Finite element modelling of a cold curved steel plate girder

Jihad Rishmany* and Issam Tawk

Department of Mechanical Engineering,
University of Balamand,
P.O. Box 100, Lebanon
Email: jihad.rishmany@balamand.edu.lb
Email: issam.tawk@balamand.edu.lb
*Corresponding author

Antoine Gergess

Department of Civil Engineering,
University of Balamand,
P.O. Box 100, Lebanon
Email: tony.georges@balamand.edu.lb

Abstract: Cold bending is a cost-effective solution that is sometimes used for curving structural steel girders. Current usage for bridge structures is limited to projects that fall outside the jurisdiction of American Association of State Highway and Transportation Officials (AASHTO) because of the lack of technical knowledge surrounding this technique. This paper presents results from a three-dimensional finite element model to assess the structural behaviour exhibited by steel girders during bending for a proprietary cold curving system. A non-linear FE model is validated against measured data obtained from a previously tested girder. The FE model is extended to explore the performance of all structural components of the girder during bending such as deformations in flanges and web, residual stresses and plastic strains. Findings from this paper provide a framework for accurately predicting the cold bent geometry and how to incorporate residual stresses and plastic strains in the design of curved girders.

Keywords: cold bending; steel girder; curving; finite element; non-linear; lateral offsets; residual stresses; web; flange; plastic strains.


Biographical notes: Jihad Rishmany received his Mechanical Engineering Diploma from the Lebanese University in 2002. He obtained a PhD from ENSICA, France, in 2007. He did a Postdoctoral research in Tribology at Ecole Des Mines in Albi, France in 2008. In 2009, he was a part-time Lecturer in several universities in Lebanon. He joined University of Balamand, Lebanon, in 2010. Since that time, he has been with the Faculty of Engineering as an Assistant Professor. His main research areas are renewable energy, finite element analysis and multibody dynamics.
Issam Tawk received his Mechanical Engineering Diploma from the Lebanese University in 2003. He obtained a PhD from the University of Toulouse III, France, in 2008. He worked as Stress Engineer for the A350XWB program at AIRBUS, France for the period (September 2008–December 2009). He joined the University of Balamand, Lebanon, in 2010. Since that time, he has been with the Faculty of Engineering as an Assistant Professor. He is currently an Associate Professor since September 2015. His main areas of research interest are composites materials, helicopter blades, steel girders, modelling of aircraft structures and composites delamination.

Antoine Gergess received a Bachelor of Engineering from the American University of Beirut in 1987, a Master of Science in Civil Engineering in 1988 and a PhD in 2001 from the University of South Florida in Tampa. He is recognised for his expertise in complex bridge design, and some of his projects include the Leonard P. Zakim Bunker Hill cable-stayed bridge over Boston’s Charles River; a post-tensioned voided slab bridge over the Tamiami Canal in Miami and a tunnel in Hong Kong. Currently, he is a Professor and Dean of Students at the University of Balamand in Lebanon.

1 Introduction

Cold bending practice in the steel bridge industry is ambiguous. For example, AASHTO specifications on a minimum radius for cold bending steel plates to avoid cracking and limit plastic strains (AASHTO, 2010a) are not applicable for fracture critical members such as steel bridge girders (AASHTO, 2010b) (Figure 1(a)). Therefore, cut curving and heat curving are widely used instead to fabricate curved bridge girders. Fabricators prefer cold bending as the heat curving process is time consuming due to the several heat/cool cycles required while cut curving flange plates result in significant scrap.

In the absence of cold bending guidelines, cold curving fabrication techniques in the US remain proprietary and applicable only for projects that fall outside the jurisdiction of AASHTO. Recently, a US steel fabricator (Klobuchur, 2002) developed a simple cold curving method that consists of separately bending the top and bottom flange of the girder at various sections along its length by applying mechanical forces in the plastic range. The desired curved shape develops as a series of short straight segments (Figure 1(b)) and in the absence of guidelines and specifications, the accuracy of the process is based on trial and error as it depends on measurements in the steel shop. In order to validate the process, a full-scale steel girder was tested and a two-dimensional analytical solution was developed to monitor the geometric shape of the deformed steel girder during bending (Gergess and Sen, 2007). As measurements from the test girder were only recorded for lateral deformations at the bottom flange level, calibration of the analytical solution was based on these bottom flange movements. Several papers resulted from the study (Gergess and Sen, 2005a, 2005b, 2006, 2007, 2008).

As the top and bottom flanges of the girder are separately bent, three-dimensional effects are evident, specifically relative movements of the top and bottom flange and residual stresses and plastic strains that build up after each load application. Such effects were not obvious to the analytical solution that is essentially an approximate two-dimensional mathematical procedure (Gergess and Sen, 2005a). This paper conducts a three-dimensional finite element analysis for examining these effects as they directly
affect the structural performance of the girder in service (Keating and Christian, 2009). The finite element analysis is non-linear as it accounts for both geometric and material non-linearity.

**Figure 1** Curved steel plate girders: (a) horizontally curved stiffened steel girders, Courtesy Bill McEleney, NSBA and (b) idealisation of the desired curved shape due to cold bending

*Geometric non-linearity:* The finite element model accurately idealises the girder web, flanges and stiffeners using shell elements that have in plane bending stiffness capabilities. Since the bending loads are applied at intervals, residual deformations, residual stresses and plastic strains are calculated for each load application and set up the basis for the subsequent load application in the finite element model.
Material non-linearity: In the previously developed analytical solution (Gergess and Sen, 2005a), the steel stress-strain curve was idealised as elastic-perfectly plastic and the maximum strain was determined based on plastic hinging which made it possible to conduct mathematical derivations and provide closed form solutions. The finite element analysis is able to overcome this simplification by using the exact stress-strain curve of the material idealising the ‘plastic’ and ‘strain hardened’ region (Figure 2). It should be noted that the girder under study is made of Grade 50W steel which has a flat yield plateau that ranges from 10 to 20 times the yield strain $\varepsilon_y$ as shown in Figure 2 (Chajes et al., 1963).

