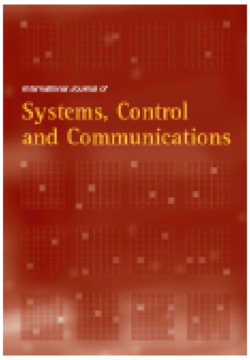

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Synchronisation among multiple chaotic systems connected in chain and star configuration using backstepping strategy

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Abstract: The present paper addresses synchronisation of multiple chaotic systems using backstepping methodology. In this approach, multiple chaotic systems belonging to a proposed class of nonlinear systems and connected in chain and star configuration are considered. The controllers are designed by developing generalised error dynamics between consecutive systems in the networks and systematic stage by stage procedure blended with Lyapunov theory is used to obtain the structure of controllers. The procedure, thus, is a generalised synchronisation method for interconnection of a class of systems with single nonlinearity. Numerical simulations on chain and star interconnection of chaotic Chua's systems are carried out in order to demonstrate the efficacy and feasibility of this proposed approach. The proposed approach can provide a method, which is applicable for designing combinational synchronisation among multiple chaotic systems in various other configurations.

Keywords: synchronisation; backstepping method; chain configuration; star configuration; Lyapunov stability criteria.

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1 Introduction

Chaos theory is a branch of nonlinear science with great significance. Due to their irregular attractors, whose trajectories do not cut through each other, the exploration of chaotic systems has become an interesting research topic. Chaotic systems exhibit high sensitivity to initial conditions, i.e., a minor change in initial conditions leads to drastic changes in the system states leading to unpredictable behaviour, which makes them unique in nature. Chaos phenomenon has been widely observed in many systems. Various chaotic systems, which include Lorenz system (Lorenz, 1963), Chen system (Chen and Ueta, 1999), Chua system (Chua et al., 1986; Jang et al., 2002), Rossler (1976) system, Lu system (Lu and Chen, 2002), Genesio system (Ramezanpour et al., 2012), etc., have been developed over last few decades. Although the primary experimentation on chaotic system was done in meteorology, applications of control and synchronisation are widely observable in engineering fields. As the systems are sensitive to initial conditions, the synchronisation of chaotic systems without losing chaotic behaviour was a major challenge to the researchers. Significant efforts have been put into making the set of chaotic systems synchronised. Since introduction of pioneering work by Pecora and Carrol (1990), synchronisation of chaotic systems has been thoroughly investigated by many researchers. We can find application of synchronisation in many fields including secure communications, robotics, chemical reactions, ecological systems, and engineering sciences.

Synchronisation has been explored in different perspectives in available literature. Since last few decades, numerous research studies have been conducted to achieve various types of synchronisations, e.g., complete synchronisation (Mahmoud and Mahmoud, 2010; Vaidyanathan, 2013), phase synchronisation (Pikovsky et al., 2000, 2001), projective synchronisation (Ghosh, 2009; Li, 2007), generalised synchronisation (He and Cao, 2009; Juan and Xingyuan, 2008), etc. Majority of these research studies, focus on synchronisation of a couple of systems, of which one is master system and another is slave system. Runzi et al. (2011) found out a different kind of synchronisation called combination synchronisation, which was utilised to study synchronisation of multiple drive systems with single response system. Thereafter, many researchers have addressed synchronisation challenges of multiple chaotic systems. Various kinds of combination synchronisation have been explored in recent literature. The adaptive function projective combination synchronisation was recommended by Xi et al. (2015). Multiswitching combination synchronisation of chaotic systems was developed by Vincent et al. (2015). Zhou et al. (2014) proposed synchronisation between two combinations of chaotic systems. Combination-combination synchronisation of hyperchaotic systems was developed by Khan and Singh (2018). Combination synchronisation of Lotka-Volterra chaotic systems was proposed by Jahanzaib et al. (2020). Several methods have been developed till date to attain stabilisation and synchronisation of nonlinear systems including chaotic systems in various configurations. Synchronisation of chaotic systems in ring configuration was proposed by Iqbal et al. (2018). Synchronisation of complex network of chaotic systems was developed by Lopez-Mancilla et al. (2019). Anand and Sharma (2020) proposed synchronisation of a class of chaotic systems in chain configuration. The methods used for synchronisation include techniques like backstepping control (Shukla and Sharma, 2017; Tao et al., 2001), active control (Mahmoud et al., 2007; Tang et al., 2009), active backstepping control (Wu and Fu, 2013; Yu et al., 2011), adaptive control (Noroozi et al., 2009; Sharma and Kar, 2008), sliding mode control (Chiang et al., 2007; Singh and Sharma, 2012), observer based synchronisation (Teh-Lu and Nan-Sheng, 1999; Sharma and Kar, 2011), variable structure control (Liaoying et al., 2006), and contraction theory based approach (Chauhan and Sharma, 2020; Solís-Perales et al., 2012; Sharma and Kar, 2009), etc. Backstepping technique is a popular method for recursive design of controllers to stabilise nonlinear systems in strict-feedback form. Here, in present work, backstepping control methodology is extended to attain synchronisation of combinations of multiple chaotic systems in a network. This procedure helps in designing a single controller using step by step systematic procedure. Lyapunov stability theory has been used for this purpose. In this method, an appropriate Lyapunov function is required to be constructed at each stage of the procedure to determine the suitable virtual control input which ensures the stability of each subsystem. The virtual control input derived in the last stage by following the procedure is the true controller and that globally stabilises the equilibrium point of the parametric system.

As so far, majority of research on combinational synchronisation has been used to synchronise a combination of drive systems with response systems in master-slave fashion. Existing recent research work (Wang et al., 2019) on synchronisation of multiple chaotic systems is proposed by considering some example systems as drive and response systems. The research work on chain configuration using contraction was addressed by Anand and Sharma (2020). In the work presented here, synchronisation of a group of chaotic systems altogether has been addressed by using backstepping strategy blended with Lyapunov stability theory. A variety of chaotic systems having single nonlinearity has been taken into consideration. The chain and star configurations have been considered for deriving the results and a generalised procedure is developed to achieve error dynamics between consecutive systems. With applications of derived backstepping based controller, error dynamics between different consecutive systems have been shown to be convergent, thus, leading to overall synchronisation. For representative case, example is taken involving four similar Chua's chaotic systems in required chain and star configurations, and efficacy of the proposed strategy is established using extensive simulations. The proposed method may have extensive applications in robotics and communication systems, where multiple chaotic systems need to be synchronised in proposed configurations.

