Simple tuning of modified Smith predictor for unstable FOPTD processes

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Abstract: A simple tuning for modified Smith predictor is reported here to control unstable first-order plus time delay (FOPTD) processes. In practice, a good number of chemical processes (like combustion, evaporation, distillation, etc.) are unstable as well as delay dominated in nature. Modified Smith predictor technique is a well-accepted methodology for its simplicity and efficacy to regulate delay dominated processes to ascertain desirable closed-loop response. Two-degree-of-freedom (2-DOF) control scheme of the proposed multi-loop controller provides performance enhancement during both set point tracking and load recovery phases. To mitigate tuning complexity of more than one controller involved in modified Smith predictor, the proposed simple guideline involves the same set of tuning parameters for all three controllers. Distinctive feature of the proposed tuning scheme is that there is no overshoot during set point tracking as well as smooth and improved recovery is observed subsequent to uncertain load changes. Superiority of the proposed methodology is established through performance evaluation and stability analysis with well-accepted modified Smith predictor-based tuning relations.

Keywords: large dead time process; dead time compensator; modified Smith predictor; 2-DOF control; FOPTD processes.

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1 Introduction

Controlling of unstable processes with large dead time is a difficult task. Hence, to ensure their desirable closed-loop response during set point change and load variation is an open challenge for the process engineers (Dwyer, 1996). A number of industrial processes often provide unstable behaviour due to improper control strategy as well as inappropriate choice of tuning parameters (Normey-Richo and Camacho, 2007). In comparison to conventional feedback control methodology, Smith (1959) predictor control technique is an effective and widely accepted scheme for controlling processes with large dead time. However, for unstable process with dead time, maintaining the process output at desired value in presence of undesired disturbances is beyond the scope of the conventional Smith (1959) predictor. In literature, a good number of research findings are reported (Watanabe and Ito, 1981; De Paor and Egan, 1989; Aström et al., 1994; Matušek and Micić, 1996; Majhi and Atherton, 1998; Kaya and Atherton, 1999; Kaya, 2003; Liu et al., 2005; Rao et al., 2007; Padhan and Majhi, 2011; Wang et al., 2016) towards modification and extension of the conventional Smith predictor control technique. Among them, works reported by De Paor and Egan (1989), Majhi and Atherton (1998), Kaya (2003), Liu et al. (2005) and Wang et al. (2016) are quite well-accepted. But, in most of the cases, they fail to eliminate the process overshoot during set point tracking and load recovery is also sluggish enough to be acceptable. Moreover, the reported tuning methodologies are relatively complicated as more than one controller is required to be tuned. Hence, there is a scope for finding out some simplified tuning methodology which is capable to provide performance enhancement of modified Smith predictor technique.

Here, we suggest a hassle-free tuning scheme for modified Smith predictor-based multi-loop control structure. Novelty of this reported finding is that a sole tuning guideline is employed for all three controllers involved and a low pass filter (LPF) is incorporated in the outermost control loop. Two-degree-of-freedom (2-DOF) structure

(Rao and Chidambaram, 2008) of the proposed controller is capable to provide performance enhancement during both set point tracking and load rejection phases. Efficacy of the proposed scheme is verified through simulation study of well-known unstable first-order plus time delay (FOPTD) process models resembling the behaviour of chemical reactor, distillation column and combustion chamber (Harriott, 2003). To verify the robustness of the reported controller perturbation is introduced in the process parameters during simulation study to apprehend the model uncertainty. Superiority of the proposed method is verified through quantitative estimation of the performance indices – integral error criterion (IAE, ISE), integral time multiplied error criterion (ITAE, ITSE) (Bissel, 1994). To have quantitative estimation of control action smoothness for the proposed scheme in comparison with modified Smith predictor technique by Padhan and Majhi (2011) total variation (TV) in control action is also computed.

In Section 2, a brief description will be provided on conventional and modified Smith predictor methodologies. Our proposed designing scheme along with other reported controllers will be discussed in Section 3 and Section 4, respectively. Stability and robustness features will be studied in Section 5. Moreover, simulation results related to various unstable FOPTD processes will be provided in Section 6 and at the end conclusion is incorporated in Section 7.

2 Conventional and modified Smith predictor

Modified Smith predictor technique reported by Majhi and Atherton (1998) is an enhanced form of conventional Smith predictor as depicted in Figure 1. Designing of modified Smith predictor scheme consists of three controllers towards achieving the desired closed-loop response. Out of these three controllers, $G_{C1}(s)$ is incorporated in forward path whereas the other two controllers $G_{C2}(s)$ and $G_{C3}(s)$ are provided in feedback path. Plant model is realised by $G_m(s)e^{-\theta_m s}$, where $e^{-\theta_m s}$ is the estimated dead time of the process. Disturbance signal D(s) is introduced in the feed-forward path. Among the three controllers of modified Smith predictor (Figure 1), $G_{C1}(s)$ is a PI type controller responsible for set point tracking, $G_{C2}(s)$ is P type controller which helps to restrict the process overshoot, and $G_{C3}(s)$ is also a P controller to minimise oscillations due to undesired disturbances.

