
Efficiency-based optimisation of a 2-DOF robotic fish model

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Abstract: One of the most attractive swimming characteristics of fishes is their efficient propulsion. In order to fully understand the kinematic, dynamic and fluid dynamic relationships that determine a high efficient fish, a mathematical model needs to be developed and analysed. Accordingly, in this paper, a fish robot with two degrees of freedom (DOF) is modelled. The robot's dynamic equations of motion are presented and discussed. And the constant parameters of the equations are optimised using particle swarm optimisation (PSO) algorithm. Finally, the effects of optimised and non-optimised parameters on the robot behaviour is observed and analysed.

Keywords: biomimetic fish robot; mathematical model; particle swarm optimisation; PSO.

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1 Introduction

Undersea operation, oceanic supervision, aquatic life-form observation, pollution search and military detection are just a few examples that demand development of underwater robots to replace humans (Yu et al., 2004). Since the best solutions are always inspired from nature, for development of an underwater robot, the inspiration from nature has been taken into account. Accordingly, a number of bio-inspired robots such as fish robots have been developed so far (Paulson, 2004).

A fish robot could be defined as a fish-like aquatic vehicle which propels through undulatory or oscillatory motion of either body or fins (Hu et al., 2006). The first fish robot, RoboTuna, was built at MIT in 1994 (Triantafyllou and Triantafyllou, 1995). Three years later, vorticity control unmanned undersea vehicle (VCUUV) was developed based on RoboTuna with some improvement and more capabilities such as avoiding obstacles and having up-down motion (Anderson and Chhabra, 2002; Liu and Hu, 2004). Afterwards, a number of institutes and universities developed their own fish robots with more capabilities such as cruising and turning by pectoral fins (Lachat et al., 2005), cruising by undulating anal fins (Low, 2009) and so on. A comprehensive review of various types of fish robots is provided in Masoomi et al. (2013).

Robots inspired by various types of fishes could be highly efficient, manoeuvrable and noiseless in marine environment (Hu et al., 2006). For instance, the propulsion system for some types of fishes is up to 90% efficient, while a conventional screw propeller is around 40% to 50% efficient (Yu and Wang, 2005).

Nevertheless, the existing fish robots show deficiencies regarding their swimming behaviours. The fish robots have been developed to have a specific gait of swimming. For instance, some fish robots are able to swim through undulation of their tail while some other fishes swim using their pectoral fins. These two types of robots demonstrate different swimming performances, the former is more suitable for cruising long distances while the latter is more suitable for hovering and slow swimming. This causes a problem since, e.g., for navigation long distances, a robot needs to have both former and latter capabilities. In order to address the aforementioned problem, the authors are

developing a novel fish robot which is able to have different swimming gaits.

The first step of developing a fish robot is the simulation of robot dynamic behaviour. Primarily, the dynamic equations of robot must be derived. So far, various attempts on mathematical modelling of fish robots including hydrodynamic forces acting on the fins and dynamic behaviour of fish robot have been done such as Lighthill (1970), Harper et al. (1998), Morgansen (2003), Yu et al. (2006) and Xu and Niu (2011). However, fishes have different mechanical structure and, therefore, each one of them could have their own dynamic equations. Once the dynamic equations of motion are derived, the constant parameter of the equation should be optimised.

In the process of developing a novel fish robot, the first step is the simulation of the robot dynamic behaviour. Initially the dynamic equations of the robot motion are derived. Then the constant parameters of the equations are substituted into the equations and the dynamic behaviour of the robot is analysed. The constant parameters include sizes of different parts of the robot, amplitude and so on. At this stage, an important issue arises: how to decide on the best values for those parameters. To address this problem, several approaches could be applied.

Some robotic fish developers have analysed different parts of the robot separately. For instance, they experimentally test different sizes of the fish fins and select the best one after the comparison of them together. Similarly, Kodati et al. (2008) have taken hydrodynamic experiment into account to design caudal fin of their fish called MARCO while the pectoral fins are analogous to a real boxfish.

This method is not ideal as overall effectiveness of the fish robot for a specific task is required not the effectiveness of one part of fish. For example, to have an efficient swimming, a manoeuvrable angelfish required fins with large surface area with low aspect ratio whereas a fast swordfish needs a caudal fin with small surface area and high aspect ratio. Both aforementioned types of fins are efficient but with respect to their tasks, manoeuvrability and high-speed swimming.

