
Optimal lot sizing policy with power demand and composed shortages under trade credits

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Abstract: This paper deals with an optimal ordering policy for non-deteriorating item with power demand under trade credits. The shortages are allowed and combination of backlogged and lost sales. Mathematical model is developed under two different situations, i.e., case 1: the permissible delay period is less than time to finish positive inventory and case 2 trade credit period is greater than or equal to time to finish positive inventory. Numerical examples are provided to illustrate the algorithm and theoretical results. The sensitivity analysis is provided on the optimal solution. The second order approximations are used for exponential terms to find the complexity on the optimal solution.

Keywords: inventory; composed shortages; stock-dependent demand; credit period; order quantity.

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1 Introduction and literature review

In modern days, an industry has to fulfil so any other objectives along with the basic objective of profit maximisation. Maximisation of sales, revenue growth of the industry, long run survival and others are some objectives for which a firm has to deal with cost is one factor which is directly related to production. Cost analysis helps in achieving optimal high quality production at low cost. The main goal of an industry is to achieve maximum profit. Price is the most important factor affecting the demand for a product. Demand for a product increases when items price falls. Yang (2014) established an EOQ model under stock-dependent demand and stock-dependent holding cost with relaxed terminal condition under shortages. Ghiami and Williams (2015) considered a single manufacturer multi-buyer model for a deteriorating product with finite production rate. Jiangtao et al. (2014) proposed a multi-product EOQ model for perishable item for stock-

dependent demand and two level trade credits. Sarkar et al. (2014) proposed an EMQ model for the selling price and the time dependent demand pattern in an imperfect production process. Wu et al. (2014) proposed an EOQ model for a seller by incorporating the facts:

- 1 deteriorating items not only deteriorate continuously but also have their maximum life time
- 2 credit period increases not only demand but also default risk. Qin et al. (2014) established an inventory model for quality dependent demand.

At present scenario, it is often seen that a buyer permits some grace period before settling the account. This process is beneficial for both vendor and buyer. The trade credit produces two benefits to the producer:

- 1 it motivates customers who want a discount
- 2 it may be applied as an alternative to a discount price.

Inventory models with trade credit were first developed by Goyal (1985). Teng et al. (2012) developed an EOQ model under trade credit financing with increasing demand. Khanra et al. (2011) established an inventory model for a deteriorating item having time induced demand when credit period is allowed. Sarkar (2012) presented a model in which demand rate and deterioration both are time varying. Abad (1996) extended the model of Goyal (1985) with joint price and lot size deterioration with incremental quantity discount offered by the supplier. Saiedy and Moghadam (2011) studied on inventory problem where the shortages were combination of backlogged and lost sales and the permissible delay in payments depends on the order quantity. Aggarwal and Jaggi (1995) extended Goyal's (1985) model for exponential deterioration rate under the condition of permissible delay in payments. Hwang and Shinn (1997) developed an EOQ model under permissible delay in payment with discount for perishable products. Shah (1993) established probabilistic inventory model for deteriorating items under the conditions of permissible delay in payments. Jamal et al. (2000) considered an EOQ model where the retailer can pay the wholesaler either at the end of the credit period or later, including interest charges on the unpaid balances for the overdue periods. Chung and Huang (2003) extended Goyal's (1985) model to consider the case that the units are replenished at a finite rate under permissible delay in payments and developed solution procedure to determine the retailer optimal ordering policy. Chang et al. (2003) developed an EOQ model for deteriorating items under supplier credits linked to order quantity. Chang (2004) proposed an inventory model under a situation in which the supplier provides the purchaser a permissible delay of payments if the purchaser orders a large quantity. There are several interesting papers related to permissible delay in payments such as Liao et al. (2000), Khouja and Mehrez (1996), Huang and Hsu (2008), Ouyang et al. (2006), Huang (2003), Chang and Teng (2004), Ouyang and Chang (2013), Chang et al. (2010), Li et al. (2014), Lou and Wang (2013) and Teng et al. (2013).

All the above researchers established their EOQ or EPQ models under trade credit financing by assuming that the demand rate is either constant or variable. However, in real life situations, for certain types of goods, the consumption rate is influenced by the inventory level. It is observed that a large pile of commodities on shelf in a supermarket will cause the customer to purchase more than generate higher demand. This occurs because of its popularity, visibility or variety or quality. Thus, building up inventory has

a positive impact on the sales as well as profit. Yang et al. (2010) developed “inventory lot size model for deteriorating items with stock-dependent demand rate”. Teng et al. (2011) established inventory model for stock-dependent demand under progressive payment scheme. Alfares (2007) established the inventory policy for an item with a stock-level dependent demand rate and a storage time dependent holding cost. Goh (1992) considered the inventory model in which the demand rate is inventory induced and the holding cost is time varying. Goyal and Chang (2009) provided an inventory model to determine the retailer’s optimal order quantity and the number of transfers per order from the warehouse to the display area. Hou (2006) proposed an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon. Teng et al. (2005) formulated EOQ model for deteriorating items and non-zero ending inventories. Other related analysis on inventory system with stock-dependent consumption rate have been performed by Soni and Shah (2008), Tripathi (2015, 2017), Tripathi and Singh (2015), Shah et al. (2013) and Wang et al. (2014).

