Modelling for combat task allocation problem of aerial swarm and its solution using wolf pack algorithm

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Abstract: To cope with the combat task allocation problems of aerial swarm, firstly, the assumption of battlefield environment was made. And the two multi-attribute principal parts of battlefield, swarm aerocraft (SA) and attacked target (AT) were analysed respectively. Secondly, based on comprehensive consideration of stealth and anti-stealth, attack and counter attack, multi-vehicle cooperation, etc., the objective function was established after analysing the costs and benefits that SA taking the task of reconnaissance, attack and assessment. Then, the task allocation model with the characteristics of multi-target, multi-task, multi-constrain, heterogeneous multi-aerocraft was established after considering some constrains, such as ammunition limit, etc. Based on binary wolf pack algorithm and a novel integer coding method which can better convey the information of the model, an integer coding wolf pack algorithm (ICWPA) was proposed to solve the task allocation model. The simulation results show that the model and the algorithm can effectively solve the combat task allocation problem of aerial swarm.

Keywords: cooperative air combat; aerial swarm; task allocation; integer coding wolf pack algorithm; ICWPA.


Biographical notes: Hao Li is a PhD candidate with Equipment Development and Application Research Center, Air Force Engineering University, China. His current research interest focuses on swarm intelligence, emergent computation and task allocation.

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1 Introduction

The US Air Force Scientific Advisory Committee made it clear that multi-aircraft swarm operations will become the main style of the future air combat (Shima and Rasmussen, 2008). At present, the multi-aircraft cooperation has become an important research topic of the military scientific research institutions, both for unmanned aerial vehicles (UAVs) and manned ones (Niu et al., 2013; Diao et al., 2014; Liu and Zhang, 2010). Further studies found that the aircrafts’ behaviours of formation flight, reconnaissance, detection, attack, cooperative defence, etc., have a lot of similarities with the collective behaviour of nature. Based on the similarity of the two, the concept of aerial swarm is put forward. aerial swarm, composed of a certain number of manned or UAVs [collectively known as swarm aerocraft (SA)], is a kind of flying system which can cooperate with each other and has the characteristics of capabilities emergence, the combat task allocation problem of which is to study the optimal configuration between SAs and arms and ammunition under the circumstance of multi targets and multi tasks.

The method of efficient task allocation is a key way to improve the effectiveness and survivability of aerial swarm in the future battlefield. Related research mainly focuses on the two aspects of task allocation model and its solution algorithm. For the former, the main research ideas are based on the classical model, such as: model of the multiple travelling salesman problem (MSTP) (Secrest, 2001; Ran et al., 2014), mixed integer programming model (Alighanbari, 2004; Bayrak and Polat, 2013) and vehicle routing problem model (Mariam and Mostafa, 2010; Li and Li, 2013) as well as the hybrid model (Tian, 2007; Chen et al., 2013) and so on. However, the task allocation of aerial swarm has greater complexity, which is characterised by multi-target, multi-tasking, multi-constraint and heterogeneous multi-aircraft. Most of the existing allocation models are for the allocation of different tasks among aircrafts, but lacking description of aerial swarm task allocation (Yao et al., 2009; Xu et al., 2013). For solving the model, the heuristic method of calculating the time and the quality of the solution is mainly used (Shen et al., 2014). Usually, it can be subdivided into the traditional heuristic algorithms such as genetic algorithm, particle swarm optimisation algorithm, etc., and the intelligent optimisation algorithms such as simulated annealing, tabu search and so on (Shima et al., 2006; Zhang and Guo, 2013). Because of its characteristics of easier to implement and stronger searching ability, the intelligent optimisation algorithm is widely used in the model solution of task allocation (Edison and Shima, 2011; He and Zhou, 2013), but its problems of being easier to faced with local extreme value, premature and Shima, 2011; He and Zhou, 2013), but its problems of wide use in the model solution of task allocation (Edison searching ability, the intelligent optimisation algorithm is its characteristics of easier to implement and stronger

2 Aerial swarm combat task allocation model

The problem can be described as: How does aerial swarm which is made up of NS heterogeneous SAs cooperatively perform combat tasks of reconnaissance, attack and assessment on NA attacked targets (ATs)?

2.1 Battlefield environment hypothesis and subject attribute analysis

Due to the high complexity of aerial swarm combat, in order to indentify the problem, let’s make these assumptions:

1. there are not any no-fly zones, obstacles and burst-threat on the battlefield
2. battlefields are limited so that communication is expedite among SAs, and fuel (or other energy like electricity, etc.) is sufficient enough for the SA to traverse those ATs, putting aside the route
3. suppose each AT is a ground target and has plenty of ammunition, their attack capability will not diminish even if multi SAs cooperatively perform combat tasks on them, as well
4. take the effectiveness and the gain of multi-SA when they cooperatively perform tasks of reconnaissance and attack into account, but put aside the effectiveness and the gain of their collaborative stealth and joint assessment, etc.
5. suppose that the enemy AT are independent of one another, that is, to put aside the tasks of joint detection, collaborative anti reconnaissance and collaborative anti-aircraft firepower, etc., that AT will perform on SA.

SA and AT are two main bodies against each other in the battlefield, SA’s attributes are represented by nine elements < IDS, STAS, VALS, PCS, PDS, PAS, PVS, TSS, Wmax >, AT’s attributes are represented by < IDA, STAA, VALA, PVS, TSS, Wmax >. The meaning of each element is detailed in Tables 1 and 2.