Relations between process output with set point change and load disturbance are given by equation (1) and equation (2), respectively

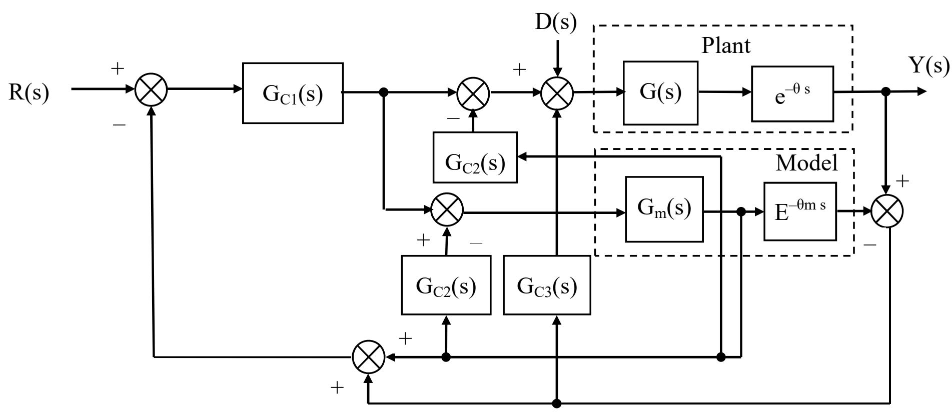
$$\frac{Y(s)}{R(s)} = \frac{G(s).G_{C1}(s)}{1+G_m(s)\left(G_{C1}(s)+G_{C2}(s)\right)} e^{-\theta_m s} \tag{1}$$

$$\frac{Y(s)}{D(s)} = \frac{G(s)e^{-\theta_m s}}{1+G_m(s)\left(G_{C2}(s)+G_{C3}(s)\right)} \frac{1+G_m(s).\left(G_{C1}(s)+G_{C2}(s)-G_{C1}(s)G_{C3}(s)e^{-\theta_m s}\right)}{1+G_{C3}(s)G(s)e^{-\theta_m s}} \tag{2}$$

In equation (1), i.e., the relation between process output and set point change does not contain any delay term in denominator. Here, two controllers $G_{C1}(s)$ and $G_{C2}(s)$

accomplish an effective role towards achieving the desired set point response. Whereas, from equation (2), it is found that all three controllers $G_{C1}(s)$, $G_{C2}(s)$, and $G_{C3}(s)$ are responsible for providing the desired load recovery behaviour. Here, it is to note that both numerator and denominator expressions of equation (2) contain the delay term. The major challenge for working with modified Smith predictor technique (Majhi and Atherton, 1998) is that all three controllers are required to be tuned individually. In the following section, a simple guideline will be provided for hassle-free tuning of all three controllers $G_{C1}(s)$, $G_{C2}(s)$, $G_{C3}(s)$ without sacrificing the closed-loop response.

Figure 1 Modified Smith predictor-based closed-loop structure



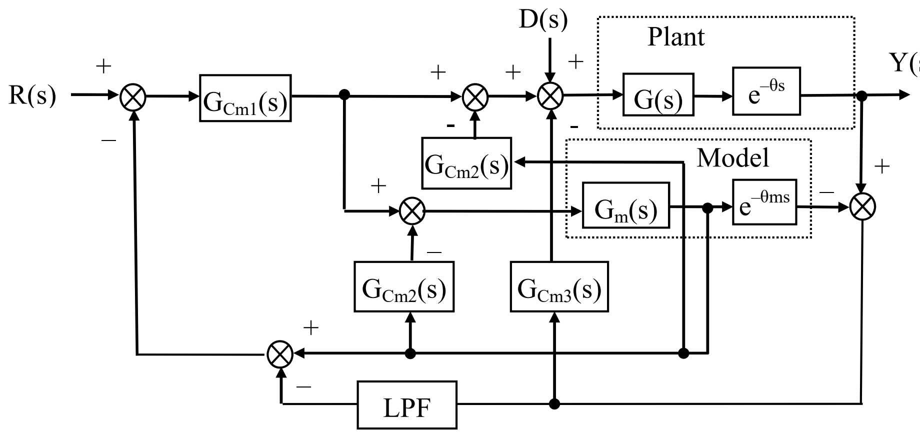
Source: Majhi and Atherton (1998)