The remaining of the paper is framed as follows: The notations and assumptions are used Section 2 followed by mathematical formulations. Optimal solution and some useful theoretical results are carried out in Section 4. Algorithm is discussed in Section 5. Numerical examples are provided in Section 6 to illustrate the theory and the procedure followed by managerial implication for the variation of parameters in Section 7. Finally, we draw a conclusion in the last section.

2 Notations and assumptions

2.1 Notations

$I_1(t)$ inventory level at time ‘ t ’, $0 \leq t \leq T_1$

$I_2(t)$ inventory level at time ‘ t ’, $T_1 \leq t \leq T$

m period of permissible delay in settling account ($0 < m < T$)

Q order quantity in completely backlogged

B the maximum of shortages

b backlogged shortage cost per unit

s lost sale shortage cost per unit

h holding cost per unit

p purchase cost per unit

A ordering cost per order

$D \equiv D\{I(t)\} \equiv \begin{cases} \alpha\{I(t)\}^\beta, & 0 < t < T \\ \alpha, & T_1 < t < T \end{cases}$ the demand rate, where α and β represent the scale parameter and shape parameter respectively, $\alpha > 0, 0 \leq \beta \leq 1$

I_e interest rate/\$/unit

I_r delays payment penalty rate $I_r > I_e$

γ the fraction of backlogged $0 \leq \gamma \leq 1$

$k = \{\alpha(1 - \beta)\}^{1/(1-\beta)}$ a constant

T_1 time to finish positive inventory

T length of the cycle time

$Z(T_1, T)$ total cost per cycle time

$$Z(T_1, T) = \begin{cases} Z_1(T_1, T), & \text{for } T_1 \geq m \\ Z_2(T_1, T), & \text{for } T_1 < m \end{cases}$$

2.2 Assumptions

- 1 The demand rate is stock-level dependent.
- 2 Shortages are allowed including backlogged and lost sale.
- 3 Planning horizon is infinite.
- 4 The inventory system involves only one item.
- 5 The lead time is negligible.
- 6 Ordered item is received fully.

3 Mathematical formulation

The inventory system can be considered according to the above assumptions as follows:

At time $t = 0$, the retailer orders and receives Q units of a single item from the producer. The level of inventory decreases during $[0, T_1]$ due to power demand only. During time T_1 to T shortages are allowed. The inventory equations are:

$$\frac{dI_1(t)}{dt} = -\alpha \{I_1(t)\}^\beta \quad (1)$$

$$\frac{dI_2(t)}{dt} = -\alpha \quad (2)$$

Under the condition $I(0) = Q$ and $I(T) = -S$.

The solution of the above differential equations are given by

$$I_1(t) = k(T_1 - t)^{1/(1-\beta)}, \quad (3)$$

and

$$I_2(t) = \alpha(T - t), \quad (4)$$

order quantity $Q - B = kT_1^{1/(1-\beta)}$ and $B = \alpha(T - T_1)$.

$$Q = kT_1^{1/(1-\beta)} + \alpha(T - T_1) \quad (5)$$

Figures 1 to 5 can be explained as follows.

In Figure 1, bold vertical line shows completely backlogged, i.e., $\gamma = 1$, it means that shortage is compensable. In Figure 2, dotted vertical line indicates that completely lost sales i.e., $\gamma = 0$, it means that shortage is not compensable. Figures 3 and 4 show cases 1 and 2, respectively. In Figure 5, partial shortage is compensable.

Based on the above assumption the planning horizon is infinite, we have presented and studied the model over a reorder during $[0, T]$. Two cases may arise that are shown in Figures 3 and 4, respectively. In Figure3, inventory level is completed after the time of permissible delay (i.e., $m < T_1$) to met for delay penalty. In Figure 4, customers does not pay any charges.

The components of total average cost per unit time of the system are as follows:

- 1 The ordering cost per unit

$$\text{time} = \frac{A}{T} \quad (6)$$

- 2 The inventory holding cost per unit time

$$HC = \frac{h}{T} \int_0^{T_1} I_1(t) dt = \frac{hk(1-\beta)T_1^{(2-\beta)/(1-\beta)}}{(2-\beta)T} \quad (7)$$

- 3 The shortage cost per unit time (for backlogged mode) is

$$SC_1 = \frac{b}{T} \int_{T_1}^T I_2(t) dt = \frac{\alpha b(T - T_1)^2}{2T} \quad (8)$$

- 4 The shortage cost per unit time (for lost sale mode) is

$$SC_2 = \frac{\alpha s(T - T_1)}{T} \quad (9)$$

Case 1 $M \leq T_1$

Since $m \leq T_1$, the interest earned is given by:

- 1 The earned interest is obtained at two models separately that both are similar

$$IE_1 = \frac{pI_e}{T} \int_0^{T_1} I_1(t) dt = \frac{kpI_e(1-\beta)}{T(2-\beta)} T_1^{(2-\beta)/(1-\beta)}. \quad (10)$$

