Methodology for developing a neural network leaf spring model

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Abstract: This paper describes the development of a neural network that is able to emulate the vertical force-displacement behaviour of a leaf spring. Special emphasis is placed on aspects that affect the predictive capability of a neural network such as type, structure, inputs and ability to generalise. These aspects are investigated in order to enable the effective use of it to model leaf spring behaviour. The results show that with the correct selection of inputs and network architecture, the neural network’s ability to generalise can be improved and also reduce the required training data. The resulting 2-15-1 feed-forward neural network is shown to generalise well and requires minimal data to be trained. Experimental data was used to train and validate the network. The methodology followed is not limited to the application of leaf springs only but should apply to various other applications especially ones with similar nonlinear characteristics.

Keywords: leaf spring modelling; multi-leaf spring; neural networks; generalisation; experimental training data; experimental validation.

Reference to this paper should be made as follows: Kat, C-J., Johrendt, J.L. and Els, P.S. (2017) ‘Methodology for developing a neural network leaf spring model’, Int. J. Vehicle Systems Modelling and Testing, Vol. 12, Nos. 1/2, pp.94–113.
1 Introduction

Virtual prototyping, or simulation, has been playing an increasing role in the vehicle development process and is used in combination with physical prototyping and testing. In order to use virtual prototyping successfully in the development process, accurate and efficient simulation models of the various systems, subsystems and components are required. This paper will focus on the component level with attention given to the modelling of the vertical behaviour of a multi-leaf spring for use in vehicle dynamic simulations.

Even though leaf springs are frequently used in practice they still hold great challenges in creating accurate mathematical models. Many different methods exist that can be used to model the leaf spring’s force-displacement characteristics. Fancher et al. (1980) and Cebon (1986) present an analytical model that uses algebraic equations that are able to fit the behaviour of the experimentally obtained characteristics of the leaf-spring. An equivalent model which models the leaf spring as an equivalent system...
using a vertical spring (or a combination of series and parallel springs) with damper and/or friction elements is used in studies by Hoyle (2004) and ElMadany (1987). Another method discretises the leaf spring into rigid elements. The rigid elements are connected by torsional springs and dampers. This method is known as the discrete or finite segment method and examples of its use in vehicle simulations can be found in Huhtala et al. (1994), Yang et al. (2009), Jayakumar et al. (2005) and Ekici (2005). A similar technique to the discrete method is the finite element method which models the leaf spring using finite element techniques. The use of finite element leaf spring models can be found in Moon et al. (2007), Li and Li (2004) and Qin et al. (2002). Other techniques that exist are the elasto-plastic leaf spring model (Kat and Els, 2011) and models that use neural networks (NN) to emulate the behaviour of the leaf spring (Ghazi et al., 2000). These models have various advantages and disadvantages. The comparison of these models, w.r.t. accuracy, computational efficiency and ease of parameterisation, is outside the scope of this paper. This study will consider the NN technique. It is expected that this technique will be computationally efficient and allow for easy parameterisation.

The study by Ghazi et al. (2000) showed that a NN can be trained to accurately emulate the typical nonlinear, hysteretic behaviour of a leaf spring. The leaf spring data considered in their study was derived from an analytical model (Fancher et al., 1980) representing the typical force-displacement characteristics of a leaf spring. No experimental leaf spring data was used. They used two recurrent NNs with similar architecture; one emulating the loading behaviour and the other one the unloading behaviour of the leaf spring. A switching algorithm was used to determine which one of the networks should be used depending on whether the spring is loaded or unloaded. Each of the loading and unloading NNs has an architecture of $3 \times 10 \times 20 \times 1$. The NN was shown to approximate the analytical model well.

In the current study, experimental data obtained from a physical multi-leaf spring is used to train and validate a NN model. This data contains typical experimental noise. Aspects that affect the predictive capability of the NN are investigated in order to enable the effective use of this approach to model the vertical behaviour of leaf springs. Particular focus is placed on the methodology for developing a suitable NN including type, structure, appropriate inputs and ability to generalise. An aim of the methodology is to develop a NN model that requires minimal (experimental) data to train it.

