The Galerkin least squares MLPG method for convection-dominated problems

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Abstract: The meshless method has been widely applied in the computational mechanics, materials science, computational heat transfer and fluid flow. However, the development of a high efficient method for convection term in the computational fluid dynamics is difficult. In this paper, a truly meshless method of the meshless local Petrov-Galerkin (MLPG) method was applied to solve convection-diffusion problem. Meanwhile, the Galerkin least squares (GLS) approximation method was proposed to overcome the influence of convection term. The accuracy and efficiency of the present method were validated by some cases with benchmark solutions. The computational results showed that the GLS method could be used in dealing with convection-diffusion problems with high computational precision.

Keywords: upwind scheme; GLS method; SUPG method; MLPG method; meshless method.

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1 Introduction

FVM or FEM method depends closely on the meshes or elements and has the inherent limitations in case of large deformations, impact, penetration, and flow pass obstacles. Therefore, the meshless method has gained increasing attention in the past decades. It is mainly applied in computational mechanics in the early stage. In recent years, some researchers have applied this method to compute the fluid flow and heat transfer problems (Tao et al., 2010). As a truly meshless method, the meshless local Petrov-Galerkin method has been widely applied in computational mechanics (Dai et al., 2017; Moghaddam and Baradaran, 2017; Sladek et al., 2013b), fluid and heat transfer problems (Rahmat et al., 2018; Sun et al., 2018; Sinnott et al., 2015; Wu and Tao, 2008; Li et al., 2011). However, its utilisation in computational fluid flow is rare considering the computational time and oscillatory behaviour produced by the convection term and the pressure term. Some various applications of the Meshless local Petrov-Galerkin (MLPG) method were introduced by Sladek et al. (2013a).

To avoid oscillatory behaviour and numerical diffusion encountering in convection-dominated flows, some upwind techniques for convection term were adopted in the MLPG method. Wu et al. (2012a) applied the MLPG method, with no upwind schemes, to compute convection-diffusion problems, the results showed that the false diffusion occurred for high Re numbers. Lin and Atluri (2000) proposed two kinds of upwind schemes of the MLPG method to solve convection-diffusion and incompressible flow problems (Lin and Atluri, 2001). However, the proposed upwind scheme was proved to be incapable of obtaining rapid convergence in the cases of high Re numbers. Arefmanesh et al. (2008) applied the MLPG method to compute non-isothermal fluid flow problem with vorticity-stream function method. Mohammadi (2008) developed a new upwind scheme based on the meshless finite volume method to calculate incompressible flow problems with vorticity-stream function formulation. In this meshless method, the Heaviside step function was used as test functions. Avila and Atluri (2009) applied MLPG method to compute the unsteady state incompressible fluid flow problem. An upwind scheme of the MLPG method was developed by Wu et al. (2010) and applied in computing incompressible flow and convection-diffusion problems (Wu et al., 2012b, 2012c; Chen et al., 2018a, 2018b; Li et al., 2019). Najafi et al. (2012) applied the MLPG method to compute incompressible laminar fluid flow, using the local weak form of the Navier-Stokes equations obtained based on the framework of the characteristic-based split (CBS) algorithm. Nikfar and Mahmoodi applied the MLPG method to solve the problem of laminar natural convection of nanofluid in a cavity with wavy side walls (Nikfar and Mahmoodi, 2012). In their computation, the governing equations were determined by the stream function–vorticity formulation. Arefmanesh et al. (2013) applied the MLPG method to simulate the buoyancy-driven flow and heat transfer in a differentially-heated enclosure with a baffle attached to its higher temperature side wall. They also applied the stream function-vorticity formulation. CBS scheme was applied by Enjilela et al. (2016) to overcome the stability of the MLPG-VF-based method and to solve the high Reynolds and Rayleigh numbers problems. Thamareerat et al. (2016) used the moving Kriging interpolation method based on the MLPG method to solve the time fractional Navier-Stokes equations.

The purpose of present paper was to solve convection-dominated problems using the MLPG method. The Galerkin least squares (GLS) was developed and applied in the MLPG method to overcome oscillations produced by convection term. Three cases were considered to verify the stability and accuracy of the present method.

2 MLPG/GLS for convection-diffusion

The dimensionless two-dimensional convection-diffusion equation and boundary conditions in the Cartesian coordinate system are as follows:

$$u_j \frac{\partial T}{\partial x_j} = \frac{1}{Pe}\frac{\partial^2 T}{\partial x_j{}^2} + \dot{\Phi} \ (j = 1,2) \text{ in } \Omega \tag{1}$$

where the Peclet number is defined as following:

$$Pe = \frac{u_{ref} L_{ref}}{a} \tag{2}$$

The Dirichlet boundary condition is:

$$T = \overline{T}_1 \ on\ \Gamma_u. \tag{3}$$

The Neumann boundary condition is:

$$-\lambda \frac{\partial T}{\partial x_j} n_j = \overline{q}_1 \ \text{ on } \ \Gamma_t. \tag{4}$$

where T represents the temperature, $\overline{T}_1$ is the given temperature, a is the thermal diffusive coefficient, λ is the thermal conductivity, $\dot{\Phi}$ is the source term, n_j is the outward unit normal vector to $\Gamma, \overline{q}$ is the given heat flux, u_j is the velocity, Γ_t and Γ_u are subsets of Γ satisfying $\Gamma_t \cap \Gamma_u = \varnothing$ and $\Gamma_t \cup \Gamma_u = \Gamma$.