Overall contribution made in this paper can be highlighted as below:

- To obtain general control design framework for synchronisation of multiple chaotic systems belonging to proposed class of systems in chain and star configuration.
- To derive analytical procedure for controller design by blending Lyapunov stability theory with backstepping technique for each pair of consecutive systems.

- To numerically validate the results for establishing efficacy of the proposed procedure.

This paper is organised into seven sections of which Section 2 describes the basic configuration of proposed scheme and design strategy, Section 3 deals with the mathematical descriptions of proposed chain and star network configurations, Section 4 shows backstepping technique based generalised procedure for deriving controller, controller design for example chaotic systems in required configurations has been discussed in Section 5. Section 6 exhibits numerical simulations for justifying the efficacy of proposed analytical methodology, and concluding remarks related to the research work are presented in Section 7.

2 Proposed scheme for synchronisation of multiple chaotic systems

General configurations of chaotic systems utilised to address goal of synchronisation are elaborated in this section. Here, according to the proposed scheme, a family of chaotic systems has to be made synchronised to each other. The arrangement of the systems may be in any arbitrary configurations which include some particular arrangements like chain, star and ring configurations. Here, for a network of chaotic systems forming chain and star arrangements, backstepping methodology based on Lyapunov theory is utilised for synchronisation purpose. The arrangements of chaotic systems in the form of chain and star are depicted in Figure 1(a) and Figure 1(b), respectively, where S_i; $i = 1, 2, 3, \ldots\ldots,$ m represents the individual chaotic systems and u_1, u_2., u_3, ……, u_{m-1} are the associated control inputs required to be appropriately designed to fulfil the requirement of synchronisation. Here, individual system S_i may represent i^{th} system of network having n-dimensional dynamics.

3 Mathematical descriptions of proposed system configurations

Let us consider the case where each configuration consists of m-similar individual systems with single nonlinearity and every system individually comprises of n-states.

The system-dynamics, which are part of chain and star network, are assumed to be described as following:

- First system (S_1):

$$\begin{aligned} \dot{s}_1 &= f_1(s_1) + g_1 s_2 \\ \dot{s}_2 &= f_2(s_1, s_2) + g_2 s_3 \\ \dot{s}_3 &= f_3(s_1, s_2, s_3) + g_3 s_4 \\ &\vdots \\ \dot{s}_n &= f_n(s_1, s_2, s_3, \ldots, s_n) \end{aligned} \tag{1}$$

- Second system (S_2):

$$
\begin{aligned}
\dot{s}_{n+1} &= f_1\left(s_{n+1}\right) + g_1 s_{n+2} \\
\dot{s}_{n+2} &= f_2\left(s_{n+1}, s_{n+2}\right) + g_2 s_{n+3} \\
\dot{s}_{n+3} &= f_3\left(s_{n+1}, s_{n+2}, s_{n+3}\right) + g_3 s_{n+4} \\
&\vdots \\
\dot{s}_{2n} &= f_n\left(s_{n+1}, s_{n+2}, s_{n+3}, \ldots, s_{2n}\right) + u_1
\end{aligned}
\tag{2}
$$

The dynamics describing i^{th} system (S_i) is written as:

$$
\begin{aligned}
\dot{s}_{(i-1)n+1} &= f_1\left(s_{(i-1)n+1}\right) + g_1 s_{(i-1)n+2} \\
\dot{s}_{(i-1)n+2} &= f_2\left(s_{(i-1)n+1}, s_{(i-1)n+2}\right) + g_2 s_{(i-1)n+3} \\
\dot{s}_{(i-1)n+3} &= f_3\left(s_{(i-1)n+1}, s_{(i-1)n+2}, s_{(i-1)n+3}\right) + g_3 s_{(i-1)n+4} \\
&\vdots \\
\dot{s}_{in} &= f_n\left(s_{(i-1)n+1}, s_{(i-1)n+2}, s_{(i-1)n+3}, \ldots, s_{in}\right) + u_{(i-1)}
\end{aligned}
\tag{3}
$$

On the similar lines, dynamics of j^{th} system (S_j) can be expressed as follows:

$$
\begin{aligned}
\dot{s}_{(j-1)n+1} &= f_1\left(s_{(j-1)n+1}\right) + g_1 s_{(j-1)n+2} \\
\dot{s}_{(j-1)n+2} &= f_2\left(s_{(j-1)n+1}, s_{(j-1)n+2}\right) + g_2 s_{(j-1)n+3} \\
\dot{s}_{(j-1)n+3} &= f_3\left(s_{(j-1)n+1}, s_{(j-1)n+2}, s_{(j-1)n+3}\right) + g_3 s_{(j-1)n+4} \\
&\vdots \\
\dot{s}_{jn} &= f_n\left(s_{(j-1)n+1}, s_{(j-1)n+2}, s_{(j-1)n+3}, \ldots, s_{jn}\right) + u_{(j-1)}
\end{aligned}
\tag{4}
$$

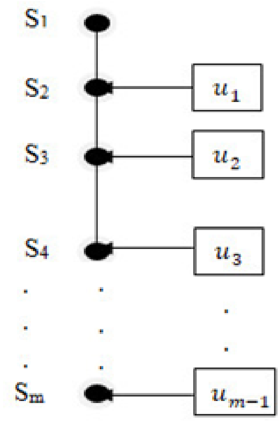
Finally, the dynamics of last system, i.e., m^{th} system (S_m) in proposed configurations is written as:

$$
\begin{aligned}
\dot{s}_{(m-1)n+1} &= f_1\left(s_{(m-1)n+1}\right) + g_1 s_{(m-1)n+2} \\
\dot{s}_{(m-1)n+2} &= f_2\left(s_{(m-1)n+1}, s_{(m-1)n+2}\right) + g_2 s_{(m-1)n+3} \\
\dot{s}_{(m-1)n+3} &= f_3\left(s_{(m-1)n+1}, s_{(m-1)n+2}, s_{(m-1)n+3}\right) + g_3 s_{(m-1)n+4} \\
&\vdots \\
\dot{s}_{mn} &= f_n\left(s_{(m-1)n+1}, s_{(m-1)n+2}, s_{(m-1)n+3}, \ldots, s_{mn}\right) + u_{(m-1)}
\end{aligned}
\tag{5}
$$

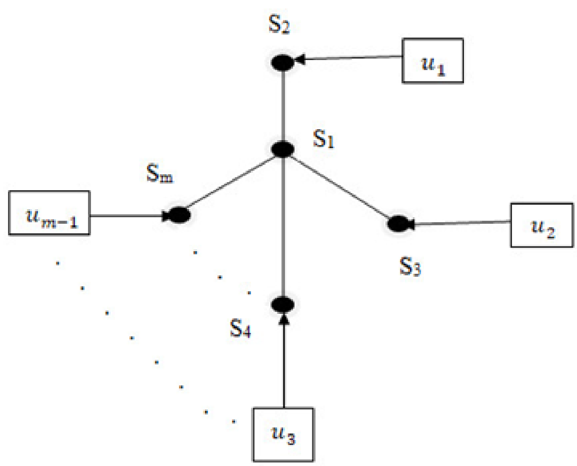
Here, in equations (1)–(5), f_i is a linear state-function for, $i = 1, 2, 3, \ldots, n - 1$ and function is f_n considered to involve nonlinearity. u_k; k = 1, 2, 3, …, (m – 1) is the control input associated with the (k + 1)$^{\text{th}}$ system($S_{(k+1)}$).