2 Multi-leaf spring characteristics

The multi-leaf spring considered in this study consists of eight blades with uniform cross-section. The total weight of the multi-leaf spring is 49 kg. Figure 1 shows the in-service setup of the spring. Figure 2 shows a typical force-displacement characteristic of a multi-leaf spring in compression and tension. In general, the multi-leaf spring will seldom be in tension as this occurs when the wheels lose contact with the road. This situation may have a higher possibility of occurring in off-road and very rough road conditions than on smooth on-road conditions. The two major aspects of the leaf spring that a model has to capture for the compression part of the cycle are:

1. the spring stiffness
2. the hysteresis behaviour.
According to Fancher et al. (1980) the force exerted by the leaf spring when deflected, is a function of the loading on the spring and the amplitude of the imposed displacement. Other factors include leaf beam bending stiffness, friction between each leaf, loaded length, as well as boundary friction (between the leaf and the supporting structure) and leaf material structural damping. Fancher et al. (1980) and Cebon (1986) found that the spring force is not dependent on the frequency of the imposed displacement for excitation frequencies up to 18 Hz. This implies that the force displacement characteristics do not need to be obtained at different input frequencies. Figure 3 shows the force-displacement characteristics of the multi-leaf spring used in this study when the amplitude of the displacement input is kept the same but the excitation frequency is swept from 0.05 Hz to 4 Hz. From this figure it can be observed that the force-displacement characteristic stays the same irrespective of the excitation frequency in that frequency range. This is similar to the findings of Fancher et al. (1980) and Cebon (1986).
Figure 3 Force-displacement characteristic of a multi-leaf spring subjected to a sinusoidal displacement with frequencies ranging from 0.05 Hz to 4 Hz

3 Artificial NNs

Artificial NNs are inspired by the biological networks found in the brain. Artificial NNs are mathematical simplifications of the biological counterparts on which they are based. For details on biological neurons and the networks they form the reader is referred to the book by Müller et al. (1995). A model of a simple artificial neuron with one input and one output is shown in Figure 4 which has the following mathematical representation for the output of the neuron:

\[ a = f(wp + b) \]

Similar to neurons being the building blocks of biological networks, the artificial neurons are the building blocks of artificial NNs. The neuron input \( p \) is multiplied by the weight \( w \) to form the term \( wp \) which is sent to the summer. The bias value, \( b \), is also sent to the summer and added to the term \( wp \) to form the value \( n \) which is sent to the transfer function (or activation function) \( f \) and produces the neuron output \( a \). The transfer function \( f \) may be a linear or nonlinear function with the most common transfer functions being a hard limit transfer function, a linear transfer function and a log-sigmoid transfer function. Transfer functions that are continuously differentiable are desirable in NNs as they allow for the back propagation of the error during the training phase which is necessary for weight and bias adjustments to achieve convergence. Modelling nonlinear behaviour requires the use of nonlinear transfer functions. The neuron shown in Figure 4 is a single-input neuron and can be extended to have multiple inputs. These multiple input neurons can be connected in parallel to form layers and layers connected in series to form various network architectures (Hagan et al., 1996). The network architecture that will be used in this study is a feed-forward multilayer network. Other networks that incorporate a feedback of an output to an input are called recurrent networks.
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The combination of various neurons into networks will result in a set of weights ($w$) and biases ($b$). The parameters $w$ and $b$ are the variables of the NN that are adjusted by a learning rule so that the network's input/output relationship reflects that of the data used for training the network. The error between the network output ($a$) and the targets that are given to the NN via the training set is quantified using a performance index such as the mean square error (MSE). The MSE is minimised by means of a back propagation algorithm wherein the network parameters [i.e., the weights ($w$) and biases ($b$)] are adjusted in order to minimise the error between the network output and the target values. Further details about this procedure can be found in Hagan et al. (1996), Dreyfus (2005) and Johrendt et al. (2008).