Generally the MLPG method integrates in a local sub-domain Ω_x, the weighted integral form of equation (1) is given as:

$$\int_{\Omega_\mathbf{x}} \left(u_j \frac{\partial T}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 T}{\partial x_j^2} - \dot{\Phi} \right) v d\Omega_\mathbf{x} = 0 \tag{5}$$

Similarly, integral form of GLS method in a local sub-domain Ω_x is given as:

$$\int_{\Omega_\mathbf{x}} \left[u_j \frac{\partial v}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 v}{\partial x^2{}_j} \right] \left[u_j \frac{\partial T}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 T}{\partial x_j^2} - \dot{\Phi} \right] d\Omega_\mathbf{x} = 0 \tag{6}$$

So, integral form of MLPG/GLS method can be expressed as follows:

$$\int_{\Omega_\mathbf{x}} \begin{matrix} \left\{ v + \tau \left[u_j \frac{\partial v}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 v}{\partial x_j^2} \right] \right\} \\ \left[u_j \frac{\partial T}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 T}{\partial x_j^2} - \dot{\Phi} \right] \end{matrix} d\Omega_\mathbf{x} = 0 \tag{7}$$

By using the Gauss theorem in equation (7), we can obtain the following local weak formulation equation:

$$\begin{aligned} &\int_{\Omega_\mathbf{x}} \left(u_j \frac{\partial T}{\partial x_j} v + \frac{1}{Pe}\frac{\partial T}{\partial x_j}\frac{\partial v}{\partial x_j} - \dot{\Phi} v \right) d\Omega \\ &+\int_{\Omega_\mathbf{x}} \left\{ \begin{matrix} \tau[u_j \frac{\partial v}{\partial x_j} u_j \frac{\partial T}{\partial x_j} - \frac{1}{Pe} u_j \frac{\partial v}{\partial x_j}\frac{\partial^2 T}{\partial x_j^2} \\ -\frac{1}{Pe}\frac{\partial^2 v}{\partial x_j^2} u_j \frac{\partial T}{\partial x_j} + \frac{1}{Pe^2}\frac{\partial^2 v}{\partial x^2{}_j}\frac{\partial^2 T}{\partial x_j^2} \end{matrix} \right\} d\Omega \\ &-\int_{\Omega_\mathbf{x}} \left[\tau \left(u_j \frac{\partial v}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 v}{\partial x_j^2} \right) \dot{\Phi} \right] d\Omega - \\ &-\int_{\Gamma_I} \frac{1}{Pe}\frac{\partial \phi}{\partial x_j} n_j v d\Gamma \\ &-\int_{\Gamma_u} \frac{1}{Pe}\frac{\partial \phi}{\partial x_j} n_j v d\Gamma - \int_{\Gamma_t} \frac{1}{\lambda Pe} \overline{q} v_I d\Gamma = 0 \end{aligned} \tag{8}$$

where Γ_I is the part of sub-domain boundary included in the global domain.

To obtain the discretised equation of each sub-domain, the unknown function can be approximated using the moving least square (MLS) method (Atluri, 2004):

$$T^h(\mathbf{x}) = \mathbf{\Phi}^T(\mathbf{x})\cdot\mathbf{T} = \sum_{I=1}^{N}\Phi_I(\mathbf{x})\hat{T}_I \qquad x\in\Omega_{\mathbf{X}} \tag{9}$$

where $\hat{T}$ represents the fictitious nodal value, but not the value of the unknown function. The characteristic of MLS has been discussed widely in literatures (Atluri and Zhu, 1998; Jin et al., 2001) and will not be restated herein. Substituting equation (9) into equation (8), we can obtain the following discretised system of linear equations:

$$\begin{aligned}&\int_{\Omega_x}\left(u_j\frac{\partial\Phi^J\hat{T}^J}{\partial x_j}v_I+\frac{1}{Pe}\frac{\partial\Phi^J\hat{T}^J}{\partial x_j}\frac{\partial v_I}{\partial x_j}-\dot{\Phi}v_I\right)d\Omega\\&+\int_{\Omega_x}\left\{\tau\begin{bmatrix}u_j\frac{\partial v_I}{\partial x_j}u_j\frac{\partial\Phi^J\hat{T}^J}{\partial x_j}-\frac{1}{Pe}u_j\frac{\partial v_I}{\partial x_j}\frac{\partial^2\Phi^J\hat{T}^J}{\partial x_j^2}\\-\frac{1}{Pe}\frac{\partial^2 v_I}{\partial x_j^2}u_j\frac{\partial\Phi^J\hat{T}^J}{\partial x_j}+\frac{1}{Pe^2}\frac{\partial^2 v_I}{\partial x_j^2}\frac{\partial^2\Phi^J\hat{T}^J}{\partial x_j^2}\end{bmatrix}\right\}\\&-\int_{\Omega_x}\left[\tau\left(u_j\frac{\partial v_I}{\partial x_j}-\frac{1}{Pe}\frac{\partial^2 v_I}{\partial x_j^2}\right)\dot{\Phi}\right]d\Omega\\&-\int_{\Gamma}\frac{1}{Pe}\frac{\partial\Phi^J\hat{T}^J}{\partial x_j}n_jv_Id\Gamma=0\end{aligned}\tag{10}$$

where M is the total number of nodes in the entire domain Ω. Equation (16) can be rewritten as:

$$\mathrm{K}\cdot\hat{\mathrm{T}}=\mathrm{F} \tag{11}$$

K and F are the global stiffness matrix and the global vector, respectively. Which are defined as:

$$\begin{aligned}K_{IJ}=&\int_{\Omega_x}\left(u_j\frac{\partial\mathbf{\Phi}^J}{\partial x_j}v_I+\frac{1}{Pe}\frac{\partial\mathbf{\Phi}^J}{\partial x_j}\frac{\partial v_I}{\partial x_j}\right)d\Omega\\&-\int_{\Gamma_I}\frac{1}{Pe}\frac{\partial\mathbf{\Phi}^J}{\partial x_j}n_jv_Id\Gamma\\&+\int_{\Omega_x}\left\{\tau\begin{bmatrix}u_j\frac{\partial v_I}{\partial x_j}u_j\frac{\partial\mathbf{\Phi}^J}{\partial x_j}-\frac{1}{Pe}u_j\frac{\partial v_I}{\partial x_j}\frac{\partial^2\mathbf{\Phi}^J}{\partial x_j^2}\\-\frac{1}{Pe}\frac{\partial^2 v_I}{\partial x_j^2}u_j\frac{\partial\mathbf{\Phi}^J}{\partial x_j}+\frac{1}{Pe^2}\frac{\partial^2 v_I}{\partial x_j^2}\frac{\partial^2\mathbf{\Phi}^J}{\partial x_j^2}\end{bmatrix}\right\}d\Omega\end{aligned}\tag{12}$$

$$\begin{aligned}\mathrm{F_I}=&\int_{\Gamma_t}\frac{1}{\lambda Pe}\bar{q}_1v_Id\Gamma\\&+\int_{\Omega_x}\left[\tau\left(u_j\frac{\partial v_I}{\partial x_j}-\frac{1}{Pe}\frac{\partial^2 v_I}{\partial x_j^2}\right)\dot{\Phi}\right]d\Omega\end{aligned}\tag{13}$$

where the velocity is constant. Equation (11) obtains the fictitious nodal value $\hat{\mathrm{T}}$, and temperature can be obtained by equation (9).

3 Patch tests

In order to validate the accuracy and efficiency of the MLPG/GLS, the numerical results obtained by the present method are compared with that from the MLPG/SUPG method. In all numerical calculations, quadratic spline weight function is adopted in the meshless computation and the transformation method is applied to deal with the essential boundary conditions.

3.1 Thermal boundary layer problem

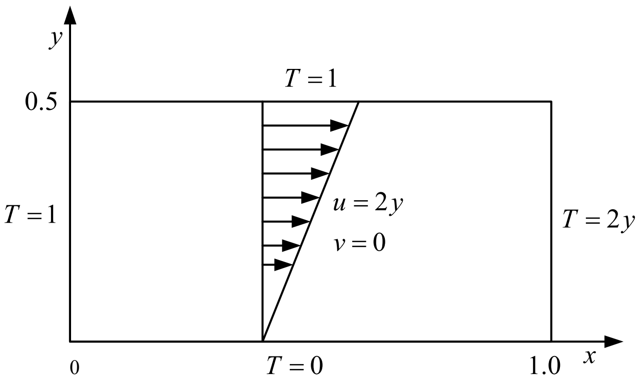
The thermal boundary layer problem proposed by Franca et al. (1992) is used to validate the stability and accuracy of scheme for the convection term. A statement of this problem is shown in Figure 1 and non-dimensional boundary conditions are defined as:

$$T=1 \qquad \begin{cases}x=0 & 0\le y\le 0.5\\ y=0.5 & 0\le x\le 1\end{cases} \tag{14}$$

