Comparative study of MCDM methods under different levels of uncertainty

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Abstract: Often, data in MCDM problems are imprecise and changeable due to the mandatory participation of human judgement, which is often unclear and vague. Hence, the selection of an appropriate MCDM method is crucial to the optimal decision-making. All the MCDM methods are heavily affected by individual or group preferences and therefore even a small change in the data can cause rank-reversal. With the regular proliferation of such methods and their modifications, it is important to carry out a comparative study that provides comprehensive insight into their performances under uncertain conditions. In this paper, we use the Monte Carlo simulation approach to empirically compare the results of five well-known and widely applied MCDM methods, WSM, WPM, TOPSIS, GRA, and MULTIMOORA under different levels of uncertainty. The findings of this paper will assist decision-makers in the selection of most robust and reliable MCDM methods for different decision scenarios. The results of this research are significant additions to the current repository of knowledge in the multi-criteria decision analysis as well as the literature pertaining to the information systems. It also provides insights for many managerial applications of these MCDM methods.

Keywords: multi-criteria decision-making; MCDM; comparative analysis; decision sciences; MCDM methods; Monte Carlo simulation; rank-reversal; rank-correlation; uncertainty.


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1 Introduction

Multi-criteria decision-making (MCDM) is a process that supports decision-makers to evaluate a number of decision alternatives over several qualitative and quantitative criteria. In MCDM, the objective is to rank the different alternatives across multiple and often contradicting criteria on the foundation of subjective opinions of decision-makers (Zavadskas et al., 2014). In order to solve MCDM problems, several approaches and techniques, also called MCDM methods have been developed. In recent years, many MCDM methods have been applied to solve real-world decision problems such as sustainable energy selection, supply chain management, quality management, recommender systems, river basin development, construction and project management, safety and risk management, technology and information management, and tourism management (Duckstein and Opricovic, 1980; Brauers and Zavadskas, 2010; Adomavicius et al., 2011; Deng and Chan, 2011; Mardani et al., 2015a; Hinduja and Pandey, 2017).

Real-world decision problems always contain unknown facts that are unfeasible to identify and in order to reach an optimal solution, human judgement is frequently required, which is often unclear and vague. Also, different criteria can cause conflicts due to their degree of completeness, mutuality, and independence, which can further affect the quality of the decision (Kolios et al., 2016a). Therefore, the real-world decision problems are prone to risks and unreliable results as the quality of inputs can be a significant source of ambiguity. For that reason, the earlier researchers and practitioners applied two approaches to address uncertainty in decision-making: fuzzy set logic and stochastic methods.

Fuzzy logic incorporates the notion of fuzziness as linguistic variables, which represent the human opinion more effectively. Fuzzy sets’ blur boundaries are the foundation of fuzzy logic, which capture the vagueness of subjective decision and handle inherent uncertainty in decision problem (Zadeh, 1965). In recent years, fuzzy logic has been extensively used in numerous real-world decision-making applications and has been incorporated with many MCDM methods (e.g., fuzzy TOPSIS, fuzzy GRA, fuzzy AHP, etc.). Mardani et al. (2015a) provided a wider review on fuzzy MCDM applications and fuzzy decision-making methods.

On the other hand, the stochastic modelling is based on ‘a number of potential outcomes’ characterised by their probability under certain constraints. ‘Randomness’ is the heart of the stochastic modelling (Taylor and Karlin, 1994). A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. The random variation is usually based on the inherent constraints of the method and requirements of the experiment. A stochastic method can be more informative because it takes the uncertainty into account using varying behavioural characteristics of the target method. Stochastic modelling has been applied in many MCDM applications such as selection of offshore wind turbines support structures (Kolios et al., 2016b), sustainable biomass crop selection (Mardani et al., 2015a) and ranking dental quality attributes (Hsu and Pan, 2009).

Since the imprecision and uncertainty are the inseparable parts of complex decision-making and all the MCDM methods exhibit rank reversal when even small changes are made on inputs (Triantaphyllou and Sanchez, 1997; Wang and Luo, 2009; Garcia-Cascales and Lamata, 2012), it is imperative to investigate the behaviour of the MCDM methods under different levels of uncertainty. There are no clear guidelines on
which MCDM method should be applied to a particular decision problem under specific conditions. Furthermore, there are quite a few instances where these methods exhibit disagreement on their ranking when applied to the same problem with unchanged data. Besides, most of the recent comparative studies (Ameri et al., 2018; Baudry et al., 2018; Lee and Chang, 2018; Mathew and Sahu, 2018; Pätäri et al., 2018; Valipour et al., 2018; Munier et al., 2019) are based on particular applications, such as risk assessment in PPP projects, ranking renewable energy sources, and equipment selection; however they do not provide any generic information regarding performances of MCDM methods. Therefore, we seek to perform a comparative analysis among some MCDM methods in an attempt to gain a comprehensive insight into their similarity and differences and their tendency for rank reversal under different levels of uncertainty.

There are several MCDM methods described in the literature of decision sciences, many of them are widely applied in industrial and management decision-making. However, we selected five methods: WSM, WPM, TOPSIS (Hwang et al., 1993), MULTIMOORA (Brauers and Zavadskas, 2012) and grey relational analysis (GRA) (Kuo et al., 2008) for our comparative study because all five methods take single evaluation matrix as input. This similarity among selected methods ensures that all methods are being evaluated using the same set of data and thus eliminates the chances of biases in the comparative analysis.

In order to carry out the investigation, in this paper, we develop an algorithm based on Monte Carlo simulation (Butler et al., 1997) that will record the behaviour of all the selected methods under different levels of uncertainty. This will provide clear insight about the tendency of selected MCDM methods for rank reversal. Our research provides an empirical guideline to support decision-makers in the selection of appropriate MCDM method. The findings of this research would contribute to decision sciences and information systems. To the best of our knowledge, this is the first work that compares the MCDM methods for their tendencies of rank-reversal under a wide range of uncertainty.

The rest of the paper is organised as follows: Section 2 shed light on selected methods for the comparative analysis and Monte Carlo method. In Section 3, we explain the mathematical model and develop simulation algorithm. Section 4 displays empirical results and provides insights on relevant significant issues. Finally, Section 5 concludes the paper.

2 Background

2.1 An overview of selected MCDM methods

2.1.1 Weighted sum model (WSM)

The weighted sum model (WSM) is probably the simplest but most commonly used MCDM method. The additive utility assumption is the foundation of this method, which implies that overall utility of an alternative is equal to aggregation of products of criteria weight and alternatives’ respective scores.