- 2 The delayed payment charged (penalty) is obtained at two modes separately where both are similar

$$IP = \frac{pI_r}{T} \int_m^{T_1} I_1(t) dt = \frac{kpI_r(1-\beta)}{T(2-\beta)} (T_1 - m)^{(2-\beta)/(1-\beta)} \quad (11)$$

Therefore, the total average cost/unit time is

$$Z_1(T_1, T) = [OC + HC + SC_1 + SC_2 + IE_1 - IP]$$

$$Z_1(T_1, T) = \frac{1}{T} \left[A + \frac{k(1-\beta)}{(2-\beta)} (h + pI_e) T_1^{(2-\beta)/(1-\beta)} + \frac{\alpha\gamma b(T - T_1)^2}{2} + \alpha(1-\gamma)s(T - T_1) - \frac{kpI_r(1-\beta)}{(2-\beta)} (T_1 - m)^{(2-\beta)/(1-\beta)} \right] \quad (12)$$

Figure 1 Completely clogged

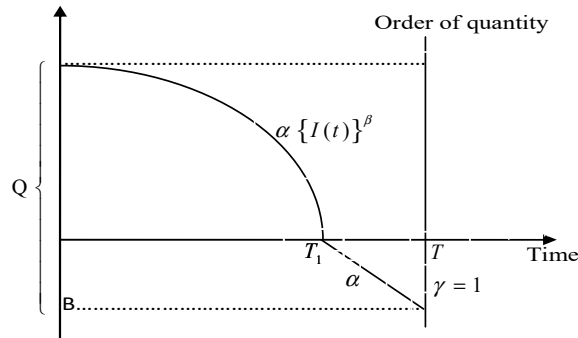


Figure 2 Completely lost sale

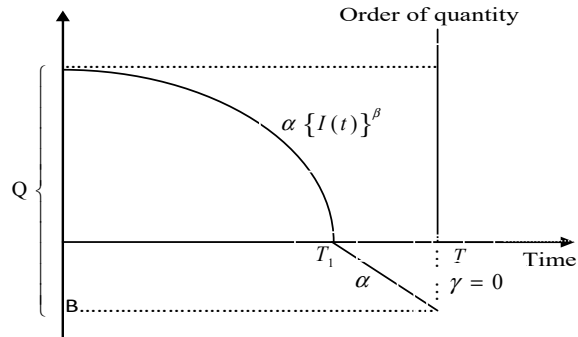


Figure 3 $m \leq T_1$

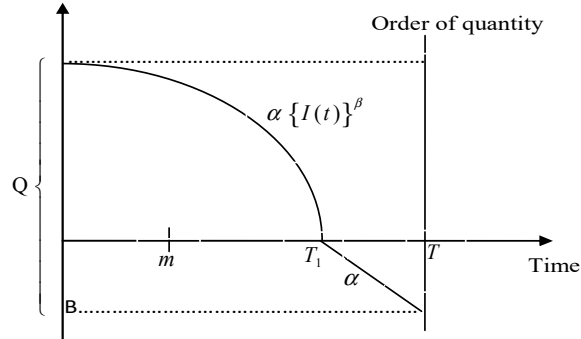


Figure 4 $m \leq T_1$

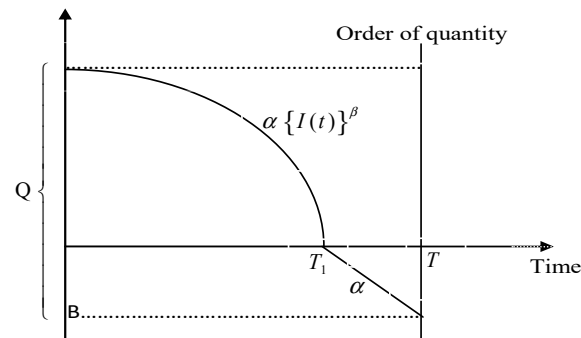
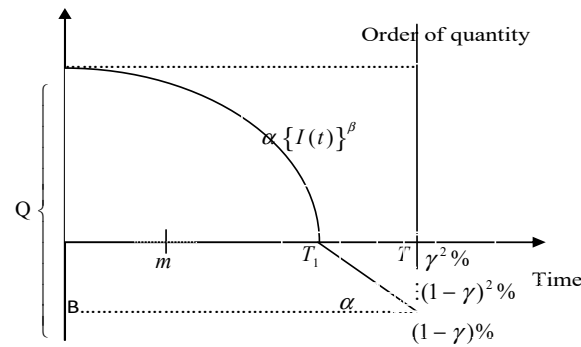


Figure 5 % γ backlogged % $(1 - \gamma)$ lost sale



Case 2 $M > T_1$

In this case, ordering cost per unit time, holding cost per unit time, and shortage cost (both modes) are same as previous case 1. Since $m > T_1$, the buyer pays no delayed payment penalty. The interest earned is obtained by taking sum of average inventory in interval $[0, T_1]$ and average sales revenue in $[T_1, m]$.

