A simple method for study of effect of Kerr nonlinearity on effective core area, index of refraction and fractional modal power through the core of monomode graded index fibre

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Abstract: Using the simple power series expression for fundamental modal field derived by Chebyshev technique, we present investigation of some important propagation parameters like effective core area, index of refraction and fractional modal power guided through the core of single-mode graded index fibre in presence of Kerr nonlinearity. In absence of nonlinearity, the said power series expression leads to prescription of analytical expressions of the concerned propagation parameters. Employing those analytical expressions, we apply iterative method in order to evaluate the said parameters in presence of Kerr nonlinearity. Choosing some typical single-mode step and parabolic index fibres for our study, we verify that the results obtained by our simple formalism match excellently with the exact results which are obtainable by applying rigorous finite element technique. This excellent agreement attests to the accuracy of our formalism. Further, our formalism requires little computation in the context of evaluation of the said parameters. Accordingly, our formalism can be considered as a simple but accurate alternative to the existing complicated methods available in literature. Thus, this user-friendly but accurate formalism will benefit the system engineers in respect of selection of suitable fibre in which modal noise due to nonlinearity is minimum.

Keywords: graded index fibre; cladding decay parameter; *V* number; effective core area; index of refraction; confinement of modal power; Kerr nonlinearity; Chebyshev technique.

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1 Introduction

Single-mode optical fibre has emerged as the most effective medium of communication of signal on account of its large bandwidth. The study of influence of nonlinearity on propagation parameters of monomode optical fibre in the nonlinear region (Tomlinson et al., 1984; Tai et al., 1986; Snyder et al., 1990; Goncharenko, 1990; Sammut and Pask, 1990; Agrawal and Boyd, 1992) has immense importance in the field of nonlinear optics. Further, intensity of optical beam as well as nature of the transmitting medium are the two parameters which are responsible for generation of different kinds of nonlinearity namely third order, fifth order, etc. (Agrawal, 2013). It is relevant to mention in this connection that there is broadening of the pulse due to dispersion while compression of the pulse takes place due to nonlinearity. Thus, interplay of dispersion and nonlinearity together results in propagation of optical beam as such (Agrawal and Boyd, 1992) and this is known as propagation of optical soliton. It deserves mentioning in this connection that nonlinearity generates noise which affects the communication of signal through the fibre. Accordingly, the study of noise due to different kinds of nonlinearity has generated tremendous interest among the research scientists (Antonelli et al., 2017). Again, the study of high quality micro resonator in presence of intense Kerr nonlinearity produces the platform of integrated nonlinear photonics (Lu et al., 2014). Further, the effect of Kerr type nonlinearity on opto-mechanical ring resonator has also been investigated (Yu et al., 2012).

The effective area of the core of the fibre controls the distortion of the signal owing to nonlinearity (Streckert and Wilezewski, 1996; Namihara, 1997a, 1997b; Hallam, 1994). Accordingly, the power of signal transmitted through the core of the fibre is also influenced by nonlinearity and its study has also emerged as a potential problem in the field of optical devices. Thus, the effect of nonlinearity on the fractional modal power guided through the core of the fibre is also an important matter in the field of communication and sensors.

Moreover, third order nonlinearity can generate four-wave mixing, self-phase modulation, stimulated Brillouin scattering and stimulated Raman scattering and thus, one is required to take care so that nonlinearity can not decrease the output power of high power fibre laser (Ghatak and Thygarajan, 1999; Taverner et al., 1997; Jiang and Marciante, 2006; Li et al., 2009). Accordingly, effective core area has to be controlled by proper change of refractive index distribution and the radius as well. In the process, one need to take care so that the single-mode region is maintained, thus prediction of effective core area in presence of nonlinearity is a potential problem. However, the estimation of effective area of the fibre needs accurate expression of fundamental modal field. Side by side, the accurate knowledge effective index of refraction is required in various fields like system modelling of system, packaging of optical device, selection of proper index matching gel which will minimise back reflection and joint losses. The value of effective index of refraction (n_{eff}) can be used for evaluation of the net delay of propagation for the particular mode at a particular wavelength. Further, in case of dual mode optical fibre, the interaction between the fundamental and first higher order mode can be assessed on the basis of knowledge of index of refraction (Savolinen et al., 2012). Moreover, the knowledge of effective refractive index can be utilised to develop suitable model for investigation of different propagation parameters properties of important devices like fibre Bragg grating (Patrick et al., 1996), and photonic crystal fibre (Knight et al., 1998a, 1998b). It can be mentioned in this context that one needs to know accurate value of cladding decay parameter for accurate estimation of effective refraction index.