In Table 1, PCS, PDS, PAS, PV5 ∈ [0, 1) are separately equal to the relative stealth capability ratio of individual single sortie SA, the probability of accurate detection for targets, the probability of attacking and destroying targets, the probability of accurate assessment. If PCS = PDS = PAS = PV5 = 0, it means that SA separately do not have the capabilities of stealth, reconnaissance, attack and assessment. Wmax is the single maximum arms and ammunition payload of SA, which means SA can attack Wmax unit targets at most. In Table 2, PCA, PDA, PAA, PVA, PRA, TSA all belong to [0, 1), and respectively mean the probability of AT finding SA, the probability of SA making wrong detection because of disturbing, the probability of
shooting down SA, and the probability of SA making wrong assessment because of the factors like the AT location, etc., the probability of AT avoiding being destroyed because the SA damage effect is affected by factors like AT defense works, etc.

**Table 1** The meaning of the nine elements in SA’s attributes

<table>
<thead>
<tr>
<th>Element</th>
<th>The meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA</td>
<td>The only identification</td>
</tr>
<tr>
<td>STA</td>
<td>Survival state and STA ∈ {alive, destroyed}</td>
</tr>
<tr>
<td>VAL</td>
<td>Value</td>
</tr>
<tr>
<td>PCA</td>
<td>Stealth capability index</td>
</tr>
<tr>
<td>PDA</td>
<td>Individual single sortie reconnaissance capability index</td>
</tr>
<tr>
<td>PAS</td>
<td>Individual single sortie attack capability index</td>
</tr>
<tr>
<td>PV</td>
<td>Individual single sortie assessment capability index</td>
</tr>
<tr>
<td>TS</td>
<td>List of tasks to be perform</td>
</tr>
<tr>
<td>Wmax</td>
<td>Maximum arms and ammunition payload</td>
</tr>
</tbody>
</table>

**Table 2** The meaning of the nine elements in AT’s attributes

<table>
<thead>
<tr>
<th>Element</th>
<th>The meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDA</td>
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<td>Value</td>
</tr>
<tr>
<td>PCA</td>
<td>Detection capability index</td>
</tr>
<tr>
<td>PDA</td>
<td>Anti-reconnaissance capability index</td>
</tr>
<tr>
<td>PA</td>
<td>Anti-aircraft fire treat index</td>
</tr>
<tr>
<td>PV</td>
<td>Assessment difficulty index</td>
</tr>
<tr>
<td>PR</td>
<td>Anti-attack capability</td>
</tr>
<tr>
<td>TS</td>
<td>List of the performed tasks on the target by SA</td>
</tr>
</tbody>
</table>

### 2.2 Costs and benefits analysis of performing reconnaissance tasks

Set the collection of SA to be $S = \{1, 2, ..., N_s\}$, and the collection of AT to $G = \{1, 2, ..., N_g\}$. Tentatively set the collection of task to be $W = \{D, A, V\}$, $D$, $A$, and $V$ represents reconnaissance, attack and assessment respectively. Obviously, $D \rightarrow A \rightarrow V$ should be assigned in order. Then each battle can be described as task $W$ in which NS aircrafts cooperatively perform tasks on NA targets. The costs and benefits when SA perform tasks will be detailed below.

**2.2.1 Costs and benefits of performing reconnaissance tasks**

Accurate reconnaissance that SA carry out on AT will be conducive to subsequent attacks and assessment tasks, and the more valuable AT become, the greater the effect; at the same time, due to the complexity of high-tech battlefield, the enemy target may also use a variety of anti-reconnaissance techniques to influence SA reconnaissance; in addition, SA are also possible to use plasma stealth technology to avoid AT detection, but once SA are detected by AT and their identities are revealed, they will be treated by the anti-aircraft fire from AT.

Based on the above considerations, the benefits of SA performing reconnaissance task relate to the SA reconnaissance capabilities, AT’s anti reconnaissance capabilities and the value of AT. The costs of performing tasks relate to the value of SA, their stealth capabilities, as well as AT’s detection and anti-aircraft ability. The two cases of single aircraft and multi aircrafts respectively reconnoitering on the same target will be discussed below.

1. **Single aircraft reconnaissance.** Simply mark SA numbered as SA$i$, mark AT numbered $J$ as $AT_j$, set $VAL(i)$, $PC_A(i)$ and $PD_A(i)$ separately to be the value of $SA_i$, stealth capability index and Individual single sortie reconnaissance capability index, and set $VAL(j)$, $PC_A(j)$, $PD_A(j)$, $PA_A(j)$ separately to be the value of $AT_j$, detection capability index, anti reconnaissance capability index and anti-aircraft fire treat index. Then the probability $PD(i, j)$ of $SA_i$ accurately reconnoitering on $AT_j$ is shown as follows:

$$PD(i, j) = PD_A(i) \times [1 - PD_A(j)]$$

The benefits $I_{ij}^P$ and the costs $C_{ij}^P$ of SA$i$ performing reconnaissance tasks on $AT_j$ are shown in formulas (2) and (3):

$$I_{ij}^P = PD(i, j) \times VAL_A(j)$$

$$C_{ij}^P = PCA_A(j) \times [1 - PC_A(i)] \times PA_A(j) \times VAL_A(i)$$

In formula (3), the probability of SA$i$ is found by $AT_j$ expressed as $PC_A(j) \times [1 - PC_A(i)]$, the probability of $SA_i$ destroyed by $AT_j$ is expressed a $PC_A(j) \times [1 - PC_A(i)] \times PC_A(i)$.