This brief introduction on NNs has discussed the concept, the different architectures and how they are trained. It has not discussed how to design a NN. The application of a NN to emulate the behaviour of the multi-leaf spring is essentially the approximation of a nonlinear function. It is known that NNs can approximate any nonlinear function (Dreyfus, 2005). Dreyfus (2005) states that the following aspects have to be considered in designing a NN that will be able to approximate a given function:

1. Find the relevant inputs
2. Collect data necessary for training and testing of the NN
3. Train the network
4. Find appropriate complexity of the network (i.e., number of layers, number of neurons per layer)
5. Assess generalisation ability of NN

These guidelines will be followed in order to obtain a NN that is able to emulate the vertical force-displacement characteristics of a multi-leaf spring. The five aspects are discussed in the following paragraphs.

3.1 Relevant inputs and network architecture

It was mentioned in Section 2 that the force exerted by the leaf spring is a function of various factors. The NN is not a physics-based approach to modelling of a physical system, so the most effective use of the NN is to use it to relate the input of interest (i.e., displacement) to the output of interest (i.e., spring force) of the leaf spring. For the purpose of this research, the leaf spring displacement will be used to predict the spring
force, as the single model output. The combined effect of the various factors is taken into account by the NN, even if they’re not explicitly used as network inputs.

The inputs used by Ghazi et al. (2000) were the deflection at the current time step, the absolute value of the deflection change and the force at the previous time step (this was the recurrent input). The selection of these input parameters was based on the analytical equation of Fancher et al. (1980). Deflection, or displacement, of the spring will also be used as one of the input parameters to the NN in this study. Velocity will be used as the other input parameter. The choice of using the displacement as input is obvious as any spring develops a force due to it being deflected. The velocity is chosen as it is expected to indicate to the NN whether it’s being loaded or unloaded, as we know that the hysteretic nature of the spring force-displacement would make it possible to associate one or more values of spring force for each spring deflection point.

There is no clear method by which the architecture of a NN should be determined, but there are some general guidelines which can be followed. For instance, the number of neurons that should be used in order to obtain good generalisation from the NN should be enough to capture the behaviour but no more (Hagan et al., 1996; Dreyfus, 2005). The nonlinear function observed in the force-displacement characteristic is not that complex and it is assumed that a single hidden layer of 35 neurons will enable the NN to emulate the nonlinear behaviour and will therefore serve as a starting point. A higher number of neurons (and layers) will result in a higher level of nonlinearity in the network.

The higher nonlinearity may be needed to capture the nonlinear relationship between the inputs and outputs of the system being emulated. However, a higher level of nonlinearity may result in the NN having too many parameters [i.e., weights \( w \) and biases \( b \)], causing the output of the network to fit the training data very accurately, including the noise in the data, but provide an inaccurate output to inputs not in the training data (Dreyfus, 2005). This is known as overfitting. Overfitting can affect the ability of the network to generalise. Generalisation refers to the ability of the NN to give a sufficiently accurate prediction of the system behaviour for situations that were not present in the training data (Dreyfus, 2005). This will be discussed further in subsequent sections.

The initial network is a feed-forward NN with architecture of two inputs, one hidden layer with 35 neurons and one output being the spring force (referred to as a 2-35-1 network). The transfer functions used in the hidden layer are tan-sigmoid (tansig) functions with the output layer using a linear (purelin) transfer function. This is a much simpler NN than the one used by Ghazi et al. (2000) described earlier. An advantage of using a feed-forward network over a recurrent network is that the training of the feed-forward network is faster.