Some others have tried to scale the real fish size and tail beat. For example, in Triantafyllou et al. (2000), Anderson and Chhabra (2002), Yu et al. (2007) and Gao et al. (2009), the shape and geometry of fish robots are mimicked. Even

more, Salume (2010) has developed a trout robot that resembles the morphology of the real fish in geometry, stiffness and stiffness distribution of the body and caudal fin.

Although preferable compared to the previous method, this method ignores the fact that the fish robots are developed based on their locomotion types for limited purposes like coastal monitoring, pollution search and so on. Whereas in nature the exercise physiology of swimming animals is compromised according to their natural demands like respiration, digestion and reproduction which are not issues of developing fish robots. For instance, bird wrasses have an elongated beak which is useful for its type of catching prey. The size of this elongated beak does not affect bird wrasse locomotion.

Yet there are many other aquatic robot developers who have not explained explicitly why specific shape and values for, e.g., the sizes and the tail beat of their fish robot are selected. For instance, BoxyBot (Lachat et al., 2005) is a well-known robot for its manoeuvrability. However, BoxyBot could have superior performance if it had optimised values for the shape and size of its body and fins. The majority of robotic fishes like Kato et al. (2000), Low and Willy (2005), Epstein et al. (2006), Morgansen et al. (2007) and Liang et al. (2011) could be classified in this group since they are mainly mimicking the swimming mode of real fishes rather than their morphologies.

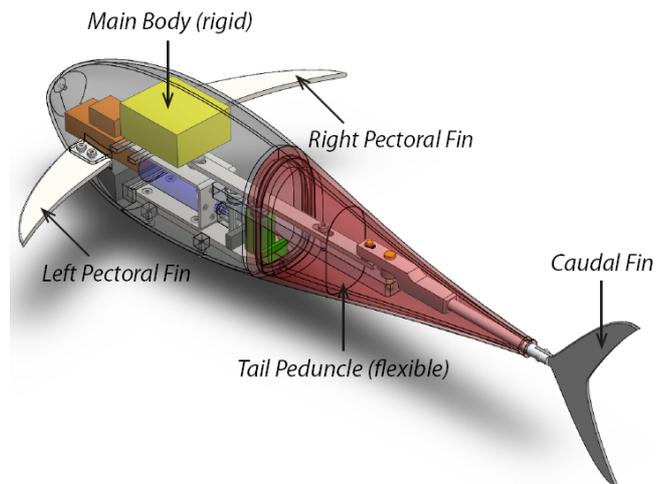
Despite the previous approaches, the authors have introduced a method of determining the best and optimal values for the sizes of the fish dimensions and also its tail beat. This method includes optimising the constant parameters of the mathematical model of the fish robot using PSO algorithm. In this method, all different parts of the robot are optimised simultaneously. Moreover, the robot could be optimised for dissimilar purposes. For example, the fish robot could be optimised for efficient or fast swimming.

The rest of paper includes five sections. Next section introduces the mechanical design and swimming modes of fishes. In section 3, kinematics, hydrodynamic forces and equation of motion of the fish robot are introduced. Section 4 explains PSO algorithm and its application for the fish robot. In section 5, the results of optimisation and simulation of the robot are presented. And finally, the paper is summarised.

2 Mechanical design

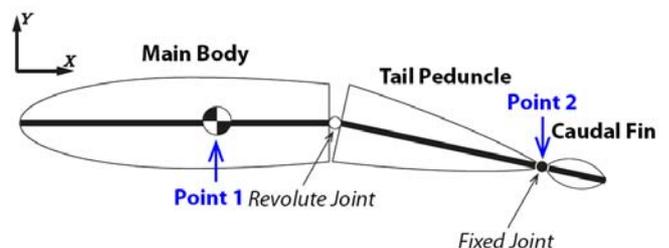
The main element which distinguishes fish robots from other types of underwater vehicles is their propulsion system. Fish robots generally have either undulatory or oscillatory swimming. When a fish passes a travelling wave along its body or its fins, its swimming method of fish is referred to as undulatory. On the other hand, in oscillatory mode the fish generates propulsion by oscillating a certain part of its body around its base (Sfakiotakis et al., 1999). The proposed fish robot by the authors shown in Figure 1 propels using an undulatory tail.