The goal is to develop a suitable control strategy to achieve synchronisation of systems in Figures 1(a) and 1(b) to each other.

Figure 1 Different arrangements of multiple chaotic systems in a network, (a) systems in chain structure and (b) systems in star structure



(a)



(b)

Remark 1: Numerous nonlinear oscillators fall in the category of system mentioned in (1). These include Duffing oscillator, Van der Pol oscillator, etc. Further, these could be category of systems which can be brought to the category of systems in equation (1) by suitable transformation. For example, cubic Chua's system can be presented in the form of equation (1) by applying suitable transformation.

4 Generalised procedure for deriving controller

To synchronise the systems altogether in chain and star arrangement as exhibited in Figure 1(a) and Figure 1(b), the error between the consecutive pairs of systems should be made to zero by designing appropriate control function.

From the equations mentioned in (3) and (4), the error between i^{th} and j^{th} system can be defined as:

$$e_{i,k} = s_{(i-1)n+k} - s_{(j-1)n+k};\ k = 1, 2, 3, \ldots\ldots\ldots, n \tag{6}$$

By using the dynamics given in equations (1) to (5), the error dynamics for the errors described in equation (6), can be presented as:

$$\begin{aligned} \dot{e}_{i,1} &= \varphi_1\left(e_{i,1}\right) + g_1 e_{i,2} \\ \dot{e}_{i,2} &= \varphi_2\left(e_{i,1}, e_{i,2}\right) + g_2 e_{i,3} \\ \dot{e}_{i,3} &= \varphi_3\left(e_{i,1}, e_{i,2}, e_{i,3}\right) + g_3 e_{i,4} \\ &\vdots \\ \dot{e}_{i,n} &= \varphi_n\left(e_{i,1}, e_{i,2}, \ldots, e_{i,n}, s_{(j-1)n+1}, s_{(j-1)n+2}, \ldots, s_{jn}\right) + u_{i-1} - u_{j-1} \end{aligned} \tag{7}$$

where φ_i; $i = 1, 2, 3, \cdots, (n-1)$ will be linear function of error states as f_i; $i = 1, 2, 3, \cdots, (n-1)$ are linear state-functions of respective system states, whereas φ_n is a function involving nonlinearity.

Here, for synchronisation, the control term $(u_{i-1} - u_{j-1})$ should be designed such that the error dynamics in equation (7) converges to zero as $t \to \infty$.

The backstepping technique-based controller design procedure for this purpose is presented in the form of following theorem:

Theorem 1: For the systems given in equations (3) and (4), with error given in equation (6), the error dynamics in equation (7) will converge to zero as $t \to \infty$, if the controller $(u_{i-1} - u_{j-1})$ is designed as:

$$\begin{aligned} (u_{i-1} - u_{j-1}) = -\alpha_n v_{i,n} - \varphi_n &\left[v_{i,1}, v_{i,2} + \theta_1(v_{i,1}), \cdots, v_{i,n}\right. \\ &\left. + \theta_{n-1}(v_{i,n-1}), s_{(i-1)n+1}, s_{(i-1)n+2}, \cdots, s_{in}\right] \\ &- g_{n-1} v_{i,n-1} + \dot{\theta}_{n-1}\left(v_{i,1}, v_{i,2}, \cdots, v_{i,n-1}\right) \end{aligned} \tag{8}$$

where θ_k; $k = 1, 2, 3, \cdots, (n-1)$; represents the estimative virtual control input of $e_{i,k+1}$ in terms of auxiliary variables; $i = 1, 2, 3, \cdots, m$ and $k = 1, 2, 3, \cdots, (n-1)$.

Here, $v_{i,k}$; $k = 1, 2, 3, \cdots, n$ and $i = 1, 2, 3, \cdots, m$; represents the auxiliary variables for $e_{i,k}$. Further, α_k; $k = 1, 2, 3, \cdots, n$; is a constant having positive value.

Proof: For the error dynamics given in equation (7), if auxiliary variable $v_{i,1}$ is chosen as, $e_{i,1} = v_{i,1}$ and estimate of $e_{i,2}$ is assumed as $e_{i,2} = \theta_1(v_{i,1})$ then, the first subsystem can be represented as:

$$\dot{v}_{i,1} = \varphi_1\left(v_{i,1}\right) + g_1\theta_1\left(v_{i,1}\right) \tag{9}$$

For this subsystem in equation (9), the Lyapunov function candidate can be expressed as:

$$L_1 = \frac{1}{2} v_{i,1}^2 \tag{10}$$

Time derivative of this energy like function is given as:

$$\dot{L}_1 = v_{i,1}\left[\varphi_1\left(v_{i,1}\right) + g_1\theta_1\left(v_{i,1}\right)\right] \tag{11}$$

If $\theta_1(v_{i,1})$ is chosen such that:

$$\theta_1(v_{i,1}) = -\frac{1}{g_1}\left[\varphi_1(v_{i,1}) + \alpha_1 v_{i,1}\right] \tag{12}$$

then, derivative of the Lyapunov function in equation (10) comes out as:

$$\dot{L}_1 = -\alpha_1 v_{i,1}^2 \tag{13}$$

The negative definite nature of $\dot{L}_1$ ensures the stability of the $v_{i,1}$ subsystem equation (9) for the virtual control input $\theta_1(v_{i,1})$ estimated in equation (12).