$$T=0 \qquad y=0,\quad 0\le x\le 1.0 \tag{15}$$

$$T=2y \qquad x=1,\quad 0\le y\le 0.5 \tag{16}$$

Figure 1 Thermal boundary layer problems



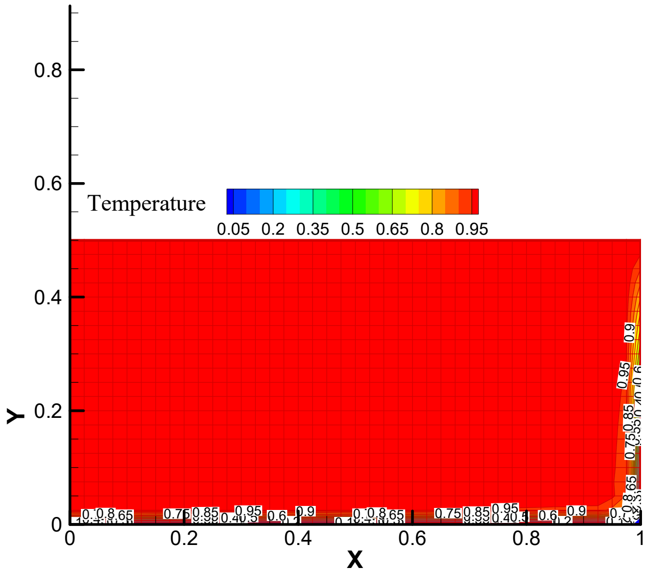
The velocity components are:

$$u=2y,\quad v=0 \tag{17}$$

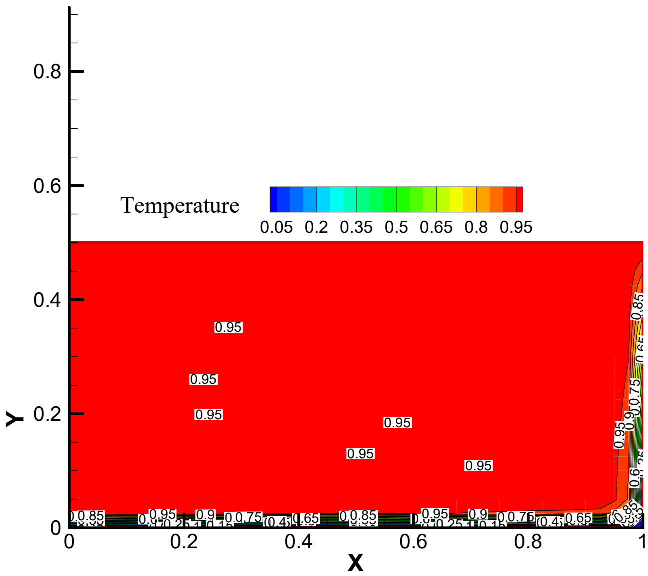
This problem can be described as a thermal boundary layer problem on a fully developed flow between parallel plates, where a uniform velocity field (equal to one) is given to the top plate and the bottom plate is fixed. In the Peclet number definition, the top plate velocity is taken as the characteristic flow velocity and the length of y-direction is taken as the characteristic length. We use a mesh consisting of 41 equally spaced nodes in the x-direction, 21 nodes uniformly distributed in the y-direction.

Figure 2 and Figure 3 give the temperature distribution of the MLPG/GLS and MLPG/SUPG methods at Pe = 10^6, respectively. From these figures, we can see that the results of the MLPG/GLS method are in good agreement with that of the MLPG/SUPG method, suggesting the good computational accuracy of the present method.

Figure 2 Temperature distribution at Pe = 10^6, (a) MLPG/GLS (b) MLPG/SUPG (see online version for colours)

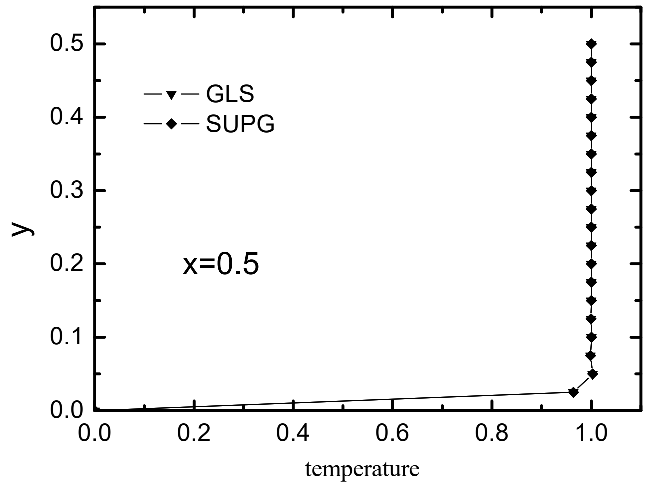


(a)