Figure 2  Stress-strain curve of Grade 50W steel

*Yield plateau: length varies from 10 to 20 times the yield strain $\varepsilon_y$.

**Typical value used by fabricators before steel properties are reduced (Jayadevan et al., 2004).

Source:  Chajes et al. (1963)

2 Objective

The main objective of this paper is to develop a three-dimensional non-linear finite element model that accurately idealises the steel plate girder geometry, material properties and bending loads (Figures 3 and 4) and investigate the resulting cumulative effects, specifically permanent deformations, residual stresses and plastic strains that build-up in the web and flanges after each load application. Such results could not be obtained from the analytical solution and are essential to quantify the residual effects that need to be considered in the design of the steel girder.
The finite element model is built based on the test girder geometry and material properties and comparisons are made with available measured values to ensure its validity. Three-dimensional effects, that are the relative web/flange deformations, residual stresses and plastic strains are then determined for each load application. As the test girder is stiffened (Figures 3(a), (b) and 4(a)), the effect of stiffeners on the bending operation is also considered. Recommendations for incorporating residual effects due to cold bending in the design of the curved girder are finally made.

Figure 3  Full-scale specimen details (Not to Scale): (a) elevation view and (b) cross-section (intermediate stiffener on one side, thickness = 2.54 cm (1”))

3 Background

A summary of the key parameters of the girder used for developing the finite element model is presented. Additional information may be found elsewhere (Gergess and Sen, 2007) (only data required for the finite element analysis is shown). SI units are the main units and US units are provided in parentheses.
Figure 4  Test girder supports (lock-in plates of loading frame also shown): (a) bottom flange bending using the top hydraulic jack and (b) isometric view of the test girder (top flange bending)

*20.32 cm (8") wide × 5.08 cm (2") thick for the bottom jack (top flange bending); 30.48 cm (12") × 5.08 cm (2") for the larger size top jack (bottom flange bending).

Longitudinal arms of loading frame are spaced at 2.1 m (7’) apart.

3.1 Test girder details

The cold bending procedure was conducted to a full-scale stiffened, unsymmetrical steel girder of length $L' = 12.88$ m (42’–3") and depth $d = 1.11$m (43’/8") (Figure 3(a)). The girder was curved in two stages, first to a radius $R$ equal to 227 m (745’) and then to
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a radius \( R = 115.5 \text{ m (379')} \). Comparisons are only made for the first stage as it contains sufficient data to highlight on the three-dimensional residual effects that result from cold bending and fulfil the objective of the paper.

The actual yield stress \( (F_y) \) of the Grade 50W (weathering) steel girder was 405 MPa (58 ksi), Figure 2. The top flange was 30.48 cm (12") wide \( \times \) 1.91 cm (\( \frac{3}{4} \)"") thick, the bottom flange was 30.48 cm (12") wide \( \times \) 3.18 cm (1¼"") thick and the web plate was 106 cm (41¾") deep \( \times \) 2.22 cm (\( \frac{7}{8} \)"") thick, Figure 3(b). Full-depth 2.54 cm (1") thick plate stiffeners were welded to one side of the web at two symmetrically located points (2.29 m (7.5') from mid-span), Figure 3(b). The web and flange plates were connected by a 1.27 cm (\( \frac{1}{2} \)"") full fillet penetration weld. The girder tapered at both ends to a shallower depth and end bearing stiffeners were provided on both sides of the web to strengthen the ends (Figure 3(a)).

The girder was placed symmetrically on wide flange steel beams spaced 9.75 m (32') apart (Figure 3(a)). To ensure the stability of the girder during bending, the bottom flange was clamped to the wide flange beam (Figure 4(b)) at both ends.

3.2 Cold bending details

Bending loads were applied to a movable frame with built-in hydraulic jacks that exerted lateral loads to the top and bottom flanges separately at designated intervals along its length (Figure 4(b)). During bending, the jack pinches the front side of the flange to create a deformation while the longitudinal arms (spaced at 2.13 m (7')) of the frame clamp on to the flange at the back (Figure 4(b)). Loads were applied to a 9.14 m (30') long section of the girder sub-divided into six equal segments of length \( L_i = 1.52 \text{ m (5') } \) each (Sections 1–7 were set at 0.3 m (1') from the end supports, Figure 3(a)). The bending loads magnitude (determined from pressure gauge readings) and sequence of loading are presented in Table 1. The loads were 800 kN (180 kips) for the bottom flange (except at Section 2, a load of 670 kN (150 kips) was used) and 445 kN (100 kips) for the top flange. The smaller load used in the initial bending operation at Section 2 of the bottom flange was selected by the fabricator based on his experience to ensure that the lateral offsets that develop after loading the subsequent sections are not excessive (this is discussed in details later on in Section 6).

As lateral offsets dictate the curved shape of the girder, they were measured using a string-line that connects the end sections (Sections 1 and 7) after bending, Figure 4(b). Loads were applied sequentially from left to right starting at Section 2 and ending at Section 6 alternating between the top and bottom flange (Table 1). Steel angles and steel square plates were used to distribute the load to the girder (Figure 4(b)).

4 Finite element model

The finite element analysis was conducted using MSC SimXpert computer software (MSC Software, The MacNeal-Schwendler Corporation, 2013) in which material and geometric non-linearity were considered. The model can accurately idealise the girder geometry, stiffness, support conditions, residual stresses and bending loads.
In the three-dimensional model, the flanges and webs were modelled using four-noded isoparametric shell elements (CQUAD4) with in-plane bending stiffness.