Analytical expression for fundamental modal field for step index fibre in absence of nonlinearity is available in literature. But, for fibres having other kinds of refractive index profiles, one has to apply numerical techniques or approximate methods in order to predict the fundamental modal fields in absence of nonlinearity. The variational technique involving two parameter Gaussian trial functions can estimate the fundamental modal field of graded index fibre and associated propagation characteristics over a sufficiently long range of V number in absence of nonlinearity (Ankiewicz and Peng, 1992; Mishra et al., 1984). But to have more accuracy in such prediction in absence of nonlinearity, one needs to apply variational technique involving Gaussian-exponential-Hankel function (Hosain et al., 1982). Thus, formulation of a simple but accurate expression of fundamental modal field of linear graded index fibre is proliferating in literature.

Application of Chebyshev formalism in correct estimation of the first higher order mode cut-off frequency of optical fibres in presence of Kerr nonlinearity has been already added to Roy and Sarkar (2016). Moreover, Chebyshev technique has been also found to be predicting excellently the propagation parameters of single-mode fibres in presence of Kerr type nonlinearity (Sadhu et al., 2014). Further, Chebyshev formalism has been excellent in predicting fundamental modal field for Kerr type nonlinear monomode step and parabolic index fibres in simple but accurate fashion (Chakraborty et al., 2017). The excellent match between the found results and the simulated exact results found by finite element method (Hayata et al., 1987) were also reported in Chakraborty et al. (2017).

In this communication, we use the fundamental modal field as well as cladding decay parameter of Kerr type nonlinear graded index fibres found by Chebyshev formalism in order to predict the effective area of the core, effective index of refraction and fractional modal power guided through the core for some fibres having typical V numbers. Again, we also present here the excellent agreement between our results and the exact results obtainable by quite rigorous finite element method (Hayata et al., 1987).

The rest of the paper has been organised as follows: Section 2 contains theory while results and discussions have been presented in Section 3. Sections 4 comprise conclusions.

2 Theory

The refractive index profile for a circular core fibre is presented as below:

$$n^2(R) = \begin{cases} n_1^2\left(1-2\delta f(R)\right), & R \le 1 \\ n_2^2, & R > 1 \end{cases} \quad (1)$$

where a = core radius, $R = r / a$ and $\delta = (n_1^2 - n_2^2) / 2n_1^2$.

Here, n_1 and n_2 denote the axial and cladding refractive indices respectively and $f(R)$ presents the shape of refractive index profile inside the core. The expression relating in case of fibre having the power law profile, $f(R)$, can be expressed as follows:

$$f(R) = R^q \quad R \le 1 \quad (2)$$

Here, q stands for the profile exponent the values of which for step and parabolic index fibres are ∞ and 2, respectively. The refractive index distribution $n(R)$ of the fibre in presence of Kerr-type nonlinearity can be presented as below (Mondal and Sarkar, 1996):

$$n^2(R) = n_L^2(R) + \frac{n_2^2 n_{NL}(R)}{\eta_0}\psi^2(R) \quad (3)$$

Further, μ_0 and ε_0 represent the permeability and permittivity of free space respectively with η_0 being equal to $\left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}}$. Again, $n_L(R)$ and $n_{NL}(R)$ denote the linear refractive index and the radial distribution of nonlinear Kerr coefficient (m^2/W), respectively. The nonlinear fundamental modal field $\psi(R)$ satisfies the following scalar wave equation (Mondal and Sarkar, 1996):

$$\frac{d^2\psi(R)}{dR^2}+\frac{1}{R}\frac{d\psi(R)}{dR}+\left[V^2\left(1-f(R)\right)-W^2\right]\psi(R)+V^2g(R)\psi^3(R)=0 \tag{4}$$

where $V\lfloor = k_0 a(n_1^2-n_2^2)^{1/2}\rfloor$ and $W\lfloor = a(\beta^2-n_2^2k_0^2)^{1/2}\rfloor$ stand for the normalised frequency and cladding delay parameter, respectively. Here, β and k_0 represent the propagation constant and free space wave number, respectively.