The interest earned is obtained at two modes separately where both are similar

$$\begin{aligned}
IE_2 &= \frac{pI_e}{T} \left[\int_0^{T_1} \alpha \{I(t)\}^\beta .tdt + (m - T_1) \int_0^{T_1} \alpha \{I(t)\}^\beta dt \right] \\
&= \frac{kpI_e T_1^{1/(1-\beta)}}{T(2-\beta)} \{(2-\beta)m - T_1\}
\end{aligned} \tag{13}$$

Therefore, the total average cost per unit time is

$$\begin{aligned}
Z_2(T_2, T) &= [\text{Ordering cost} + \text{Holding cost} + \text{Shortage cost (at backlogging mode)} \\
&\quad + \text{Shortage cost (at lost sales mode)} - \text{Interest earned}] \text{ per unit time}
\end{aligned}$$

i.e.,

$$\begin{aligned}
Z_2(T_1, T) &= [OC + HC + SC_1 + SC_2 - IE_2] \\
Z_2(T_1, T) &= \frac{1}{T} \left[A + \frac{hk(1-\beta)}{(2-\beta)} T_1^{(2-\beta)/(1-\beta)} + \frac{\alpha\gamma b(T-T_1)^2}{2} \right. \\
&\quad \left. + \alpha(1-\gamma)s(T-T_1) - \frac{kpI_e T_1^{1/(1-\beta)}}{(2-\beta)} \{(2-\beta)m - T_1\} \right]
\end{aligned} \tag{14}$$

4 Determination of optimal solution

Differentiating (12) and (14) partially two times with respect to T_1 and T , we get

$$\begin{aligned}
\frac{\partial Z_1(T_1, T)}{\partial T_1} &= \frac{1}{T} \left\{ k(h + pI_e) T_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) \right. \\
&\quad \left. - \alpha(1-\gamma)s - kpI_r(T_1 - m)^{1/(1-\beta)} \right\},
\end{aligned} \tag{15}$$

$$\frac{\partial Z_1(T_1, T)}{\partial T} = -\frac{N}{T^2} + \frac{\alpha\gamma b(T - T_1)}{T} + \frac{\alpha(1-\gamma)s}{T}, \tag{16}$$

where

$$\begin{aligned}
N &= A + \frac{k(1-\beta)}{(2-\beta)} (h + pI_e) T_1^{(2-\beta)/(1-\beta)} + \frac{\alpha\gamma b(T - T_1)^2}{2} + \alpha(1-\gamma)s(T - T_1) \\
&\quad - \frac{kpI_r(1-\beta)}{(2-\beta)} (T_1 - m)^{(2-\beta)/(1-\beta)} \\
\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} &= \frac{1}{T} \left\{ \frac{k(h + pI_e) T_1^{\beta/(1-\beta)}}{(1-\beta)} + \alpha\gamma b - \frac{kpI_r(T_1 - m)^{\beta/(1-\beta)}}{(1-\beta)} \right\} > 0,
\end{aligned} \tag{17}$$

$$\frac{\partial^2 Z_1(T_1, T)}{\partial T \partial T_1} = -\frac{1}{T^2} \left\{ k(h + pI_e)T_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) - \alpha(1-\gamma)s - kpI_r(T_1 - m)^{1/(1-\beta)} \right\} - \frac{\alpha\gamma b}{T} \quad (18)$$

$$\frac{\partial^2 Z_1(T_1, T)}{\partial T^2} = \frac{2N}{T^3} - \frac{2\alpha}{T^2} \{ \gamma b(T - T_1) + (1-\gamma) \} + \frac{\alpha\gamma b}{T} > 0 \quad (19)$$

$$\frac{\partial Z_2(T_1, T)}{\partial T_1} = \frac{1}{T} \left\{ hkT_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) - a(1-\gamma)s - \frac{kpI_e T_1^{\beta/(1-\beta)}}{(1-\beta)} (m - T_1) \right\} \quad (20)$$

$$\frac{\partial Z_2(T_1, T)}{\partial T} = -\frac{N_1}{T^2} + \frac{1}{T} \{ \alpha\gamma b(T - T_1) + \alpha(1-\gamma)s \}, \quad (21)$$

where

$$N_1 = A + \frac{hk(1-\beta)}{(2-\beta)} T_1^{(2-\beta)/(1-\beta)} + \frac{\alpha\gamma b(T - T_1)^2}{2} + \alpha(1-\gamma)s(T - T_1) - \frac{kpI_e T_1^{1/(1-\beta)}}{(2-\beta)} \{ (2-\beta)m - T_1 \}$$

$$\frac{\partial^2 Z_2(T_1, T)}{\partial T_1^2} = \frac{1}{T} \left\{ \frac{hkT_1^{\beta/(1-\beta)}}{(1-\beta)} + \alpha\gamma b - \frac{k\beta pI_e T_1^{(2\beta-1)/(1-\beta)}}{(1-\beta)^2} (m - T_1) \right\} \quad (22)$$

$$\frac{\partial^2 Z_2(T_1, T)}{\partial T^2} = \frac{2N_1}{T^3} - \frac{2\{ \alpha\gamma b(T - T_1) + \alpha(1-\gamma)s \}}{T^2} + \frac{\alpha\gamma b}{T} > 0 \quad (23)$$

$$\frac{\partial^2 Z_2(T_1, T)}{\partial T \partial T_1} = -\frac{1}{T^2} \left\{ hkT_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) - \alpha(1-\gamma)s - \frac{kpI_e}{(1-\beta)} T_1^{\beta/(1-\beta)} (m - T_1) + \alpha\gamma bT \right\} < 0 \quad (24)$$