4.1 Mesh sensitivity analysis

Prior to the nonlinear analysis, a linear static analysis was conducted to select an appropriate element size. Three models were tested with different elements size, 76 × 130 mm (3″ × 5\frac{1}{8}″) for model 1 (aspect ratio of 2), 76 × 76 mm (3″ × 3″) for model 2 (aspect ratio of 1) and 51 × 51 mm (2″ × 2″) for model 3 (aspect ratio of 1). The accuracy of the models was confirmed by calculating the lateral offset at mid-span due to a concentrated lateral load of 45 kN (10.1 kips) applied to the top flange. This was calculated as 86.12 mm (3 \frac{3}{8}″) in model 1, 84.91 mm (3 \frac{5}{16}″) in model 2 and 84.9 mm (3 \frac{5}{16}″) in model 3. Since model 2 and model 3 gave close correlations, the mesh from model 2 was selected in this study.

Table 1 Loads (P) and lateral offsets (\delta_{ij})b (corresponding to Figure 1(b))

<table>
<thead>
<tr>
<th>Loaded flange</th>
<th>Loaded section (Figure 3(a))</th>
<th>Load kN (kips)</th>
<th>Offset (\delta_{ij})b*</th>
<th>Lateral displacement mm (″)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>2</td>
<td>670 (150)</td>
<td>(\delta_{22})</td>
<td>3.2 (\frac{1}{8})</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>670 (150)</td>
<td>(\delta_{23})</td>
<td>6.4 (\frac{1}{4})</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>800 (180)</td>
<td>(\delta_{23})</td>
<td>11.1 (\frac{7}{16})</td>
</tr>
<tr>
<td>Top</td>
<td>2</td>
<td>445 (100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>445 (100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>445 (100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>800 (180)</td>
<td>(\delta_{24})</td>
<td>20.6 (\frac{3}{16})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{25})</td>
<td>28.6 (\frac{1}{4})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{34})</td>
<td>30.2 (\frac{3}{4})</td>
</tr>
<tr>
<td>Bottom</td>
<td>5</td>
<td>800 (180)</td>
<td>(\delta_{33})</td>
<td>23.8 (\frac{5}{16})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{35})</td>
<td>34.9 (\frac{1}{4})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{45})</td>
<td>38.1 (\frac{1}{2})</td>
</tr>
<tr>
<td>Bottom</td>
<td>6</td>
<td>800 (180)</td>
<td>(\delta_{36})</td>
<td>25.4 (1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{46})</td>
<td>38.1 (\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{56})</td>
<td>46.0 (\frac{13}{16})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\delta_{66})</td>
<td>20.6 (\frac{3}{8})</td>
</tr>
<tr>
<td>Top</td>
<td>5</td>
<td>445 (180)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>445 (180)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*(\delta_{ij})b is the offset that develops at section i when the bending load is applied at section j at the bottom flange level.

In the test, Section 3 was first loaded to 670 kN (150 kips) and then to 800 kN (180 kips). In the finite element model, the load is taken as 800 kN (180 kips).
4.2 Finite element mesh

The finite element mesh consisted of a total of 3939 nodes and 3752 elements (Figure 5), 2352 elements for the web, 672 elements for each flange (the same number of elements was used since the top and bottom flanges have the same width) and 28 elements for each intermediate stiffener. For simplicity, the tapered ends (Figure 3(a)) were taken as full depth sections in the finite element model without end stiffeners (Figure 3(b)) as they are located outside the supporting ends and the points of bending. Therefore, their contribution to the stiffness and overall behaviour of the girder during bending is considered to be minimal. The welding joints (Figure 3(b)) were not considered in the model, instead, the flanges are assumed to be rigidly connected to the web.

![Figure 5 MSC SimXpert finite-element mesh (3939 nodes, 3752 elements) (see online version for colours)](image)

4.3 Modelling of bending loads

The loading assembly that consisted of the load frame, the hydraulic jacks, the steel plates and angles between the girder flanges and hydraulic jacks (Figure 4) were idealised by equivalent loads e.g., a load of magnitude \( P \) representing the force exerted by the hydraulic jacks, and a load of magnitude \( P/2 \) representing the reactions from each of the supporting arms of the loading frame at the back of the flange. The pinching force was equally distributed over a distance equivalent to the width of the distribution plates e.g., 203 mm (8") for the top flange (the load is designated as \( P_t \)) and 304 mm (12") for the bottom flange (the load is designated as \( P_b \)). Based on the shell element size of 76 mm (3"), this resulted in 5 load applications for the bottom flange (\( P_b/5 \) each) and 3 load applications for the top flange (\( P_t/3 \) each) (as shown in Figure 6 for the top flange). The bending load was assumed to be equally distributed since the distribution plates have a large thickness (5.08 cm (2"), Figure 4(a)).
4.4 Modelling of end supports

During the test, end supports consisted of clamping the bottom flange of the girder to a wide flange steel beam at one side of the flange only (at about 1/6 of the flange width from the flange edge, Figure 4(b)) to prevent vertical movement and ensure stability (overturning) during bending. Note that the clamps were provided at opposite side of the bending loads (Figure 4).

Modelling of the clamp supports in the finite element analysis was not obvious as they prevent vertical movements and allow rotation in the plane of bending that is somehow reduced due to friction that develops between the clamp and the bottom flange of the steel girder (steel to steel surface). In absence of precise information on the friction coefficient/force, the idealisation of the clamps in the finite element model was determined by trial and error based on finding a suitable end supports configuration that gives comparable results between numerical and measured offsets (comparisons are presented in Table 2 for the bottom flange). The best idealisation was observed when each clamp was represented by two pin supports that only allowed rotations as shown in Figure 7. A spacing of 5 cm (2") between these nodes gave a close match between measured and calculated offsets (comparisons are shown in Table 2 after each load application). For this purpose, the mesh of the bottom flange at the clamps’ side had to be refined to accommodate the 5 cm (2") spacing (element size reduced from 76 × 76 mm (3" × 3") to 76 × 50 mm (3" × 2")). Consequently, the number of elements in the bottom flange increased from 336 to 504 and the adjusted finite element model consisted of a total of 4108 nodes and 3920 elements.