Figure 1 The CAD model of UC-Ika 1 (see online version for colours)



As previously mentioned, in this paper a simplified version of the robot illustrated in Figure 1, called UCIKA I, is considered for optimisation. As Figure 2 shows, the robot includes main body and tail connected by a revolute joint. The main body is an ellipsoid shape body. The main body is designed to contain all electronics including microcontroller, batteries, sensors and buoyancy control tank system. The tail consists of two parts: the tail peduncle and the caudal fin. The tail peduncle is like a bar while the caudal fin is shaped as a rectangular fin. The span of the caudal fin is to be greater than its chord. The caudal fin is attached to the tail peduncle with a fixed joint.

Figure 2 The structure of 2-DOF fish robot (see online version for colours)

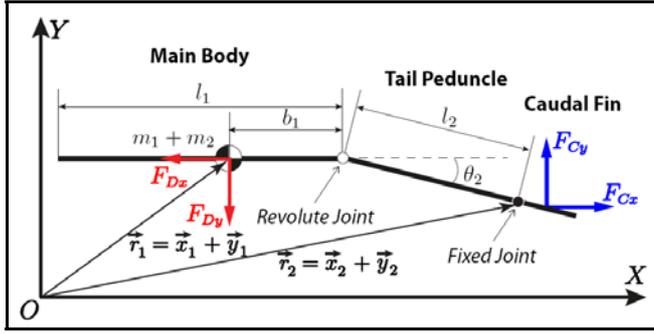


The fish robot is designed to simulate its planar cruising by the forces generated by the caudal fin. In the other words, the tail is actuated around its base on revolute joint. The actuation of the tail helps the caudal fin to create thrust. The thrust then propels the robot. Provided that the lateral projection area of the tail peduncle is much less than that of caudal fin, the thrust is assumed to be merely produced by the caudal fin.

3 Dynamic equations of motion

The governing equations of the fish robot model are derived based on Newton's law. The schematic sketch of the robot model is illustrated in Figure 3.

Figure 3 Schematic sketch of fish robot (see online version for colours)



3.1 Kinematics

The fish robot model consists of two masses, m_1 for the main body and m_2 for the tail. The robot swims in $X - Y$ plane. Accordingly, no vertical up-down motion of the robot in Z direction is considered. It is assumed that the rotational motion of the main body is zero. Accordingly, the robot has three DOFs, x_1 , y_1 and θ_2 , which are chosen to be translational motion of the centre of mass of the robot, point 1, and rotation of the tail peduncle. However, in the first analysis the actuation of the tail peduncle, θ_2 , is modelled as a kinematic variable and assumed a harmonic input signal expressed by

$$\begin{aligned} \theta_2 &= -A \sin(2\pi ft), \\ \dot{\theta}_2 &= -(2\pi f) A \cos(2\pi ft), \\ \ddot{\theta}_2 &= (2\pi f)^2 A \sin(2\pi ft), \end{aligned} \quad (1)$$

where A and f are the amplitude and frequency of pitching motion. This reduces the 3-DOF fish robot to the final 2-DOF model presented in this paper. From kinematics point of view, fixed joint between the tail peduncle and the caudal fin, point 2, is an important point since its speed plays an important role in calculations of the hydrodynamic forces generated at the caudal fin. Displacements and velocities of points 1 and 2 with respect to global reference frame, $O - XY$, are obtained as

$$\begin{aligned} x_2 &= x_1 - b_1 - l_2 \cos \theta_2 \\ y_2 &= y_1 - l_2 \sin \theta_2 \end{aligned} \quad (2)$$

$$\begin{aligned} \dot{x}_2 &= \dot{x}_1 + l_2 \dot{\theta}_2 \sin \theta_2 \\ \dot{y}_2 &= \dot{y}_1 - l_2 \dot{\theta}_2 \cos \theta_2 \end{aligned} \quad (3)$$

where b_1 is the distance between point 1 and the tail peduncle and l_2 is the length of the tail connector. θ_2 is the rotation of the tail connector.