3.2 Collect necessary data for training and testing of NN

Numerical data was generated from an analytical function (Fancher et al, 1980) in order to train and test the NN in Ghazi et al. (2000). In this study experimental data (with its inherent noise) was obtained and used to train, test and validate the NN. The experimental setup used to obtain the force-displacement characteristic of the multi-leaf spring in compression is shown in Figure 5. The leaf spring is connected to the dummy axle which is connected through an interface plate and U-joint to the actuator via a load cell. The leaf spring is simply supported on bearings by the front and rear supports. This
is a simplified version of the in-service setup of the leaf spring shown in Figure 1. The actuator imposes a predefined displacement onto the multi-leaf spring to which the leaf spring then exerts a corresponding force. The force exerted by the leaf spring is measured by the load cell that is located between the actuator and the multi-leaf spring.

**Figure 5** Experimental setup of the multi-leaf spring (see online version for colours)

![Experimental setup of the multi-leaf spring](image)

**Figure 6** Three different loops shown on (a) the force-displacement characteristic and (b) force and displacement versus data point plots (see online version for colours)

![Force-displacement characteristic and force-displacement plots](image)

Figure 6(a) shows the force-displacement characteristic of the multi-leaf spring that is obtained when subjected to the displacement signal shown in Figure 6(b). The frequency of the displacement signal used during testing was 0.25 Hz. The inertial loads resulting from the maximum acceleration and mass of components, was negligible. The displacement signal results in three observable loops on the force-displacement characteristic, i.e., an outer, inner and end loop. Each closed loop corresponds to the force produced by the leaf spring when subjected to a reciprocating deflection input of different amplitudes. The three loops can be associated with three different vertical loads with
different displacement amplitudes. The training set that will be used to train the NN will be constructed from the experimental data as discussed in the following section.

### 3.3 Training the network

Two training sets were constructed from the experimental data. Training set 1 used data that consisted of only the outer force-displacement loop. Training set 2 used data that consisted of all the loops in the force-displacement characteristic (see Figure 6). The Levenberg-Marquardt training algorithm is used in MATLAB® as it is regarded as being the fastest algorithm for small to medium sized networks (Mathworks, R2016a). The performance index used is the MSE. Early stopping was also used during the training of the network to avoid overfitting and ensure that good generalisation is achieved by the network.

**Figure 7** Comparison of experimental data and NN predictions, NN 2-35-1 trained (a) with outer loop and (b) all loops of force-displacement characteristic (see online version for colours)

Two 2-35-1 NNs were trained, one using Training set 1 and the other using Training set 2. The two NNs will be referred to as 2-35-1 TS1 and 2-35-1 TS2, respectively. After the two networks have been trained, both are simulated by giving them the full displacement input signal shown in Figure 6(b). Figure 7(a) shows the result of the 2-35-1 TS1 NN compared to the experimental data of the actual leaf spring subjected to the same displacement. It can be observed that the NN emulates the leaf spring well for the outer loop for which it was trained. However, for the other loops it gives inaccurate predictions. When we use the NN that was trained using all the loops in the force-displacement characteristic (i.e., 2-35-1 TS2) and compare its prediction to the experimentally measured force-displacement characteristic [see Figure 7(b)], it can be observed that this NN is able to better predict the force for all the loops. From the results in Figure 7 it can be concluded that the NN has difficulty in generalising. In other words, it is not able to correctly predict the force for inputs (in this case displacements and velocities) it was not trained with. This implies that in order for the NN to be able to emulate the behaviour of the leaf spring it has to be trained with data over its entire working range. It would be more advantageous to have a network with a better ability to
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3.4 Appropriate complexity of the NN

The ability of the NN to generalise and the noise on its predictions may be linked to the complexity of the network. In this case reducing the number of hidden neurons (i.e., degrees of freedom of the network) may address both issues. The 35 hidden neurons in the network were reduced to 15, 9 and 4. Reducing the number of neurons did not reduce the noise on the network’s predictions.