The remaining nodes along the end supports were restricted to move in the vertical direction (Figure 7) in order to account for the girder self-weight (that was not included in the model) and ensure vertical stability.

Comparison of results between calculated and measured offsets (Table 2) is discussed in details later on in the paper (Section 5). It will be shown that end supports play a key role in the bending operation as they can greatly affect the magnitude of the induced offsets and deformed shapes.
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Figure 7  Boundary conditions used in the finite element model

Table 2  Lateral offsets ($\delta_{ij}$), at the bottom flange level (measured vs. numerical)

<table>
<thead>
<tr>
<th>Location* ($\delta_{ij}$)</th>
<th>Lateral** Offsets (measured) mm (&quot;)</th>
<th>Lateral** Offsets (numerical) mm (&quot;)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>($\delta_{21}$)</td>
<td>3.2 ($\frac{1}{8}$&quot;)</td>
<td>0.05 (0)</td>
<td>-98%</td>
</tr>
<tr>
<td>($\delta_{23}$)</td>
<td>11.1 ($\frac{5}{16}$&quot;)</td>
<td>8.15 ($\frac{1}{16}$&quot;)</td>
<td>-27%</td>
</tr>
<tr>
<td>($\delta_{24}$)</td>
<td>20.6 ($\frac{3}{16}$&quot;)</td>
<td>15.09 ($\frac{1}{8}$&quot;)</td>
<td>-27%</td>
</tr>
<tr>
<td>($\delta_{34}$)</td>
<td>28.6 ($\frac{1}{8}$&quot;)</td>
<td>32.66 ($\frac{1}{16}$&quot;)</td>
<td>14%</td>
</tr>
<tr>
<td>($\delta_{44}$)</td>
<td>30.2 ($\frac{3}{16}$&quot;)</td>
<td>32.72 ($\frac{1}{16}$&quot;)</td>
<td>8%</td>
</tr>
<tr>
<td>($\delta_{45}$)</td>
<td>23.8 ($\frac{5}{16}$&quot;)</td>
<td>17.14 ($\frac{3}{16}$&quot;)</td>
<td>-28%</td>
</tr>
<tr>
<td>($\delta_{35}$)</td>
<td>34.9 ($\frac{1}{8}$&quot;)</td>
<td>37.90 ($\frac{1}{8}$&quot;)</td>
<td>9%</td>
</tr>
<tr>
<td>($\delta_{46}$)</td>
<td>38.1 ($\frac{1}{2}$&quot;)</td>
<td>42.06 ($\frac{1}{8}$&quot;)</td>
<td>10%</td>
</tr>
<tr>
<td>**($\delta_{26}$)</td>
<td>25.4 (1&quot;)</td>
<td>18.4 ($\frac{3}{4}$&quot;)</td>
<td>-28%</td>
</tr>
<tr>
<td>**($\delta_{36}$)</td>
<td>38.1 ($\frac{1}{2}$&quot;)</td>
<td>41.1 ($\frac{1}{8}$&quot;)</td>
<td>8%</td>
</tr>
<tr>
<td>**($\delta_{46}$)</td>
<td>46 ($\frac{1}{4}$&quot;)</td>
<td>47.05 ($\frac{1}{8}$&quot;)</td>
<td>2%</td>
</tr>
<tr>
<td>**($\delta_{56}$)</td>
<td>38.1 ($\frac{1}{2}$&quot;)</td>
<td>37.51 ($\frac{1}{8}$&quot;)</td>
<td>-2%</td>
</tr>
<tr>
<td>**($\delta_{66}$)</td>
<td>20.6 ($\frac{3}{16}$&quot;)</td>
<td>20.03 ($\frac{1}{16}$&quot;)</td>
<td>-3%</td>
</tr>
</tbody>
</table>

*($\delta_{ij}$): offset at section $i$ due to the load applied at section $j$ at the bottom flange level.

**Offsets that dictate the final deformed shape.

4.5 Non-linear analysis

The non-linear finite element analysis was carried out in steps to allow successive loading and unloading of the applied bending loads at Sections 2–6 of the girder (Figure 3(a)). The loading sequence was set identical to that from the test girder data (Table 1). 20 increments were used per load application with a convergence tolerance
of 0.1 for displacements and 0.5 for loads. The program automatically combines results (e.g., residual deformations, residual stresses and plastic strains) from the individual steps to provide both intermediate and final results.

4.6 Modelling of material properties

The flat yield plateau (taken at 10 times the yield strain (Chajes et al., 1963), Figure 2) and the strain hardened region of the stress-strain curve of the Grade 50W steel girder ($F_y = 405$ MPa (58 ksi)) (Figure 2) were both modelled in the finite element analysis. Based on the range of bending loads applied ($P_b = 800$ kN (180 kips) for the bottom flange and $P_t = 445$ kN (100 kips) for the top flange), it will be shown later in the paper (Section 7) that the plastic strains that develop during bending will remain within the yield plateau. Noted that the yield stress of 405 MPa (58 ksi) was input as a von Mises stress in the finite element model.

5 Finite element results

Results from the three-dimensional finite element model were first compared against available test data (Gergess and Sen, 2007) which relate primarily to lateral displacements ($\delta_{ij}$) at the bottom flange level as recorded in Table 1 (that is the only recorded data). The finite element analysis directly provides permanent deformations as they build-up after each bending operation as well as residual stresses and plastic strains.

5.1 Deformed shapes

The deformed shape of the steel girder flanges after each load application is shown in Figure 8 (Figure 8(a) for the bottom flange and Figure 8(b) for the top flange). Lateral offsets in Figure 8 correspond to a straight-line that connects the end supports at the bottom flange level (spaced at 9.75 m (32′), Figure 3(a)) while measured values ($\delta_{ij}$, Tables 1) were taken with respect to a string-line that connects Sections 1 and 7 (spaced at 9.14 m (32′)) at the bottom flange level, Figure 3(a)). For the sake of comparisons, lateral offsets from the finite element model are translated to the string-line that connects Sections 1 and 7 of the bottom flange as presented in Table 2 ($\delta_{ij}$, bottom flange lateral offsets) and in Figures 9–15 for the final deformed shapes of the bottom and top flange. In Figures 9–15, the abscissa axis also shows the distance of Sections 2–6 from Section 1.