As $\theta_1(v_{i,1})$ is an estimated function of $e_{i,2}$, so the auxiliary variable for the second subsystem is defined as:

$$v_{i,2} = e_{i,2} - \theta_1(v_{i,1}) \tag{14}$$

Along with this, $e_{i,3}$ is estimated as:

$$e_{i,3} = \theta_2(v_{i,1}, v_{i,2}) \tag{15}$$

As a result, the second subsystem becomes as follows:

$$\begin{aligned} \dot{v}_{i,1} &= -\alpha_1 v_{i,1} + g_1 v_{i,2} \\ \dot{v}_{i,2} &= \varphi_2\left(v_{i,1}, v_{i,2} - \frac{\varphi_1(v_{i,1})}{g_1} + \frac{\alpha_1}{g_1} v_{i,1}\right) + g_2\theta_2(v_{i,1}, v_{i,2}) - \dot{\theta}_1(v_{i,1}) \end{aligned} \tag{16}$$

Lyapunov function for the description mentioned in equation (16) can be assumed as:

$$L_2 = \frac{1}{2}\left(v_{i,1}^2 + v_{i,2}^2\right) \tag{17}$$

Using dynamics given in equation (16), the first derivative of the above function in equation (17) can be expressed as:

$$\begin{aligned} \dot{L}_2 &= -\alpha_1 v_{i,1}^2 + g_1 v_{i,1} v_{i,2} \\ &\quad + v_{i,2}\left[\varphi_2\left(v_{i,1}, v_{i,2} - \frac{\varphi_1(v_{i,1})}{g_1} + \frac{\alpha_1}{g_1} v_{i,1}\right) + g_2\theta_2(v_{i,1}, v_{i,2}) - \dot{\theta}_1(v_{i,1})\right] \end{aligned} \tag{18}$$

If $\theta_2(v_{i,1}, v_{i,2})$ is chosen such that:

$$\theta_2(v_{i,1}, v_{i,2}) = \frac{1}{g_2}\left[-\varphi_2\left(v_{i,1}, v_{i,2} - \frac{\varphi_1(v_{i,1})}{g_1} + \frac{\alpha_1}{g_1} v_{i,1}\right) - \alpha_2 v_{i,2} - g_1 v_{i,1} + \dot{\theta}_1(v_{i,1})\right] \tag{19}$$

then, derivative of the Lyapunov function in equation (17) becomes as:

$$\dot{L}_2 = -\alpha_1 v_{i,1}^2 - \alpha_2 v_{i,2}^2 \tag{20}$$

It is evident from equation (20) that $\dot{L}_2 < 0$ which implies the fact that the virtual control input estimated in equation (19) can make the $v_{i,1}$, $v_{i,2}$ subsystem asymptotically stable.

As $\theta_2(v_{i,1}, v_{i,2})$ is estimated virtual control input of $e_{i,3}$, the auxiliary variable $v_{i,3}$ for next subsystem can be written as:

$$v_{i,3} = e_{i,3} - \theta_2(v_{i,1}, v_{i,2}) \tag{21}$$

If $e_{i,4}$ is estimated as:

$$e_{i,4} = \theta_3(v_{i,1}, v_{i,2}, v_{i,3}) \tag{22}$$

then, the third subsystem becomes as following:

$$\begin{aligned} \dot{v}_{i,1} &= -\alpha_1 v_{i,1} + g_1 v_{i,2} \\ \dot{v}_{i,2} &= -\alpha_2 v_{i,2} - g_1 v_{i,1} + g_2 v_{i,3} \\ \dot{v}_{i,3} &= \varphi_3\left(v_{i,1}, v_{i,2} + \theta_1\left(v_{i,1}\right), v_{i,3} + \theta_2\left(v_{i,1}, v_{i,2}\right)\right) + g_3\theta_3\left(v_{i,1}, v_{i,2}, v_{i,3}\right) - \dot{\theta}_2\left(v_{i,1}, v_{i,2}\right) \end{aligned} \tag{23}$$

Lyapunov function candidate for the descriptions of subsystem in equation (23) can be assumed as:

$$L_3 = \frac{1}{2}\left(v_{i,1}^2 + v_{i,2}^2 + v_{i,3}^2\right) \tag{24}$$

which leads to:

$$\begin{aligned} \dot{L}_3 = & -\alpha_1 v_{i,1}^2 - \alpha_2 v_{i,2}^2 + g_2 v_{i,2} v_{i,3} + v_{i,3}\Big[\varphi_3\left(v_{i,1}, v_{i,2} + \theta_1\left(v_{i,1}\right), v_{i,3} + \theta_2\left(v_{i,1}, v_{i,2}\right)\right) \\ & + g_3\theta_3\left(v_{i,1}, v_{i,2}, v_{i,3}\right) - \dot{\theta}_2\left(v_{i,1}, v_{i,2}\right)\Big] \end{aligned} \tag{25}$$

If $\theta_3(v_{i,1}, v_{i,2}, v_{i,3})$ is chosen as:

$$\begin{aligned} \theta_3\left(v_{i,1}, v_{i,2}, v_{i,3}\right) = & \frac{1}{g_3}\Big[-\varphi_3\left(v_{i,1}, v_{i,2} + \theta_1\left(v_{i,1}\right), v_{i,3} + \theta_2\left(v_{i,1}, v_{i,2}\right)\right) \\ & -\alpha_3 v_{i,3} - g_2 v_{i,2} + \dot{\theta}_2\left(v_{i,1}, v_{i,2}\right)\Big] \end{aligned} \tag{26}$$

then, the time derivative of Lyapunov function in equation (24) for this subsystem can be derived as:

$$\dot{L}_3 = -\alpha_1 v_{i,1}^2 - \alpha_2 v_{i,2}^2 - \alpha_3 v_{i,3}^2 \tag{27}$$

The result in equation (27) ensures the stability of ($v_{i,1}$, $v_{i,2}$, $v_{i,3}$) for the estimation mentioned in equation (26).