So, the net benefits of $SA_i$ performing reconnaissance task on $AT_j$ can be calculated by the following formula:

$$E_{ij}^P = I_{ij}^P - C_{ij}^P$$

2. **Multi aircrafts cooperative reconnaissance.** Suppose there are $Q$ SA performing reconnaissance task on $AT_j$. Because of the multi aircrafts cooperation, the information obtained by multi platforms and multi type detection equipments can be integrated, then, the target location, size and firepower distribution can be measured quickly and accurately (Deng et al., 2013). The obtained reconnaissance benefits will be so great that no other single aircraft can compare. So we set cooperative reconnaissance capability gain parameter $\mu$ to reflect the capability gain of the multi aircrafts collaboration. Set the number of SA that participate in the cooperative reconnaissance to be $z = i_1, i_2, ..., i_Q$, then the probability $CPD(Q, j)$ of $Q$ SA performing
accurate reconnaissance task on AT can be calculated by the following formula:

\[
CPD(Q, j) = (1 + \mu) \times \left[1 - \prod_{z=1}^{q_j} [1 - PD(z, j)] \right]
\]

(5)

In the above formula, the probability of SA individually single sortie performing accurate reconnaissance task on AT is expressed as PD(z, j), whose counting method is the same as formula (1).

Set the costs of SA cooperatively performing reconnaissance task to be the one in formula (3). The benefits \( CI^D_{0,j} \) and costs \( CC^D_{0,j} \) of Q SA cooperatively performing reconnaissance task on AT are separately shown in formulas (6) and (7).

\[
CI^D_{0,j} = CPD(Q, j) \times VAL_A(j)
\]

\[
=(1 + \mu) \times \left[1 - \prod_{z=1}^{q_j} [1 - PD(z, j)] \right] \times VAL_A(j)
\]

(6)

\[
CC^D_{0,j} = \sum_{z=1}^{q_j} \left[PC_A(j)[1 - PC_S(z)] \times PA_A(j) \times VAL_S(z)\right]
\]

(7)

The net benefits of the Q SA cooperatively performing reconnaissance task on AT is shown in the following formula:

\[
CE^D_{0,j} = CI^D_{0,j} - CC^D_{0,j}
\]

(8)

### 2.2.2 Costs and benefits of performing attack tasks

After the effective reconnaissance that SA have performed on AT, the attack against AT will be carried out. Now the attack benefits relate to the capability of SA attacking AT, the value of AT and the capability of anti-attack as well. If SA attacks AT after performing reconnaissance task on AT, then the threat of AT to SA is not repeated, that is, \( C_{i,j}^A = 0 \).

If not, the costs that SA pay are calculated according to formula (3), that is \( C_{i,j}^A = C_{i,j}^C \).

The costs of SA's attacking AT are as follows:

\[
C_{i,j}^A = J \left(x^A_{i,j} \neq 1\right) \times C_{i,j}^A
\]

(9)

In the above formula, \( x^A_{i,j} \) is decision variable, and if \( x^A_{i,j} = 1 \), it means that SA is performing reconnaissance tasks on AT, and vice versa. \( J(\cdot) \) is decision function, if the condition is satisfied, then \( J(\cdot) = 1 \), otherwise \( J(\cdot) = 0 \).

The two cases of single aircraft and multi aircrafts will be discussed respectively below.

1. **Single aircraft attack.** Set \( PA_A(i) \) to be the single sortie attack capability index of SA, and set \( PR_A(j) \) to be anti-attack capability index of AT, then the probability \( PD(i, j) \) of SA destroying AT is shown as the following formula:

\[
PA_A(i, j) = PA_A(i)[1 - PR_A(j)]
\]

(10)

\( VAL_A(j) \) to be the value of AT, then the net benefits of SA attacking AT are described in the following formula:

\[
E_{i,j}^A = I_{i,j}^A - C_{i,j}^A = PA_A(i) \times VAL_A(j) - C_{i,j}^A
\]

\( = PA_A(i)[1 - PR_A(j)] \times VAL_A(j) - J \left(x^A_{i,j} \neq 1\right) \times C_{i,j}^A
\]

(11)

2. **Multi aircrafts cooperative attack.** If there are Q SA cooperatively attacking the same target AT, then various types of weapons can be used to suppress fire against targets, whose cooperative attack effect is obviously stronger than the sum of the single ones. So we set cooperative attack capability gain parameter \( \eta \) to reflect the capability gain of the multi aircrafts collaboration. Set \( z = i_1, i_2, \ldots, i_q \), to be the number of SA that participate in the cooperative attack task, then the probability \( CPA(Q, j) \) of destroying AT after Q SA cooperatively attacking it is described in the following formula:

\[
CPA(Q, j) = (1 + \eta) \times \left[1 - \prod_{z=1}^{q_j} [1 - PA_A(z, j)] \right]
\]

(12)

In formula (12), \( PA_A(z, j) \) represents the probability of SAZ individually single sortie performing attack task on AT, which can be calculated according to formula (10). Based on the aforementioned assumptions, the costs of single SA’s attack in condition of cooperation can be calculated according to formula (9).

