demand', *International Journal of Production Economics*, Vol. 170, Part A, pp.136–145.