Suppose there is a decision problem, consisting $N$ criteria and $M$ alternatives. The total score of $i^{th}$ alternative $S_i$ can be calculated as:
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\[ S_i = \sum_{j=1}^{N} a_{ij} \cdot w_j \]  

(1)

where \( a_{ij} \) is a score of \( i^{th} \) alternative over \( j^{th} \) criterion and \( w_j \) represents the weight of \( j^{th} \) criterion. And, the sum of all criteria should be equal to one; that is \( \sum_{j=1}^{N} w_j = 1 \).

WSM can easily be applied in single dimension decision problem that means the problems where all the criteria can be expressed in identical units for quantities attributes. But when the decision problem involves, the quantitative as well as qualitative criteria that can be measured in different units, the problem becomes difficult to handle. In such conditions, normalisation should be employed; it implies that all the performance scores over different criteria should belong to a certain range (e.g., [1–100], [0.0–1.0], [1–10]).

2.1.2 Weighted product model

The weighted product model (WPM) is akin to WSM. The key difference is that rather than absolute addition, WPM is based on the relative multiplication. It implies that each alternative is compared through a multiplication of ratios, one for each criterion. The WPM is also called dimensionless its formation eliminates any unit of measure. Therefore, the WPM can be fit into applying on single-dimension as well as multi-dimension decision problems (Triantaphyllou and Mann, 1989).

The method compares all the alternatives selected for the decision-making process. One alternative \( A_i \) is considered more preferred than other, suppose \( A_k \), if the ratio \( R\left( \frac{A_k}{A_i} \right) > 1 \) (for maximisation case). The ratio can be calculated as:

\[ R\left( \frac{A_k}{A_i} \right) = \prod_{j=1}^{N} \left( \frac{a_{kj}}{a_{ij}} \right)^{w_j} \]  

(2)

where as previously, \( a_{ij} \) is the score of \( i^{th} \) alternative over \( j^{th} \) criterion and \( w_j \) is the relative weight of \( j^{th} \) criterion.

2.1.3 The technique for order preference by similarity to ideal solution

The technique for order preference by similarity to ideal solution (TOPSIS) was developed by Hwang and Yoon in 1981. The basic concept of this method is that the selected alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution in a geometrical sense. TOPSIS assumes that each attribute has a tendency of monotonically increasing or decreasing utility. Therefore, it is easy to locate the ideal and negative-ideal solutions. The Euclidean distance approach is used to evaluate the relative closeness of alternatives to the ideal solution. Thus, the preference order of alternatives is yielded through comparing these relative distances (Hwang et al., 1993).

TOPSIS – step by step

Step 1  Create a decision matrix \( X = (x_{ij})_{M \times N} \) consisting of \( M \) alternatives and \( N \) criteria. Where, \( x_{ij} \) is the performance score of \( i^{th} \) alternative over \( j^{th} \) criteria.
Step 2 Construct a normalised decision matrix \( R = (r_{ij})_{M \times N} \), corresponding to original decision matrix \( X \). Normalised value \( r_{ij} \) can be calculated as:

\[
 r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} 
\]  

(3)

Step 3 Construct \( T = (t_{ij})_{M \times N} \), as the weighted normalised decision matrix. The evaluation scores are multiplied by their respective criteria weights. Hence,

\[
 t_{ij} = r_{ij} \times w_j 
\]  

(4)

where, \( w_j \) is weight of \( j \)th criteria and \( \sum_{j=1}^{N} w_j = 1 \).

Step 4 Determine the ideal and the negative-ideal solutions (NIS). The ideal solution [or positive-ideal solution (PIS)] is a hypothetical optimum alternative, which does not exist but used to compare other existing alternatives. Similarly, negative-ideal solutions is the worst possible alternative, used to compare other existing alternatives. PIS and NIS can be determined as:

\[
 \text{PIS}_j = \{ \max \{ t_{ij} | i = 1, 2, \ldots, M \} \}; \quad \text{for all } j = 1, 2, 3, \ldots, N 
\]  

(5)

\[
 \text{NIS}_j = \{ \min \{ t_{ij} | i = 1, 2, \ldots, M \} \}; \quad \text{for all } j = 1, 2, 3, \ldots, N 
\]  

(6)

where \( \text{PIS}_j \) is the performance score of ideal-solution over \( j \)th criterion and similarly \( \text{NIS}_j \) is the performance score of negative-ideal solution. We assume that all criteria are larger-the-better, for smaller-the-better criteria that the formula of \( \text{PIS}_j \) and \( \text{NIS}_j \) will be interchanged.

Step 5 Calculate the separation measure. At this step, the distance of all alternatives, with PIS and NIS, is calculated. It indicates the dissimilarity of target alternatives with the ideal solution and the negative-ideal solution. The separation measures can be calculated as:

\[
 S_{i}^+ = \sqrt{\sum_{j=1}^{N} (t_{ij} - \text{PIS}_j)^2}; \quad \text{for all } i = 1, 2, \ldots, M 
\]  

(7)

\[
 S_{i}^- = \sqrt{\sum_{j=1}^{N} (t_{ij} - \text{NIS}_j)^2}; \quad \text{for all } i = 1, 2, \ldots, M 
\]  

(8)

where \( S_{i}^+ \) is the distance between \( i \)th alternative and ideal solution PIS, and \( S_{i}^- \) is the distance between \( i \)th alternative and negative-ideal solution NIS.

Step 6 Calculate the relative closeness to the ideal solution. At this step, the proximity is calculated between all alternatives and ideal solution. The relative closeness can be estimated as:

\[
 C_i = \frac{S_{i}^-}{S_{i}^+ + S_{i}^-}; \quad \text{for all } i = 1, 2, \ldots, M 
\]  

(9)
The values of $C_i$ lie between 0 and 1. If $C_i = 1$ then, $i^{th}$ alternative is PIS and if $C_i = 0$ then, $i^{th}$ alternative is NIS. In most cases the alternatives lie between PIS and NIS. The alternative with maximum value of $C_i$ should be considered as best available alternative.

Step 7  Rank the preference order according to the $C_i$ values.

### 2.1.4 Grey relational analysis

The grey system theory proposed by Deng has been broadly applied in numerous applications. GRA combines every alternative’s performance over each criterion to solve MCDM problems, and then transforms the original problem into a single criterion decision-making problem. Therefore, alternatives with multiple criteria can be evaluated easily after the GRA process. The details of the GRA procedure are given below (Kuo et al., 2008).