3 Proposed tuning

Major drawback of the modified Smith predictor scheme reported by Majhi and Atherton (1998) is that more than one controller is involved in the design and they need to be tuned individually. On the contrary, in the proposed modified Smith predictor-based controller designing as depicted in Figure 2, three controllers with a LPF is associated for enhanced closed-loop performance using identical and simple tuning approach. Here, two controllers $G_{Cm1}(s)$ and $G_{Cm2}(s)$ are incorporated in feed-forward path towards achieving better servo response in terms of faster set point tracking along with overshoot elimination to ensure desired closed-loop performance. Moreover, the other controller $G_{Cm3}(s)$ and a LPF are incorporated in the feedback path to eliminate undesired disturbances and fluctuations. The proposed controllers and LPF are so designed that the enhanced closed-loop performance may be achieved by using simple tuning rules. Controllers $G_{Cm1}(s)$, $G_{Cm2}(s)$, and $G_{Cm3}(s)$ and filter are tuned based on Routh (1905) stability criterion. LPF is incorporated in the outermost feedback path (Figure 2) to achieve improved closed-loop response by eliminating the undesired fluctuations present in the process output. In the proposed structure, $G_{Cm1}(s)$ is PI type controller present in the feed-forward path helps to achieve faster set point tracking, and $G_{Cm2}(s)$ is P type controller which facilitates to minimise oscillations. In addition, another P controller $G_{Cm3}(s)$ attempts to eliminate undesired oscillations during load recovery. Consequently, an improved set point tracking as well as faster load rejection with lesser oscillation may be achieved from our proposed modified Smith predictor.

The notable feature of our proposed tuning with 2-DOF control scheme (Rao and Chidambaram, 2008) is that, all the three controllers $G_{Cm1}(s)$, $G_{Cm2}(s)$, $G_{Cm3}(s)$ and LPF are tuned using sole tuning guideline based on Routh (1905) stability analysis of the closed-loop transfer functions. Here, LPF present in the outermost feedback path helps to reduce the undesired fluctuations present in the process output.

Figure 2 Proposed modified Smith predictor-based closed-loop structure



3.1 *Controller designing*

Expression for all three controllers $G_{Cm1}(s)$, $G_{Cm2}(s)$, $G_{Cm3}(s)$, and the proposed LPF are provided by the following relations:

$$G_{Cm1}(s) = K_p\left(1+\frac{1}{T_i s}\right) \tag{3}$$

$$G_{Cm2}(s) = \beta K_p \tag{4}$$

$$G_{Cm3}(s) = \gamma K_p \tag{5}$$

$$LPF = \frac{1}{(0.05\theta_m)s+1}. \tag{6}$$

In equation (3), K_p is the proportional gain, T_i is the integral time for the PI controller $G_{Cm1}(s)$. P type controllers $G_{Cm2}(s)$ and $G_{Cm3}(s)$ contain additional tuning parameters β and γ in their individual expression as depicted in equations (4) and (5). Both β and γ are obtained from Routh (1905) stability analysis of the closed-loop transfer functions.

Now, to obtain the expression of our proposed modified Smith predictor, we consider a typical unstable FOPTD model as given by equation (7)

$$G(s)e^{-\theta s} = G_m(s)e^{-\theta_m s} = \frac{K_m}{-\tau_u s+1}e^{-\theta_m s} = \frac{K_m}{\tau_u s-1}e^{-\theta_m s}. \tag{7}$$

Pole position $\left(\frac{1}{\tau_u}\right)$ of the FOPTD model is on the right side of the imaginary plane signifying its unstable behaviour. As per the tuning guideline by Majhi and Atherton (1998), unity proportional gain K_p and a smaller value of integral time T_i are recommended for unstable FOPTD processes. However, to restrict the overshoot during set point tracking, set point weighting (Rao et al., 2007) mechanism is incorporated in designing of PI controller [as given by equation (3)] present in the feed-forward path of the modified Smith predictor.

$$G'_{Cm1}(s) = K_p\left[\{b.R(s) - Y(s)\} + \frac{1}{T_i s}\right] \tag{8}$$

Here, the value of the weighting factor (b < 1) is multiplied with the actual set value and the resulting controller $G'_{Cm1}(s)$ is given by equation (8).

4 Tuning guideline

In this section, tuning guideline for all three controllers $G_{Cm1}(s)$, $G_{Cm2}(s)$, $G_{Cm3}(s)$ along with additional tuning parameters β and γ are provided by employing Routh (1905) stability criterion for the transfer functions related to set point tracking and load rejection responses.

4.1 Stabilising controller $G_{Cm2}(s)$

In the proposed scheme, $G_{Cm2}(s)$ operates as a stabilising controller so that an improved set point tracking may be obtained without having any overshoot. Here, $G_{Cm2}(s)$ is a P controller [equation (4)] with gain βK_p where K_p is the proportional gain and β is an additional tuning parameter. The value of β may be obtained from the stability analysis in relation with set point tracking transfer function of the proposed controller structure as given by

$$\frac{Y(s)}{R(s)} = \frac{G(s).G_{Cm1}(s)}{1 + G_m(s)\left(G_{Cm1}(s) + G_{Cm2}(s)\right)} e^{-\theta_m s} \tag{9}$$

As per the guideline of Majhi and Atherton (1998), in nominal case, it can be as written as

$$\frac{Y(s)}{R(s)} = \frac{G(s).G_{Cm1}(s)}{1 + G_m(s)\left(G_{Cm1}(s) + G_{Cm2}(s)\right)} \tag{10}$$