Considering (3), U , the velocity of point 2 and α , the instantaneous angle of attack of the caudal fin are given by

$$U = (\dot{x}_2^2 + \dot{y}_2^2)^{1/2} \quad (4)$$

$$\alpha = \theta_2 + \arctan \left(\frac{\dot{y}_2}{\dot{x}_2} \right). \quad (5)$$

$$\dot{x}_1, \dot{y}_1, \dot{\theta}_2, \ddot{x}_1, \ddot{y}_1 \text{ and } \ddot{\theta}_2.$$

3.2 Hydrodynamic forces

The derivation of the hydrodynamic forces acting on the fish robot are associated with the following assumptions:

- the rotation of the main body is negligible
- the propulsion thrust is generated only by the caudal fin
- the fluid around the tail is inviscid while drag forces act on the main body.

3.2.1 Drag forces at main body

In general, two main hydrodynamic forces act on the fins, which are lift and drag forces, respectively. However, lift forces are not taken into account since the main body is symmetric relative to $X - Z$ plane and its rotation is neglected. The drag force of the main body could be obtained by

$$F_{Dx} = C_{Dx} \frac{\rho A_{bx} \dot{x}_1^2}{2} \quad (6)$$

$$F_{Dy} = C_{Dy} \frac{\rho A_{by} \dot{y}_1^2}{2} \quad (7)$$

where A_{bx} and A_{by} are main body projected area, C_{Dx} and C_{Dy} are drag coefficient of main body in X and Y direction, respectively. ρ is the water density.

3.2.2 Lift and inertial forces at caudal fin

Since the caudal fin is undulating, the motion of gliding wings are very different and thus new equation must be applied. Based on the aforesaid assumptions, the quasi-steady wing theory has been employed to calculate total forces acting on the caudal fin (Nakashima et al., 2003).

Considering the quasi-steady condition, lift and inertial forces of the caudal fin are described by (8) and (9).

$$F_L = 2\pi\rho LC_C U^2 \sin \alpha \cos \alpha \quad (8)$$

$$F_I = \pi\rho LC_C^2 (\dot{U} \sin \alpha + \dot{\alpha} U \cos \alpha) \quad (9)$$

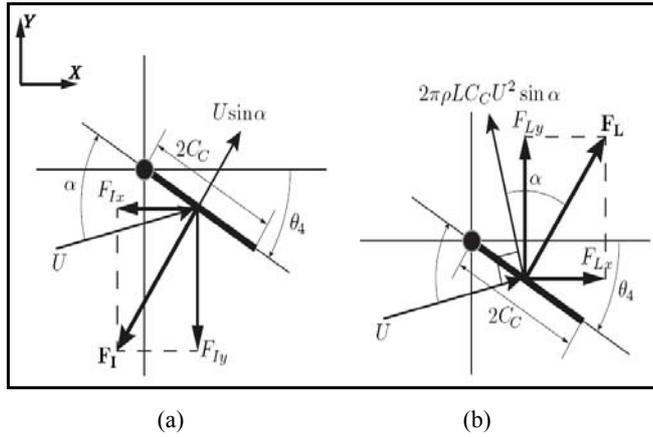
where L and $2C_C$ are the span and the chord of the caudal fin, respectively. Figure 4 illustrates the model of inertial and lift forces acting on the caudal fin.

With (8) and (9), F_{Cx} and F_{Cy} are obtained as

$$F_{Cx} = (F_L - F_I) \sin \theta_2 \quad (10)$$

$$F_{Cy} = (F_L - F_I) \cos \theta_2 \quad (11)$$

Figure 4 Forces acting on the caudal fin, (a) fluid inertial force and (b) lift force



3.3 Equations of motion

In kinematics section, it is explained that the robot is a 2-DOF mechanism with x_1 and y_1 as its DOFs. Accordingly, there exist two dynamic equations of motion in X and Y direction as

$$(m_1 + m_2) \ddot{x}_1 = F_{Cx} - F_{Dx} \quad (12)$$

$$(m_1 + m_2) \ddot{y}_1 = F_{Cy} - F_{Dy} \quad (13)$$

where F_{Dx} , F_{Dy} , F_{Cx} and F_{Cy} are previously defined by (6), (7), (10) and (11).

In (12) and (13) both F_{Cx} and F_{Cy} are functions of x_1 , y_1 , \dot{x}_1 , \dot{y}_1 , $\dot{\theta}_2$, \ddot{x}_1 , \ddot{y}_1 and $\ddot{\theta}_2$. As previously mentioned, θ_2 is assumed to be a kinematic variable expressed by (1). F_{Dx} and F_{Dy} are also functions of \dot{x}_1 and \dot{y}_1 .