Then the benefits \( CI^D_{0,j} \) and costs \( CC^D_{0,j} \) of Q SA cooperatively attacking AT are shown in formulas (13) and (14).

\[
CI^A_{0,j} = CPA(Q, j) \times VAL_A(j)
\]

\[
= (1 + \eta) \times \left[1 - \prod_{z=1}^{q_j} [1 - PA_A(z, j)] \right] \times VAL_A(j)
\]

(13)

\[
CC^A_{0,j} = \sum_{z=1}^{q_j} C^A_{i,j} = \sum_{z=1}^{q_j} \left[J \left(x^A_{i,j} \neq 1\right) \times C^D_{i,j}\right]
\]

(14)

The net benefits of the Q SA cooperatively performing attack tasks on AT is shown in the following formula:

\[
CE^A_{0,j} = CI^A_{0,j} - CC^A_{0,j}
\]

(15)
Of course, the premise of $S_A$, attacking $A_T$ should meet the attacking conditions that the weapon and ammunition of $S_A$ are adequate enough. Set $W_{\text{max}}(i)$ to be the maximum limit of the weapon and ammunition, and with each target being attacked, a unit of ammunition is consumed, then the amount of its ammunition $W(i)$ is reduced by 1, until $W(i) = 0$, it withdraws from fight or turns into the tasks of reconnaissance or assessment.

### 2.2.3 Costs and benefits of performing assessment tasks

When the target is attacked, it needs to send $S_A$ to evaluate the effect of attack and the damage of target. At this point, the task benefits relate to the ability of $S_A$’s assessment, the assessment difficulty of the target AT and the value of AT. The costs relate to the value of $S_A$, the threat of target to $S_A$, and the damage of AT as well. The threat index of AT after $S_A$ has attacked $A_T$ is calculated as follows:

$$PA_A(i, j) = [1 - PA_A(i) \times (1 - PR_A(j))] \times PA_A(j)$$  \hspace{1cm} (16)

In formula (16), $1 - PA_A(i) \times (1 - PR_A(j))$ represents the survival probability of $A_T$ after $S_A$ has attacked $A_T$.

If $S_A$ continues to assess damage after attacking $A_T$, then the threat of $A_T$ is not considered, that is $C^{s}_{i,j} = 0$. If not, the costs of $S_A$ performing evaluation task is shown in the following formula:

$$C^{e}_{i,j} = PA_A(i, j) \times VAL_A(i)$$

$$= [1 - PA_A(i) \times (1 - PR_A(j))] \times VAL_A(i)$$  \hspace{1cm} (17)

Set $PV_A(i)$ to be the assessment capability index of $S_A$, and $PV_A(j)$ to be the difficulty index of $A_T$, then the net benefits of assigned $S_A$ to perform evaluation task on $A_T$ is shown in the following formula:

$$E^{e}_{i,j} = PV_A(i) \times [1 - PV_A(j)] \times VAL_A(j) - J \times x^{e}_{i,j} \times C^{e}_{i,j}$$  \hspace{1cm} (18)

In the above formula, $x^{e}_{i,j}$ is decision variable, and if $x^{e}_{i,j} = 1$, it means that $S_A$ is performing attack tasks on $A_T$, and vice versa.

### 2.3 Establishment of model

Let the net benefits of each task $S_A$ have performed to meet the additivity, defining variable $x^{e}_{i,j} = 1$ represents that the task $K$ of $A_T$ is assigned to $S_A$, otherwise, $x^{e}_{i,j} = 0$, $i \in S$, $j \in G$. $k = 1, 2, 3$ respectively represents the reconnaissance task of $D$, the attack task of $A$ and the assessment task of $V$ in set $M = \{D, A, V\}$. Then the assignment problem of the cluster operations task is described as follows:

$$\max \ f = \sum_{i=1}^{N_A} \sum_{j=1}^{N_M} \sum_{k=1}^{3} x^{e}_{i,j} \cdot E^{e}_{i,j}$$  \hspace{1cm} (19)

Mainly consider the following constraints:

a The constraint of arms and ammunition payload for each $S_A$:

$$\sum_{j=1}^{N_M} x^{e}_{i,j} \leq W_{\text{max}}(i), \quad \forall i \in S$$  \hspace{1cm} (20)

b The capacity constraint of each $S_A$:

$$\begin{cases}
\forall j \in G, \ x^{e}_{i,j} = 0, & \text{if } PD_A(i^*) = 0 \\
\forall j \in G, \ x^{e}_{i,j} = 0, & \text{if } PD_A(i^*) = 0 \\
\forall \in G, \ x^{e}_{i,j} = 0, & \text{if } PD_A(i^*) = 0
\end{cases}$$  \hspace{1cm} (21)

c If it needs $t$ different $S_A$ to cooperatively reconnoiter $A_T$, then when $x^{t}_{i,j} = 1$, $x^{t}_{i,j} = 1$, ..., $x^{t}_{i,j} = 1$, $i, t, \ldots, i_t$ shall not be equal with one another at the same time. But there is no such restriction when attacking $A_T$.

d The constraints on each objective of the task:

$$\sum_{i=1}^{N_A} x^{t}_{i,j} \geq |M(j,k)|, \quad \forall j \in G$$  \hspace{1cm} (22)

The left side of the inequality in the above formula represents the number of flight that all $S_A$ perform the task $k$ on the target $j$, $|M(j,k)|$ represents the number of flight needed to perform the task $k$ on the target $j$. In formula (22) the number of flight for tasks of reconnaissance, attack, and assessment on the target $J$ is ensured, so that the combat task can be finished.

### 2.4 Design of coding

Task decision variable $x^{e}_{i,j}$ includes three dimensions, from which the task allocation scheme are not intuitive and inconvenient to use intelligent algorithms to optimise the scheme. Therefore, this paper designs a new kind of two-dimensional integer encoding $X$ to represent the task allocation scheme.