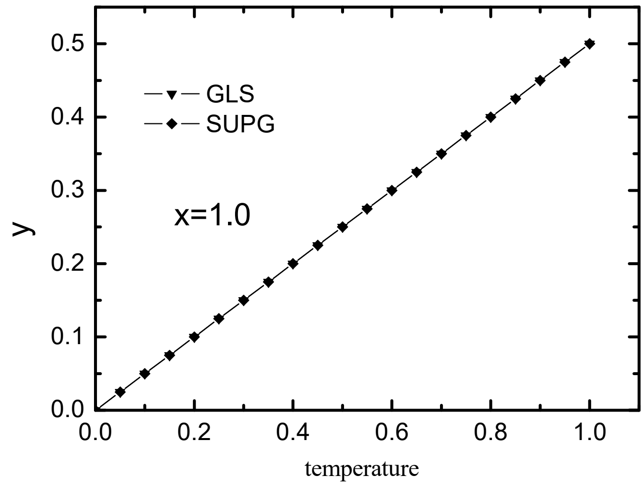


(b)

Figure 3 Local temperature distribution at x = 0.5, 1.0 under Pe = 10^6, (a) x = 0.5 (b) x= 1.0 (see online version for colours)



(a)



(b)

3.2 *Smith-Hutton problem*

Smith and Hutton (1982) proposed a convection-diffusion problem which is shown in Figure 4. The given velocity field and boundary conditions are provided below:

$$u = 2y\left(1-x^2\right), v = -2x\left(x-y^2\right) \tag{18}$$

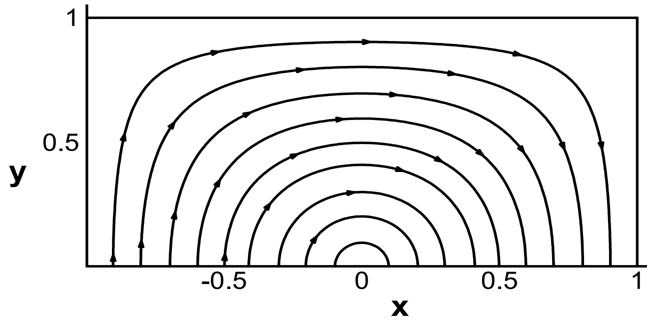
$$T_{in}(x) = 1 + \tanh\left[\alpha(1+2x)\right] \text{ on } -1 \le x \le 0, y = 0 \tag{19}$$

$$\frac{\partial T}{\partial y} = \bar{q} = 0 \text{ on } 0 \le x \le 1, y = 0 \tag{20}$$

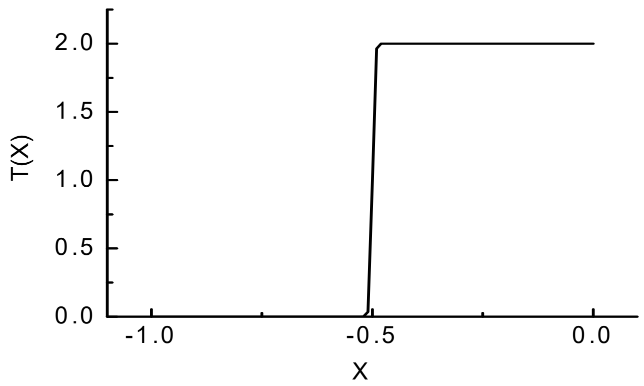
$$T = 0 \text{ on other boundaries} \tag{21}$$

where α is the sharp transition coefficient. The inlet temperature distribution is a stepwise profile, and the bigger the value of α, the sharper is the inlet temperature distribution. In the present paper, α = 100 is selected, representing a very sharp transition. In this computation, the outlet profile is studied

Figure 4 Simth-Hutton problem at $\alpha = 100$, (a) velocity field (b) Inlet temperature

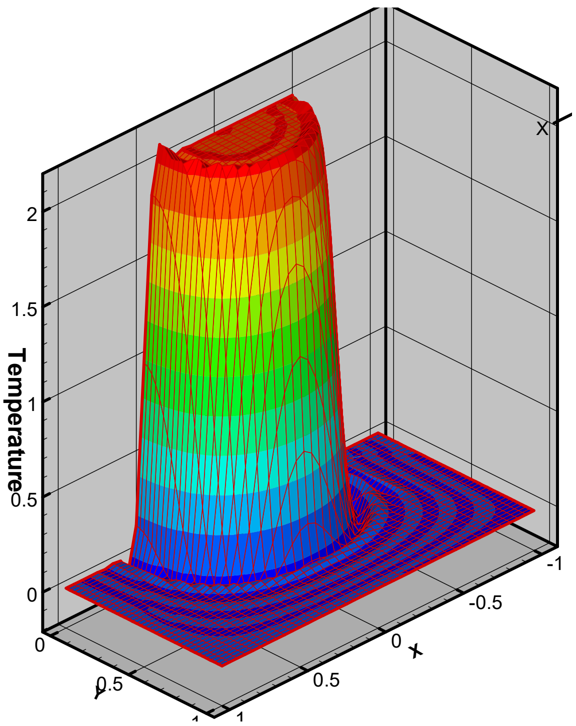


(a)