Note that lateral offsets are residual offsets that permanently build-up after the bending loads are released and the plots shown in Figures 9–15 are based on numerical values from the finite element model recorded at each node along the length of the girder and not only at Sections 2–6.

5.2 Comparison

Figure 8 shows the progression of the deformed shapes during bending for the bottom flange (Figure 8(a)) and top flange (Figure 8(b)). Based on plastic hinging
(as the bending loads were based on the full plastic moment capacity of the steel
girder (Gergess and Sen, 2005a), it can be seen that the deformed shapes develop as a
series of straight segments. In Figure 8, bending loads are labelled per section per flange
(for example $P_{2,b}$ corresponds to the bending load applied at Section 2 of the bottom
flange).

**Figure 8** Residual offsets and deformed shapes after each bending application: (a) bottom flange and (b) top flange (see online version for colours)
The permanent offsets that build-up when the bending loads are applied at Section 6 of the bottom flange ($P_{6,b}$) and top flange ($P_{6,t}$) are of particular importance in this paper as they dictate the final (desired) curved shape of the girder, Figure 1(b). As offsets were only measured for the bottom flange, comparisons with numerical offsets from the finite element model were only made for the bottom flange.

The final deformed shape of the bottom flange after bending Section 6 of the bottom flange ($P_{6,b} = 800$ kN (180 kips)) is plotted in Figure 9 and measured and calculated offset values ($\delta_{ij}$) are presented in Table 2. Table 2 shows comparable results between numerical and measured offsets except at Section 2 after the first bending operation ($P_{2,b}$). $\delta_{22,b}$ calculated is $0.05$ mm ($0''$) compared to $3.2$ mm ($\frac{1}{8}''$) measured. This large discrepancy between measured and calculated offsets at Section 2 reduces to $28\%$ in the subsequent load applications ($\delta_{23,b}$, $\delta_{24,b}$, $\delta_{25,b}$, $\delta_{26,b}$) for the loading sequence presented in Table 1. The differences between measured and numerical offsets for the remaining sections are even smaller; they vary from $14\%$ at Section 3 after bending Section 4 ($\delta_{34,b} = 28.6$ mm ($\frac{11}{16}''$) measured compared to $32.66$ mm ($\frac{11}{16}''$) numerical) to $2\%$ at Sections 4 and 5 after bending Section 6 (the maximum offset at midspan ($\delta_{6b}$) is $46$ mm ($\frac{11}{16}''$) measured compared to $47.05$ mm ($\frac{11}{16}''$) numerical). It should be noted that this close correlation was obtained based on adjusting the end support conditions as discussed in Section 4.4.

On the other hand, Figure 9 shows asymmetry in the deformed shapes of the bottom flange based on both numerical and measured offsets. The difference between the numerical offsets at Sections 2 and 6 ($\delta_{26,b} = 18.4$ mm ($\frac{1}{4}''$) and $\delta_{6b} = 20.03$ mm ($\frac{1}{16}''$)) and sections 3 and 5 ($\delta_{36,b} = 41.1$ mm ($\frac{11}{16}''$) and $\delta_{56,b} = 37.51$ mm ($\frac{1}{2}''$)) is $9\%$. Based on measured offsets, the difference is more noticeable at Sections 2 and 6 ($\delta_{26,b} = 25.4$ mm ($1''$) compared to $\delta_{6b} = 20.6$ mm ($\frac{13}{16}''$), a difference of $20\%)$ and nil at Sections 3 and 5 ($\delta_{36,b}$ and $\delta_{56,b}$ are both equal to $38.1$ mm ($\frac{11}{2}''$)).
Deformed shapes of the top and bottom flanges due to bending at Section 6:
(a) bending load applied at Section 6 at the bottom flange level ($P_{6b}$) and (b) bending load applied at Section 6 at the top flange level ($P_{6t}$). The deformed shape based on measured offsets is included in Figure 10(b) for comparisons although it is based on loading the bottom flange ($P_{6b}$).

Differential lateral offsets ($\Delta \delta_{ij}$) and rotations ($\Delta \theta_{ij}$) between the top and bottom flanges after bending Section 6 of the top flange ($P_{6t}$).
5.3 Discussion

At first glance, asymmetry in the deformed shapes (both measured and numerical) would be attributed to the fact that the initial bending load at Section 2 is smaller \((P_{2,b}) = 670\, \text{kN} \, (150 \, \text{kips})\) compared to \(800\, \text{kN} \, (180 \, \text{kips})\) for the remaining sections. However, that is not the case as shown later (Section 6.3) when the bending load is increased. In any case, the asymmetry noted in the deformed shapes based on numerical offsets (9%) and measured offsets (20% between Sections 2 and 6) is not of particular importance.
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importance for steel bridge girders as the required curved shape of the structure can be easily smoothened during casting of the concrete deck slab.

Figure 13 Deformed shapes based on changing the loading sequence

![Deformed shapes based on changing the loading sequence](image1)

Figure 14 Deformed shapes for stiffened and unstiffened cases – final load applied at Section 6 at the top flange level ($P_{6,t}$)

![Deformed shapes for stiffened and unstiffened cases – final load applied at Section 6 at the top flange level ($P_{6,t}$)](image2)

On the other hand, the large discrepancy between measured and calculated offsets at Section 2 in the initial bending operation is probably caused by inaccuracy in the tape measurements at Section 2 as no LVDT’s were used and a small value was recorded (3.2 mm (1/8″)). In any case, this discrepancy is not critical, especially that it greatly reduced after bending the subsequent sections (as discussed above).