Optimal values of $T_1 = T_1^*$ and $T = T^*$ (for case 1) and $T_1 = T_1^{**}$ and $T = T^{**}$ (for case 2) is obtained by solving equations $\frac{\partial Z_i(T_1, T)}{\partial T_1} = 0$, and $\frac{\partial Z_i(T_1, T)}{\partial T} = 0$, for $i = 1, 2$ respectively. Since $\frac{\partial^2 Z_i(T_1, T)}{\partial T_1^2} > 0$, $\frac{\partial^2 Z_i(T_1, T)}{\partial T^2} > 0$ and $\left\{ \frac{\partial^2 Z_i(T_1, T)}{\partial T_1^2} \right\} \left\{ \frac{\partial^2 Z_i(T_1, T)}{\partial T^2} \right\} - \left\{ \frac{\partial^2 Z_i(T_1, T)}{\partial T \partial T_1} \right\}^2 > 0$; for $i = 1, 2$, the values obtained by solving, $i = 1, 2$ will be minimum. Putting (15) and (16) equal to zero for case 1 and (20), (21) equal to zero for case 2, we get

$$\begin{cases} k(h + pI_e)T_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) - \alpha(1-\gamma)s - kpI_r(T_1 - m)^{1/(1-\beta)} = 0, \\ A + \frac{k(1-\beta)}{(2-\beta)}(h + pI_e)T_1^{(2\beta-1)/(1-\beta)} - \frac{\alpha\gamma b(T^2 - T_1^2)}{2} \\ - \alpha(1-\gamma)sT_1 - \frac{kpI_r(1-\beta)}{(2-\beta)}(T_1 - m)^{(2\beta-1)/(1-\beta)} = 0. \end{cases} \quad (25)$$

and

$$\begin{cases} hkT_1^{1/(1-\beta)} - \alpha\gamma b(T - T_1) - \alpha(1-\gamma)s - \frac{kpI_e}{1-\beta}T_1^{\beta/(1-\beta)}(m - T_1) = 0, \\ A + \frac{hk(1-\beta)}{(2-\beta)}T_1^{(2-\beta)/(1-\beta)} - \frac{\alpha\gamma b(T^2 - T_1^2)}{2} - \alpha(1-\gamma)sT_1 \\ - \frac{kpI_eT_1^{1/(1-\beta)}}{2-\beta}\{(2-\beta)m - T_1\} = 0. \end{cases} \quad (26)$$

4.1 Special case on optimal solution

If $\beta = 0$, equations (25) and (26) reduces to

$$\begin{cases} [k\{h + p(I_e - I_r)\} + \alpha\gamma b]T_1 = \alpha\gamma bT + \alpha(1-\gamma)s - kpI_r m, \\ [k\{h + p(I_e - I_r) + \alpha\gamma b\}]T_1^2 - 2\{\alpha(1-\gamma)s - kpI_r m\}T_1 \\ - \{\alpha\gamma bT^2 + kpI_r m^2 - 2A\} = 0. \end{cases} \quad (27)$$

and

$$\begin{cases} \{k(h + pI_e) + \alpha\gamma b\}T_1 = \alpha\gamma bT + \alpha(1-\gamma)s + kpI_e m, \\ [\{k(h + pI_e) + \alpha\gamma b\}]T_1^2 - 2\{\alpha(1-\gamma)s + kpI_e m\}T_1 - \{\alpha\gamma bT^2 - 2A\} = 0. \end{cases} \quad (28)$$

4.2 Determination of optimal solution for complete lost sale ($\gamma = 0$)

In case of complete lost sale, the optimal policy is to have either no stock outs or all stock outs. If $\gamma = 0$, we do not order, therefore model is solved without any shortages.