4 PSO algorithm

In this section an optimisation method called particle swarm optimisation (PSO) is introduced and its application for optimisation of the robot parameters is described.

4.1 Background

Optimisation is one of the most important problems in engineering. Optimisation cannot be always addressed through analytical methods like gradient decent. For instance, multimodal functions cannot be optimised using gradient method since there are a number of local optimal solutions. To address the optimisation problems, evolutionary algorithms (EA) are one of the most attractive approaches since they are capable of solving large and complex problems (Yu and Gen, 2010a).

EAs are stochastic-search algorithm inspired by the natural evolution. EAs initiate optimisation by a group of solutions called population. The performance of each individual in the population is then evaluated using a fitness function. The population behaviour is evolved in analogy to nature (Yu and Gen, 2010b). EAs like genetic algorithm (GA), PSO and differential evolution (DE) all go through the same algorithms.

So far various types of EAs have been proposed started from 1960 (Yu and Gen, 2010a). All have their own pros and cons. Among the proposed methods, the PSO algorithm that relies on swarm intelligence (SI) has shown more suitability for complex optimisation purposes. SIs are inspired by the social behaviour of animal species like birds, fish and ants. Due to self-organising nature of SIs, they are suitable for dynamic problems. Note that, in addition to PSO, there is another main SI-based algorithm called ant colony optimisation (ACO). ACO is not discussed in this paper since ACO is apt to discrete optimisation problems. A more detailed review on ACO can be found in Yu and Gen (2010b).

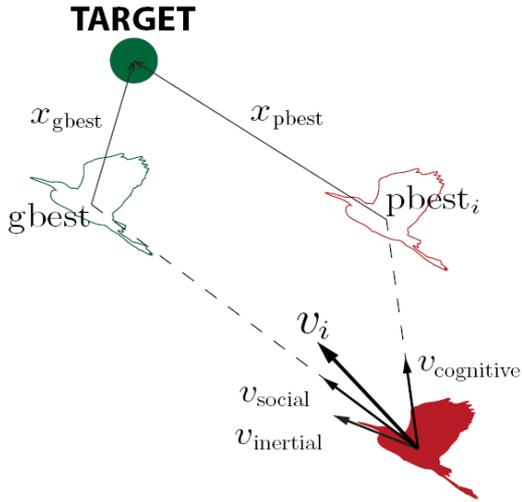
Kennedy and Eberhart (1995) invented the PSO algorithm. Many authors have compared the performance of PSO with many other EAs and global optimisation methods. For instance, PSO is compared with GA in Abraham et al. (2006), Kachitvichyanukul (2012) and Hassan et al. (2005). The results point out that PSO with its simple conceptual structure is comparable favourably with other global optimisation methods in means of its speed of convergence, its computational efficiency and its capability to optimise complex and non-linear functions.

4.2 The algorithm

As a SI-based algorithm, PSO is inspired by the social behaviour of birds within a flock. In other words, each bird in the swarm modifies its motion with the information obtained from other members of the swarm, its own experience and its current direction of motion. This makes the basic intuitive ideology of the PSO algorithm. The birds are defined as particles in the algorithm.

In PSO, each particle has a position, $x_i(t)$, which represents a solution to the fitness function. In each iteration, the particles' positions are updated with the particles' velocities of that iteration, $v_i(t)$, which show the directions of motion of the particles. The velocities of the particles are computed considering three factors: velocities of the particles toward the best experienced position of the swarm called *gbest*, velocities of the particles toward the best experienced position of each particle called *pbest* and the previous particles' velocities. Note that the position and the velocity in the algorithm do not have their physical properties. Figure 5 shows the behaviour of each particle within a swarm conceptually.