In Figure 1, $X$ represents the number of $S_A$, $X \in S$, $t = 1, 2, \ldots, MD, \ldots, MD + MA, \ldots, MD + MA + MV$. $MD, MA$ and $MV$ respectively represents the total sorties of $S_A$ needed in the tasks of reconnaissance, attack and assessment on all targets.

**Figure 1** Assignment scheme

$$Y = \{Y_{1}, Y_{2}, \ldots, Y_{MD}, Y_{MD+1}, \ldots, Y_{MD+MA}, Y_{MD+MA+1}, \ldots, Y_{MD+MA+MV}\}$$

$$X = \{X_{1}, X_{2}, \ldots, X_{MD}, X_{MD+1}, \ldots, X_{MD+MA}, X_{MD+MA+1}, \ldots, X_{MD+MA+MV}\}$$
At the same time, there is a group of target code vector \( Y \) which is determined by the target task table corresponding to \( X \). If \( t \in \{1, 2, \ldots, MD\} \), then it means that the aircraft \( X_t \) is reconnoitering the target \( Y_t \). If \( t \in \{MD + 1, MD + 2, \ldots, MD + MA\} \), it means that the aircraft \( X_t \) is attacking the target \( Y_t \). If \( t \in \{MD + MA + 1, \ldots, MD + MA + MV\} \), it means that the swarm aircrafts \( X_t \) are performing assessment task on the target \( Y_t \). In order to understand the above encoding, a numerical example is introduced to illustrate the examples shown in Table 3.

### Table 3 Task list of AT

<table>
<thead>
<tr>
<th>No. of AT</th>
<th>Reconnaissance sortie</th>
<th>Attack sortie</th>
<th>Assessment sortie</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Single task sortie</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Target code vector \( Y \) can be obtained from Table 3. As is shown in Figure 2, MD, MA and SA respectively represents the SA sorties needed in the tasks of reconnaissance, attack and assessment on three targets. \( MD = 4, MA = 6, MV = 4 \), respectively.

**Figure 2** Diagram of encoding

\[
Y = (1, 1, 2, 3, 1, 1, 1, 2, 3, 1, 2, 3, 3)
\]

\[
X = (2, 4, 1, 1, 1, 3, 4, 2, 4, 1, 3, 2, 2, 2, 3, 1)
\]

In Figure 2, \( X \) is the number of SA, then the task allocation scheme can be obtained, that is, numbers 2, 4, 1, 1 of SA respectively performs reconnaissance task on numbers of 1, 1, 2, 3 of AT, which correspond to the decision variable \( x_{i,j}^R \) as \( x_{1,1}^R = 1, x_{1,2}^R = 1, \ldots \). Numbers 3, 4, 2, 4, 1, 3 of SA respectively performs attack task on numbers of 1, 1, 2, 3, 3 of AT, which correspond to the decision variable \( x_{i,j}^A \) as \( x_{1,1}^A = 1, x_{1,2}^A = 1, \ldots \). Numbers 2, 2, 3, 1 of SA respectively performs assessment task on numbers of 1, 2, 3, 3 of AT, which correspond to the decision variable \( x_{i,j}^V \) as \( x_{1,1}^V = 1, x_{1,2}^V = 1, \ldots \). Thus, the two-dimensional integer encoding \( X \) can be used to express the meaning of three-dimensional decision variable \( x_{i,j}^R \). So it can not only save the computing storage space but also conveniently make use of intelligent optimisation algorithm to search optimisation.

## 3 Integer coding wolf pack algorithm

Based on BWPA, ICWPA is put forward on the basis of changing binary coding to integer coding and redefining moving operator and motion operator. Relevant description will be discussed below in detail.

### 3.1 Relevant definitions

Set the solution space to be a European space of \( N \times m \), the position \( X_t \) of the artificial wolf \( i \) is expressed as an integer encoding \((x_{1,i}, x_{2,i}, \ldots, x_{M,i}, x_{N,i})\), \( i = 1, 2, \ldots, N, j = 1, 2, \ldots, m \), \( N \) represents the total number of artificial wolves, \( m \) represents the length of encoding. Element \( x_{ij} \) is the value of \( j^{th} \) coding bit of \( X_t \) and \( x_{ij} \in \mathbb{Z} \).

\[
Z = \{1, 2, \ldots, T_t, \ldots, N_t\} \quad \text{represents the set of integers of the possible value for } x_{ij}. \quad \text{The odor concentration of prey which the artificial wolf perceive is described as the target function value } Y = f(X). \quad \text{Define the distance between the artificial wolves } p \text{ and } q \text{ as the Manhattan distance between their codings, which is described as follows:}

\[
L(p,q) = \sum_{j=1}^{m} |x_{ij} - x_{iq}|, \quad p, q \in \{1,2,\ldots,N\} \tag{23}
\]

**Definition 1**: Movement. The moving \( x_{ij} \) is to assign the Value of \( j^{th} \) coding bit of \( X_t \) in the position \( X_t = (x_{1,i}, x_{2,i}, \ldots, x_{N,i}) \) of the artificial wolf \( i \), according to formula (24), that is to randomly select from \( Z \) an integer which is different from the original value of \( x_{ij} \) to reassign \( x_{ij} \).

\[
x_{ij} = \nu, \quad \nu \in Z, \quad \nu \neq T, \quad \text{if } x_{ij} = T \tag{24}
\]

**Definition 2**: Motion operator. Set \( X_t = (x_{1,i}, x_{2,i}, \ldots, x_{N,i}) \) to be the position of the artificial wolf \( i \). \( M \) represents the moving coding bits set and not the empty set which can be described as the movable range of the artificial wolf. \( r \) represents the number of encoding bits inverted, that is the walking steps of the artificial wolf. The implementation of a motion operator \( \Theta(X_t, M, r) \) represents the process of randomly selecting \( r \) coding bits from \( M \) and moving their value to get a new interpretation.