Again,

$$g(R)=\frac{n_2 n_{NL}P}{\pi a^2\left(n_1^2-n_2^2\right)} \tag{5}$$

Moreover, the boundary condition at core cladding interface is given by

$$\left[\frac{1}{\psi}\frac{d\psi}{dR}\right]_{R=1}=-\left[\frac{WK_1(W)}{K_0(W)}\right] \tag{6}$$

Here, K_1 and K_0 represent the modified Bessel functions of first and zero order, respectively.

The field inside the cladding is, however, found as below:

$$\psi(R)\sim K_0(WR), \qquad R>1 \tag{7}$$

Application of Chebyshev formalism leads to the following power series expression for the fundamental modal field in case of graded index fibre (Gangopadhyay et al., 1997; Patra et al., 2000)

$$\begin{aligned}\psi(R)&=\left(a_0+a_2R^2+a_4R^4+a_6R^6\right), && R\le 1\\ &=\left(a_0+a_2+a_4+a_6\right)\frac{K_0(WR)}{K_0(W)}, && R>1\end{aligned} \tag{8}$$

where a_0, a_2, a_4 and a_6 stand for multiplicative constants.

Following Chen (1982), the Chebyshev points can be given as follows:

$$R_m=\cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right) \qquad m=1,2,\ldots,(M-1) \tag{9}$$

Here, equation (8) corresponds to M = 4 and accordingly, three values of R found from equation (9) are as follows:

$$R_1=0.9749,\ R_2=0.7818 \text{ and } R_3=0.4338 \tag{10}$$

Employing equation (8) and equation (4), we obtain

$$\begin{aligned}&a_0\left[V^2\left(1-f\left(R_i\right)\right)-W^2+V^2g\psi^2\left(R_i\right)\right]\\ &+a_2\left[4+R_i^2\left\{V^2\left(1-f\left(R_i\right)\right)-W^2+V^2g\psi^2\left(R_i\right)\right\}\right]\end{aligned}$$

$$+a_4\left[16R_i^2+R_i^4\left\{V^2\left(1-f\left(R_i\right)\right)-W^2+V^2g\psi^2\left(R_i\right)\right\}\right]$$
$$+a_6\left[36R_i^4+R_i^6\left\{V^2\left(1-f\left(R_i\right)\right)-W^2+V^2g\psi^2\left(R_i\right)\right\}\right]=0 \quad (11)$$

where $i = 1, 2, 3$.

Moreover, least square fitting in the interval $0.6 \leq W \leq 2.5$ gives the following expression (Gangopadhyay et al., 1997):

$$\frac{K_1(W)}{K_0(W)}=\alpha+\frac{\beta}{W} \quad (12)$$

where $\alpha = 1.034623$ and $\beta = 0.3890323$.

Using equations (8) and (12) in equation (6), we obtain and the following expression:

$$a_0(\alpha W+\beta)+a_2(\alpha W+2+\beta)+a_4(\alpha W+4+\beta)+a_6(\alpha W+6+\beta)=0 \quad (13)$$

The condition of non-trivial solution of the multiplicative constants present in equations (11) and (13) is given as follows:

$$\begin{vmatrix}\alpha_1 & \beta_1 & \gamma_1 & \delta_1\\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2\\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3\\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4\end{vmatrix}=0 \quad (14)$$

where

$$\begin{aligned}\alpha_i &= V^2\left[1-f\left(R_i\right)\right]-W^2+V^2g\psi^2\left(R_i\right)\\ \beta_i &= 4+R_i^2\left[\left(V^2\left(1-f\left(R_i\right)\right)-W^2\right)+V^2g\psi^2\left(R_i\right)\right]\\ \gamma_i &= 16R_i^2+R_i^4\left[\left(V^2\left(1-f\left(R_i\right)\right)-W^2\right)+V^2g\psi^2\left(R_i\right)\right]\\ \delta_i &= 36R_i^4+R_i^6\left[\left(V^2\left(1-f\left(R_i\right)\right)-W^2\right)+V^2g\psi^2\left(R_i\right)\right]\end{aligned} \quad (15)$$

with $i = 1, 2, 3$ and

$$\alpha_4=\alpha W+\beta,\ \beta_4=2+\alpha_4,\ \gamma_4=4+\alpha_4,\ \delta_4=6+\alpha_4$$

The presence of the term $\psi^2(R)$ makes the solution of equation (14) quite complicated. Accordingly, we first use $g = o$ in order to solve equation (14) for the linear region. Thus, one can obtain W value for a particular value of V. Then, this W value and the corresponding V value are used in equation (11) to get a_2, a_4 and a_6 in terms of a_0 in the linear region. Then, for particular value of g, the method of iteration is applied a number of times till convergent value of W corresponding to a particular V value is obtained. This W value results in evaluation of values of a_2, a_4 and a_6 in terms of a_0 (Chakraborty et al., 2017).