Figure 5 The concept of a particle's behaviour in PSO algorithm (see online version for colours)



PSO algorithm includes the following steps (Shi and Eberhart, 1999):

- 1 Initialise the swarm by randomly dedicating $x_i(t)$ and $v_i(t)$ of each particle.
- 2 Evaluate the performance of each particle based on its current position using a fitness function, F .
- 3 Compare the value of the fitness function of each particle to its best, $pbest_i$. If the new value is better than $pbest_i$, then

$$pbest_i = (x_i(t)) \quad (14)$$

$$x_{pbest_i} = x_i(t) \quad (15)$$

- 4 Compare $pbest_i$ to the global best performance, $gbest$. If $pbest_i$ is smaller than $gbest$, then

$$gbest = F(x_i(t)) \quad (16)$$

$$x_{gbest} = x_i(t) \quad (17)$$

- 5 Update the velocity of each particle based on following function:

$$v_i(t) = wv_i(t-1) + c_1r_1(x_{pbest_i} - x_i(t)) + c_2r_2(x_{gbest_i} - x_i(t)), \quad (18)$$

where w , called inertia weight, represents the influence of the previous velocity of the particle on the new velocity. Large w helps the swarm to explore new areas in the search space; however, it slows down the speed of convergence. Inertia weight can be constant, variable and adaptive using some approaches like fuzzy controllers (Abraham et al., 2006).

c_1 and c_2 are cognitive and social constants which do not have an essential impact on the convergence of the algorithm. Nevertheless, they might change the speed of convergence. By default $c_1 = c_2 = 2$ but experiments show that $c_1 = c_2 = 1.49$ facilitates the algorithm with

better results. Yet there is a study that reveals that $c_1 + c_2 \leq 4$ provides better results if $c_1 > c_2$ (Abraham et al., 2006). r_1 and r_2 are random constants.

- 6 Update the position of each particle according to following function:

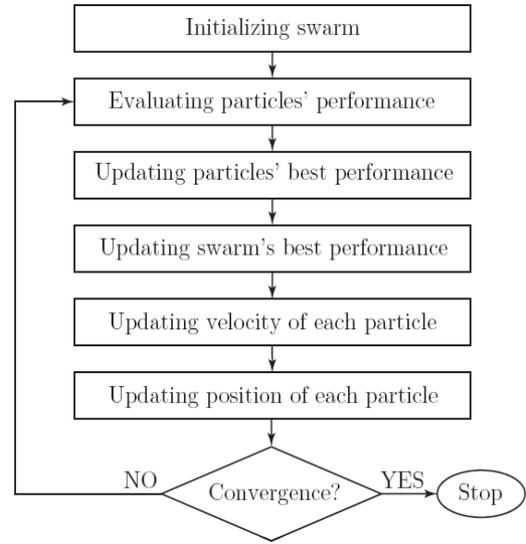
$$x_i(t) = x_i(t-1) + v_i(t) \quad (19)$$

$$t = t + 1$$

- 7 Go back to step 2. This process is repeated until convergence has been reached.

The flowchart shown in Figure 6 illustrates the aforementioned steps.

Figure 6 PSO flowchart



Instead of $gbest$, the best experienced position of the local particles called $lbest$ could be also employed to update the particles' velocities. Accordingly, steps 4 and 5 of the PSO algorithm should be modified. To introduce the neighbourhood of each particle, a number of topologies have been recommended. Using $lbest$ decreases the possibility of premature convergence for PSO algorithm; however, it slows down the speed of convergence considerably. Therefore, in this project $gbest$ is applied for optimisation.

4.3 PSO application for the fish robot

In order to apply the PSO algorithm for the optimisation of the fish robot parameters, two main elements of the algorithm, the particles and the fitness function, are defined.

In (12) and (13), seven parameters need to be optimised. The parameters are A , f , a , b , l_2 , L and C_C which are the maximum pitching motion, the pitching frequency, the vertical semi-axis of ellipsoid main body, the horizontal semi-axis of ellipsoid main body, the length of tail peduncle, the span of the caudal fin and the half chord of the caudal fin, respectively. These parameters are the dimensions of a particle's position. In this particular application of PSO algorithm, five particles are defined

whose positions have seven dimensions which are the unknown parameters of (12) and (13). Hereafter, the particles and accordingly their parameters are optimised using a fitness function.

The length of the main body, l_1 , is not selected since it is the reference size. In the other words, other parameters are modified with respect to 11.

As indicated, the PSO algorithm includes a fitness function. The fitness function is the criterion of optimisation. In case of the fish robot, the propulsive performance is the paramount criterion known as efficiency. The efficiency of fish robot is calculated by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\overline{F_c} \overline{\dot{x}}}{\overline{P}} \quad (20)$$

where $\overline{F_c}$ is the average value of the thrust and $\overline{\dot{x}}$ is the average speed (Nakashima et al., 2003). $\overline{F_c}$ and $\overline{\dot{x}}$ must be calculated under steady state condition. \overline{P} is also average consumed power by DC motor described by

$$\overline{P} = T\overline{\dot{\theta}_2}, \quad (21)$$

where T is a constant torque created by the motor.