Now, substituting the expressions of $G_{Cm1}(s)$, $G_{Cm2}(s)$ from equations (3) and (4) and G(s) from equation (7) in equation (10), we get

$$\frac{Y(s)}{R(s)} = Y_r = \frac{\frac{K_m}{-\tau_u s + 1}.K_p\left(1 + \frac{1}{T_i s}\right)}{1 + \frac{K_m}{-\tau_u s + 1}\left(K_p\left(1 + \frac{1}{T_i s}\right) + \beta K_p\right)} \tag{11}$$

Through simplification, equation (11) can be written as

$$Y_r = \frac{K_m\left(K_p.s+\frac{K_p}{T_i}\right)}{\left(-\tau_u s^2 + s + \beta K_m K_p s\right) + K_m\left(K_p.s+\frac{K_p}{T_i}\right)} = \frac{1}{\frac{s\left(-\tau_u s + 1 + \beta K_m K_p\right)}{K_m\left(K_p.s+\frac{K_p}{T_i}\right)}+1}. \tag{12}$$

As the concerned unstable process is first-order in nature, hence to eliminate the higher-order term of s in equation (12), we consider

$$\left(-\tau_u s + 1 + \beta K_m K_p\right) = \left(K_p.s + \frac{K_p}{T_i}\right). \tag{13}$$

Now, from equation (13), the sole pole position can be obtained as given by

$$s = \frac{\beta K_m K_p - \frac{K_p}{T_i} + 1}{\tau_u + K_p}. \tag{14}$$

However, from equation (7), the unstable pole location is found to be $s = P_u = \frac{1}{\tau_u}$ which should be identical to the expression of equation (14)

$$\frac{\beta K_m K_p - \frac{K_p}{T_i} + 1}{\tau_u + K_p} = \frac{1}{\tau_u}. \tag{15}$$

From equation (15), we can calculate the value of the unknown tuning parameter β as given by equation (16)

$$\beta = \frac{1 + \frac{\tau_u}{T_i}}{K_m \tau_u}. \tag{16}$$

Now, substituting the value of β from equation (16) in equation (12), we get

$$Y_r = \frac{K_m}{s + K_m}. \tag{17}$$

From equation (17), it can be found that for any positive value of the open-loop gain ($K_m > 0$) of the unstable FOPTD process one should get stable behaviour.

4.2 *Disturbance suppressor $G_{Cm3}(s)$*

$G_{Cm3}(s)$ is a P type controller with gain γK_p which plays vital role towards suppressing oscillations due to undesired disturbances. Here, the unknown parameter γ is obtained from the Routh (1905) stability analysis of the closed-loop transfer function obtained due to load response of the unstable FOPTD process as given by

$$\frac{Y(s)}{D(s)} = \frac{G(s)e^{-e^{-\theta_m.s}}}{1+G_m(s)\left(G_{Cm1}(s)+G_{Cm2}(s)\right)} \qquad (18)$$

$$\frac{1+G_m(s).\left(G_{Cm1}(s)+G_{Cm2}(s)-G_{Cm}(s)G_{Cm3}(s)\left(\frac{1}{(0.05\theta_m)s+1}\right)e^{-\theta_m s}\right)}{1+G_{Cm3}(s)G(s)e^{-e^{-\theta_m.s}}}.$$

Hence, the characteristic equation of load rejection transfer function [equation (18)] is defined as

$$1+G_{Cm3}(s)G(s)e^{-\theta_m s} = 0. \qquad (19)$$

Now, substituting the values of $G_{Cm3}(s)$ from equation (5) and G(s) from equation (7) in equation (19), we get

$$1+\gamma K_p.\frac{K_m}{\tau_u s-1}.e^{-\theta_m s} = 0. \qquad (20)$$

Time delay part $e^{-\theta_m s}$ is approximated using the first-order Pade's relation (Boyce and Diprima, 1992) provided in equation (21)

$$1+\gamma K_p.\frac{K_m}{\tau_u s-1}.\frac{-0.5\theta_m s+1}{0.5\theta_m s+1} = 0. \qquad (21)$$

Now, Routh array is obtained from equation (21) and to ensure the stability, first element of each row of the array should be positive as given by the following relations:

$$0.5\theta_m\tau_u > 0 \qquad (22)$$

$$\tau_u - 0.5\theta_m - 0.5\theta_m\gamma K_m K_p > 0 \qquad (23)$$

$$\gamma K_m K_p - 1 > 0. \qquad (24)$$

Positive value for θ_m and τ_u satisfies equation (22). For having positive value for the expressions as given by equations (23) and (24), the unknown parameter γ can be defined as

$$\frac{1}{K_m K_p} < \gamma < \frac{\tau_u - 0.5\theta_m}{0.5\theta_m K_m K_p}. \qquad (25)$$