The effective core area A_{eff} is given as follows (Agrawal, 2013):

$$A_{eff} = 2\pi a^2 \frac{\left(\int_0^\infty \psi^2(R)RdR\right)^2}{\int_0^\infty \psi^4(R)RdR} \tag{16}$$

Using equation (8) in equation (16), we get

$$A_{eff} = 2\pi a^2 \frac{\left(\int_0^1 \left(1+A_2R^2+A_4R^4+A_6R^6\right)^2 RdR + \int_0^\infty \left(1+A_2+A_4+A_6\right)^2 \frac{K_0^2(WR)}{K_0^2(W)} RdR\right)^2}{\int_0^1 \left(1+A_2R^2+A_4R^4+A_6R^6\right)^4 RdR + \int_0^\infty \left(1+A_2+A_4+A_6\right)^4 \frac{K_0^4(WR)}{K_0^4(W)} RdR} \tag{17}$$

where $A_{2j} = a_{2j} / a_0$ with $j = 1, 2$ and 3.

Equation (17) is evaluated following Gradshteyn and Ryzhik (2014), Watson (1995) and Abramowitz and Stegun (2012), whereby the following expression is obtained:

$$A_{eff} = 2\pi a^2 \frac{\left[0.5\left\{S_1 + 2S_2 + S_3\left(S_4^2 - 1\right)\right\}\right]^2}{S_5 + S_3^2 S^6} \tag{18}$$

where

$$\begin{aligned}
S_1 &= \left(1 + \frac{A_2^2}{3} + \frac{A_4^2}{5} + \frac{A_6^2}{7}\right) \\
S_2 &= \left(\frac{A_2}{2} + \frac{A_4}{3} + \frac{A_6}{4} + \frac{A_2A_4}{4} + \frac{A_2A_6}{5} + \frac{A_4A_6}{6}\right) \\
S_3 &= \left(1 + A_2 + A_4 + A_6\right)^2 \\
S_4 &= \left(\alpha + \frac{\beta}{W}\right) \\
S_5 &= \frac{1}{2} + A_2 + \frac{1}{3}\left(3A_2^2 + 2A_4\right) + \frac{1}{2}\left(A_3^2 + 3A_2A_4 + A_6\right) \\
&\quad + \frac{1}{10}\left(6A_4^2 + 12A_2A_6 + 12A_2^2A_4 + A_2^4\right) + \frac{1}{3}\left(3A_2A_4^2 + 3A_2^2A_6 + A_2^3A_4 + 3A_4A_6\right) \\
&\quad + \frac{1}{7}\left(3A_6^2 + 12A_2A_4A_6 + 3A_2^2A_4^2 + 2A_2^3A_6 + 2A_4^3\right) \\
&\quad + \frac{1}{4}\left(3A_2A_6^2 + 3A_2^2A_4A_6 + 3A_4^2A_6 + A_2A_4^3\right) \\
&\quad + \frac{1}{18}\left(6A_2^2A_6^2 + 12A_2A_4^2A_6 + 12A_4A_6^2 + A_4^4\right) + \frac{1}{5}\left(3A_2A_4A_6^2 + A_6^3 + A_4^3A_6\right) \\
&\quad + \frac{1}{11}\left(3A_4^2A_6^2 + 2A_2A_6^3\right) + \frac{1}{6}A_4A_6^3 + \frac{1}{26}A_6^4 \\
S_6 &= \frac{1}{K_0^4(W)} \times \frac{1}{W^2}\left[\exp\left(0.2076W^2 - 5.0785W - 0.1185\right)\right]
\end{aligned} \tag{19}$$

Thus, A_{eff} can be obtained by using the obtained values of A_2, A_4, A_6 and W.

The effective refractive index (n_{eff}) of an optical fibre is given as

$$n_{eff} = \frac{\beta}{k_0} = \left(n_2^2 + \frac{W^2}{a^2 k_0^2} \right)^{1/2} \tag{20}$$

Here, n_2 and k_0 denote the refractive index of the cladding and the propagation constant, respectively. Moreover, the cladding decay parameter W, the waveguide parameter U and the normalised frequency V are connected by the relationship, $U^2 = (V^2 - W^2)$.