For example, \( X_t = (2, 3, 1, 1, 3), Z = \{1, 2, 3\}, M = \{2, 4\}, r = 1 \), then \( \Theta(X_t, M, r) \) is any one of \( (2, 1, 1, 1, 3) \), \( (2, 1, 1, 3, 1) \), \( (2, 3, 1, 2, 3) \), \( (2, 1, 1, 3) \). If \( |M| \) is the number of coding bits included in the set \( M \), \( N_S \) is the number of \( x_{ij} \)'s possible values in the set \( Z \). Then, there will be \( r \times 1 \times (N_S - 1)^{|M|} \) kinds of new interpretations after performing motion operator \( \Theta(X_t, M, r) \) on \( X_t \). In this case, we can not only achieve a large space search with the help of small algorithms, but also to maintain the diversity of the solution as well as enhance the probability of jumping out of local extremum. Ultimately it is helpful to the global search of algorithm.

### 3.2 Algorithm description

The same as BWPA, ICWPA also consists of three kinds of intelligent behaviours like rules generation by the leader wolf, wolves update mechanism and wolves’ walk, call and laid siege. The calculation steps of ICWPA are as follows:

**Step 1** Numerical initialisation. Initialise the position \( \{X_t\} \) of the artificial wolf and its number \( N_t \), maximum iteration \( k_{\text{max}} \), maximum migration times \( T_{\text{max}} \), judge distance \( d_{\text{max}} \), and step factor \( S \). Update proportion factor \( \beta \).
Step 2 Walk behaviour. Select the optimal artificial wolf as the leader wolf, the others act as exploring wolves performing walking behaviour. The exploring wolf \( i \) makes one step forward towards \( h \) directions. That is, according to the formula below, \( h \) new positions are obtained by performing motion operator.

\[
\{ X_{1}^{\text{new}}, X_{2}^{\text{new}}, ..., X_{p}^{\text{new}}, ..., X_{h}^{\text{new}} \} = \Theta^h ( X_{1}, M_a, \text{step}_a )
\]  

(25)

In formula (25), set \( M_a = \{1, 2, ..., M\} \), \( \text{step}_a \) is the walk step size. \( Y_0 \) and \( Y_p = f( X_p^{\text{new}} ) \) are respectively the odor concentration of prey which the artificial wolf \( i \) has perceived before and after walking forward towards the direction of \( p \), \( p \in H \), \( H = \{1, 2, ..., h\} \). Because of individual differences, \( h \) generally takes a random integer within a limited range. Select the direction of the \( p^* \) forward and update \( X_i \), repeat the above process until \( Y_i > Y_{\text{lead}} \) or when the migration times \( T \) is over the limited \( T_{\text{max}} \), then turn to step 3. The selected direction of \( p^* \) is as below:

\[
\begin{aligned}
 & p^* \in \max \{ Y_p, p \in H \} \\
 & Y_{p^*} > Y_0
\end{aligned}
\]  

(26)

Step 3 Call behaviour. Treat all the other artificial wolves except the leader one as fierce wolves. Through howling, the leader wolf directs fierce wolves to quickly come closer to the position \( X_{\text{leader}} \) where it is, fierce wolves, then raid with relatively large raid step \( \text{step}_b \). The position \( X_i \) of the fierce wolf \( i \) will transfer according to the following formula:

\[
X_i^{\text{new}} = \Theta^i ( X_i, M_b, \text{step}_b )
\]  

(27)

The set \( M_b \) in the above formula is obtained by the following one.

\[
M_b(k) = \begin{cases} 
  j, k = k + 1, j = j + 1, & \text{if } x_{ik} = x_{ij} \\
  \text{null}, k = j, j = j + 1, & \text{if } x_{ik} \neq x_{ij}
\end{cases}
\]  

(28)

In formula (28), \( j = 1, 2, ..., m \), while the initial value of \( k \) is 1, \( \text{null} \) is empty. Now set \( M_b \) is actually the collection of the position \( X_i \) of fierce wolves and the position \( X_{\text{leader}} \) of the leader one. Do not move the values of the same code bits so as to reflect fierce wolves’ hunting basis and to keep their own advantage. The movement of the values of \( \text{step}_b \) different code bits not only represents the trend of artificial wolves gradually gathering towards the leader one, but also reflects the leader wolf’s command. But if \( M_b \) is null, then perform motion operator \( \Theta^i ( X_i, M_b, r ) \). Set \( Y_i \) to be the odor concentration of prey that the fierce wolf \( i \) has perceived. If \( Y_i > Y_{\text{lead}} \) then \( Y_{\text{lead}} = Y_i \), the fierce wolf \( i \) replaces the leader one. If \( Y_i < Y_{\text{lead}} \), the fierce wolf \( i \) continues to raid until the time when \( d_{a_i} < d_{\text{near}} \), then it turns to besiege prey. Among them, \( d_{a_i} \) is the distance between the fierce wolf \( i \) and the leader one, \( d_{\text{near}} \) is the judge distance.