Now, continuing the procedure up to nth term, we get:

$$\begin{aligned} \dot{L}_n = & -\alpha_1 v_{i,1}^2 - \alpha_2 v_{i,2}^2 - \alpha_3 v_{i,3}^2 \cdots - \alpha_{n-1} v_{i,n-1}^2 + g_{n-1} v_{i,n-1} v_{i,n} \\ & + v_{i,n}\Big[\varphi_n\left(v_{i,1}, v_{i,2} + \theta_1\left(v_{i,1}\right), \cdots, v_{i,n} + \theta_{n-1}\left(v_{i,n-1}\right), s_1, s_2, \cdots, s_n\right) \\ & -\dot{\theta}_{n-1}\left(v_{i,1}, v_{i,2}, \cdots, v_{i,n-1}\right) + u_{i-1} - u_{j-1}\Big] \end{aligned} \tag{28}$$

If the control input associated with the error dynamics is chosen as:

$$\begin{aligned} u_{i-1} - u_{j-1} = & -\alpha_n v_{i,n} - \varphi_n\left(v_{i,1}, v_{i,2} + \theta_1\left(v_{i,1}\right), \cdots, v_{i,n}\right. \\ & \left. + \theta_{n-1}\left(v_{i,1}, v_{i,2}, \cdots, v_{i,n-1}\right), s_1, s_2, \cdots, s_n\right) \\ & - g_{n-1} v_{i,n-1} + \dot{\theta}_{n-1}\left(v_{i,1}, v_{i,2}, \cdots, v_{i,n-1}\right) \end{aligned} \tag{29}$$

then, the equation (29) consisting of the description of the derivative of the energy-like function assumed for nth subsystem becomes as:

$$\dot{L}_n = -\alpha_1 v_{i,1}^2 - \alpha_2 v_{i,2}^2 - \cdots - \alpha_n v_{i,n}^2 \tag{30}$$

which shows negative definiteness, hence, establishing the stability of the error dynamics as $v_{i,1}, v_{i,2}, \cdots, v_{i,n}$ converge to zero with time $t \to \infty$. As $v_{i,1} = e_{i,1}$ so $e_{i,1}$ converges to zero. Also, as $v_{i,l} = e_{i,l}(v_{i,1}, v_{i,2}, \cdots, v_{i,l-1})$ with $l = 2, 3, \cdots, n$, convergence of $v_{i,l} \to 0 \Rightarrow e_{i,l} \to 0$ with $t \to \infty$. Hence, stabilisation of error dynamics in equation (7) is achieved leading to synchronisation of overall system in assumed network configurations. Thus, synchronisation conditions are established using the proposed control configuration in equation (29) for any network with structure highlighted in Sections 2 and 3.

5 Controller design for example systems

Here, in this section, the controllers have been designed for obtaining synchronisation of example systems connected in the form of chain and star by using proposed analytical procedure. For this purpose, systematic procedure using backstepping based on Lyapunov criteria is fully utilised to constitute the control functions. Example of Chua's system with cubic nonlinearity has been used for this purpose. For representative case, four such systems have been considered. The dynamics of the systems can be presented as:

First system:

$$\begin{aligned} \dot{x}_1 &= -kx_2 \\ \dot{x}_2 &= x_3 - x_2 + x_1 \\ \dot{x}_3 &= m\left(x_2 - x_3^3 - lx_3\right) \end{aligned} \tag{31}$$

Second system:

$$\begin{aligned} \dot{y}_1 &= -ky_2 \\ \dot{y}_2 &= y_3 - y_2 + y_1 \\ \dot{y}_3 &= m\left(y_2 - y_3^3 - ly_3\right) + u_1 \end{aligned} \tag{32}$$

Third system:

$$\begin{aligned} \dot{z}_1 &= -kz_2 \\ \dot{z}_2 &= z_3 - z_2 + z_1 \\ \dot{z}_3 &= m\left(z_2 - z_3^3 - lz_3\right) + u_2 \end{aligned} \tag{33}$$

Fourth system:

$$\begin{aligned} \dot{w}_1 &= -kw_2 \\ \dot{w}_2 &= w_3 - w_2 + w_1 \\ \dot{w}_3 &= m\left(w_2 - w_3^3 - lw_3\right) + u_3 \end{aligned} \tag{34}$$

The uncontrolled systems display chaoticity for the parameter-values $k = 16$, $m = 10$, and $l = -0.143$, respectively. Here, u_1, u_2 and u_3 are control functions associated with corresponding systems which are to be designed suitably to make them synchronised with each other.

5.1 Chain configuration

In this case, first system is connected with second system; second system is connected with third system; and third is connected with fourth system in configuration as presented in Figure 1(a).

The errors between the states of first system and second system are defined as:

$$\begin{aligned} e_{1,1} &= y_1 - x_1 \\ e_{1,2} &= y_2 - x_2 \\ e_{1,3} &= y_3 - x_3 \end{aligned} \tag{35}$$

Similarly, errors between the states of second and third system can be written as:

$$\begin{aligned} e_{2,1} &= z_1 - y_1 \\ e_{2,2} &= z_2 - y_2 \\ e_{2,3} &= z_3 - y_3 \end{aligned} \tag{36}$$

Finally, errors between the states of third and fourth system can be expressed as:

$$\begin{aligned} e_{3,1} &= w_1 - z_1 \\ e_{3,2} &= w_2 - z_2 \\ e_{3,3} &= w_3 - z_3 \end{aligned} \tag{37}$$

Using the above-mentioned definitions of errors and substituting the dynamics of Chua's system given in equations (31)–(34), we can obtain error dynamics between consecutive systems in chain configuration as:

$$\begin{aligned} \dot{e}_{1,1} &= -ke_{1,2} \\ \dot{e}_{1,2} &= e_{1,3} - e_{1,2} + e_{1,1} \\ \dot{e}_{1,3} &= me_{1,2} - m\left(e_{1,3}^3 + 3x_3^2 e_{1,3} + 3x_3 e_{1,3}^2\right) - mle_{1,3} + u_1 \end{aligned} \tag{38}$$

$$\begin{aligned} \dot{e}_{2,1} &= -ke_{2,2} \\ \dot{e}_{2,2} &= e_{2,3} - e_{2,2} + e_{2,1} \\ e_{2,3} &= me_{2,2} - m\left(e_{2,3}^3 + 3y_3^2 e_{2,3} + 3y_3 e_{2,3}^2\right) - mle_{2,3} + u_2 - u_1 \end{aligned} \tag{39}$$

$$\begin{aligned} \dot{e}_{3,1} &= -ke_{3,2} \\ \dot{e}_{3,2} &= e_{3,3} - e_{3,2} + e_{3,1} \\ \dot{e}_{3,3} &= me_{3,2} - m\left(e_{3,3}^3 + 3z_3^2 e_{3,3} + 3z_3 e_{3,3}^2\right) - mle_{3,3} + u_3 - u_2 \end{aligned} \tag{40}$$

The control inputs u_1, u_2 and u_3 have to be designed by applying Lyapunov-based backstepping strategy so that error dynamics can be made convergent as time increases.

As a result, the different systems which are part of chain network will be synchronised with each other.