Using equation (20), one can easily obtain the value of n_{eff} in the nonlinear region from the knowledge of core radius, wavelength of light, refractive index of cladding and the value of cladding decay parameter evaluated in the nonlinear region.

The fractional power f_{co} of the fundamental modal field propagating through the fibre-core is given by (Gangopadhyay and Sarkar, 1997)

$$f_{co} = \frac{\int_0^1 |\psi(R)|^2 \, RdR}{\int_0^\infty |\psi(R)|^2 \, RdR} \tag{21}$$

Employing $\psi(R)$ given in equation (8), we obtain (Gradshteyn and Ryzhik, 2014; Watson, 1995; Abramowitz and Stegun, 2012)

$$f_{co} = \frac{T_2 + 2T_3}{T_2 + 2T_3 + T_1 \left(T_4^2 - 1 \right)} \tag{22}$$

where

$$\begin{aligned} T_1 &= \left(1 + A_2 + A_4 + A_6\right)^2 \\ T_2 &= 1 + \frac{A_2^2}{3} + \frac{A_4^2}{5} + \frac{A_6^2}{7} \\ T_3 &= \frac{A_2}{2} + \frac{A_4}{3} + \frac{A_6}{4} + \frac{A_2 A_4}{4} + \frac{A_2 A_6}{5} + \frac{A_4 A_6}{6} \\ T_4 &= 1.034623 + \frac{0.3890323}{W}. \end{aligned} \tag{23}$$

3 Results and discussion

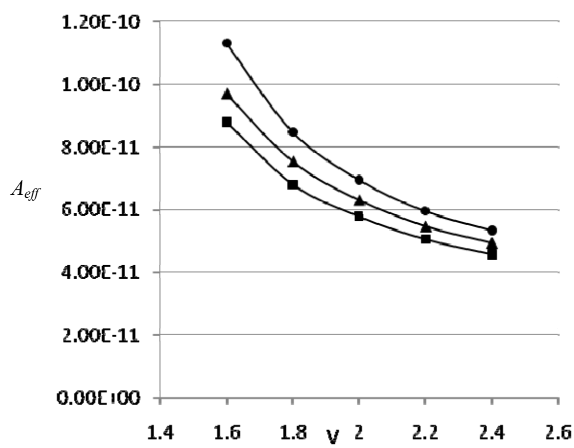
Here, we find out values of effective core area (A_{eff}) and effective refraction index (n_{eff}) for both step and parabolic index single-mode fibres having some typical values of normalised frequency (V) in presence as well as in absence of Kerr nonlinearity. Here, the radius of the fibre is taken as 3.65 μm, refractive index of cladding as 1.45 and the wavelength of light as 1.3 μm. It deserves mentioning in this connection that we use the method of iteration in order to evaluate the concerned parameters. Moreover, we have also found out the values of the parameters by finite element method (Hayata et al., 1987) for the purpose of verification of the accuracy of our formalism. In respect of execution of variational technique involving finite element method, we have used MATLAB as simulator. Further, change of refractive index due to Kerr nonlinearity is dependent on both nonlinear coefficient $n_{nL}(R)$ (m^2/W) and the power $P(W)$ of incident light. This has led us to take care of the term $n_{nL}(R)P$ (m^2) for the nonlinear study. Here, the positive value of $n_{nL}P$ causes self-focussing while negative value of $n_{nL}P$ leads to self-defocusing. Taking into consideration this behaviour, we carry on our investigation for $n_{NL}P = -1.5 \times 10^{-14}$ m^2 and $n_{NL}P = +1.5 \times 10^{-14}$ m^2 (Chakraborty et al., 2017; Mondal and Sarkar, 1996).

In Table 1, we have shown found values of effective core areas in case of step index fibres for the typical positive and negative nonlinearity along with effective core area in absence of nonlinearity. We have conducted our investigation for step index fibres having V numbers 1.6, 1.8, 2.0, 2.2 and 2.4.