Mode 1 If $m < T_1$, $B = 0$ and $T = T_1$. Thus,

$$Z_1(T_1) = \frac{A}{T_1} + \frac{k(1-\beta)}{(2-\beta)}(h + pI_e)T_1^{1/(1-\beta)} - \frac{kpI_r(1-\beta)}{(2-\beta)}\frac{(T_1 - m)^{(2-\beta)/(1-\beta)}}{T_1} \quad (29)$$

Differentiating $Z_1(T_1)$ with respect to T_1 , we get

$$\begin{aligned} \frac{dZ_1(T_1)}{dT_1} = & -\frac{A}{T_1^2} + \frac{k}{(2-\beta)}(h + pI_e)T_1^{\beta/(1-\beta)} \\ & - \frac{kpI_r(T_1 - m)^{\beta/(1-\beta)}}{(2-\beta)T_1^2} \{T_1 + (1-\beta)m\} \end{aligned} \quad (30)$$

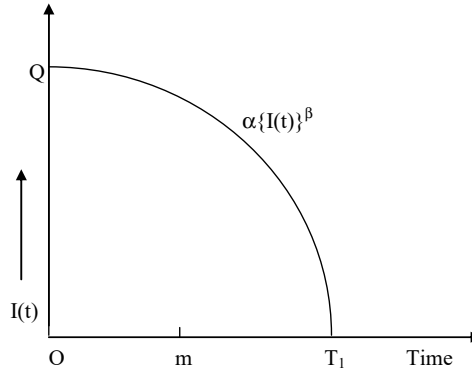
The optimal value of $T_1 = T_1^*$ is obtained by solving $\frac{dZ_1(T_1)}{dT_1} = 0$, we get

$$k(h + pI_e)T_1^{(2-\beta)/(1-\beta)} - kpI_r(T_1 - m)^{\beta/(1-\beta)} \{T_1 + (1-\beta)m\} - (2-\beta)A = 0. \quad (31)$$

If $\beta = 0$, we get

$$T_1 = T_1^* = \sqrt{\frac{2A - kpI_r m^2}{k\{h - p(I_r - I_e)\}}} \quad (32)$$

Figure 6 $m < T_1, B = 0$ at $T = T_1$ (without shortages)



Mode 2 If $m = T_1, B = 0$ and $T = T_1$, hence

$$Z_2(m) = \frac{A}{m} + \frac{k(1-\beta)}{2-\beta} \{h + p(I_e - I_r)\} m^{\beta/(1-\beta)} \quad (33)$$

Differentiating (35) with respect to m , we get

$$\frac{dZ_2(m)}{dm} = -\frac{A}{m^2} + \frac{k}{2-\beta} \{h + p(I_e - I_r)\} m^{\beta/(1-\beta)} \quad (34)$$

The optimal value of $T_1 = m^*$ is obtained by solving $\frac{dZ_2(m)}{dm} = 0$, we get

$$T_1 = m^* = \left[\frac{(2-\beta)A}{k\{h + p(I_e - I_r)\}} \right]^{(1-\beta)/(2-\beta)} \quad (35)$$

If $\beta = 0$, we get

$$T_1 = m^* = \sqrt{\frac{2A}{k\{h + p(I_e - I_r)\}}} \quad (36)$$

5 Algorithm

The following algorithm is used to find optimum cost and economic order quantity:

- Step 1 Obtain T_1^* from equation (25), if $T_1^* \geq m$, and evaluate $Z_1(T_1, T)$ from equation (12).
- Step 2 Obtain T_1^{**} from equation (25), if $T_1^{**} < m$ and evaluate $Z_2(T_1, T)$ from equation (14) else go to step 4 otherwise go to Step 5.
- Step 4 If the condition $T_1 \geq m > T_2$ and find minimum cost.
- Step 5 Compare $Z_1^*(T_1^*, T^*)$ and $Z_2^*(T_1^{**}, T^{**})$ and find minimum cost.
- Step 6 If $T_1^* \geq m$ is satisfied but $T_1^{**} > m$ then $Z_1(T_1, T)$ is the minimum cost else $T_1^* < m$ but $T_1^{**} < m$ then $Z_2(T_1, T)$ is the minimum cost.
- Step 7 Compute Q_1^* and Q_2^* from the respective minimum costs.

6 Numerical examples

6.1 Example 1: case 1: $m \leq T_1$

Let us take the parameter values of the inventory system as $A = \$250$ per order, $h = 10$ per unit, $b = 40$ unit per year, $s = 60$ unit per year, $I_e = 0.1$ per year, $I_r = 0.15$ per year, $p = \$100$ per unit, $m = 0.02$ years, $\alpha = 2,000$, $\beta = 0.5$, $\gamma = 0.9$. Putting these values in equations (25), we get $T = T^* = 0.047217$ year, $T_1 = T_1^* = 0.0265857$ year, and minimum total cost per cycle time $Z_1(T_1, T) = Z_1^*(T_1^*, T^*) = \$13,485.5$ and $Q = Q_1^* = 748.062$ units, this proves case 1 since $m \leq T_1$.

6.2 Example 2: case 2: $m > T_1$

Let us take the parameter values of the inventory system as $A = \$500$ per order, $h = 20$ per unit, $b = 40$ unit per year, $s = 60$ unit per year, $I_e = 0.1$ per year, $I_r = 0.15$ per year, $p = \$50$ per unit, $m = 0.02$ years, $\alpha = 5,000$, $\beta = 0.5$, $\gamma = 0.9$. Substituting these values in equations (26), we get $T = T^{**} = 0.0393356$ year, $T_1 = T_1^{**} = 0.0154702$ year, and minimum total cost per cycle time $Z_2(T_1, T) = Z_2^*(T_1^{**}, T^{**}) = \$ 34295.8$ and $Q = Q_2^* = 1,615.12$ units, this proves case 1 since $m \leq T_1$.