From the result mentioned in equation (8) and with error dynamics in equation (38), the controller can be derived as:

$$\begin{aligned} u_1 &= -\alpha_3 v_{1,3} + me_{1,2} - m\left(e_{1,3}^3 + 3x_3^2 e_{1,3} + 3x_3 e_{1,3}^2\right) - mle_{1,3} \\ &\quad - v_{1,2} + \dot{\theta}_2(v_{1,1}, v_{1,2}) \end{aligned} \tag{41}$$

By taking $\alpha_3 = 1$ and putting value of $\theta_2(v_{1,1}, v_{1,2})$, the controller can be rewritten as:

$$\begin{aligned} u_1 &= v_{1,1}\left(-\frac{m}{k} + ml(k-1) - (2k-1)\right) \\ &\quad + v_{1,2}\left(-m - ml - k(k-1)\right) \\ &\quad + v_{1,3}(ml-2) + m\left(y_3^3 - x_3^3\right) \end{aligned} \tag{42}$$

Remark 2: $\alpha_3; k = 1, 2, 3, \cdots, n$ can be chosen as any arbitrary positive value. Here, for convenience of computations α_3 is chosen as 1.

If the values of $v_{1,1}$, $v_{1,2}$ and $v_{1,3}$ are substituted in terms of (x_1, x_2, x_3) and (y_1, y_2, y_3) then the controller structure comes out as:

$$\begin{aligned} u_1 &= \left(y_1 - x_1\right)\left(k + \frac{2}{k} - 2\right) \\ &\quad + \left(y_2 - x_2\right)\left(-2 - m - k(k-1)\right) \\ &\quad + \left(y_3 - x_3\right)(ml-2) + m\left(y_3^3 - x_3^3\right) \end{aligned} \tag{43}$$

Following similar procedure in case of 2nd error dynamics equation (39), the control function is derived as:

$$\begin{aligned} u_2 &= u_1 + \left(z_1 - y_1\right)\left(k + \frac{2}{k} - 2\right) \\ &\quad + \left(z_2 - y_2\right)\left(-2 - m - k(k-1)\right) \\ &\quad + \left(z_3 - y_3\right)(ml-2) + m\left(z_3^3 - y_3^3\right) \end{aligned} \tag{44}$$

Similarly, in case of third error dynamics equation (40), controller structure comes out as:

$$\begin{aligned} u_3 &= u_2 + \left(w_1 - z_1\right)\left(k + \frac{2}{k} - 2\right) \\ &\quad + \left(w_2 - z_2\right)\left(-2 - m - k(k-1)\right) \\ &\quad + \left(w_3 - z_3\right)(ml-2) + m\left(w_3^3 - z_3^3\right) \end{aligned} \tag{45}$$

So, using controllers in equations (43), (44), and (45), the error between the consecutive systems in chain configuration can be made convergent to zero. The error subsystems, which are involved between different successive pairs, can be made stabilised by using the control input obtained from the result mentioned in equation (8). It leads to overall synchronisation of all the systems forming chain structure, thus meeting out the desired objective.

5.2 Star configuration

The same Chua's system having third order nonlinearity as mentioned in equations (31)–(34) are considered here as part of star network and appropriate controllers are designed using the procedure highlighted in Section 4.

In this case, first system is connected with second, third and fourth system individually in configuration as presented in Figure 1(b).

The errors between states of first and second system for this configuration can be defined as:

$$\begin{aligned} e_{1,1} &= y_1 - x_1 \\ e_{1,2} &= y_2 - x_2 \\ e_{1,3} &= y_3 - x_3 \end{aligned} \tag{46}$$

Similarly, errors between the states of first and third system can be expressed as:

$$\begin{aligned} e_{2,1} &= z_1 - x_1 \\ e_{2,2} &= z_2 - x_2 \\ e_{2,3} &= z_3 - x_3 \end{aligned} \tag{47}$$

Finally, errors between the states of first system and fourth system can be written as:

$$\begin{aligned} e_{3,1} &= w_1 - x_1 \\ e_{3,2} &= w_2 - x_2 \\ e_{3,3} &= w_3 - x_3 \end{aligned} \tag{48}$$

Using the above definitions of errors and exploiting the dynamics of systems given in (31) to (34), we can obtain error dynamics of the systems in star configuration as:

$$\begin{aligned} \dot{e}_{1,1} &= -ke_{1,2} \\ \dot{e}_{1,2} &= e_{1,3} - e_{1,2} + e_{1,1} \\ \dot{e}_{1,3} &= me_{1,2} - m\left(e_{1,3}^3 + 3x_3^2 e_{1,3} + 3x_3 e_{1,3}^2\right) - mle_{1,3} + u_1 \end{aligned} \tag{49}$$

$$\begin{aligned} \dot{e}_{2,1} &= -ke_{2,2} \\ \dot{e}_{2,2} &= e_{2,3} - e_{2,2} + e_{2,1} \\ \dot{e}_{2,3} &= me_{2,2} - m\left(e_{2,3}^3 + 3y_3^2 e_{2,3} + 3y_3 e_{2,3}^2\right) - mle_{2,3} + u_2 \end{aligned} \tag{50}$$

$$\begin{aligned} \dot{e}_{3,1} &= -ke_{3,2} \\ \dot{e}_{3,2} &= e_{3,3} - e_{3,2} + e_{3,1} \\ \dot{e}_{3,3} &= me_{3,2} - m\left(e_{3,3}^3 + 3z_3^2 e_{3,3} + 3z_3 e_{3,3}^2\right) - mle_{3,3} + u_3 \end{aligned} \tag{51}$$

From the results mentioned in equation (8) and with error dynamics in equation (49), the controller u_1 can be derived as mentioned in equation (41). By taking $\alpha_3 = 1$ and putting value of $\dot{\theta}_2(v_{1,1}, v_{1,2})$, the controller can be rewritten as mentioned in equation (42). If the values of $v_{1,1}$, $v_{1,2}$ and $v_{1,3}$ in equation (42) are substituted in terms of (x_1, x_2, x_3) and (y_1, y_2, y_3), then the controller structure comes out as mentioned in equation (43).