#### Grey relational generating

Consider a multi-criteria decision problem, if there are $m$ alternatives and $n$ criteria, the $i^{th}$ alternative can be expressed as $Y_i = (y_{i1}, y_{i2}, \ldots, y_{in})$, where $y_{ij}$ is the performance score of alternative $i$ over criterion $j$. The vector $Y_i$ can be translated into the comparability sequence $X_i = (x_{i1}, x_{i2}, \ldots, x_{in})$ as:

$$x_{ij} = \frac{y_{ij} - \text{Min}\{y_{ij}, i = 1, 2, \ldots, m\}}{\text{Max}\{y_{ij}, i = 1, 2, \ldots, m\} - \text{Min}\{y_{ij}, i = 1, 2, \ldots, m\}}; \quad \text{for all } i \text{ and } j$$  \hspace{1cm} (10)

#### Reference sequence definition

After generating the comparability sequence $X_i = (x_{i1}, x_{i2}, \ldots, x_{in})$ for all $m$ alternatives, the performance values will be ranged into $[0, 1]$, that means $x_{ij} \in [0, 1]$ for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The closer the value of $x_{ij}$ to 1, the better the performance of the alternative $i$ over criteria $j$. Therefore an alternative is considered as the best choice, if all of its performance values are closest to or equal to 1. However, this kind of alternative does not exist in a real decision problem. As a solution a virtual comparability sequence, called reference sequence is created which represent the optimal reference alternative. Reference sequence $X_0$ is defined as $(x_{01}, x_{02}, \ldots, x_{0n}) = (1, 1, \ldots, 1, \ldots, 1)$. The reference sequence is then compared with each comparability sequence. The alternative, whose comparability sequence has the closest proximity to the reference sequence, is considered as the best alternative among all alternatives.

#### Grey relational coefficient calculation

The grey relational coefficient is intended to determine proximity of $x_{ij}$ to $x_{0j}$. Larger grey relational coefficient $\xi_{ij}$ reflects the better performance of alternative $i$ among others over criteria $j$. The grey relational coefficient can be calculated as:

$$\xi_{ij} = \frac{\delta_{ij} + \rho\delta_{ij}}{\delta_{ij} + \rho\delta_{max}}, \quad \text{for all } i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n$$  \hspace{1cm} (11)
where $\xi_{ij}$ is the grey relational coefficient between $x_{ij}$ and reference sequence $x_0$, and $\delta_{ij} = d(x_0, x_j)$, $\delta_{\text{min}} = \min \{\delta_{ij}, i, j \in M\}$, $\delta_{\text{max}} = \max \{\delta_{ij}, i, j \in M\}$ · $\rho$ is the distinguishing coefficient. The idea behind distinguishing coefficient is to enlarge or shrink the range of the grey relational coefficient, the smaller the $\rho$, the greater the distribution range.

Grey relational grade calculation

The purpose of calculating grey relational grade is to aggregate the each vector of grey relational coefficient $\{\xi_{i1}, \xi_{i2}, \ldots, \xi_{im}\}$, and transform into a single-valued score, which represents the overall performance of alternative $i$. The grey relational grade can be calculated as:

$$\gamma(X_0, X_i) = \gamma_i = \sum_{j=1}^{n} w_j \xi_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \quad \sum_{j=1}^{n} w_j = 1. \quad (12)$$

In equation above, $\gamma(X_0, X_i)$ is the grey relational grade between $X_0$ and $X_i$. It reflects the correlation between the comparability sequence and the reference sequence. Term $w_j$ represents the weight of criteria $j$. The grey relational grades $\gamma_i$ represent the degree of similarity between the $i$th alternative and the reference alternative. The alternative with maximum value of grey relational grade should be considered as the best available alternative.

2.1.5 MULTIMOORA

The MULTIMOORA method was developed from MOORA method. Brauers and Zavadskas (2010), the developer of MOORA, added the full multiplicative form to the MOORA and the new method was called MULTIMOORA. It requires a performance matrix of alternative as input and a vector of criteria weights to produce the ranking of the alternatives. The MULTIMOORA method is, in fact, an arrangement of three different ranking methods: ratio system, reference point approach and full multiplicative form. The MULTIMOORA method functions on the dominance theory and aims to produce final ranking from three ranks obtained by the aforementioned three methods.

The MULTIMOORA method has been extensively applied on many real-world decision problems because of its simplicity and robustness (Brauers and Zavadskas, 2012). Some of its recent applications are wireless network selection (Obayiuwana and Falowo, 2015), laptop selection (Aytaç Adalı and Tuş Işık, 2016), and automobile selection problem of a marble company (Kundakci, 2016). The three methods that compose MULTIMOORA are explained briefly as follows.

Ratio system

The first two steps of the ratio system are similar to the first two steps of TOPSIS: normalisation of decision matrix utilising equation (3) and then construction of weighted normalised decision matrix using equation (4). Final preference values are calculated with the help of equation (13)
\begin{equation}
\gamma^*_i = \sum_{j=1}^{g} t_{ij} - \sum_{j=g+1}^{n} t_{ij}
\end{equation}

where \( j = 1, 2, \ldots, g \) indicates the criteria, meant to be maximised and \( j = g + 1, g + 2, \ldots n \) indicates the criteria meant to be minimised.

The next step is to rank the alternatives in accordance with their preference value \( \gamma^*_i \). The greater the \( \gamma^*_i \) value, the better the alternative is.

The reference point approach

The referenced point approach utilises the normalised performance of \( i \)th alternative over \( j \)th criterion, which is calculated with the help of equation (4). The best performance score over each criterion is determined as reference point \( r_j \) among normalised performance. It is defined as \( r_j = \max_{j} \{ t_{ij} \} \) the criteria is meant to maximised, and, \( r_j = \min_{j} \{ t_{ij} \} \) if the criteria is meant to minimised. This reference point is considered as more realistic and non-subjective (Brauers et al., 2008). Brauers and Zavadskas (2006) highlighted that the Min–Max metric is the most appropriate for the estimation of the deviation from reference point and formulated as:

\begin{equation}
\min_i \left\{ \max_j \{ r_j - t_{ij} \} \right\}
\end{equation}

Afterward, the alternatives are ranked as the lower the deviation value, the better the alternative is.

The full multiplicative form

The full multiplicative form is a pure multiplicative utility function that consists both the criteria to be maximised and the criteria to be minimised. The key characteristics of this form are nonlinearity, non-additive and not using criteria weights (Kracka et al., 2010). The utility of \( i \)th alternative can be calculated as:

\begin{equation}
U_i = \frac{A_i}{B_i}
\end{equation}

In this formula, \( A_i = \prod_{j=1}^{g} x_{ij} \) and \( B_i = \prod_{j=g+1}^{n} x_{ij} \). If \( x_{ij} = 0 \) then, it should be withdrawn from the calculation as it turns whole calculation into zero. After calculating the utilities, all the alternatives are ranked.