In conclusion, based on the correlations obtained between measured and numerical offsets, the finite element model is used to investigate three-dimensional effects that were not provided by the test. It should be noted that the test girder may have had initial residual stresses which would affect its structural behaviour during bending. In absence of such data, these were not considered in the analysis.
6 Three-dimensional effects

The finite element model is used to monitor three-dimensional effects which consist of the top and bottom flange deformations, maximum plastic strains and residual stresses that build-up after bending. Such information was not available in the test data but is important for the design of the curved girder.

6.1 Top and bottom flange deformation

The residual offsets from the finite element analysis were plotted in Figure 8(a) for the bottom flange and Figure 8(b) for the top flange after each load application. The final deformed shapes that develop after bending Section 6 at the bottom flange level ($P_{6,b}$) and top flange level ($P_{6,t}$) are also plotted in Figure 10(a) and (b), respectively (the deformed shape based on measured offsets of the bottom flange is shown for comparisons as Figures 10a and b both relate to bottom flange deformations). All offsets for the top and bottom flange are shown with respect to a straight-line that connects Sections 1 and 7 at the bottom flange level. This explains why lateral offsets at the top flange level in Figure 10 (and Figures 11–15 later) are not zeros at Sections 1 and 7.

Figure 10(a) shows that the deformed shape of the top flange that develops after bending Section 6 of the bottom flange ($P_{6,b}$) is not symmetrical (the maximum offset occurs at Section 3 ($\delta_{36}^t$) = 51 mm (2″)) and that the top flange deformed shape becomes more symmetrical (as for the bottom flange) after bending Section 6 of the top flange ($P_{6,t}$) as shown in Figure 10(b). Moreover, the final maximum residual offset is larger in the top flange ($\delta_{64}^t$ at Section 4 of the top flange is equal to 61.6 mm (27/16″) compared to ($\delta_{64}^b$) = 48.5 mm (115/16″) in the bottom flange, Figure 10(b)). It is also noted that the top flange bending at Section 6 ($P_{6,t}$) increases the bottom flange residual offset only by 4% (from 47.05 mm (17/8″), Figure 10(a) to 48.5 mm (115/16″), Figure 10(b)).

The relative displacements and rotations of the top flange and bottom flange based on residual offsets and rotations that developed after completing the last bending operation (when the load is applied to Section 6 of the top flange) are shown in Figure 11 for Sections 1–7 (labelled as $\Delta\delta_{ij}$ for displacements and $\Delta\theta_{ij}$ for rotations). Figure 11 shows
that the maximum relative movements occur at Section 2 ($\Delta \delta(26) = 25.45 \text{ mm (}1''\text{), }\Delta \theta(26) = 0.0189 \text{ rad (}1.1''\text{)}$) although the maximum offsets develop at Section 4.

6.2 Summary

Based on comparisons of the top flange and bottom flange lateral offsets, two conclusions can be drawn:

The top flange offsets are larger compared to the bottom flange (the maximum offset at Section 4 is larger by 13.1 mm ($1\frac{5}{16}''$), Figure 10(b)). This is attributed to the fact that only the bottom flange is supported (clamped) at its ends during bending (Figures 4b, 7) and the 1.11 m ($43\frac{3}{4}''$) deep web provides some flexibility for the unsupported top flange during bending.

Asymmetry in the final deformed shape is mostly noted in Section 2 of the top flange, that is ($\delta_{26t}$) is 41.5 mm ($1\frac{5}{8}''$) compared to ($\delta_{66b}$) = 33 mm ($1\frac{3}{16}''$) at Section 6, a difference of 20% compared to 9% for the bottom flange, while the offset at Section 3 ($\delta_{36b}$) is 58 mm ($2\frac{1}{8}''$) compared to 53 mm ($2\frac{1}{16}''$) at Section 5 ($\delta_{56b}$), a difference of 9% same as for the bottom flange.

While the results presented so far are based on the actual loading sequence/magnitude used in the test, further investigations are conducted using the finite element model to show the impact of changing some parameters on the deformed shapes. The first trial consists of increasing the initial bending load at Section 2 ($P_{2-b}$) from 670 kN (150 kips) to 800 kN (180 kips) as for Sections 3–6. Then, the effects of changing the loading sequence and end support conditions are investigated.

6.3 Effect of increasing the initial bending load ($P_{2-b}$)

The finite element analysis is performed for the case where the initial bending load at Section 2 of the bottom flange ($P_{2-b}$) is taken as 800 kN (180 kips) instead of 670 kN (150 kips) (as for the remaining bottom flange sections). The final deformed shapes of the bottom flange and top flange are shown in Figures 12(a) and (b), respectively and compared with the deformed shapes from the finite element model based on the actual load $P_{2-b} = 670$ kN (150 kips). Figure 12(a) shows the final deformed shapes of the bottom flange after bending Section 6 of the bottom ($P_{6-b}$) (that is the last bending application for the bottom flange) and top flange ($P_{6-t}$) (that is the last bending application for the girder) while Figure 12(b) shows the final deformed shapes of the top flange after bending Section 6 of the top flange only ($P_{6-t}$).

Figure 12(a) shows that using a bottom flange bending load ($P_{2-b}$) equal to 800 kN (180 kips) increases the offset at Section 2 of the bottom flange ($\delta_{26b}$) from 18.4 mm ($1\frac{3}{4}''$) to around 22.5 mm ($1\frac{7}{8}''$) while it insignificantly reduces the offset at Section 6 ($\delta_{66b}$) from 20.03 mm ($1\frac{3}{16}''$) to around 19.8 mm ($1\frac{3}{16}''$). Its effect on the offsets at the remaining sections ($\delta_{36b}$, $\delta_{46b}$ and $\delta_{56b}$) of the bottom flange is also minor. On the other hand, Figure 12(b) shows that the deformed shape of the top flange becomes more asymmetrical as the offset at Section 2 ($\delta_{26t}$), increases from 41.5 mm ($1\frac{5}{8}''$) to around 46 mm ($1\frac{3}{16}''$) while the offset at Section 6 ($\delta_{66t}$) does not change (around 33 mm ($1\frac{3}{16}''$)). The effect of the load increase on the offsets at the remaining sections (3, 4 and 5) is very minor (for example the offset at Section 4 ($\delta_{46t}$) is slightly increased from 59 mm ($2\frac{5}{8}''$) to around 60.5 mm ($2\frac{7}{8}''$)).
In conclusion, increasing the bending load at Section 2 was not in favour of the deformed shape as it caused more non-symmetry especially in the top flange. This justifies why a smaller initial bending load at Section 2 of the bottom flange was recommended by the fabricator. In absence of precise analytical procedures, the magnitude of this reduced load relied on trial and error in the steel shop and correction loads were used at the end of the bending operation to make up for excessive offsets.