Here, it is to mention that the value of γ as suggested by De Paor and Egan (1989) also conform the value provided in equation (25). As per the guideline of De Paor and Egan (1989), the expression of γ is given by the following relation:

$$\gamma = \frac{1}{K_p}\sqrt{\frac{\tau_u}{\theta K_m^2}}. \qquad (26)$$

Efficacy of the proposed methodology is established through performance comparison with the well-known modified Smith predictor schemes reported by Majhi and Atherton (1998), Kaya (2003), and Wang et al. (2016) during set point tracking and load variation phases. To have quantitative estimation rise time (t_r), percentage peak overshoot ($\%M_p$), settling time (t_s), integral error indices (IAE and ISE) as well as integral time error indices (ITAE and ITSE) are calculated. In addition, to estimate the smoothness in control action, TV in control action is also computed for each case.

5 Stability and robustness

Efficacy of a controller is analysed by the stability and robustness study of the closed-loop systems in presence of uncertainties in process parameter and load changes. Plant model is nothing but the approximation of true dynamics of the actual system, which is used for designing the controllers. Uncertainties may be reflected in terms of process gain and time delay. Robust stability analysis is performed using well-known and widely accepted small gain theorem for multiplicative uncertainty represented by M – Δ structure (Morari and Zafiriou, 1989). The closed-loop system is said to be robustly stable if and only if the constraint (Morari and Zafiriou, 1989) satisfies the small gain theorem.

$$\text{i.e., } \left\| \Delta_m(j\omega)C(j\omega) \right\| < 1 \qquad \forall\omega(-\infty, \infty) \tag{27}$$

where $C(s = j\omega)$ is the complimentary sensitivity function and $\Delta_m(s = j\omega)$ is the bound on the process multiplicative uncertainty. Complementary sensitivity function for proposed scheme can be defined as

$$C(j\omega) = \frac{G_{Cm1}G_m e^{-\theta s}}{1 + (G_{Cm2} + G_{Cm1})G_m}. \tag{28}$$

The bound on complimentary sensitivity function can be represents as

$$\Delta_m(j\omega) < \left| \frac{G(j\omega)e^{-\theta s} - G_m(j\omega)e^{-\theta_m s}}{G_m(j\omega)e^{-\theta_m s}} \right|. \tag{29}$$

Here, $G(j\omega)e^{-\theta s}$ is the actual process and $G_m(j\omega)e^{-\theta_m s}$ is the plant model.

If uncertainty exists in time delay, then the tuning parameter must be selected as

$$\left\| C(j\omega) \right\|_\infty < \frac{1}{|e^{-\Delta\theta s} - 1|}. \tag{30}$$

If the uncertainty exists in process gain, then the tuning parameter must be selected as

$$\left\| C(j\omega) \right\|_\infty < \frac{1}{\frac{|\Delta K_m|}{K_m}}. \tag{31}$$

Hence, if the uncertainty exists in process gain and time delay, then the tuning parameter must be selected as

$$\left\| C(j\omega) \right\|_\infty < \frac{1}{\left| \left(\frac{\Delta K_m}{K_m} + 1 \right) e^{-\Delta\theta s} - 1 \right|}. \tag{32}$$

Moreover, the sensitivity and complementary sensitivity function must satisfy the condition for robust performance of the closed-loop system (Morari and Zafiriou, 1989) as given by equation (33)

$$\left\| \Delta_m(j\omega)C(j\omega) + w_m\left(1 - C(j\omega)\right) \right\| < 1. \tag{33}$$

Here, w_m is considered as the uncertainty bound in the sensitivity function s(jω) = 1 – C(jω).

6 Simulation results

For simulation study, three well-known unstable FOPTD models are considered during closed-loop performance evaluation during set point change and load variation. Performance robustness is also verified in comparison with reputed modified Smith predictor schemes reported by Majhi and Atherton (1998), Kaya (2003) and Wang et al. (2016) with +10% perturbations in dead time and open-loop gain of the process. Initially, step set point change is provided and once process reaches the steady state condition, pulsed nature disturbance is introduced in the process input. Here, for all three unstable process models, same set of tuning parameters are considered for $G_{Cm1}(s)$, $G_{Cm2}(s)$, and $G_{Cm3}(s)$ other than the values of two additional tuning parameters β and γ. $G_{Cm1}(s)$ is a PI type controller tuned with proportional gain $K_p = 1$ and the integral time $T_i = 0.1$ s. $G_{Cm2}(s)$ and $G_{Cm3}(s)$ are purely proportional controllers and they are tuned as per equation (4) and equation (5) with the same proportional gain $K_p = 1$. The value for β and γ present in the expressions of $G_{Cm2}(s)$ and $G_{Cm3}(s)$ are obtained from equation (16) and equation (26), respectively.