In order to produce the final ranking of MULTIMOORA, a ‘summary of rankings’ is calculated utilising the ‘theory of dominance’ (Brauers and Zavadskas, 2012).

2.2 Monte Carlo method

Monte Carlo simulation is, fundamentally, the generation of random objects or processes with the help of a computer. These objects can arise ‘naturally’ as part of the modelling of a real-life system, such as a complex road network, the transport of neutrons, or the evolution of the stock market. However, in many cases, the random objects in Monte Carlo techniques are introduced ‘artificially’ in order to solve purely deterministic
problems. In this case, the Monte Carlo method merely involves random sampling from certain probability distributions. In both modes of the Monte Carlo method, the natural and the artificial, the idea is to repeat the experiment numerous times to obtain many quantities of interest using the law of large numbers and other methods of statistical inference (Kroese et al., 2014).

The Monte Carlo simulation is a useful tool to MCDM as it increases the confidence of the decision-makers in dealing with uncertainty and imprecision, involved in complex decision problems. Essentially, the Monte Carlo method is an approach to represent arbitrary nature of stochastic modelling. The fundamental parts of Monte Carlo method are, to generate random numbers as input sets, which are drawn arbitrary and the numerous iteration. Algorithms that implement the Monte Carlo simulation consist of certain steps of that particular algorithm and the random number generated using probability distribution under some constraints required for empirical study or to imitate the inherent characteristics of the algorithm (Kroese et al., 2011).

The Monte Carlo simulation method has been extensively used in decision-making for analysis of MCDM method using stochastic modelling and sensitivity analysis. In recent years, many practitioners applied the Monte Carlo simulation method with MCDM such as: ranking dental quality attributes (Hsu and Pan, 2009), wind turbine support structure (Kolios et al., 2016a), analysis of MCDM methods (Carmone et al., 1997; Bertsch et al., 2006; Yaraghi et al., 2015), and strategic environment decisions making (Mosadeghi et al., 2013).

3 Simulation algorithm

To generate reliable data for a statistical analysis in MCDM, simulation has been extensively used in prior research (Carmone et al., 1997; Bertsch et al., 2006; Mosadeghi et al., 2013; Yaraghi et al., 2015). We also rely on simulation to generate required inputs for experiments.

In the following algorithm, we generate a random score matrix for the decision alternatives’ performance with regard to each decision criteria as well as a random vector to represent the weight of each decision criteria. Based on these two sets of values, the overall scores and ranks of the decision alternatives are calculated for all the MCDM methods, selected for the comparative study. However, we try to avoid judgement bias and achieve more natural results by creating these sets of values based on pseudorandom numbers automatically generated by the computer. Steps 1 through 7 are assigned to this purpose.

We introduce uncertainty in the algorithm by creating uniform distributions based on the elements of ‘score matrix’ and ‘criteria weight vector’ created in previous steps. In other words, for each element of the score matrix and weight matrix, instead of fixed value, an arbitrary value is picked from a uniformly distributed range of values, based on the element. As we gradually increase the range of the uniform distribution, the uncertainty increases as well. After that, we calculate the ranks based on newly formed score matrix and criteria weight matrix. These ranks indicate the deviation on ranks under a certain level of uncertainty. For each MCDM method, we derive the final ranks for 1,000 different iterations at each level of uncertainty and compare them with the ranks that we first created.
To ensure the accuracy of the outputs, the aforementioned procedure is repeated for 1,000 times. This implies that the simulation algorithm creates 1,000 randomly generated decision problems and for each decision problem the ranks produced by all the methods are examined utilising previous steps. The algorithm is formally described in the seven steps and graphically shown in Figure 1.

**Figure 1**  Simulation algorithm

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**Step 1**  Suppose we have \( N \) decision criteria and \( M \) alternatives. We create a random criteria vector \( W \) having dimension \( N \times 1 \) which contains the randomly generated criteria weights. We also create a random performance matrix \( X \) of dimension \( M \times N \) which contains performance scores of all \( M \) alternatives over each criterion.

**Step 2**  Since all methods, under this comparative study – TOPSIS, GRA, WSM, WPM and MULTIMOORA, derive priorities of alternative based on one criteria vector and one performance matrix, we apply all the methods one by one on our criteria vector \( W \) and performance matrix \( X \). The ranks produced by different methods are stored in five different \( M \times 1 \) vectors respectively – \( R_{TOPSIS} \), \( R_{GRA} \), \( R_{WSM} \), \( R_{WPM} \) and \( R_{MM} \). As discussed in Section 2, all the aforementioned methods employ different steps to calculate scores of alternatives and to determine their ranks accordingly. In this section, we formally denote the entire process of the aforementioned methods for priority derivation as – \( \text{derive}_\text{priority}^{\text{method}}(\text{criteria}_\text{vector}, \text{performance}_\text{matrix}) \). For instance, the priority derivation of TOPSIS will be denoted as \( \text{derive}_\text{priority}^{\text{TOPSIS}}(\text{criteria}_\text{vector}, \text{performance}_\text{matrix}) \).
Step 3 We proceed to generate a weight vector for criteria in correspondence with \( W \) and a score matrix in correspondence with \( X \) such as:

\[
\mathbf{\bar{w}}_i = \text{Uniform}\left[ w_i - \text{deviation}(\mathbf{\bar{w}}_i), w_i + \text{deviation}(\mathbf{\bar{w}}_i) \right]
\]

and

\[
\mathbf{\bar{x}}_j = \text{Uniform}\left[ x_{ij} - \text{deviation}(\mathbf{\bar{x}}_j), x_{ij} + \text{deviation}(\mathbf{\bar{x}}_j) \right]
\]

where

\[
\text{deviation}(\mathbf{\bar{x}}_j) = \left( x_{ij} \right) \times \frac{\mu}{100}
\]

The parameter \( \mu \) points to the level of uncertainty and initially \( \mu \) is set to 1 and the bar notation in \( \mathbf{\bar{w}}_i \) and \( \mathbf{\bar{x}}_j \) is used to differentiate the elements of the new vector/matrix from its correspondence initial vector/matrix. This also indicates at the same time that elements of the new vector/matrix, \( \mathbf{W} \) and \( \mathbf{X} \) are created based on the initial values of the correspondence vector/matrix. In other words, \( \mathbf{\bar{w}}_i \)s are created as random draws from uniform distributions, which are based on \( w_i \)s. Similarly \( \mathbf{\bar{x}}_j \)s are created as random numbers from uniform distributions, which are based on \( x_{ij} \)s.