Using above algorithm, minimum total average cost per unit time is $Z_1(T_1, T) = Z_1^*(T_1^*, T^*) = \$13,485.5$ and minimum order quantity is $Q = Q_1^* = 748.062$ units.

7 Sensitivity analysis

We now study the effects of variations in the parameters A , h , b , s , p , and α , on the optimal total cost/cycle time. In this discussion, taking one parameter at a time and keeping all the remaining parameters the same. The results of sensitivity analysis are summarised in Tables 1 to 12.

Case 1

Table 1 Change of $Z_1(T_1, T)$ with respect to 'A'

A	$T = T^*$	$T_1 = T_1^*$	$Z_1^*(T_1^*, T^*)$	Q_1^*
260	0.0503020	0.0268223	13,690.5	766.395
280	0.0559625	0.0272548	14,067.0	800.240
300	0.0610991	0.0276455	14,408.7	831.234
350	0.0723569	0.0284963	15,158.0	899.760
400	0.0826788	0.029225	15,805.6	961.008
500	0.986776	0.0304565	16,911.8	1,064.04

Table 2 Variation of $Z_1(T_1, T)$ with respect to 'h'

h	$T = T^*$	$T_1 = T_1^*$	$Z_1^*(T_1^*, T^*)$	Q_1^*
11	0.0483386	0.0259516	13,611.9	718.26
14	0.0511476	0.0243348	13,930.5	645.808
15	0.0519414	0.0238712	14,021.1	625.975
20	0.0551896	0.0219472	14,393.5	548.164
25	0.0576265	0.0204786	14,674.6	493.669

Table 3 Variation $Z_1(T_1, T)$ with respect to backlogged shortage cost 'b'

b	$T = T^*$	$T_1 = T_1^*$	$Z_1^*(T_1^*, T^*)$	Q_1^*
41	0.0468313	0.026595	13,493.4	747.767
44	0.0457629	0.0266211	13,516.0	746.967
46	0.045116	0.0266372	13,530.0	746.498
50	0.0439544	0.0266670	13,555.9	745.704
60	0.0416376	0.0267295	13,610.1	744.282

Table 4 Variation of $Z_1(T_1, T)$ with respect to lost sale shortage cost 's'

s	T^*	T_1^*	$Z_1^*(T_1^*, T^*)$	Q_1^*
61	0.0457234	0.0266843	13,570.8	750.130
63	0.0425339	0.0268656	13,728.1	753.097
64	0.0408222	0.0269471	13,799.0	753.896
65	0.0390191	0.0270216	13,863.8	754.162
70	0.0279908	0.0272400	14,054.1	743.136

Table 5 Variation $Z_1(T_1, T)$ with respect to of purchase cost/unit time ' p '

p	T^*	T_1^*	$Z_1^*(T_1^*, T^*)$	Q_1^*
110	0.0483649	0.0260086	13,609.7	772.177
130	0.0503479	0.0249392	13,829.4	672.781
150	0.0520084	0.0239786	14,018.1	631.033
200	0.0552268	0.0219940	14,392.8	550.202
250	0.0576299	0.0204823	14,674.6	493.820

Table 6 Variation of $Z_1(T_1, T)$ with respect to ' α '

α	T^*	T_1^*	$Z_1^*(T_1^*, T^*)$	Q_1^*
2,100	0.0445059	0.0256953	14,022.1	767.426
2,300	0.0394843	0.0241375	15,070.7	805.811
2,400	0.0371372	0.0234519	15,582.4	824.833
2,500	0.0348780	0.0228186	16,085.3	843.724
3,000	0.0244188	0.0202498	18,450.3	935.129

*Case 2***Table 7** Variation of ordering cost ' A '

A	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
550	0.0458210	0.0158151	35,401.1	1,713.26
600	0.0515510	0.0161161	36,378.3	1,800.48
750	0.0659677	0.0168595	38,839.5	2,020.06
800	0.0701498	0.0170715	39,554.1	2,086.87
900	0.0778720	0.0174589	40,874.4	2,207.15
1,000	0.0849221	0.0178081	42,080.5	1,315.62

Table 8 Variation of holding cost ' h '

h	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
21	0.0404645	0.0151043	34,564.8	1,552.68
25	0.0441571	0.0138896	35,448.1	1,357.09
30	0.047522	0.0127502	36,258.9	1,189.91
40	0.0519964	0.0111686	37,349.0	983.749
50	0.0548977	0.0100901	38,065.4	860.351
100	0.0616062	0.00736967	39,762.6	610.633

Table 9 Variation of backlogged shortage cost 'b'

b	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
42	0.0385320	0.0154889	34,355.1	1,614.63
44	0.0377876	0.0155066	34,411.6	1,614.25
46	0.0370959	0.0155235	34,465.5	1,613.98
48	0.0364511	0.0155396	34,516.9	1,613.80
50	0.0358485	0.0155550	34,566.0	1,613.71
60	0.0333376	0.0156228	34,783.0	1,614.02