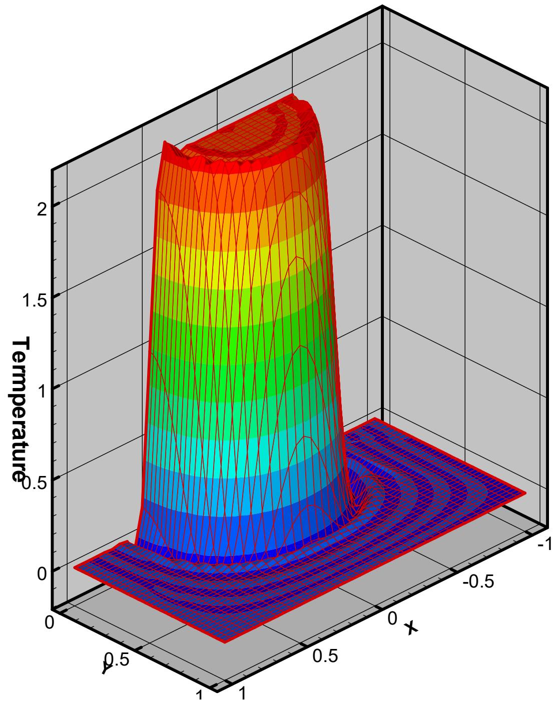
(b)

Figure 5 Temperature distribution at Pe = 10^6, (a) MLPG/GLS method (b) MLPG/SUPG method (see online version for colours)



(a)

Figure 5 Temperature distribution at Pe = 10^6, (a) MLPG/GLS method (b) MLPG/SUPG method (continued) (see online version for colours)



(b)

Figure 6 Outlet temperature distribution at Pe = 10^6 (see online version for colours)

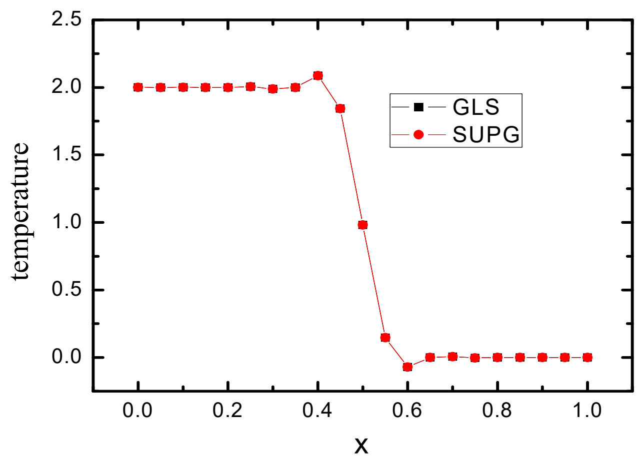


Figure 5 and Figure 6 give the temperature field and outlet temperature distribution of MLPG/GLS and MLPG/SUPG method at Pe=10^6. From these figures, we can see that the results of MLPG/GLS and MLPG/SUPG methods have produced false diffusion.

3.3 *Brezzi problem*

The Brezzi problem is suggested by Brezzi et al. (1988). The problem statement and the boundary conditions are depicted in Figure 7; the velocity field ($u = -y$, v= x) is shown in Figure 8.

Figure 7 Brezzi problem (see online version for colours)

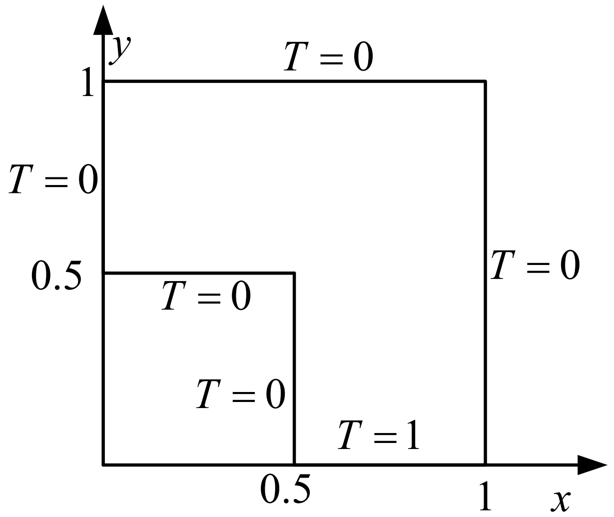
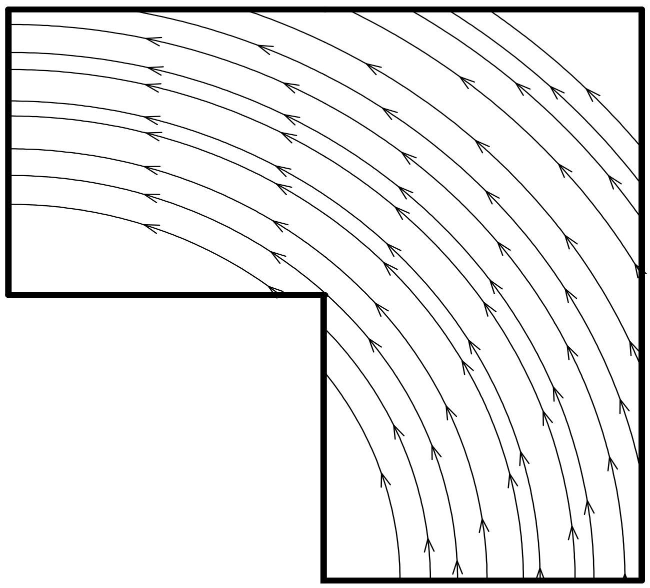