Step 4 At this step, we derive priorities, similar to Step 2. The difference is that at this step the priorities are calculated on the basis of criteria vector and performance matrix generated in Step 3. For each value of \( \mu \), we generate 1,000 random criteria vectors and 1,000 performance matrices through Steps 3 and 4.

Step 5 At this step, we aggregate ranks obtained from 1,000 iterations of Steps 3 and 4. To summarise the ranks into five single values respectively, we calculate their average score and store in five different variables \( \mathbf{\bar{R}}^{\text{TOPSIS}}, \mathbf{\bar{R}}^{\text{GRA}}, \mathbf{\bar{R}}^{\text{WSM}}, \mathbf{\bar{R}}^{\text{WPM}} \) and \( \mathbf{\bar{R}}^{\text{MM}} \). These variables represent the mean rankings of alternatives, determined by respective MCDM methods for a particular level of uncertainty.

To analyse the results of all five methods under different levels of uncertainty, we gradually increase \( \mu \) from 1 to 50, which represents the level of uncertainty from 1 percentage to 50 percentage. At each value of \( \mu \), a new average rank is calculated for all the methods as discussed previously. The gradual increase in the value of \( \mu \) permits us to explore the tendency of all five methods for rank-reversal over a wide range of uncertainties.

The degree of uncertainty in the simulation algorithm is determined, based on the range of the corresponding uniform distributions defined in equation (16) and equation (17). We believe that as the lower and upper bounds of the uniform distribution from which we draw our random values increase, the variation in the values of the random values also increases. It implies that as the range of the uniform distribution increases, the level of uncertainty in the decision-making also increases. We did not find any theory about the specific level of uncertainty in the current stock of academic literature that can provide guidelines for our study; therefore we design our simulation algorithm for a wide spectrum of
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possible uncertainty levels. That is, levels of the uncertainty from 1% to 50% are simulated with an increment of 1 percentage. Both the 1–50 percentages range and the 1 percentage increments are designed to accommodate the highest uncertainty levels so that all the probable decision-making instances can be covered by the comparative study.

Step 6 For each value of $\mu$, we estimate difference between the initial ranking $R_{\text{method}}^<$ and ranking generated by Monte Carlo simulation $R_{\text{method}}^>$. The Euclidian method is utilised to estimate the distance:

$$
\text{distance}^{\text{method}} = \left( \sum_{i=1}^{m} (R_{i}^{<\text{method}} - R_{i}^{>\text{method}})^2 \right)^{\frac{1}{2}}
$$

(19)

where $R_{i}^{<\text{method}}$ represents the initial ranking of $i^{th}$ alternative obtained by one of the MCDM methods under the study and $R_{i}^{>\text{method}}$ represents the ranking of $i^{th}$ alternative obtained by the same method using Monte Carlo simulation. Hence, at each level of uncertainty we estimate five distance$^{\text{method}}$ variables, one for each method. The distance$^{\text{method}}$ indicates the deviation on ranks under two scenarios. In the first scenario, there is no uncertainty in decision-making. In the second scenario, we introduce some level of uncertainty using $\mu$ variable. Next, we increment the value of $\mu$ by 1 and check the condition: if $\mu < 50$, we go through Steps 3–6.

Step 7 Iterate the Steps 1–6 1,000 times and calculate the mean distances for each method over different levels of uncertainty. Also, perform statistical analysis on the collected data to validate the results. This step adds an additional layer of randomisation in the simulation algorithm. Since with this step, 1,000 random decision problems are generated to be analysed, it also eradicates the chances of a biased decision problem that support a particular MCDM method.

4 Results and discussion

4.1 Methodology

In this section, we present the results of our comparative analysis. All the experiments were conducted through appropriately developed Monte Carlo algorithms. The purpose of the experiment was to compare selected MCDM method in terms of their tendency for rank-reversal under different levels of uncertainty and to gain some insight about their behaviour when certain parameters are changed. The simulation process was carried out with following parameters:

1. Number of criteria: $n \in \{3, 5, 7, 9\}$.
2. Number of alternatives: $m \in \{3, 5, 7, 9\}$.
3. Performance scores for alternatives: $x_{ij} \rightarrow$ randomly generated from a uniform distribution in range [1–100].
Criteria weights: $w_j \rightarrow$ randomly generated from a uniform distribution in range $[0.01–1.0]$. Then, normalised in order that $\sum_{j=1}^{n} w_j = 1$.

Methods selected for the study: TOPSIS, GRA, WSM, WPM and MULTIMOORA.

Number of iterations: for each method, we generated 1,000 decision matrices. For each matrix, we performed experiments for 50 different levels of uncertainty, and for each level of uncertainty 1,000 iteration were performed. Therefore number of iteration: $1,000 \times 50 \times 1,000 = 50,000,000$ iteration for each method.

4.2 Evaluation of MCDM methods

In this section, we introduce four criteria, over which all the methods were analysed. The first criterion is the deviation on ranks across different distribution ranges of the performance scores. The second criterion is the deviation on ranks across different size of decision problems. The third criterion is the total number of rank-reversal and the last criterion is the rank-reversal of the best (top) alternative.

Deviation on ranks across different distribution ranges

In order to analyse the MCDM methods over different distribution ranges, we implement the simulation algorithm discussed in Section 3 with different distribution ranges of performance scores. To facilitate this notion, we gradually changed the range of uniform distribution in the step: 1 of the algorithm from 1–100 (i.e., scores are randomly designated value between 1 and 100) to 90–100 (i.e., scores are randomly designated value between 90 and 100). The objective of this experiment was to analyse the MCDM methods over different degrees of competition among alternatives. The graphical representation of a subset of the results is depicted in Figures 2 and 3. Figure 2 shows the distances between original rankings (uncertainty = 0) and simulated rankings over different levels of uncertainty with distribution range 1–100, while in Figure 3 the distribution range is 90–100.

Figure 2 Rank difference with distribution range 1–100 (see online version for colours)
The simulation results reveal that as the distribution range shrinks the deviation on ranks increases for all the methods. We observe one more interesting pattern from simulation results that after the value of the rank-deviation reaches at 3, the rate of increment over the level of uncertainty slowdowns. Furthermore, GRA exhibits the maximum sensitivity towards uncertain changes among other MCDM methods in all the distribution ranges. On contrary, MULTIMOORA’s performance increases along with uncertainty. The result also shows that WPM is the least sensitive to the larger range of uniform distribution while the MULTIMOORA exhibits the least Rank-deviation among other methods with smaller distribution ranges. We can conclude from the results that if scores of decision matrix are close, MULTIMOORA method should be applied to the decision problem.