Similarly, in case of error dynamics mentioned in equation (50), the control input u_2 comes out as:

$$\begin{aligned} u_2 &= (z_1 - x_1)\left(k + \frac{2}{k} - 2\right) \\ &+ (z_2 - x_2)(-2 - m - k(k-1)) \\ &+ (z_3 - x_3)(ml - 2) + m\left(z_3^3 - x_3^3\right) \end{aligned} \tag{52}$$

In case of error dynamics mentioned in equation (51), the control input comes out as:

$$\begin{aligned} u_3 &= (w_1 - x_1)\left(k + \frac{2}{k} - 2\right) \\ &+ (w_2 - x_2)(-2 - m - k(k-1)) \\ &+ (w_3 - x_3)(ml - 2) + m\left(w_3^3 - x_3^3\right) \end{aligned} \tag{53}$$

Using control inputs mentioned in equations (43), (52) and (53), the error between the systems in star configuration converges to zero. The error subsystems, which are involved between different pairs, can be made stabilised by using the control input obtained from the result mentioned in equations (8) leading to overall synchronisation of all the systems forming star configuration. The above control functions clearly highlight the efficacy of the analytical results derived in Section 4.

6 Simulation results

The results presented in previous section are used to carry out numerical validation in this section. The differential equations, which have been used in this work, have been solved using numerical approximations. Here, detailed simulation results of uncontrolled Chua's system and further, of synchronisation in chain and star configuration are elaborated.

6.1 Numerical simulation of uncontrolled Chua's system

The third order Chua's system having cubic nonlinearity falls into the category of system described in equation (1). The system dynamics without controller is described as:

$$\begin{aligned} \dot{s}_1 &= -ks_2 \\ \dot{s}_2 &= s_3 - s_2 + s_1 \\ \dot{s}_3 &= m\left(s_2 - s_3^3 - ls_3\right) \end{aligned} \tag{54}$$

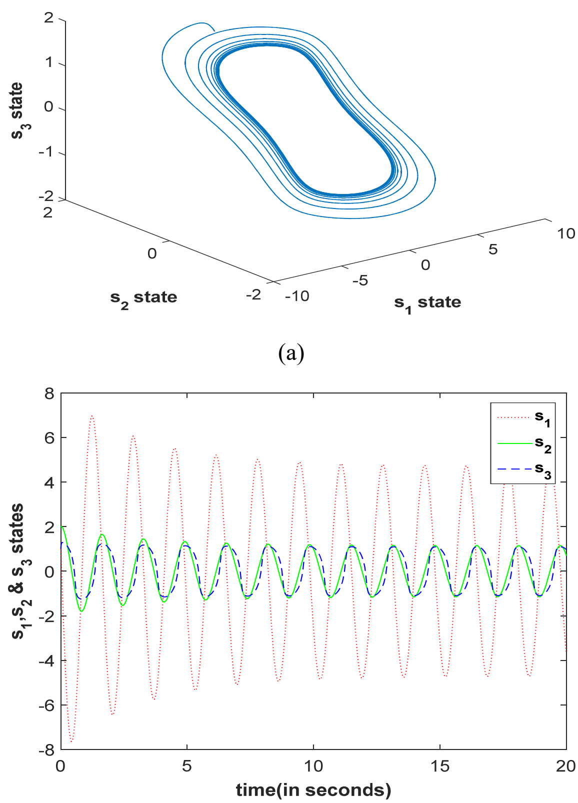
This system exhibits chaoticity for parameter-values mentioned as $k = 16$, $m = 10$, and $l = -0.143$, respectively.

To experience the behavioural response of uncontrolled Chua's system, initial conditions are taken as:

$$s_1(0) = 1;\ s_2(0) = 2;\ \text{and } s_3(0) = 1$$

The chaotic behavioural response of the Chua's system is presented in Figure 2. Phase portrait of the system is depicted in Figure 2(a), and the variation of state trajectories with time is shown in Figure 2(b).

Figure 2 Response of the uncontrolled Chua's system having cubic nonlinearity, (a) phase portrait and (b) variation of state trajectories with time (see online version for colours)



(a)

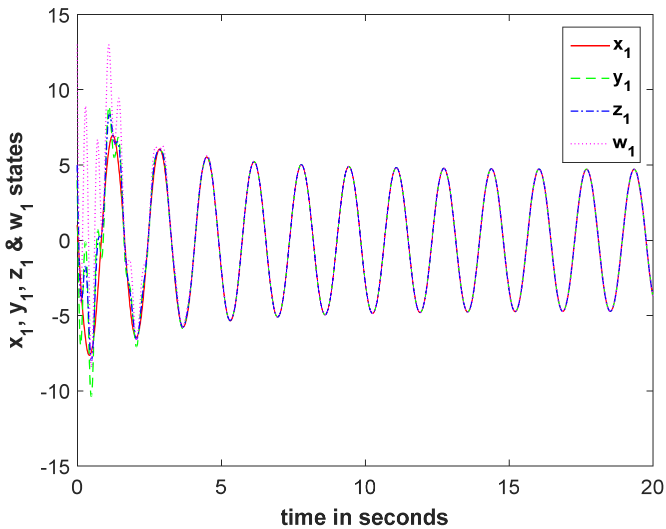
(b)

6.2 *Synchronisation of Chua's systems in chain and star configuration*

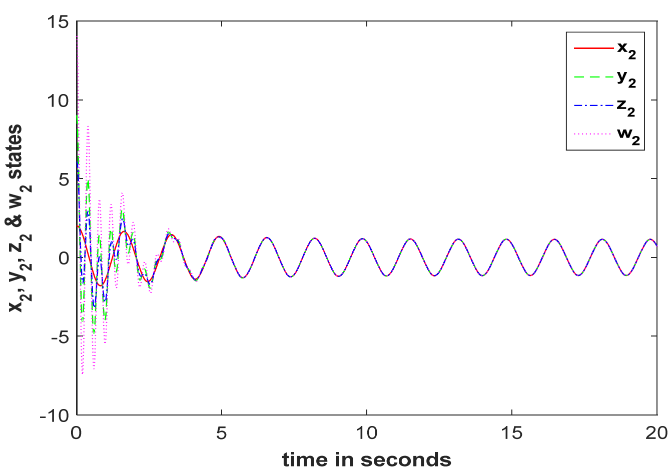
The initial conditions for different systems in chain and star network are taken as $[x_1(0), x_2(0), x_3(0)] = (1, 2, 1)$, $[y_1(0), y_2(0), y_3(0)] = (4, 9, 8)$, $[z_1(0), z_2(0), z_3(0)] = (5, 6, 12)$ and $[w_1(0), w_2(0), w_3(0)] = (13, 14, 15)$, respectively. The corresponding results obtained from simulation are presented in Figures 3–4. In Figure 3, synchronisation of corresponding states of different Chua's systems in chain arrangement is depicted. Figure 4 displays synchronisation of corresponding states in star configuration. The numerical simulations clearly justify the efficacy of proposed methodology, in achieving the goal of synchronisation among the systems forming structure like chain and star.