6.4 Changing the loading sequence

In this section, the effect of changing the loading sequence on the deformed shapes is investigated. The loading sequence is made symmetrical with respect to mid-span (e.g., Section 4), starting at Section 4 of the bottom flange \(P_{4b}\), then moved to Sections 4–5 of the top flange \(P_{4t}, P_{5t}, P_{6t}\), then to Sections 3, 5, 2 and 6 of the bottom flange \(P_{3b}, P_{5b}, P_{2b}, P_{6b}\) and finally to Sections 2 and 6 of the top flange \(P_{2t}, P_{6t}\).

Deformed shapes from this trial are presented in Figure 13 for the bottom flange and top flange for the cases where the initial bending load at Section 2 of the bottom flange \(P_{2b}\) is 670 kN (150 kips) (as used in the test) and 800 kN (180 kips) (in order to examine its effects in a symmetrical loading sequence). In either case, it is shown that the deformed shapes for the top and bottom flanges become more symmetrical (Figure 13) compared to the case of the actual loading sequence (from left to right starting at Section 2) and that the increased bending load at Section 2 \(P_{2b}\) made the deformed shape for the top and bottom flange even more symmetrical (contrarily to the case of the actual loading sequence from left to right). On the other hand, the lateral offsets that build-up are now larger, that is the offset at Section 4 of the top flange \((\delta_{4t})\), increases from 61.6 mm (2\(\frac{7}{16}\)″) to 81 mm (3\(\frac{3}{16}\)″) in the top flange and from 48.5 mm (1\(\frac{13}{16}\)″) to 71 mm (2\(\frac{1}{16}\)″) in the bottom flange \((\delta_{4b})\) for the case where \(P_{2b}\) is 670 kN (150 kips). If \(P_{2b}\) is set at 800 kN (180 kips), these offsets are even larger \((\delta_{4t}) = 88.5 \text{ mm (3\(\frac{1}{2}\)″)}\) and \((\delta_{4b}) = 80.3 \text{ mm (3\(\frac{3}{16}\)″)}\).

In summary, the symmetrical loading sequence results in symmetrical deformed shapes but on the other side it substantially increases the lateral offsets. For steel fabricators, the test loading sequence used in the test (from left to right alternating between the bottom and top flange) is more practical as:

- it makes it easier to monitor the progress of offsets (Gergess and Sen, 2005a) and make-up for errors in the initial bending stages
- the magnitude of the bending loads is predictable as it is set based on the plastic moment capacity of the steel flange sections.

In the symmetrical loading sequence, bending loads should be reduced to induce comparable offsets and in the absence of a detailed analytical procedure, the actual loading sequence remains the preferred practice (note that the symmetrical loading sequence will be thoroughly investigated by the authors and results will be presented in a future publication).
6.5 Changing the end supports

This section shows the impact of changing the end support conditions on the deformed shape of the girder. Based on the test girder data, the end supports were idealised as two pins spaced at 5 cm (2") at the side of the bottom flange opposite to where the bending loads are applied (that was actually the case for the test girder). As a first trial, these pin supports were moved to the side of the bending loads. This condition led to reducing the lateral offsets in the top and bottom flange extensively (the maximum lateral offset at Section 4 decreases from 61.6 mm (2 7/16") to 50 mm (2") in the top flange ($\delta_{46}$) and from 48.5 mm (11 5/16") to 27 mm (1 11/16") in the bottom flange ($\delta_{46}$)). Moreover, asymmetry in the deformed shape was more noticeable in this case.

In the second trial, clamps were idealised by one pin (Figure 4(a)) and various locations of the pins were investigated starting in the bottom flange opposite to the side of the bending load (labeled as support 1 in Figure 7) and moving to the side where the load is applied (labeled as supports 2 and 3 in Figure 7). Numerical results show that one pin support increases the differential movements between the top and bottom flange and induces more asymmetry in the deformed shape, especially in the top flange. Moreover, lateral offsets increase substantially (especially for the case where the pin is at the opposite side of the load e.g., support 1 in Figure 7).

In conclusion, modelling each clamp by two pins spaced at 5 cm (2") at each end is the preferred choice as it gave closer correlations between the test girder measured and numerical offsets.

6.6 Effect of stiffeners

Finally, the effect of stiffeners (Figure 5) on the bending operation was investigated by removing the stiffeners in the finite element model. Stiffeners were welded to one side of the web at two symmetrically located points from mid-span (Figure 3). The final deformed shapes of the top flange and bottom flange of the un-stiffened girder are shown in Figure 14 for the case where the final bending load is applied at Section 6 of the top flange ($P_{6t}$). It is shown that when stiffeners are removed, lateral offsets increase insignificantly by few millimetres (around 2 mm).

7 Residual stresses and maximum strains

Residual stresses and plastic strains that develop from the bending operation are finally presented. Residual stresses are flexural stresses that develop due to bending along the longitudinal axis of the girder, which should be quantified and incorporated in the design of the curved steel girder. Maximum strains correspond to the plastic strains that develop during bending which are also presented to verify where they fit in the steel stress-strain curve (Figure 2).