Deviation on ranks across different size of decision problems

In order to analyse the MCDM methods over different numbers of decision criteria and alternative, we implement the simulation algorithm discussed in Section 3 with different values of criteria, \( n \) and alternatives, \( m \). To fulfil this purpose, we run a simulation with increasing values of \( m \) and \( n \) at Step 1 of the simulation algorithm. The objective of this experiment was to analyse the behaviour of the MCDM methods over different sizes of decision problems. The results are presented in Figures 4 and 5. Figure 4 shows the distances between original rankings (zero percentages uncertainty) and simulated rankings over different levels of uncertainty applied to decision problem with three alternatives and three criteria (i.e., \( n = m = 3 \)), while in Figure 5, \( n = m = 9 \).

The simulation results reveal that the number of alternatives and criteria significantly affects the value of rank-deviations of MCDM methods. Evidently, WSM outperforms all the other methods and exhibits consistency against rank-reversal over different sizes of problems and level of uncertainty. GRA is clearly the least robust MCDM method. We also observe that MULTIMOORA exhibits an interesting behaviour over uncertainty as it gradually mitigates the problem of rank-reversal over increasing uncertainty and competition among alternatives.
Number of rank-reversal

In this section, we evaluate all the MCDM methods on the basis of ‘number of rank-reversal’ over different levels of uncertainty. This criterion reveals a very vital information about MCDM methods as the ‘number of rank-reversal’ indicates the probability to which ranks might reverse at a certain level of uncertainty. In order to perform the analysis, a simulation was conducted with $n = m = 5$, that means we set the size of decision problems to five alternatives and five criteria throughout this experiment. We also set the distribution range of performance scores to 1–100 with the aim of providing uncontrolled randomisation. We present the results in Figure 6.

The simulation results reveal the number of rank-reversal occurred out of 1,000 iterations. Evidently, the WPM method exhibits the lowest number of rank-reversal
Comparative study of MCDM methods under different levels of uncertainty

at each level of uncertainty beyond 2 percentages. Interestingly, MULTIMOORA and GRA exhibit similar performance as both methods have the maximum number of rank-reversal among the pool of MCDM methods.

**Figure 6** Number of rank-reversal in 1,000 iterations (see online version for colours)

![Graph showing number of rank-reversal in 1,000 iterations for different methods.](image)

*Number of rank-reversal of the best (top) alternative*

The objective of quite a few decision problems is ‘selection of the optimal alternative’. It implies that in many real-world decision problems, rank-reversal of the best alternative is the only matter of interest. Analogous to the previous criteria, this criterion also holds very critical information about MCDM methods as it indicates the chances of changes of top ranked alternative. In order to analyse the MCDM methods over this criterion; we conduct simulation with the same parameter, discussed in the previous section. The results are presented in Figure 7.

**Figure 7** Number of rank-reversal of best alternative in 1,000 iterations (see online version for colours)

![Graph showing number of rank-reversal of best alternative in 1,000 iterations for different methods.](image)
The graph, presented in Figure 7, reveals that the WPM is the most reliable method among all the MCDM methods. The TOPSIS’s score is slightly better than WPM’s score under 7% uncertainty, but beyond that level, WPM’s performance is significantly better than others. GRA and MULTIMOORA’s score are worst at different levels.

4.3 Similarities among rankings produced by the MCDM methods

The previous section discloses the difference in results produced by different MCDM methods under different levels of uncertainty. In this section, we compare the ranking produced by different methods using Spearman’s correlation coefficient index (Myers and Well, 2003) to gauge the similarity among them. We can define the Spearman correlation coefficient as the Pearson correlation coefficient between the ranked variables. The value of Spearman correlation coefficient lies between −1 to +1. A coefficient value closer to 1 indicates a high level of agreement between the rankings and if the coefficient value is close to 0, then there is no agreement. A coefficient value ‘−1’ indicates that the ranking is inverted.

In order to measure similarity among the rankings produced by the MCDM methods, we design a simulation in which random score matrices and criteria vector are produced as described in Section 3. After that, we measure the Spearman correlation coefficient for rankings produced by each pair of MCDM method. For the sake of accuracy, we repeated the aforementioned procedure for 100,000 times for each pair of MCDM methods and then, we calculate the mean of Spearman correlation coefficients obtained by iterations. As the simulation described in Section 3, this simulation was also conducted across different parameters (distribution ranges for scores and number of alternatives and criteria) to gain insights on the similarity among rankings over a wide range of decision-making instances. The results are presented in Figure 8 and Figure 9.

Figure 8 Mean of spearman’s correlation coefficients of each pair of methods for different distribution ranges of scores (see online version for colours)

Figure 8 reveals the similarity among the rankings of each pair of MCDM methods over different distribution ranges. We observe that with exception of TOPSIS-WSM and
TOPSIS-MM, the Spearman’s correlation coefficient increases with the contraction of the range of the performance scores; hence, we can conclude that increasing competition among alternative also increases the similarity among rankings produced by different MCDM methods. The result also provides one more noteworthy indication that the lowest Spearman’s correlation coefficient is recorded for GRA-WPM, when distribution range for scores is 1–100. This also confirms the previous findings of the paper, revealing the least and the most sensitive MCDM method in uncertain conditions (see Figure 2 and Figure 3). We also observe the notable increment on Spearman’s correlation coefficient of WSM-WPM over different distribution ranges, similar to that recorded of GRA-WPM and the high similarity of rankings produced by GRA and WSM.

Figure 9  Mean of spearman’s correlation coefficients of each pair of methods for different problem size (see online version for colours)

The results presented in Figure 9 also confirms that for different numbers of alternatives and criteria GRA-WPM is the least similar ranking producer pair among MCDM methods with Spearman’s correlation coefficients 0.77 and GRA-WSM pair produces most similar ranking with Spearman’s correlation coefficients 0.95. We also observe that changes in number of alternative and criteria do not affect the similarities among ranking derived by different MCDM methods as much as they are affected by changes in distribution ranges for performance score. The results also disclose the high similarity between rankings produced by WPM and MULTIMOORA for all the distribution ranges and problem size with Spearman’s correlation coefficients 0.90 or above.

4.4 Optimal value of distinguishing coefficient $\rho$ in GRA

As discussed in Section 2.1.4, the distinguishing coefficient $\rho$ is utilised to calculate grey relational coefficients [equation (11)] and its value can lie between 0.1 and 0.9, based on the judgement of decision-makers. The purpose of distinguishing coefficient is to expand
or shrink the range of grey relational coefficients, but it also significantly affects the final result. To best of our knowledge, there is no relevant theory on the optimal value of distinguishing coefficient in the current stock of literature on GRA method. Therefore it is imperative to investigate the effects of distinguishing coefficient on rank-reversal under different levels of uncertainty while applying GRA method.