Step 4 Siege behaviour. The leader wolf directs other wolves to besiege prey. Treat the position \( X_{\text{follower}} \) where the leader one is as the position of prey, change the position \( X_i \) of the artificial wolf \( i \) that participate in the siege according to formula (29) and make greedy decision after comparing odor concentration of prey which is perceived before and after the implementation of the siege behaviour in the new and old position.

\[
X_i^{\text{new}} = \Theta^i ( X_i, M_a, \text{step}_a )
\]  

(29)

In the above formula, \( X_i = \Theta^i ( X_i, M_a, \text{step}_a ) \) not only reflects the wolves of the information transmission and sharing mechanism, but also reflects ‘followership’ and ‘response’ which other wolves show to the outstanding one, that is the leader wolf. \( X_i^{\text{new}} = \Theta^i ( X_i, M_a, \text{step}_a ) \) can be interpreted as group movement that wolves do in prey siege process within a small range around the prey, it can not only reflect that the algorithm nicely search for the optimal solution that the algorithm in the excellent solution domain, but at the same time reduce the probability of the algorithm’s premature convergence. Steps of \( \text{step}_a \), \( \text{step}_b \) and \( \text{step}_c \) are determined by the following formula.

\[
\begin{aligned}
 & \text{step}_a = \text{rand int}[\text{step}_a, S] \\
 & \text{step}_b = \text{rand int}[\text{step}_b, 2S]
\end{aligned}
\]  

(30)

In formula (30), \( \text{step}_a = 1 \), \( S \) is the step size factor, an integer. \( \text{rand int}[\text{step}_a, S] \) represents a random integer in \([1, S]\). Compared with the setting method of step size in the BWPA algorithm, this method, on the one hand reduces the number of parameters, on the other hand, better traverses solution space and maintains a wider search area.

Step 5 Wolf pack update. Compare the objective function value of the optimal wolf after each iteration with the one of the leader wolf last time. If better, then update the position of the leader wolf. Indentify the number \( R \) of artificial wolves washed out according to the scale factor \( \beta \) of wolf pack update. Take \( R \) a random integer in \([N/(2\beta), N/\beta]\), then randomly generate \( R \) new artificial wolves.

Step 6 Judge whether it fulfils the ending condition, if yes, then output the position of the leader wolf (the optimal solution), if not, turn to step 2.
Modelling for combat task allocation problem of aerial swarm and its solution using wolf pack algorithm

Table 4  List of abilities of SA

<table>
<thead>
<tr>
<th>Number of SA</th>
<th>Reconnaissance</th>
<th>Attack</th>
<th>Assessment</th>
<th>Stealth</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>A certain type of advanced fighter</td>
</tr>
<tr>
<td>2</td>
<td>√</td>
<td>×</td>
<td>√</td>
<td>√</td>
<td>A certain type of invisible reconnaissance aircraft</td>
</tr>
<tr>
<td>3</td>
<td>√</td>
<td>√</td>
<td>×</td>
<td>×</td>
<td>A certain type of normal fighter</td>
</tr>
<tr>
<td>4</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>A certain type of invisible bomber</td>
</tr>
<tr>
<td>5</td>
<td>×</td>
<td>√</td>
<td>×</td>
<td>×</td>
<td>A certain type of normal bomber</td>
</tr>
<tr>
<td>6</td>
<td>√</td>
<td>×</td>
<td>√</td>
<td>√</td>
<td>A certain type of reconnaissance aircraft</td>
</tr>
</tbody>
</table>

Note: ‘√’ represents possessing this ability, ‘×’ represents the opposite.

Table 5  List of abilities of AT

<table>
<thead>
<tr>
<th>Ability</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anti-aircraft fire treat</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detection</td>
<td></td>
<td></td>
<td>×</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anti-reconnaissance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: ‘√’represents possessing this ability, ‘×’represents the opposite. The Arabic numerals represent the number of AT.

Table 6  List of aerial swarm combat tasks

<table>
<thead>
<tr>
<th>Task sortie</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconnaissance</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Attack</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Assessment</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: The first line numbers represent the number of AT.

Besides, the parameters of SA’s and AT’s the capability index and value are respectively shown in Tables 7 and 8. Both of the gain parameter $\mu$ of aerial swarm cooperative reconnaissance capability and the gain parameter $\eta$ are set to 0.1. The meaning of each symbol in Tables 7 and 8 is the same as the one in Tables 1 and 2.

Table 7  Parameters of SA

<table>
<thead>
<tr>
<th>Number</th>
<th>VALS</th>
<th>PDS</th>
<th>PAS</th>
<th>PVS</th>
<th>PCS</th>
<th>Arms and ammunition load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.7921</td>
<td>0.8029</td>
<td>0.7908</td>
<td>0.7929</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>0.8235</td>
<td>0</td>
<td>0.8903</td>
<td>0.8206</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>0.6516</td>
<td>0.7826</td>
<td>0.7401</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>0.6299</td>
<td>0.8638</td>
<td>0.8208</td>
<td>0.7502</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>0</td>
<td>0.8309</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>0.8859</td>
<td>0</td>
<td>0.8628</td>
<td>0.4548</td>
<td>0</td>
</tr>
</tbody>
</table>

Using ICWPA to solve the problem of aerial swarm combat task allocation, set ICWPA parameters as follows: $N = 100$, $k_{\text{max}} = 100$, $T_{\text{max}} = 10$, $d_{\text{near}} = 6$, $S = 2$, $\beta = 3$. The target number vector $Y = (1, 2, 3, 3, 4, 5, 6, 7, 8, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 7, 8, 8, 8, 1, 2, 3, 4, 5, 6, 7, 8)$which corresponds to each $X_i$ can be directly obtained from Table 6, is $(1, 2, 3, 3, 4, 5, 6, X_i)$, then the total number of the task sorties is 34, the first 11 are the reconnaissance targets, the middle 15 are the attack targets, the last 8 are the assessment targets. The position $X_i$ is an integer encoding of $(x_{i1}, x_{i2}, ..., x_{im}, i = 1, 2, ..., N, j = 1, 2, ..., m, N = 100, m = 34, and x_{ij} \in \{1, 2, 3, 4, 5, 6\}$.