Table 10 Variation of lost sale shortage cost 's'

s	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
10	0.0712747	0.00895962	16,216.7	813.293
20	0.0673574	0.0105796	20,220.0	983.439
30	0.0624445	0.0120273	24,075.1	1,156.19
40	0.0564090	0.0133255	27,755.0	1,325.22
50	0.0489420	0.0144796	31,203.2	1,482.68

Table 11 Variation of purchase cost per unit time 'p'

p	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
55	0.0383680	0.01528390	34,155.1	1,575.41
65	0.0365270	0.01488950	33,894.7	1,493.80
75	0.0348616	0.14468700	33,670.7	1,410.36
100	0.0318519	0.01333330	33,333.4	1,203.70
150	0.0319200	0.01103060	33,760.1	864.910
200	0.0370301	0.00905831	35,034.9	652.69

Table 12 Variation of ' α '

α	T^{**}	T_1^{**}	$Z_2^*(T_1^{**}, T^{**})$	Q_2^*
5,100	0.0382322	0.0152109	34,826.7	1,621.90
5,500	0.0340958	0.0142551	36,928.5	1,645.88
5,600	0.0331238	0.0140345	37,448.4	1,651.12
5,700	0.0321741	0.0138206	37,966.1	1,656.09
5,800	0.0312454	0.0136129	38,481.7	1,660.73
6,000	0.0294469	0.0132154	39,506.0	1,669.21

The following inferences can be made from results obtained from both the cases, i.e., for cases 1 and 2, respectively.

- Case 1:
 - 1 From Table 1, we see that when ordering cost ' A ' increases, T^* , T_1^* , $Z_1^*(T_1^*, T^*)$ and Q_1^* increases.
 - 2 From Table 2, we observe that, when holding cost ' h ' increases, T^* increases, T_1^* and Q_1^* decreases and $Z_1^*(T_1^*, T^*)$ increases.
 - 3 From Table 3, increase of backlogged shortages cost ' b ' results, decrease in T^* , slight increase in T_1^* , increase in $Z_1^*(T_1^*, T^*)$, and Q_1^* approximately constant.
 - 4 From Table 4, we see that increase of lost sale shortage cost ' s ' results, decrease in T^* , increase in T_1^* , $Z_1^*(T_1^*, T^*)$, and Q_1^* .
 - 5 From Table 5. We see that, increase of purchase cost ' p ' results, increase in T^* , decrease in T_1^* , increase in $Z_1^*(T_1^*, T^*)$ and decrease in Q_1^* .
 - 6 From Table 6, we observe that increase in ' α ' results fluctuation in T^* , decrease in T_1^* , increase in $Z_1^*(T_1^*, T^*)$ and Q_1^* .
- Case 2
 - 7 From Table 7, we see that when ordering cost ' A ' increases, T^{**} , T_1^{**} , $Z_2^*(T_1^{**}, T^{**})$ and Q_2^* increases.
 - 8 From Table 8, we observe that, when holding cost ' h ' increases, T^{**} , and $Z_2^*(T_1^{**}, T^{**})$ increases, T_1^{**} and Q_2^* decreases.
 - 9 From Table 9, increase of backlogged shortages cost ' b ' results, decrease in T^{**} , increase in T_1^{**} , $Z_2^*(T_1^{**}, T^{**})$ and non-uniform change in Q_2^* .
 - 10 From Table 10, we observe that, increase of lost sale shortage cost ' s ' leads, decrease in T^{**} , increase in T_1^{**} , $Z_2^*(T_1^{**}, T^{**})$ and Q_2^* .
 - 11 From Table 11, we observe that, increase of purchase cost ' p ' results, non-uniform change in T^{**} , decrease in T_1^{**} and $Z_2^*(T_1^{**}, T^{**})$ is concave function with respect to ' p ' and decrease in Q_2^* .
 - 12 From Table 12, we see that increase in ' α ' results decrease in T^{**} , T_1^{**} , increase in $Z_2^*(T_1^{**}, T^{**})$ and Q_2^* .

8 Conclusions

In this paper, we have provided an analytical formulation of the problem on the framework discussed in the manuscript. The present model is based on a stock-dependent demand rate under shortages where the shortages are a combination of backlogged and lost sales. Numerical examples have been given to illustrate the proposed model on optimal solution. We have derived several managerial phenomena. For example, a higher

value of A , h , b , s , p , and α caused a higher value of total cost per cycle only one case is different for variation of purchase cost ' p '. This model is very useful in the retail business. It can be used for fashionable cloths, domestic goods, electronic components and other products.

We may generalise the model for deteriorating items. We may also extend the model for time dependent demand. There is an ample range of several assumptions that can be imbibed in the current study to come up with better inventory control models that can help for establishing the theory further. Each model developed in the thesis can be further enriched by considering different situation in addition to already considered background. Further, preservation technology factor can be applied with various assumptions to minimise the items rate of deterioration. The fuzzification of models can help to create a more enriched study.

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