Nonetheless, prior to obtaining the steady state condition through solving the dynamic equations, (20) cannot be employed directly. Accordingly, some changes have been made to (20) that enables the comparison of the propulsive performance of fish robot with different parameters. Those changes include neglecting T and calculating $\overline{F_c}$ based on both transient and steady state conditions.

5 Results

Mathematical modelling and optimisation is carried out with Maple 14.00. The differential equations of motion of the fish robot are solved numerically using a seventh-eighth order continuous Runge-Kutta method. After 40 iterations and selecting ρ_1 and ρ_2 as random constants between 0–2, the PSO algorithm submits the best particle including the most optimal parameters. The values of optimal parameters and fixed parameters used in the dynamic equations of fish robot are given in Table 1.

The optimised parameters are substituted into dynamic equations of motion (12) and (13) with initial conditions, that are, $x = 0$, $\dot{x} = 0.05$ m/s, $y = 0$ and $\dot{y} = 0$. The simulations show stable cruising of the robot after 15 seconds while its average speed in steady state condition is 0.14 m/s in X direction.

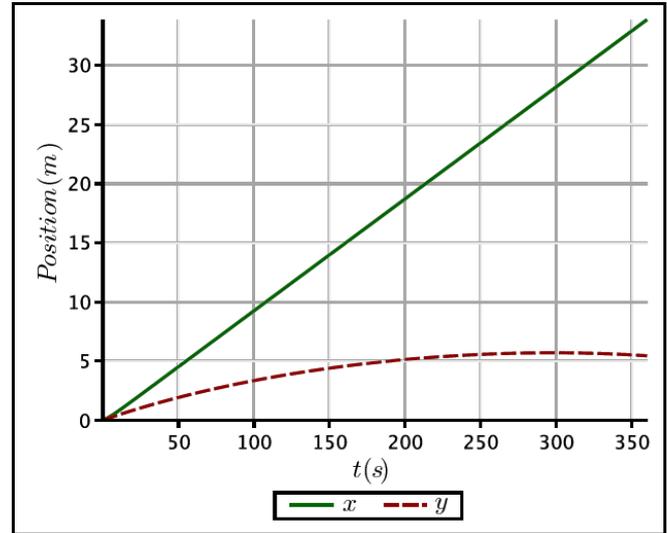
Note, as expected from the equation of motion, the robot has an offset in Y direction. In fact, when the robot starts swimming, despite \dot{y}_2 , \dot{y}_1 is zero. Accordingly, F_{Cy} is greater than F_{Dy} . However, growing \dot{y}_1 causes F_{Dy} to be equal to F_{Cy} . This helps the robot to keep its position in Y direction.