Table 1 Effective core area (A_{eff}) of step index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

V	*A_{eff} of step index fibre*		
	$n_{NL}P = +1.5 \times 10^{-14}$ m^2	$n_{NL}P = -1.5 \times 10^{-14}$ m^2	$n_{NL}P = 0$ *(Chowdhury and Gangopadhyay, 2018)*
1.6	8.80491E–11	1.13119E–10	9.69736E–11
1.8	6.82082E–11	8.48564E–11	7.54159E–11
2.0	5.78641E–11	6.95929E–11	6.30678E–11
2.2	5.06856E–11	5.95213E–11	5.46956E–11
2.4	4.59143E–11	5.3472E–11	4.93757E–11

Figure 1 Effective core area (A_{eff}) versus V number for step index fibre



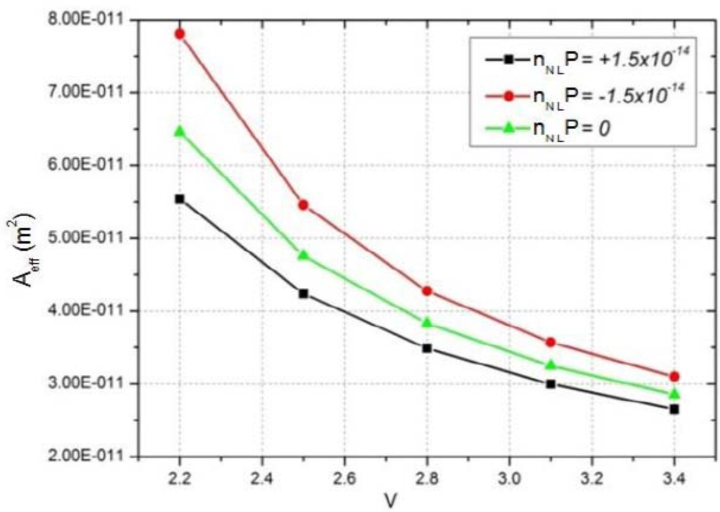
Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

Here, Figure 1 contains graphical presentation of our found values 2.2, 2.5, 2.8, 3.1 and 3.4. Table 2 contains similar observations for parabolic index fibre. Here, V numbers used are 2.2, 2.5, 2.8, 3.1 and 3.4 while Figure 2 comprises the graphical presentation of the observations in Table 2.

Table 2 Core area (A_{eff}) of parabolic index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

V	*A_{eff} of parabolic index fibre*		
	$n_{NL}P = +1.5 \times 10^{-14}\ m^2$	$n_{NL}P = -1.5 \times 10^{-14}\ m^2$	$n_{NL}P = 0$ *(Chowdhury and Gangopadhyay, 2018)*
2.2	5.54057E–11	7.80451E–11	6.45935E–11
2.5	4.23902E–11	5.45679E–11	4.76005E–11
2.8	3.48866E–11	4.27716E–11	3.83203E–11
3.1	2.99766E–11	3.57058E–11	3.25075E–11
3.4	2.6529E–11	3.10022E–11	2.8521E–11

Figure 2 Effective core area (A_{eff}) versus V number for parabolic index fibre (see online version for colours)



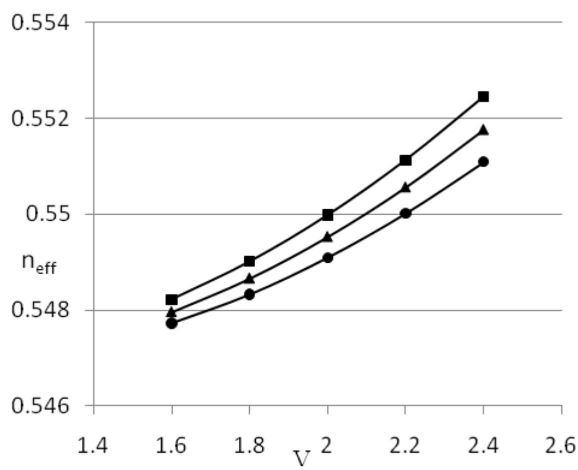
Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

Table 3 Effective index of refraction (*neff*) of step index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

V	*n_{eff} of step index fibre*		
	$n_{NL}P = +1.5 \times 10^{-14}\ m^2$	$n_{NL}P = -1.5 \times 10^{-14}\ m^2$	$n_{NL}P = 0$ *(Chowdhury and Gangopadhyay, 2018)*
1.6	0.54822	0.54773	0.54796
1.8	0.54901	0.54833	0.54866
2.0	0.54998	0.5491	0.54953
2.2	0.55113	0.55002	0.55056
2.4	0.55245	0.55109	0.55176