Figure 8  Velocity input signal before and after applying the four-point moving average

The velocity input signal is obtained by differentiating the displacement. When the velocity signal is viewed it is observed that the signal has a lot of noise present. This noise on the velocity signal seems to be the source of the noise observed in the predictions of the NN in Figure 7. The velocity signal was smoothed by applying a four point moving average to the signal. Figure 8 shows the velocity signal before and after the four point moving average is applied. Figure 9 shows the results obtained from the two 2-35-1 NNs (2-35-1 TS1 and 2-35-1 TS2) when the four point moving average is applied to the velocity signal. The prediction from the NN has a lot less noise when applying the four point moving average to the velocity input signal. Note, when comparing the results of NN 2-35-1 TS1 between Figure 7(a) and Figure 9(a) it is not only observed that there is less noise on the predictions of the NN but also a noticeable difference in the prediction of the force associated with the displacements of the inner
and end loops. This has to do with the network’s ability to generalise, which is discussed in detail in Section 3.5.

**Figure 9** Comparison of experimental data and NN predictions. Four point moving average applied to the velocity input signal. NN 2-35-1 trained with (a) outer loop and (b) all loops of force-displacement characteristic (see online version for colours)

**Figure 10** Effect of reducing neurons on the predictions of the NN (see online version for colours)
The results in Figure 9(b) show that the 2-35-1 network is able to accurately emulate the behaviour of the multi-leaf spring when it is trained with a training set containing input-output data from all the loops. It was observed in Figure 9(a) that the 2-35-1 network had difficulties in generalising the behaviour and was unable to correctly predict the response when given inputs that were not included in the training set. As mentioned, generalisation is expected to improve when the network is as simple as possible while still being able to adequately represent the training set (Hagan et al., 1996; Dreyfus, 2005).

Therefore, the 2-35-1 network’s neurons are reduced. Figure 10 shows how the predictions of the NN (trained with training set 2) compares to the experimental data as the number of neurons is reduced. It can be seen that the number of neurons can be reduced until the network has nine neurons at which point the network fails to accurately represent the training data. The network with 15 neurons results in a simple network that is still able to accurately represent the training data. This architecture should therefore give us a network that will be able to generalise better. The ability of the network to generalise is discussed further in the following section.

3.5 Assess generalisation

The results in Figure 9(b) show that the NN is able to predict the spring force due to a given displacement when it was trained with data that covered the entire working range of the inputs. In Figure 9(a) however, it was shown that the NN has difficulty in predicting the correct force for inputs not included in the training data. This was referred to as an inability of the NN to generalise. It should be noted that generalisation is often used to refer to the ability of the NN to interpolate correctly between the supplied training data, whereas, in the case shown in Figure 9(a) the NN is actually trying to extrapolate. In other words, generalisation with respect to interpolation means the ability of the NN to give the correct output value for input values that was not part of the training set but falls within the range of the training set values for the input parameter(s). Generalisation with respect to extrapolation means the ability of the NN to give the correct output value for input values that was not part of the training set and falls outside the range of the of the training set values for the input parameter(s).

Figure 11 Results of (a) NN 1-9-1 and (b) NN 1-2-1 when interpolating

Source: Similar to Hagan et al. (1996)
The difference between the generalisation ability of the NN with respect to interpolation and extrapolation is discussed using a simple example similar to the one used in Hagan et al. (1996). A 1-9-1 feed-forward NN was used to emulate an analytical function \( y = 1 + \sin \left( \frac{\pi}{4} x \right) \). After this network was trained it was given a set of inputs.

The inputs include points that were not part of the training set but are within the range of the data in the training set. This network architecture results in the NN having many more adjustable parameters (weights and biases) in comparison to the data points in the training set and therefore does not generalise well [see Figure 11(a)]. To improve the generalisation of the NN the number of neurons can be reduced to give the simplest model that is able to adequately represent the training set (Hagan et al., 1996; Dreyfus, 2005). When the network architecture is reduced to one having two neurons (i.e., 1-2-1) the generalisation is improved as shown in Figure 11(b). Figure 11(b) shows that for any input(s) lying between the data points of the training set, the NN will give good predictions. With the lower level of nonlinearity of NN 1-2-1 overfitting is avoided. Using the 1-2-1 network, the ability of the NN to extrapolate is shown in Figure 12. The 1-2-1 network was trained three separate times. From this figure it is clear that the network is not able to predict the correct response for data outside the range used in the training set. It can be noted that for the region that the network has to extrapolate the network predictions differ for each of the three training runs. This is because the training process results in different combinations of values for the weights and biases. The region where each of these networks interpolates yields similar results, but the region where the networks extrapolate show markedly different results. This example indicated that the 1-2-1 network is able to generalise well with respect to interpolation but not extrapolation.