Table 1 Parameters used in the mathematical model

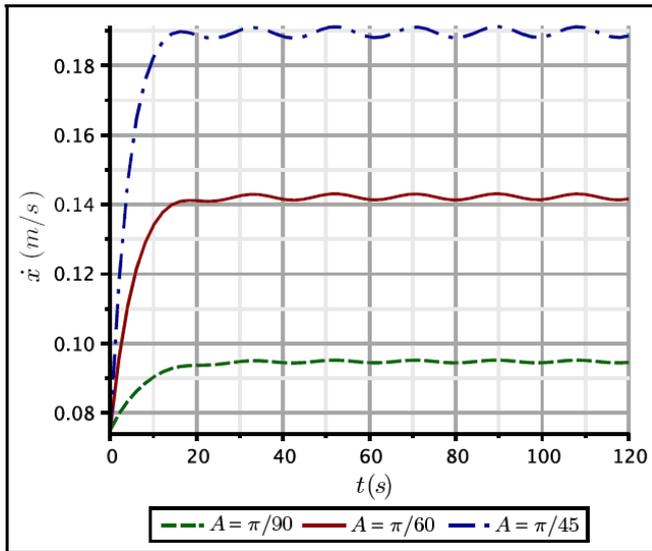
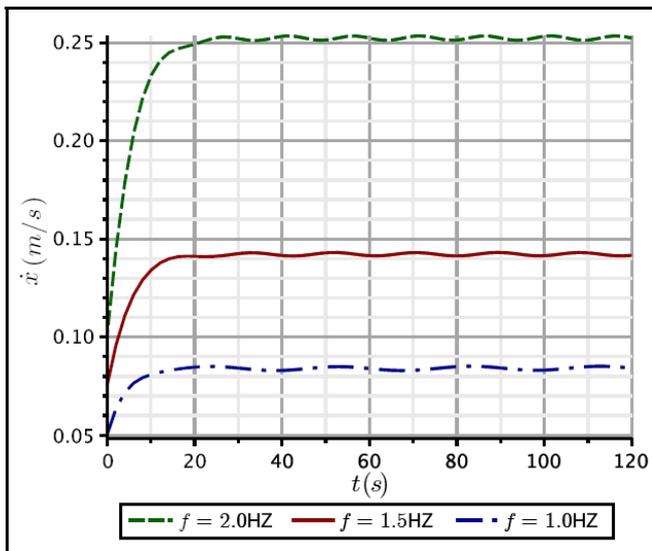
Main body	Mass (optimised using a , b and l_1)	$m_1 = 3.076$ kg
	Vertical semi-axis (optimised)	$a = 0.084$ m
	Horizontal semi-axis (optimised)	$b = 0.050$ m
	Length (fixed)	$l_1 = 0.260$ m
Tail peduncle	Mass (optimised using a , b and l_2)	$m_2 = 0.479$ kg
	Length (optimised)	$l_2 = 0.140$ m
DC motor	Amplitude (optimised)	$A = \pi/60$ rad
	Frequency (optimised)	$f = 1.502$ rad/s
Caudal fin	Span (optimised)	$L = 0.260$ m
	Chord (optimised)	$C_c = 0.070$ m
Forces	Density of water (fixed)	$\rho = 998$ kg/m ³
	Main body drag along X (fixed)	$CD_x = 0.5$
	Main body drag along Y (fixed)	$CD_y = 0.5$

Figure 7 shows displacement of the robot in X and Y directions. The displacement of the fish robot in Y direction does not rise continuously and fluctuates slightly. This is due to the fact that, F_{Dy} is growing as \dot{y}_1 is increasing.

Figure 7 Displacement of fish robot in X and Y direction (see online version for colours)



Comparison of the optimal with non-optimal parameters in simulations enlightens some other points. Principally, having a fast fish robot does not necessary mean that the robot is efficient. For example, out of the optimised parameters used in the dynamic equations, two parameters, A and f have considerable effects on the robot's behaviour. As Figures 8 and 9 illustrate, increasing A and f leads to a faster robot. However, the increased values of A and f are no longer optimal since amplitude and frequency initially influence $\dot{\theta}_2$ and averaged consumed power by motor, (21). Therefore, η expressed by (20) is reduced.

Figure 8 Speed of fish robot using different amplitudes (see online version for colours)**Figure 9** Speed of fish robot using three different frequencies (see online version for colours)

The effects of other parameter variations are also studied. Simulations illustrate that increasing l_2 , L and C_C and decreasing a and b helps the robot to move faster. Considering (3), (8) and (9), l_2 is directly proportional to \dot{x}_1 , \dot{y}_1 , F_L and F_I . L and C_C have also proportional relationship with F_L and F_I . Thus, increasing l_2 , L and C_C cause increased speed of the robot. On the other hand, enlarging the fish sizes, a and b , increases drag forces (6) and (7), and accordingly, slows down the robot.

6 Summary

To facilitate the efficient motion for the fish robot, the parameters of its dynamic equation of motion including robot dimensions, frequency and amplitude of tail oscillation need to be optimised. Accordingly, the

mathematical model of the robot is complemented by an optimisation algorithm called PSO. The robot will go to its steady state condition after 15 seconds. At steady state condition the robot has an average speed of 0.14 m/s.

Simulations show that changing the parameters affects the robot's velocity. In the other words, the speed of the fish increases as A , f , l_2 , L and C_C are increased and a , b are decreased. Nevertheless, having a faster fish robot does not necessarily mean that the robot is more efficient. This is due to the fact that η is not only the function of average value of thrust, \bar{F}_c , and average speed, \bar{x} , but also affected by the average consumed power, P .

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