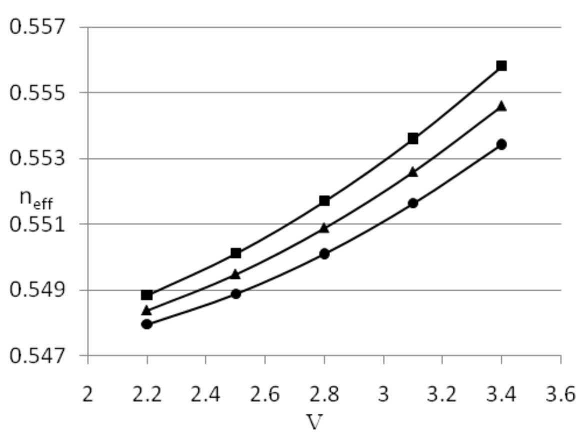
Table 3 contains our found values of effective index of refraction for step index fibre in presence of the said kinds of nonlinearity as well as in absence of the nonlinearity for *V* numbers 1.6, 1.8, 2.0, 2.2 and 2.4. The graphical presentation of our observations given in Table 3 has been shown in Figure 3. In case of parabolic index fibre, we have presented our estimated values in Table 4. Here, the *V* numbers used are same as before and those are namely 2.2, 2.5, 2.8, 3.1 and 3.4. The observations shown in Table 4 are used to make the graphical presentation in Figure 4 which contains the variation of effective refractive index varies with *V* number for parabolic index fibre.

Figure 3 Effective index of refraction (n_{eff}) versus *V* number for step index fibre



Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

Figure 4 Effective index of refraction (n_{eff}) versus *V* number for parabolic index fibre



Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

Table 4 Effective index of refraction (n_{eff}) of parabolic index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

V	n_{eff} of parabolic index fibre		
	$n_{NL}P = +1.5 \times 10^{-14}$ m^2	$n_{NL}P = -1.5 \times 10^{-14}$ m^2	$n_{NL}P = 0$ (Chowdhury and Gangopadhyay, 2018)
2.2	0.54884	0.54795	0.54837
2.5	0.55011	0.54888	0.54947
2.8	0.5517	0.5501	0.55088
3.1	0.5536	0.55163	0.55259
3.4	0.55581	0.55343	0.5546

In Table 5, we have presented evaluated values of fractional modal power guided through the core of monomode step index fibre for the said positive and negative nonlinearity as well as in absence of nonlinearity. Here, we have also carried our investigation for the typical V numbers namely 1.6, 1.8, 2.0, 2.2 and 2.4. In Figure 5, we have made graphical presentation of observations shown in Table 5. Similarly in case of parabolic fibre, we have presented evaluated values of fractional modal power guided through the core of monomode parabolic index fibre for the said positive and negative nonlinearity as well as in absence of nonlinearity in Table 6. Here, we have carried out the investigations for parabolic index fibre using six typical V numbers namely 2.2, 2.5, 2.8, 3.1 and 3.4. In Figure 6, we have made graphical presentations of the results given in Table 5.

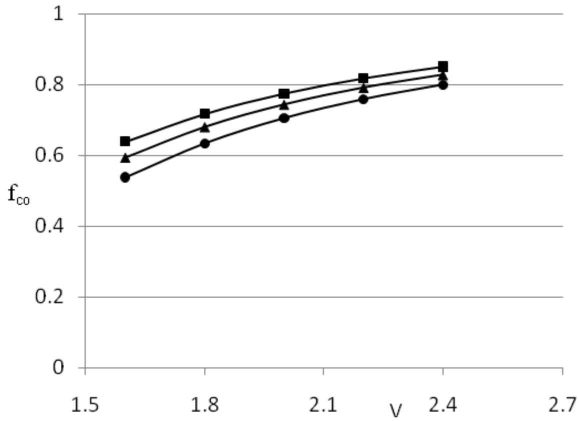
Table 5 Fractional modal power guided through core (f_{co}) of step index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

V	f_{co} of step index fibre		
	$n_{NL}P = +1.5 \times 10^{-14}$ m^2	$n_{NL}P = -1.5 \times 10^{-14}$ m^2	$n_{NL}P = 0$ (Gangopadhyay and Sarkar, 1997)
1.6	0.638875272249	0.537941188795	0.594206949645
1.8	0.717135941070	0.634023309013	0.680450629770
2	0.774395982744	0.705105767437	0.743836015180
2.2	0.817143355870	0.758470655574	0.791269739058
2.4	0.849678314899	0.799211692545	0.827423664967