Figure 3 is the net benefits evolution curve of the model obtained by using ICWPA.

Figure 3  Curve of net benefits evolution (see online version for colours)
As shown in Figure 3, the algorithm converges to the optimal solution after 38 iterations, which reflects the ability of the algorithm to efficiently solve the problem in the face of such a complex model. Finally, the optimal solution of $X_{best} = (6, 4, 2, 1, 2, 2, 4, 2, 6, 1, 4, 4, 4, 5, 1, 3, 3, 4, 5, 3, 5, 4, 1, 5, 1, 4, 1, 3, 3, 3, 4, 1)$, $f(X_{best}) = 460.45$ is obtained from ICWPA algorithm.

At the same time, as shown in Table 9, the results of the cooperative task allocation of aerial swarm.

The principle of task allocation is to ensure maximum net benefit, that is, to maximise benefits of combat, minimise the cost. As shown in Table 9, relatively speaking, SA1, SA2, SA4, all of which have better stealth capabilities, are used repeatedly. If the same SA performs a succession of tasks on the same AT, then anti-aircraft fire threat is not considered. As a result, there are many of the same SA performing reconnaissance, attack, assessment tasks on the same AT. For example, the SA4 with capabilities of stealth and attack performed the reconnaissance task, two units of attack tasks and the assessment task one by one on the AT2 with stronger capability and the AT2 with higher anti-aircraft fire treat index. In turn, the SA1 with capabilities of stealth, attack, assessment and reconnaissance performed the reconnaissance task, cooperative attack task with the SA5 and the assessment task on the AT8.Consistency and quickness of performing tasks in sequence are very important, and repeated threat is not considered in the assumption, this is consistent with the reality, and the method of task allocation is more reasonable. For attack tasks, SA1, SA3, SA4 and SA5 respectively perform three units, three units, five units and four units of attack tasks, the distribution is more balanced. At the same time, it also reflects multi-aircraft cooperation in the reconnaissance and attack tasks. For example, the SA1 and the SA2 cooperatively perform reconnaissance task on the AT3 with higher anti-reconnaissance capability index, the SA3, SA4 and SA5 cooperatively attack the AT6 with the largest value.

Table 9  Results of aerial swarm cooperative combat task allocation

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Number of AT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>Reconnaissance (D)</td>
<td>6 4 2,1 2 2,4 2,6 1</td>
</tr>
<tr>
<td>Attack (A)</td>
<td>4,4 4,5 1 3 3 4,5 3 4,5 1,5,1</td>
</tr>
<tr>
<td>Assessment (V)</td>
<td>4,4 4 1 3 3 3 4 1</td>
</tr>
</tbody>
</table>

Note: The numbers of 2nd, 3rd and 4th line are the number of SA.

Figure 4 shows the operation process of how the aerial swarm, which is composed of 6 SA, cooperatively perform tasks on eight AT.

In Figure 4, the fine horizontal lines, the coarse horizontal lines and the dotted lines respectively represent the tasks of reconnaissance, attack and assessment, the Arabic numerals above lines are the number of SA. According to Figure 4, it can direct the aerial swarm to perform operational tasks on eight targets with different characteristics. This task also begins simultaneously with the cooperative reconnaissance on the SA1 and SA2, and the reconnaissance respectively from the SA4 and the SA6 on target 2 and target 1. The operation task of aerial swarm is finish after the assessment task from the SA1 and the SA3 respectively on target 8 and target 6.

In conclusion, in the task allocation scheme, the ammunition each SA consumed is relatively balanced, the utilisation of resource is also full enough. Moreover, it has not only made full use of the combat advantage of their respective SA, but also reduced the major threat from targets.

5 Conclusions

Aiming at the problem of aerial swarm combat task allocation, this paper establishes the task allocation model of multi-target, multi-task, multi-constrain, heterogeneous multi-aerocraft. The built model considers not only the attributes of swarm aircraft value, reconnaissance capabilities, attack capabilities, assessment capabilities, stealth capabilities, the capabilities of mounting weapons and ammunition, etc., but also the attributes of ATs value, the capabilities of detecting aircrafts, anti reconnaissance capability, anti strike capability, anti-aircraft fire capability and so on. Through the establishment of mapping between combat task allocation and integer coding, integer coding wolves algorithm is proposed based on the binary wolves algorithm, and then the integer coding one is used to solve the model built. The simulation results show that in the solved task allocation schemes, the ammunition consumed by the SAV in tasks is relatively balanced, the resource utilisation is more fully, and the operational advantages of each SAV, such as the advantages of reconnaissance, attack, stealth, etc., are fully made used of to reduce the main threat from targets. Therefore, the model and the solution method
can be used as a useful reference for the task allocation and other related problems of aerial swarm combat.

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References