**Figure 12** Results of 1-2-1 network when interpolating and extrapolating (see online version for colours)

The ability of the feed-forward 2-35-1 NN to generalise the behaviour of the multi-leaf spring was shown in Figure 9(a) to be unacceptable. The network’s neurons were reduced from 35 to 15 in paragraph 3.4 and it was shown to be the simplest network which was still able to adequately capture the behaviour of the multi-leaf spring. This should
improve the generalisation ability of the network. The generalisation ability of the 2-15-1 network is now assessed with attention given to both the interpolation and extrapolation ability of the network.

3.5.1 Generalisation with respect to interpolation

Figure 13(a) shows the results for the 2-15-1 TS1 network that was trained on three occasions. It was simulated with a displacement signal having displacements associated with all the loops of the force-displacement characteristic. The displacement signal does not have any of the displacements used in the training set (training set 1). The results from this figure show that the network has good generalisation with respect to interpolation. It is able to predict the outer loop of the force-displacement characteristic well.

3.5.2 Generalisation with respect to extrapolation

The predictions of NN 2-15-1 TS1 in Figure 13(a) shows that the network is not able to predict the spring force for displacements associated with the inner and end loops. It should be noted that in using the outer loop for training, the displacement values of the inner and end loops do fall within the range of the training set. However, when the velocity of the three loops are considered the velocity of the inner and end loops fall outside the range of the velocity of the outer loop that was used for training. The network therefore has to extrapolate. Similarly, the network will have to extrapolate when given displacements associated with the outer loop at different excitation frequencies to what was used for the outer loop training data. The experimental force-displacement characteristic (see Figure 6) that is used to generate the training set was obtained using an excitation frequency of 0.25Hz. Figure 14(a) shows the results when the 2-15-1 TS1 NN is simulated with the same outer loop displacement signal but with three different excitation frequencies (0.26, 0.3 and 0.5 Hz). The network gives good predictions when it
is simulated with the displacement signal having the same excitation frequency of 0.25Hz as used for training. The predictions deteriorate as the excitation frequency of the displacement input moves away from the excitation frequency used during training. The results in Figure 13(a) are similar to what was observed in Figure 12, i.e. the network has good generalisation with respect to interpolation but not with respect to extrapolation. As stated previously, this implies that, in order for the NN to be able to emulate the behaviour of the leaf spring, it has to be trained with data over its entire working range (i.e. spring displacements and excitation frequencies). It would be more advantageous to have a network with a better ability to generalise as this would require less experimental data for training.

Figure 14 Results of 2-15-1 network using (a) displacement and velocity and (b) displacement and \( \text{sign}(x_i - x_{i-1}) \) as inputs, for different excitation frequencies (see online version for colours)

The dependency of the NN on the excitation frequency is due to the use of velocity as one of the inputs. Figure 15 shows the displacement and velocity inputs to the NN for displacement associated with the outer loop (as indicated in Figure 6) at different excitation frequencies. Note that the displacement and velocity time histories have been shifted such that the point where the loading direction on the spring changes from loading to unloading, coincides. The difference between the velocities for the displacement signal having the same amplitude but different frequencies can be seen from Figure 15. This difference in the velocities causes the network to extrapolate when given different excitation frequencies. It is interesting to note that the NN gives good force predictions at the point of maximum deflection (see Figure 14(a)). This is the moment where the loading on the leaf spring changes and the velocity is zero. At this point the velocity, for all the different excitation frequencies, is similar and therefore the force prediction of the network is good for all the excitation frequencies.
4 Effect of inputs on generalisation