In simulation, for convenience, we have taken four numbers of systems having cubic nonlinearity in proposed configuration as example. The results and synchronisation graphs show the feasibility of the proposed procedure. In general, if we take m numbers of nth order systems, then by using the result in equation (8), all the systems can be synchronised. These results present a generalised framework to address synchronisation of multiple chaotic systems connected to each other. Such framework enriches the existing literature which was lacking for the proposed class of nonlinear systems.

Figure 3 Synchronisation behaviours among (a) first states (b) second states, and (c) third states of the chaotic Chua's systems connected in chain configuration (see online version for colours)

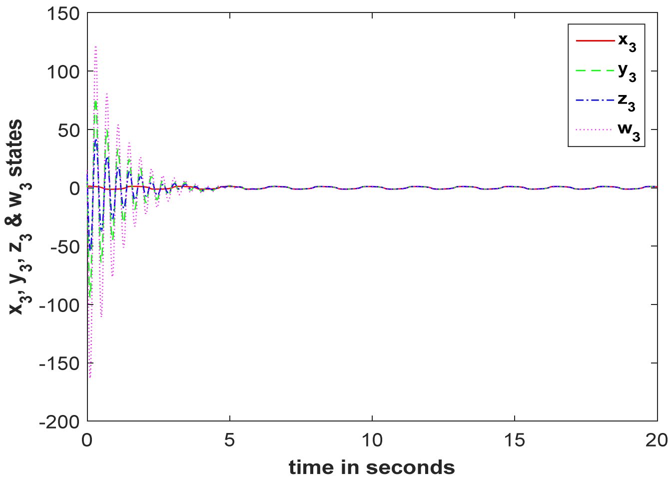


(a)



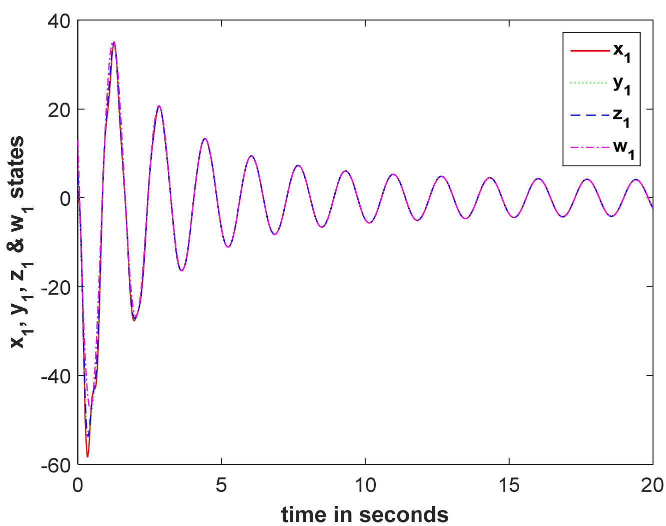
(b)

Figure 3 Synchronisation behaviours among (a) first states (b) second states, and (c) third states of the chaotic Chua's systems connected in chain configuration (see online version for colours) (continued)



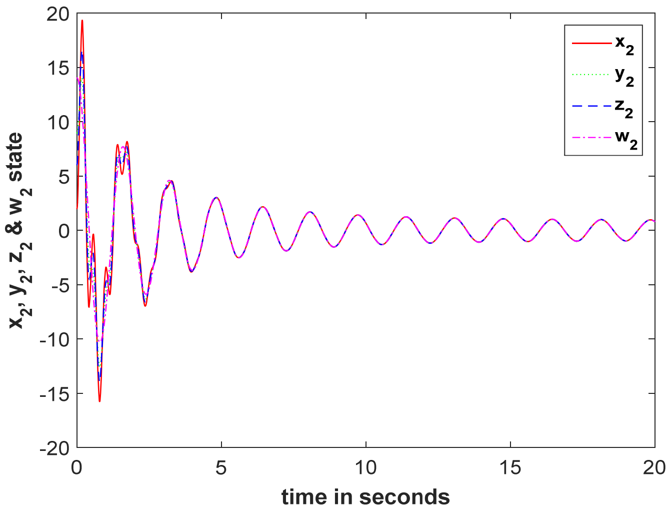
(c)

Figure 4 Synchronisation behaviours among (a) first states (b) second states, and (c) third states of the chaotic Chua's systems connected in star configuration (see online version for colours)

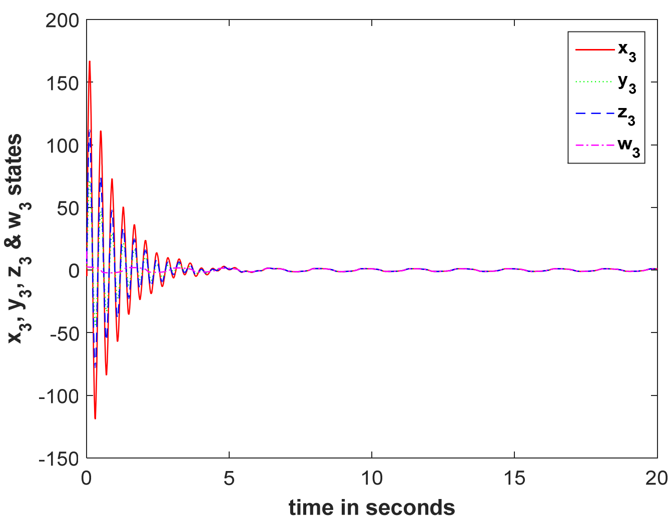


(a)

Figure 4 Synchronisation behaviours among (a) first states (b) second states, and (c) third states of the chaotic Chua's systems connected in star configuration (see online version for colours) (see online version for colours)



(b)



(c)

7 Conclusions

This paper deals with the issue of synchronising a number of chaotic systems altogether, connected in chain and star arrangements. Synchronisation of the category of chaotic systems proposed here is achieved using systematic step by step backstepping approach based on Lyapunov stability criteria. The structure for control function is designed analytically by formulating error dynamics between consecutive pairs of systems in assumed network configurations. Synchronisation of the similar type chaotic Chua's system with cubic nonlinearity in chain and star configurations is achieved using the proposed procedure. As is evident from the simulation graphs, the proposed procedure can ensure synchronisation of the systems, hence, justifying the efficacy of proposed analytical strategy. This approach deals with the issues of synchronisation and control problems for a general class of systems forming structures like chin and star. The results obtained here are eligible for addressing the problems of combinational synchronisation in many fields.

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