In order to examine effects of different values of distinguishing coefficient on rank-reversal, we use the simulation algorithm described in Section 3, but instead of comparing different MCDM methods, we compare the results produced by GRA with all the possible values $[0.1–0.9]$. We present the graphical representation of the result in Figure 10.

**Figure 10** Deviation on rank while using different value of $\rho$ (see online version for colours)

The simulation results disclose that changes on distinguishing coefficient significantly affects the ranks and the tendency of GRA for rank-reversal. We conclude from results that the optimal value of the distinguishing coefficient, for which the tendency of GRA for rank-reversal notably reduces, is ‘0.9’. Hence, throughout our comparative study, we set the value of distinguishing coefficient to 0.9 while applying GRA.

Besides the simulation method, we also analysed the impact of the distinguishing coefficient on ranks of alternatives from the mathematical perspective. It can be stated from equation (11) that a change in the value of distinguishing coefficient directly influences the values of grey relational coefficients, and changes in the values of grey relational coefficients directly impacts the final priority scores and the ranks of alternatives. It is given in the Section 2.1.4 that the values of grey relational coefficients $\delta_{ij}$ lie between 0.01 and 1, $\delta_{\text{min}}$ denotes min-valued grey relational coefficient, and $\delta_{\text{max}}$ denotes max-valued grey relational coefficient. It implies that values of $\delta_{\text{min}}$ and $\delta_{\text{max}}$ are always less and greater, respectively, than value of $\delta_{ij}$. It can be assumed that $\delta_{\text{min}} \leq \delta_{ij} - 0.01$ and $\delta_{\text{max}} \leq \delta_{ij} - 0.01$. By removing inequalities, we can rewrite equation (11) as follows:

$$\zeta_{ij} = \frac{\delta_{ij} - 0.01 + \rho \left(\delta_{ij} + 0.01\right)}{\delta_{ij} + \rho \left(\delta_{ij} + 0.01\right)}, \text{ for all } i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \quad (20)$$
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If we add one unit (0.01) to the \( \delta_{ij} \),

\[
\tilde{\xi}_{ij} = \frac{\delta_{ij} + \rho(\delta_{ij} + 0.02)}{\delta_{ij} + 0.01 + \rho(\delta_{ij} + 0.02)}, \quad \text{for all } i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \tag{21}
\]

The net effect, \( \Delta_{ij} \) of the addition on the grey relational coefficient \( \tilde{\xi}_{ij} \) will be,

\[
\Delta_{ij} = \frac{\left( \frac{\delta_{ij} + \rho(\delta_{ij} + 0.02)}{\delta_{ij} + 0.01 + \rho(\delta_{ij} + 0.02)} - \frac{\delta_{ij} - 0.01 + \rho(\delta_{ij} + 0.01)}{\delta_{ij} + \rho(\delta_{ij} + 0.01)} \right)}{\delta_{ij} - 0.01 + \rho(\delta_{ij} + 0.01)} \tag{22}
\]

To compare the net effects of different values of distinguishing coefficient on the grey relational coefficients, we evaluated equation (22) for \( \delta_{ij} = (0.01, 1.00) \), and \( \rho = [0.1, 0.9] \). We present a part of the result in Table 1.

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \delta_{ij} = 0.02 )</th>
<th>( \delta_{ij} = 0.15 )</th>
<th>( \delta_{ij} = 0.3 )</th>
<th>( \delta_{ij} = 0.45 )</th>
<th>( \delta_{ij} = 0.6 )</th>
<th>( \delta_{ij} = 0.75 )</th>
<th>( \delta_{ij} = 0.9 )</th>
<th>( \delta_{ij} = 0.99 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho = 0.1 )</td>
<td>81.30%</td>
<td>77.52%</td>
<td>74.07%</td>
<td>70.92%</td>
<td>68.03%</td>
<td>37.59%</td>
<td>33.56%</td>
<td>30.30%</td>
</tr>
<tr>
<td>( \rho = 0.3 )</td>
<td>23.20%</td>
<td>20.28%</td>
<td>18.02%</td>
<td>16.21%</td>
<td>14.73%</td>
<td>16.78%</td>
<td>14.53%</td>
<td>12.82%</td>
</tr>
<tr>
<td>( \rho = 0.5 )</td>
<td>13.14%</td>
<td>11.33%</td>
<td>9.95%</td>
<td>8.87%</td>
<td>8.01%</td>
<td>10.80%</td>
<td>9.28%</td>
<td>8.13%</td>
</tr>
<tr>
<td>( \rho = 0.7 )</td>
<td>9.17%</td>
<td>7.86%</td>
<td>6.87%</td>
<td>6.11%</td>
<td>5.50%</td>
<td>9.17%</td>
<td>7.19%</td>
<td>6.29%</td>
</tr>
<tr>
<td>( \rho = 0.9 )</td>
<td>8.40%</td>
<td>7.19%</td>
<td>6.29%</td>
<td>5.59%</td>
<td>5.03%</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The results, presented in Table 1, reveal that the net effect of a unit addition/deletion to \( \delta_{ij} \) on its grey relation coefficient is inversely proportional to the value of distinguishing coefficient. Since grey relational coefficients directly influence priorities of alternatives and subsequently, their rankings, we can conclude from the experiment that when value of distinguishing coefficient decreases, the effect of random changes in the performance values on the ranks increases. Given these facts, it can be determined that greater the distinguishing coefficient, smaller the impact of a change on the priority scores and hence, less the chances of rank-reversal. The experiment also validated the simulation results shown in Figure 10.

4.5 Summary of the results

The simulation results reveal many interesting points about the tendencies of the MCDM methods for rank-reversal and similarities among rankings. In this section, we summarise the performed experiments and their results.

First, we identified four different criteria for our experiments and evaluate all the methods under study over each of them. Over different criteria, we examined the rank-deviations of MCDM methods under different levels of uncertainty. We observe that WPM consistently exhibits lower rank-deviations than that are exhibited by others.
Next, we assess the performances of the MCDM methods over different degrees of competition among the alternatives. In order to do that, we make changes on the distribution range of the performance scores. We find that with increasing competition among the alternatives, the rate of rank-reversal also increases. Furthermore, the rate of increment in deviation on ranks slowdowns over the level of uncertainty at which the value of the rank-deviation reaches at 3. We find WPM as the most appropriate method until the distribution range 40–99, beyond that MULTIMOORA outperformed all other methods. GRA consistently exhibits its sensitivity on rankings over all the distribution range under different levels of uncertainty.