The results shown in Figure 14(a) clearly indicate the dependency of the NN on the excitation frequency of the displacement input, which is contradictory to the behaviour of the physical leaf spring. It was shown in Section 2 that the force-displacement characteristic of the leaf spring is not dependent on the excitation frequency of the displacement input to the leaf spring. This contradiction seems to indicate that velocity may not be a good input to use in order to create a NN that is able to emulate a multi-leaf spring.

The effect of the input(s) on the generalisation ability of the NN is demonstrated by using the same 2-15-1 NN as used in the previous paragraphs but with different inputs. The network considered up to now uses the displacement and velocity as inputs. The choice of displacement as input, for a spring, is obvious. Velocity was initially chosen as it indicates to the NN whether the leaf spring is being loaded or unloaded. The use of velocity causes the network to be dependent on the excitation frequency of the displacement signal. This is in contradicion to the behaviour of the physical leaf spring.
for which the characteristics have been shown to be velocity independent (see Figure 3). Velocity may therefore not be the best choice for use as an input. The velocity input is replaced by a signal that will still indicate to the network whether it is being loaded or unloaded but will not make the network dependent on the excitation frequency of the displacement signal. This signal takes the sign of the difference between the displacement at the current time step and the previous time step [i.e., \( \text{sign}(x_i - x_{i-1}) \)]. Figure 15(c) shows the characteristics of this input parameter. Comparing this to the characteristic of velocity [in Figure 15(b)], it can be seen that the outer loop at 0.25 Hz spans the range of the input parameter and has the same amplitude for different excitation frequencies. This leads to a signal that indicates to the network whether the spring is being loaded or unloaded and is not affected by different excitation frequencies.

The 2-15-1 network using displacement and \( \text{sign}(x_i - x_{i-1}) \) as inputs was trained using the outer loop of the experimental force-displacement characteristic. This network was again simulated with displacement signals having different excitation frequencies to that used for training. The results are shown in Figure 14(b). It can be observed that the network gives similar force predictions for displacement signals having different excitation frequencies. The network is therefore no longer dependent on the excitation frequency of the displacement signal. Figure 13(b) shows the results of this network when trained using the outer loop (training set 1) but simulated using all the loops. Comparing Figure 13(b) to Figure 13(a), a significant improvement in the ability of the network to generalise, particularly with respect to extrapolating beyond the range of the training set, can be observed.

The substitution of velocity with the sign of the difference in displacements at the current and previous time steps \( \text{sign}(x_i - x_{i-1}) \), greatly improved the ability of the NN to generalise. This improvement was due to the fact that the outer loop spans the range of data and has the same amplitude for different excitation frequencies for the input parameter \( \text{sign}(x_i - x_{i-1}) \). There are some aspects with respect to the force predictions of the network, using displacement and the \( \text{sign}(x_i - x_{i-1}) \) as inputs, that have to be further investigated. These aspects include the spikes observed at the points where the direction of loading changes with certain displacement inputs as well as the deviation from the experimental force-displacement characteristic in certain areas of the inner and end loops [see Figure 13(b)]. These deviations are most likely due to the input parameter \( \text{sign}(x_i - x_{i-1}) \) used and further illustrate the effect of the inputs on the results of the NN.