7.1 Residual stresses

The maximum residual stresses that build-in the girder after removal of the bending loads are shown in Figure 15. The stress distributions are provided in Sections 2–6 for the final loading case after bending Section 6 of the top flange ($P_{6t}$). Figure 15 shows that the
maximum residual stresses develop at the loaded side of the bent flange and that these stresses are tensile at the flange tips. Stresses vary from 116 MPa (16.8 ksi) in tension (that is 0.29 times the yield stress $F_y$) at the loaded side of Section 4 to 198 MPa (28.7 ksi) in compression (0.49$F_y$) at the clamps’ side of Section 3.

For completeness, stress distributions near the end supports are shown in Figure 16. The maximum stresses are noted in the bottom flange near the left end support (123 MPa (17.8 ksi) e.g., $0.3F_y$, in tension and 217 MPa (31.5 ksi) e.g., $0.54F_y$ in compression). At the right end support, stresses reduce to 70.5 MPa (10.2 ksi) e.g., $0.17F_y$ in tension and 85.6 MPa (12.4 ksi) e.g., $0.21F_y$ in compression). Figure 16 Maximum residual stresses (MPa) in end support regions (see online version for colours)

In summary, residual stresses that build up from cold bending are of large magnitude and if they are not released by heat treatment, they should be included in the service load design of the curved steel girder as they will affect the initiation of yield at the flange tips. However, they can be neglected in the fatigue check as they are already incorporated in the fatigue stress range specified (AASHTO, 2010b).

It should be noted that residual stresses are in a state of self-equilibrium and therefore they will not impact the magnitude of the bending load that is calculated based on the plastic moment capacity of the section (Gergess and Sen, 2005a). The effects of residual stresses on structural steel girders bent about their major axis may be found elsewhere (Mansouri et al., 2004).

7.2 Maximum strains

The maximum strains that develop during bending are shown in Figure 17. Strain distributions are presented for the case when the bending load is applied at Section 6 of
the bottom flange ($P_{b,b}$) (Figure 17(a)) and top flange ($P_{b,t}$) (Figure 17(c)). Residual strains that remain after removing these loads are shown in Figure 17(b) ($P_{b,b}$) and Figure 17(d) ($P_{b,t}$) respectively.

**Figure 17** Flexural strains (mm/mm) due to bending: (a) $P_{b,b}$ (bending); (b) $P_{b,b}$ (residual); (c) $P_{b,t}$ (bending) and (d) $P_{b,t}$ (residual) (see online version for colours)

The maximum strain that develops in the bottom flange during bending of the bottom flange is 0.00413 mm/mm (that is two times the yield strain ($\varepsilon_{\text{yield}}$), Figure 17(a)). The maximum strain that develops in the top flange during bending of the top flange is 0.00674 mm/mm (3.33 × $\varepsilon_{\text{yield}}$, Figure 17(c)). The residual strains that remain in the section after removal of the bending loads are 0.00149 mm/mm for the bottom flange (Figure 17(b), that is 0.74 × $\varepsilon_{\text{yield}}$) and 0.00426 mm/mm for the top flange (Figure 17(d), that is 2.1 × $\varepsilon_{\text{yield}}$). The difference between the top flange and bottom flange residual strains is attributed to the fact that the top flange maximum lateral offset at Section 4 is larger compared to the bottom flange. In all cases, the yield stress is smaller than 10 × $\varepsilon_{\text{yield}}$ (that is the limiting strain for the flat yield plateau, Figure 2) and the loss in strain during unloading is around 1.25 × $\varepsilon_{\text{yield}}$. Also, the maximum strains are much smaller than the 3% limit (Jayadevan et al., 2004), a typical value used by fabricators before steel properties are reduced (Figure 2).

### 8 Conclusion

This paper presents results from three-dimensional finite element modelling of a steel plate test girder that was curved using a proprietary cold bending system. The commercially available finite element program MSC SimXpert was used for this purpose as it has the ability to account for both material and geometric non-linearity. The steel girder flanges, web and stiffeners were modelled using four-noded isoparametric shell elements with in-plane bending stiffness. End supports (clamps) were idealised by two pins spaced at 50 mm (2") at each end of the bottom flange on the opposite side of the bending load. The validity of the finite element model was verified by comparing numerical and measured offsets at the bottom flange level (as those were the only available results from the test).
The finite element model was then used to predict offsets, residual stresses and plastic strains in the bottom and top flange which directly affect the structural performance of the girder in service (Keating and Christian, 2009). Analysis showed that the top and bottom flange permanent deformations are not necessarily equal (they are larger in the unsupported top flange) and that the deformed shapes are somehow asymmetrical. It was also indicated that the maximum strains were within the flat yield plateau of the stress-strain curve (Chajes et al., 1963) and that residual stresses were of large magnitude. The maximum residual stresses were noted in the bottom flange near the end supports as they varied from 123 MPa (17.8 ksi) in tension ($0.3F_y$) to 217 MPa (31.5 ksi) in compression ($0.54F_y$) at the flange tips.

The implications of changing the bending loads magnitude/sequence were also examined as they can greatly affect the deformed curved shape of the steel girder. It was confirmed (based on comparisons with the test data) that decreasing the magnitude of the initial bending load at the bottom flange level to around 85\% of the plastic load limit was necessary to control the deformed shape that develops based on the induced offsets. It was also shown that a symmetrical loading sequence reduces asymmetry in the deformed shapes but significantly increases the induced offsets.

The effect of stiffeners in stiffened girders on the bending operation was finally investigated. Analysis indicated that stiffeners have minimal effects on the deformed shape and induced offsets.

Despite the lack of more precise test data for this cold bending procedure, results presented in this paper provide indications of cold bending residual stresses and plastic strains that affect the design of the curved girder. Further refinements of the cold bending operation and parametric studies are planned to optimise the loading configuration and reduce trial and error.

References


Nomenclature

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<tr>
<td>d</td>
<td>Web depth</td>
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