After that, we evaluate the results produced by all the MCDM methods over different numbers of alternatives and criteria. At every level of uncertainty and every combination of number of criteria and number of alternatives, WPM outperformed all the other methods. Besides, no other variations on rank-deviations of the MCDM methods were observed except for MULTIMOORA’s rank-deviation, as under certain level of uncertainty MULTIMOORA performed worse than GRA and TOPSIS while beyond that level, it performed better than them.

Afterward, we compare the MCDM methods over ‘number of rank-reversal’ and ‘number of rank-reversal of the best alternative’. Interestingly, the number of rank-reversal produced by WPM is significantly greater than the numbers of rank-reversal produced by WSM while uncertainty is fewer than two percentages. Beyond two percentages of uncertainty, WPM’s performance is significantly better while comparing with other methods’ performances. Furthermore, number of rank-reversal of best alternatives produced by TOPSIS is fewer than numbers produced by all the other methods till six percentages of uncertainty. Beyond six percentages of uncertainty, WPM outperformed TOPSIS. MULTIMOORA and GRA produced the maximum number of rank-reversal and the maximum number of rank-reversal of best alternatives at all levels of uncertainty.

As we discussed earlier, the ‘number of rank-reversal’ and the ‘number of rank-reversal of best alternative’ are the crucial criteria as they reveal the probability of alteration of rankings. The results show that WPM’s tendency, for rank-reversal and rank-reversal of alternatives, is significantly lower than other MCDM methods. Furthermore, the estimated rank-deviation of WPM, at each level of uncertainty, over all the combinations of number of criteria and number of alternatives and across all the distribution ranges, is significantly less than the rank-deviations produced by other methods. Some instances are also observed from the results where WPM is outperformed by other methods, but those are only a few. On the whole, we infer that when a decision problem involves a significant level of uncertainty, performs appropriate MCDM methods.

After that, we design a simulation to examine the similarities among rankings derived by different MCDM methods over different courses of problem size and distribution ranges for performance scores of alternatives. To measure the similarity among rankings, we utilise Spearman’s correlation index. We find that, similarity among rankings produced by different methods are heavily affected by changes in the distribution ranges but remain unaffected over different numbers of alternatives and criteria. We also observe that GRA-WSM rankings exhibit high similarity while GRA-WPM rankings exhibit the lowest similarity. The rank correlation results also indicate that high similarities among ranking of WPM and MULTIMOORA.
We also investigate the effect of distinguishing coefficient on the tendency of GRA for rank-reversal. In order to find an optimal value for distinguishing coefficient, we design a simulation which gradually increases the value of distinguishing coefficient from 0.1 to 0.9 and record the rank-deviations produced by GRA under different level of uncertainty. The results are depicted in Figure 10, which reveals that the rank-deviation decreases as we decrease the distinguishing coefficient. Therefore ‘0.9’ is the ‘most rank-reversal safe’ value for distinguishing coefficient when applying GRA on a decision problem.

As the interpretation of the simulation results, we can create guidelines on selection of MCDM methods for decision-makers facing real-world decision problems involving a considerable amount of uncertainty:

1. When the decision problem involves a significant amount of uncertainty and the problem is supposed to be solved by only one MCDM methods, WPM should be applied.

2. If for any specific reason, WPM cannot be utilised to solve decision problem, TOPSIS can be used as it exhibits the lowest tendency for ‘number of rank-reversal’ and ‘number of rank-reversal of best alternatives’ among remain MCDM methods.

3. If the performance scores of alternatives are close, MULTIMOORA should be used as it exhibits lowest rank-deviation under such circumstances.

4. If more than one MCDM methods are used to solve a decision problem, decision-makers should choose MCDM methods with lower rank-similarity as WPM-GRA and if performance scores are close then MULTIMOORA-GRA combination should be chosen.

5. Changes on distribution ranges of performance scores heavily affect the rank-deviations produced by MCDM methods under different levels of uncertainty and similarities among results of the MCDM methods while the effect of changes on number of alternatives and criteria is relatively insignificant.

6. While applying GRA for a decision problem, the value of distinguishing coefficient should be ‘0.9’.

5 Conclusions

The contributions of our paper have both theoretical and empirical relevance. This paper addresses one of the most important issues in the field of decision-making-performances of the MCDM method under uncertainty. In this paper, we use Monte Carlo simulation to compare the performance of five well-known and broadly used MCDM methods under different levels of uncertainty. These methods are the WSM, the WPM, the technique for the order of preference by similarity to the ideal solution (TOPSIS), the GRA and the MULTIMOORA. We believe that our work is the first one that compares the MCDM methods for their tendencies of rank-reversal under a wide range of uncertainties.

In order to offer the benefit of our comparative analysis to the decision-makers for the broad range of decision-making instances, we consider different distribution ranges for scores and numerous combinations of number of criteria and alternatives while designing simulations. The experiments were carried out through appropriately developed Monte Carlo simulations in order to obtain comprehensive insights into the behavior of these methods under uncertainty.
Carlo algorithms. All the MCDM methods were compared for the deviation of ranking derived by them in two different treatments: in first treatment, there is no uncertainty present, and in the second treatment, we introduced a certain degree of uncertainty. We analysed the deviations over a broad range of uncertainty that is from 1% to 50%. The findings of the comparative study are summarised in Section 4.5. We believe that the findings of this paper will aid the decision-makers in numerous decision problems.

Our methodology to compare the MCDM methods is not affected by any biases since the criteria are assumed to be independent and the performance scores and criteria weights are generated from a uniform distribution. Also, the results have been validated through statistical analysis. Hence, the simulation results are not affected by any unspecified issues.

Since our intent behind the study is to achieve high accuracy in the results, our simulation algorithm performs much more iterations than the algorithms proposed by other researchers (Yaraghi et al., 2015; Ceballos et al., 2016) and therefore the simulation requires a huge amount of computational time. Hence, we limited the number of criteria and alternatives in our investigation to 'nine'. In order to reduce the computational time, we are working on optimisation of the simulation algorithm as it will allow us to further examine the decision problems with more criteria and alternatives. Since each method runs independently in the simulation, addition of a new MCDM method in the experiment will not affect performances of existing methods. However, it will be very interesting to incorporate recently developed MCDM methods such as COPRAS and VIKOR into the experiment to examine their tendency for rank-reversal. Another subject for our further investigation is the comparative analysis of the performances of different theories and concepts that administer uncertainty in multi-criteria decision problems with imprecise and incomplete information (Torra and Narukawa, 2009; Yager, 2009; Mardani et al., 2015b). This investigation will shed light on the significance and accuracy of the respective tools and methods while handling uncertainty.

References


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A. Hinduja and M. Pandey


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