Figure 8 Velocity field



The boundary conditions are as:

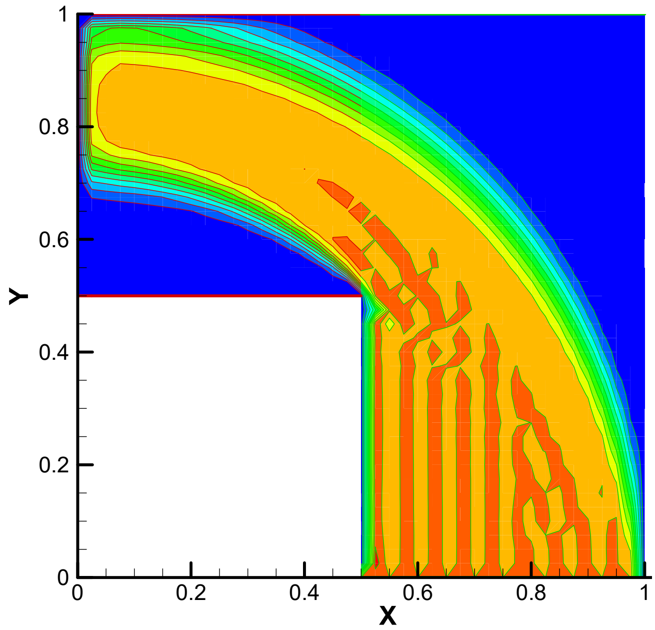
$$T = 0 \quad \begin{cases} x=0 & 0.5 \le y \le 1.0 \\ x=1 & 0.0 \le y \le 1.0 \\ x=0.5 & 0.0 \le y \le 0.5 \\ y=0.5 & 0.0 \le x \le 0.5 \\ y=1.0 & 0.0 \le x \le 1.0 \end{cases} \tag{22}$$

$$T = 1 \qquad y = 0, \quad 0.5 \le x \le 1.0 \tag{23}$$

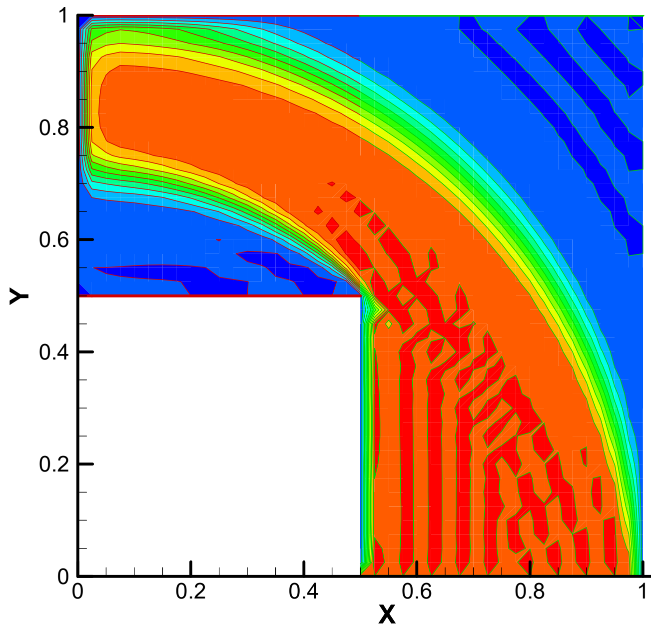
The velocity field is:

$$u = -y, \qquad v = x \tag{24}$$

Figure 9 Temperature field at Pe = 10^3, (a) MLPG/GLS (b) MLPG/SUPG (see online version for colours)



(a)



(b)

Figure 9 to Figure 12 show the temperature fields and outlet temperature distributions at Pe = 1,000 and 10^6, respectively. From these figures, it can be seen that these two methods give good results at low Pe number, but both of them produce slightly false diffusion at high Pe number. Therefore further studies are needed to improve the accuracy of present methods and decrease the false diffusion.