6.1 Model I

We consider a well-known unstable FOPTD model reported by Kaya (2003) as Model I given by equation (34)

$$G_{p1}(s) = \frac{4}{4s-1}e^{-2s}. \tag{34}$$

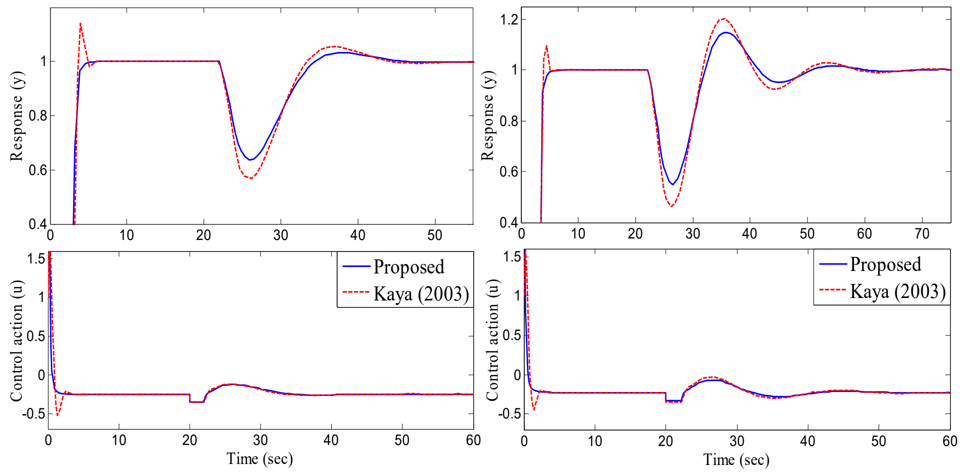
Here, the FOPTD model has a time delay of $\theta_m = 2$ sec with open-loop gain $K_m = 4$ and time constant $\tau_u = 4$ sec. Performance-based comparison is made with modified Smith predictor setting reported by Kaya (2003). For our proposed scheme, all three controllers' $G_{Cm1}(s)$, $G_{Cm2}(s)$, $G_{Cm3}(s)$ settings and LPF relation are provided in Table 1. Closed-loop responses along with control action for the nominal model [equation (34)] during set point tracking and load rejection phases are depicted in Figure 3(a) where solid line represents the proposed setting and dashed line for the reported scheme by Kaya (2003). To evaluate performance robustness of the reported controller by Kaya (2003) and our proposed scheme, responses and the corresponding control actions for the perturbed model as given by equation (35) are depicted in Figure 3(b).

$$\hat{G}_{p1}(s) = \frac{4.4}{4s-1}e^{-2.2s}. \tag{35}$$

Table 1 Controller tuning parameters of the proposed modified Smith predictor for all the unstable FOPTD models as given by equation (34), equation (36) and equation (38)

Model	*Process model*	K_p	T_i	*b*	G_{cm1}	β	G_{cm2}	γ	G_{cm3}	*LPF*
I	$\frac{4}{4s-1}e^{-2s}$ (Kaya, 2003)	1	0.1	0.5	$\frac{0.05s+1}{0.1s}$	2.56	2.56	0.35	0.35	$\frac{1}{0.02s+1}$
II	$\frac{1}{10s-1}e^{-2s}$ (Majhi and Atherton, 1998)			1.0	$\frac{0.01s+1}{0.1s}$	10.10	10.1	2.24	2.24	$\frac{1}{0.02s+1}$
III	$\frac{1}{s-1}e^{-0.4s}$ (Wang et al., 2016)			0.9	$\frac{0.09s+1}{0.1s}$	11	11	1.58	1.58	$\frac{1}{0.004s+1}$

Figure 3 Set point tracking and load recovery response along with control action for nominal and perturbed Model I as given by equation (34) and equation (35) (see online version for colours)



Nominal model responses

(a)

Perturbed model responses

(b)

Table 2 Servo responses for nominal and perturbed unstable processes Models I–III

Model	*Nominal/perturbed*	*Process model*	*Scheme*	*Nominal/perturbed system*			
				TV	t_r *(sec)*	M_P *(%)*	t_r *(sec)*
I	Nominal	$\frac{4}{4s-1}e^{-2s}$	Proposed	13.85	3.11	0	48.95
			Kaya (2003)	13.95	3.33	9.60	51.54
	Perturbed	$\frac{4.4}{4s-1}e^{-2.2s}$	Proposed	22.57	3.50	0	67.97
			Kaya (2003)	22.80	5.41	14.10	68.44
II	Nominal	$\frac{1}{10s-1}e^{-2s}$	Proposed	94.94	3.59	0	85.75
			Majhi and Atherton (1998)	95.44	4.49	3.50	89.58
	Perturbed	$\frac{1.1}{10s-1}e^{-2.2s}$	Proposed	104.60	3.47	0	79.56
			Majhi and Atherton (1998)	105.20	4.32	4.90	81.20
III	Nominal	$\frac{1}{s-1}e^{-0.4s}$	Proposed	16.73	2.20	0	17.82
			Wang et al. (2016)	16.50	2.20	4.10	18.05
	Perturbed	$\frac{1.1}{s-1}e^{-0.44s}$	Proposed	20.05	2.25	0	18.20
			Wang et al. (2016)	20.25	2.25	7.20	21.08