5 Application to other leaf springs and setups

A 2-15-1 NN, with displacement and the \( \text{sign}(x_i - x_{i-1}) \) as inputs, was derived for the multi-leaf spring setup as shown in Figure 5. The 2-15-1 NN is used to emulate the force displacement characteristics of the multi-leaf spring in the in-service setup (Figure 1) as well as of a three-blade parabolic leaf spring in the in-service setup. The blades in the parabolic leaf spring are spaced such that there is only contact between the blades at their ends. The in-service setup of the parabolic leaf spring is similar to the in-service setup of the multi-leaf spring shown in Figure 1. The 2-15-1 NN is trained with a force-displacement data set containing the maximum deflections of the spring, in both compression and extension, for the in-service setup of the multi-leaf and parabolic leaf spring, respectively. The force-displacement characteristics for these two springs in the in-service setups are obtained using an experimental setup similar to the one shown in
Figure 5. Figure 16 shows the results of the NN for the in-service setups of the multi-leaf and a parabolic leaf spring (consisting of three blades) simulated with a displacement input containing different amplitudes. A qualitative comparison between the NN and the experimental data shows good agreement.

**Figure 16** Results of 2-15-1 network using displacement and sign(xi-xi-1) as inputs, in emulating the in-service setups of (a) the multi-leaf spring and (b) a parabolic leaf spring (see online version for colours)

(a)      (b)

6 Conclusions

The methodology to develop a NN multi-leaf spring model has been presented. Aspects such as choosing relevant inputs, network architecture, network complexity training of the network and the generalisation ability of the network were investigated. The effect of the inputs on the generalisation ability of the NN was demonstrated using two 2-15-1 feed-forward NNs, one having displacement and velocity as network inputs and the other displacement and the sign of the difference in displacements at the current and previous time steps [sign(xi – xi–1)]. It was shown that the generalisation ability of the network can be improved by the selection of appropriate network inputs. The advantage of a NN with better generalisation is that less training data is required. This may not be an advantage when training data is generated from an analytical model as in the study by Ghazi et al. (2000). However, when experimental data is to be used for the training data it would be advantageous if a minimal amount of data is sufficient to train the NN. In the case of the leaf spring it means that only data of the outer loop, or in other words, force-displacement data of the leaf spring’s maximum displacement range is required. Furthermore, appropriate inputs will also focus the network toward interpolation instead of extrapolation which, from the results in this study, is a desirable situation. The generalisation of the network that has to interpolate can be improved by selecting the least amount of neurons that is still able to capture the response of the spring.

The methodology followed resulted in a simple feed-forward network with two inputs [displacement and sign(xi – xi–1)] and one hidden layer with 15 neurons that is able to emulate the vertical force-displacement behaviour of the multi-leaf spring. This NN requires minimal (experimental) data for training. The NN created in this study gives a
component level model of the leaf spring which can be extended to create more detailed models in order to represent the suspension system. From the results obtained in this study it is postulated that the NN inputs strongly affect the ‘intelligence’ of the NN and influence the generalisation ability of the network. Well-chosen inputs can improve the generalisation of the NN (especially when extrapolating) and may reduce the required range of the training set.

As stated, NNs can approximate any nonlinear function (Dreyfus, 2005). In designing a NN to approximate a given function (or emulate the characteristics of a physical component/system) certain aspects need to be considered (i.e., finding the relevant inputs, data necessary for training and testing the NN, training the network and finding the appropriate complexity of the network). By taking these aspects into account one should be able to obtain a NN that can approximate any given nonlinear function. Considering leaf spring force-displacement characteristics, various leaf spring types (multi-leaf spring, parabolic leaf spring) and configurations have similar characteristics. It is therefore expected that this methodology will be applicable to various leaf springs and configurations. Section 5 showed that the NN model developed is able to emulate the force-displacement characteristics of the in-service setups of an eight-blade multi-leaf spring and a three-blade parabolic leaf spring. This is not to say that the methodology, applied to other leaf springs (and their boundary conditions), will result in the same NN architectures as developed in this study. The methodology presented is not limited to the application of leaf springs only but should apply to various other applications especially those with similar nonlinear characteristics.

7 Future work

A thorough validation of the NN must be done by comparing the predictions of the NN model to experimental data of the leaf spring obtained from subjecting the leaf spring to white noise excitation as well as more structured road profile data containing a range of amplitudes and frequencies that span the full behaviour of the spring.

References


Methodology for developing a neural network leaf spring model