Table 6 Fractional modal power guided through core (f_{co}) of parabolic index fibre for different V numbers in presence of positive nonlinearity ($+n_{NL}P$), negative nonlinearity ($-n_{NL}P$) and linear region ($n_{NL}P = 0$)

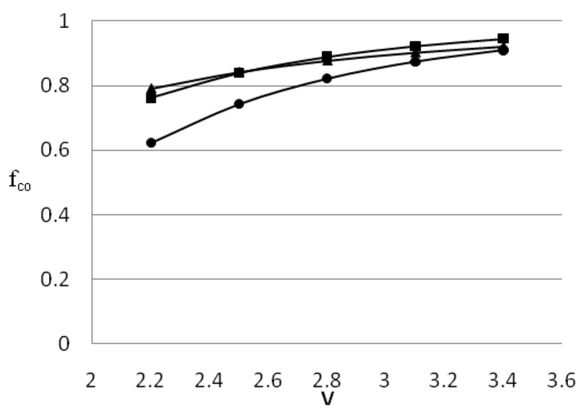
V	f_{co} of parabolic index fibre		
	$n_{NL}P = +1.5 \times 10^{-14}$ m^2	$n_{NL}P = -1.5 \times 10^{-14}$ m^2	$n_{NL}P = 0$ (Gangopadhyay and Sarkar, 1997)
2.2	0.761928288672	0.622781302876	0.791269739058
2.5	0.838881801757	0.742972841489	0.842302326472
2.8	0.889373411911	0.822152063534	0.877546566173
3.1	0.923018224673	0.874911033030	0.902706048293
3.4	0.945779428436	0.910660475093	0.921183037040

Figure 5 Fractional modal power guided through the core (f_{co}) versus V number for step index fibre



Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

Figure 6 Fractional modal power guided through the core (f_{co}) versus V number for parabolic index fibre



Note: Our results: ■ for $n_{NL}P = +1.5 \times 10^{-14}$ m^2, ● for $n_{NL}P = -1.5 \times 10^{-14}$ m^2, ▲ for $n_{NL}P = 0$ and simulated exact results (Hayata et al., 1987): ———.

In general, in all the said figures ■, ● and ▲ correspond to $n_{NL}P = +1.5 \times 10^{-14}$ m^2, $n_{NL}P = -1.5 \times 10^{-14}$ m^2 and $n_{NL}P = 0$, respectively while the solid lines represent the simulated exact values.

It has been found here that effective area as well as fractional modal power guided through the core is more affected due to nonlinearity in the low V region. Accordingly, the figures also show the dominance of nonlinear effect in low V region. However, the effect of nonlinearity on the effective refractive index is practically the same throughout the V region in case of both single-mode step and parabolic index fibres.

Thus, the present investigation leads to suitable selection of V number for a particular fibre for the purpose of reduction of modal noise generated by nonlinearity. Literature has been already enriched by reports of success of application of Chebyshev formalism in the context of estimation of different propagation characteristics of graded index fibres having Kerr type nonlinearity (Roy and Sarkar, 2016; Sadhu et al., 2014; Chakraborty et al., 2017a, 2017b). But, as per our knowledge, no such simple but accurate method involving Chebyshev formalism in order to estimate effective area, index of refraction and fractional modal power guided through the core of single-mode Kerr type nonlinear graded index fibre has been reported till date. Thus, our formalism is novel. It is relevant to mention in this connection that one needs to employ simulator for lengthy computation involved in the field of nonlinear optics (Burdin et al., 2018; Brehler et al., 2018; Nesrallah et al., 2018). On the other hand, our formalism requires little computation and it leads to accurate prediction. Accordingly, the present formalism will be user-friendly for the system engineers.

4 Conclusions

We have developed a simple and novel method based on iteration technique for accurate prediction of effective core area, index of refraction and fractional modal power guided through the core in case of monomode Kerr type nonlinear graded index fibre. The execution of our formalism is very simple. Our results will be beneficial to the technologist working in the field of nonlinear photonics and sensors as well. Again, the formalism developed can be suitably modified for extension in the low V region which is suitable for directional coupler, switches, etc. Moreover, the present formalism can be also used for study of important device like photonic crystal fibre, holy fibre, fibre Bragg grating, etc. in presence of third order nonlinearity. Conclusively, the present formalism generates ample scope for extension for study of various types of nonlinear devices of contemporary interest.

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