Performance indices for servo and regulatory responses of Model I is provided in Table 2 and Table 3, respectively. Here, it is to note that process overshoot is completely eliminated in case of our proposed scheme as well as faster load recovery is also observed in comparison to the controller reported by Kaya (2003). Quantitative estimation also substantiates the superiority of our proposed mechanism from the calculated values of performance indices as listed in Tables 2 and 3. Performance enhancement of the proposed scheme in terms of no overshoot due to set point weighting (b = 0.5) during servo response is reported in Table 2 for both the nominal as well as perturbed models. In comparison to the settings reported by Kaya (2003) TV in control action is reduced by nearly 1% for both nominal and perturbed models. Similarly for the proposed scheme rise time (t_r) is reduced by more than 50% for the perturbed model. Moreover, settling time (t_s) for nominal model is reduced by more than 5% than the corresponding setting reported by Kaya (2003). Similar to the servo response considerable performance enhancement is also found in terms of IAE, ITAE, ISE and ITSE for the proposed scheme during load recovery phase also. For the nominal model IAE and ITAE are reduced by nearly 7% and 16%, respectively. Alternatively, for the perturbed model ISE and ITSE are reduced by more than 4% and 30% compared to the setting by Kaya (2003).

Table 3 Regulatory responses for nominal and perturbed unstable process Models I–III

Model	*Nominal/perturbed*	*Process model*	*Scheme*	*Nominal/perturbed system*			
				IAE	*ITAE*	*ISE*	*ITSE*
I	Nominal	$\frac{4}{4s-1}e^{-2s}$	Proposed	0.72	8.28	0.24	0.47
			Kaya (2003)	0.77	9.58	0.25	0.54
	Perturbed	$\frac{4.4}{4s-1}e^{-2.2s}$	Proposed	0.75	14.09	0.25	0.48
			Kaya (2003)	0.88	15.25	0.26	0.64
II	Nominal	$\frac{1}{10s-1}e^{-2s}$	Proposed	0.99	4.96	0.48	0.25
			Majhi and Atherton (1998)	1.54	7.39	0.92	0.65
	Perturbed	$\frac{1.1}{10s-1}e^{-2.2s}$	Proposed	1.01	5.00	0.50	0.26
			Majhi and Atherton (1998)	1.60	7.63	0.95	0.66
III	Nominal	$\frac{1}{s-1}e^{-0.4s}$	Proposed	1.36	5.49	0.52	1.00
			Wang et al. (2016)	1.84	5.94	0.82	1.43
	Perturbed	$\frac{1.1}{s-1}e^{-0.44s}$	Proposed	1.41	5.86	0.57	1.04
			Wang et al. (2016)	1.96	6.87	0.94	1.90

6.2 Model II

Here, we consider another well-known unstable process model as given by equation (36)

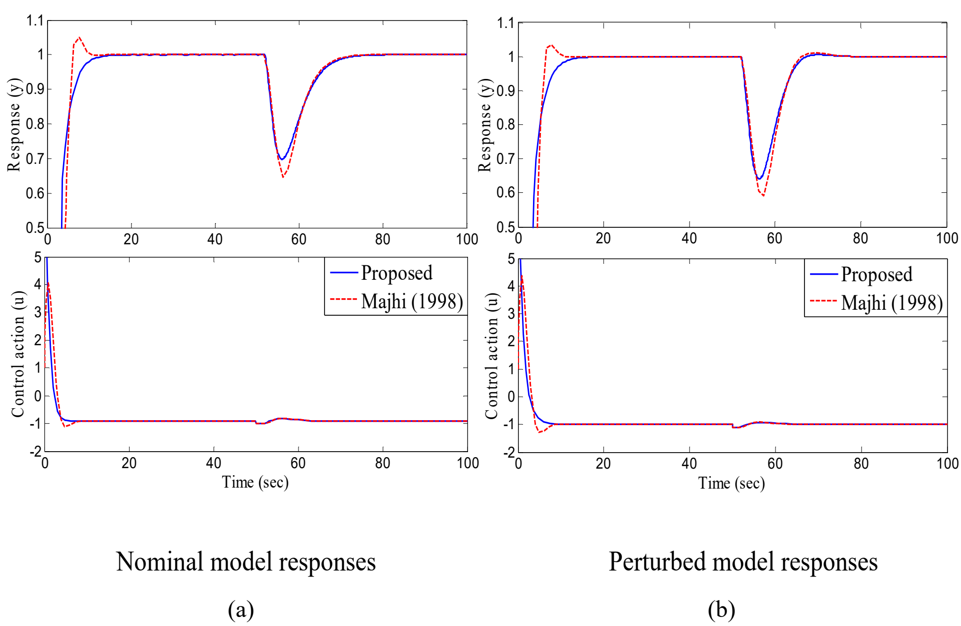
$$G_{p2}(s) = \frac{1}{10s-1}e^{-2s}. \quad (36)$$