Figure 10 Outlet temperature distribution (Pe=10^3) (see online version for colours)

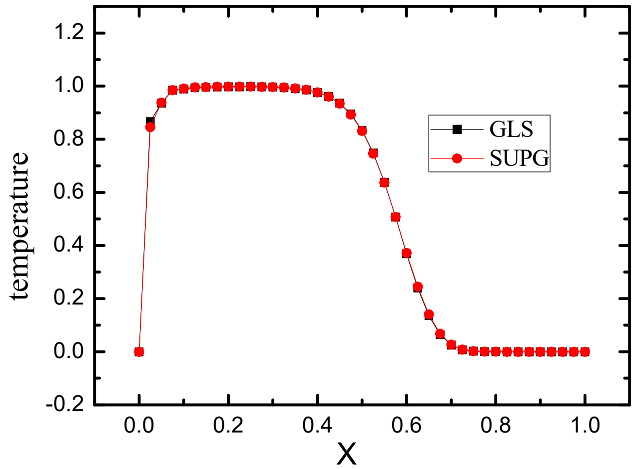
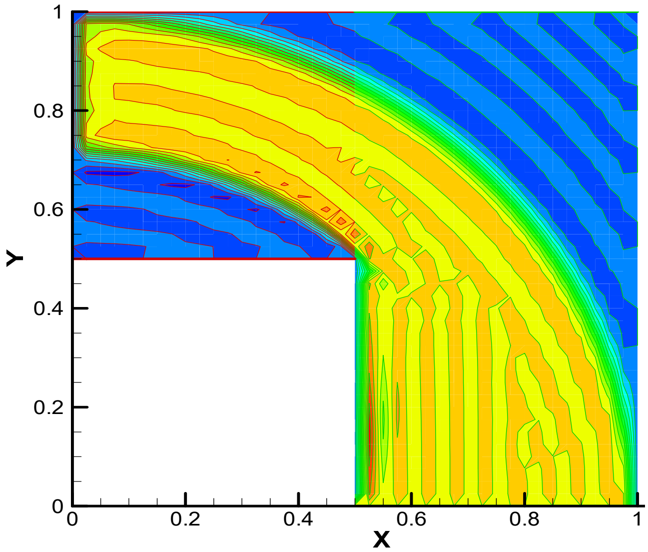
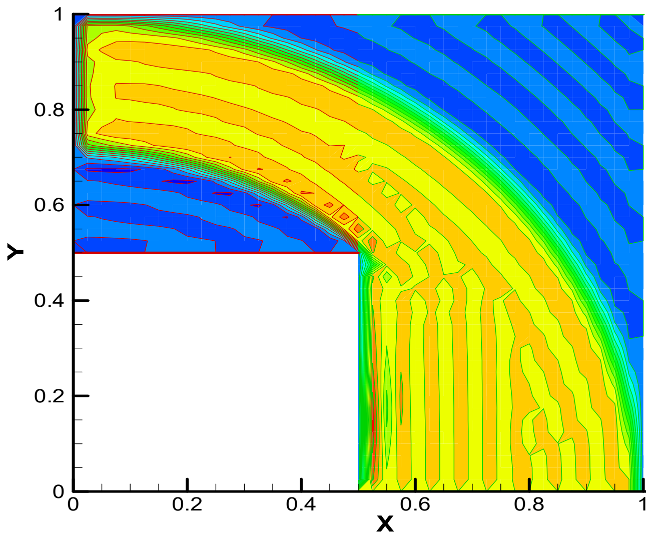


Figure 11 Temperature field at Pe = 10^6, (a) MLPG/GLS method (b) MLPG/SUPG method (see online version for colours)

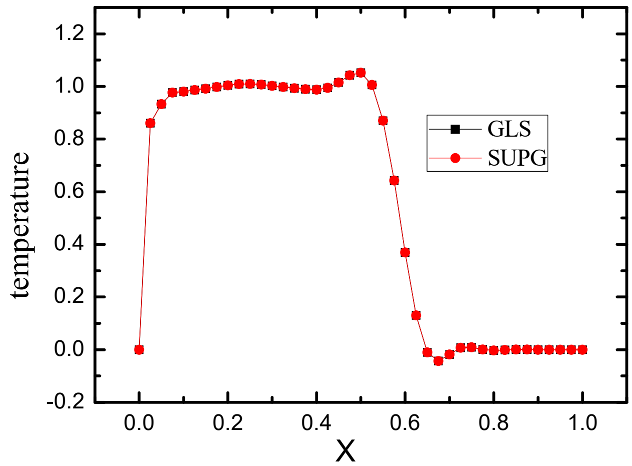


(a)



(b)

Figure 12 Outlet temperature distribution (Pe = 10^6) (see online version for colours)



4 Conclusions

In this paper, the GLS method was applied to deal with the convection term in the MLPG method. The results of the present method were compared with that of the MLPG/SUPG method to validate the efficiency and accuracy of the present method. The computational results showed that the present method can obtain good accuracy at low Pe number, but the employed two methods also produced false diffusion at high Pe number. Therefore further investigations are needed to construct a better upwind scheme for convection term.

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