This unstable FOPTD model is reported by Majhi and Atherton (1998) representing the behaviour of chemical reactor. For this model, settings of all three controllers $G_{Cm1}(s)$, $G_{Cm2}(s)$, $G_{Cm3}(s)$ and LPF relation are provided in Table 1. Closed-loop responses during set point change and load variation along with their corresponding control action are depicted in Figure 4(a). Closed-loop performance evaluation is also made for the perturbed model $\hat{G}_{p2}(s)$ also as given by equation (37) to verify the robustness of the reported controllers.

$$\hat{G}_{p2}(s) = \frac{1.1}{10s-1}e^{-2.2s}. \quad (37)$$

Corresponding responses and control action for the perturbed model are shown in Figure 4(b) and the related performance indices are listed in Table 2 and Table 3. In comparison to the controller setting reported by Majhi and Atherton (1998), considerable improvement is found for our proposed setting during both set point tracking and load recovery phases for both the nominal and perturbed process models. Rise time is reduced by nearly 25% in case of our proposed scheme. From the load rejection responses as well as listed values of performance indices, it is found that our proposed scheme is also capable to offer considerable improvement during regulatory response with more than 50% reduction in ITAE value.

Figure 4 Set point tracking and load recovery response along with control action for nominal and perturbed Model II as given by equation (36) and equation (37) (see online version for colours)



6.3 Model III

We consider another reputed unstable FOPTD model (i.e., Model III) reported by Wang et al. (2016) for performance evaluation of our proposed scheme. Model III is given by the following relation:

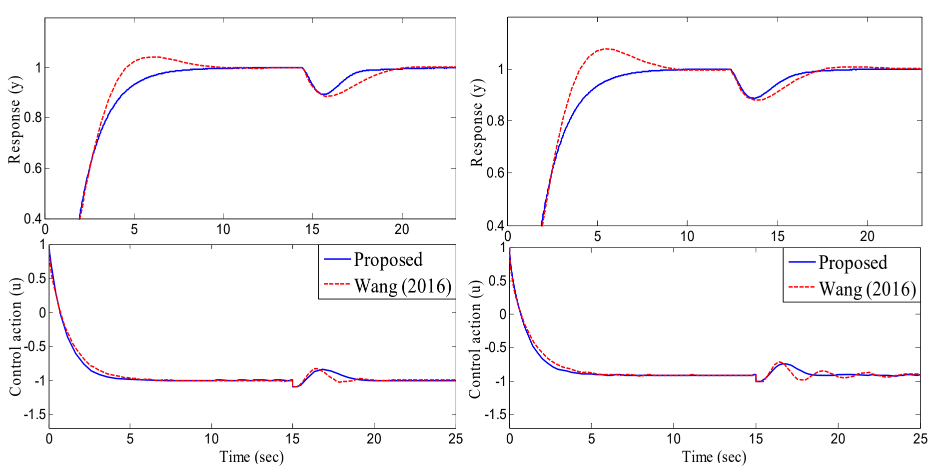
$$G_{p3}(s) = \frac{1}{s-1}e^{-0.4s}. \tag{38}$$

Responses and related variation in control action of the Model III during set point tracking and load change for our proposed control methodology and the setting (Table 1) reported by Wang et al. (2016) are depicted in Figure 5(a). Performances of the reported controllers are also tested by introducing perturbation in Model III as given by equation (39).

$$G_{p3}(s) = \frac{1.1}{s-1}e^{-0.44s}. \tag{39}$$

Responses of the perturbed model are shown in Figure 5(b). Performance indices for both the nominal and perturbed models (Model III) during set point tracking and load recovery phases are listed in Table 2 and Table 3. Closed-loop performance improvements are also calculated in terms of performance indices as depicted in Table 2 and Table 3 respectively during servo and regulatory responses. Process overshoot is completely eliminated for both the nominal and perturbed models. During load recovery phases, both the integral error criteria IAE and ISE are reduced by more than 35%. Similarly, integral time error criteria values are also considerably reduced compared to the settings offered by Wang et al. (2016).

Figure 5 Set point tracking and load recovery response along with control action for nominal and perturbed Model III as given by equation (38) and equation (39) (see online version for colours)



Nominal model responses

(a)

Perturbed model responses

(b)

7 Conclusions

A simple modified Smith predictor tuning methodology is proposed here for unstable FOPTD model representing the behaviour of a good number of chemical processes. It is found to be capable to offer superior performance compared others' reported well-known settings during both set point tracking and load recovery phases. Robustness of the proposed scheme is also established through closed-loop performance evaluation of the perturbed models with the same setting of the controller parameters as with nominal model. Another interesting feature of the proposed tuning technique is that except two additional tuning parameters same set of tuning parameters are adequate for achieving desirable closed-loop performance for all three controllers involved. Guidelines are also provided to find out the value of the two additional tuning parameters.

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