A Pareto-based optimisation algorithm for a multi-objective integrated production-distribution planning problem of a multi-echelon supply chain network design

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Abstract: This paper addresses a multi-periodic supply chain network design (SCND) problem involving suppliers, manufacturers, distribution centres (DCs), and customer zones (CZs). Logistic decisions made in each time period have a tactical nature. Location/strategic decisions are made at the beginning of the time horizon and remain unchanged until the last period. While both backorders and lost sales are considered, the aim is to design the supply chain network (SCN) under three minimisation objectives including total costs, the transfer time of products to CZs, and backorder level and lost sale of products. A multi-objective mixed-integer linear programming (MILP) model is developed and a meta-heuristic algorithm named multi-objective vibration damping optimisation (MOVDO) with tuned parameters is proposed to find non-dominated solutions. The performance of this method is compared with two popular existing algorithms called NSGA-II and NRGA when they solve some randomly generated problems.

Keywords: supply chain network design; SCND; integrated production-distribution planning; production-distribution planning; PDP; multi-objective vibration damping optimisation; MOVDO; NSGA-II; non-dominated ranking genetic algorithm; NRGA; Taguchi method.

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1 Introduction

In the 1980s, most of the manufacturing and commercial companies tried to find strategies and methods for reducing their operational and production costs. As results, different approaches such as, just in time (JIT), lean manufacturing, or total quality management (TQM) were evolved. However, in current decades, more comprehensive philosophies such as supply chain management (SCM) have found considerable attention because of the changes in the competitive environment among businesses. In these philosophies, in addition to the cost reduction, other improvement opportunities are also considered (Ballou, 2004).

SCM includes a comprehensive set of techniques for integrating SC members, including suppliers, manufacturers, distributors, retailers, and customers (Simchi-Levi et al., 2000). The main objectives of SC are to reduce the total cost of the system and increase the service level to customers (Chopra and Meindle, 2005; Shen, 2007). In a simple view to the SC, raw materials are first supplied by suppliers to manufacturers. Then, manufacturers change raw materials to products and merchandises. Finally, they deliver their goods to the warehouses or DCs to be presented to their customers (Simchi-Levi et al., 2000).

In the literature of SC, researchers focused on different concepts such as supplier selection, locating of the distribution centres (DCs) and logistic warehouses, integrated production-distribution planning (PDP), transportation and distribution, customer evaluation, and other similar problems. Among these concepts, development of the SCND models has found a great attention. These models by optimising different activities of an SC such as procurement, production, and distribution have a great impact in reducing the costs of SC and increasing the service level to customers. Integrated PDP in SCND models are ones of the most important items, where decisions related to producing the products, times and quantities of orders to meet customer needs in a production planning problem of an SC, finding a channel to deliver the products from a manufacturer to a customer in a distribution planning problem are usually made (Torabi and Hassini, 2008; Lee and Kim, 2002). Some researchers studied these concepts separately and some considered them within an integrated concept simultaneously.

Erenguc et al. (1999) and Chen (2004) presented a review of integrated production-distribution planning problems (PDPP) in SCs. Fahimnia et al. (2013) classified integrated PDP models into seven categories based on their degree of complexity, implementation in the real world, and solving methodologies, where some of these research works focused on the model as a single objective problem (Lee and Kim, 2002; Williams, 1981; Chandra and Fisher, 1994; Barbarosoglu and Ozgar, 1999; Gen and Syarif, 2005; Tsiakis and Papageorgiou, 2008; Altiparmak et al., 2009). Syarif et al. (2002) designed an SCN including suppliers, factory, and DC, which was solved using a spanning-tree-based GA. Park et al. (2007) presented a multi-product multi-period SC model with a cost minimisation objective and developed a genetic algorithm (GA) to find a near-optimum solution to the problem. Ferrio and Wassick (2008) presented a mixed-integer linear programming (MILP) model of redesigning and optimising a multi-product chemical supply network made up of production sites, an arbitrary number of echelons of DCs, and CZs. Their problem was solved by the GAMS/CPLEX solver. Tuzkaya and Önüt (2009) proposed an integrated model to minimise the cost involved in an SC. Their model aimed to determining the best strategy for distributing the sub-products from the suppliers to the warehouse and from the warehouse to the manufacturers. To make PDP decisions, Kazemi et al. (2009) studied a multi-level supply chain in two scenarios. They proposed a multi-agent solution system based on GA for each level, considering the interplay of the levels. Chang (2010) presented an SCND model in a multi-echelon SC. The objective function of his proposed model was the minimisation of total costs. He utilised a GA combined with a co-evolutionary search and a constraint-satisfaction procedure to find a solution faster. Jolayemi (2010) proposed an integrated MILP model for a production-distribution and transportation-planning problem in a three-stage SC. His model had two versions as the fully optimised version (FOV) and the less fully optimised (LFOV). Amrani et al. (2011) designed a multi-commodity production-distribution network with alternative facility configuration, formulated the problem as a mixed integer programming (MIP) model and solved the problem using a variable neighbourhood search (VNS) method. Bashiri et al. (2012) presented a new mathematical formulation for the strategic and tactical planning of a multi-product multi-echelon SC network. They solved small and medium-sized problems using the CPLEX solver and employed heuristics to have less solution times. Sadjady and Davoudpour (2012) considered a two-echelon SCND problem in deterministic, single period, and multi-commodity contexts. The problem was designed in both strategic and tactical levels of SC planning and was formulated as a MIP model to minimise the total SC cost. An efficient Lagrangian-based heuristic solution algorithm was proposed to solve the problem. To make strategic and tactical decisions, Badri et al. (2013) proposed a new mathematical model for a multiple echelon, multiple commodity SCND. The objective function in this model was to maximise the total net income over the time; computed by subtracting the total cost (including the fixed costs of opening facilities, adding facility options, operating facilities, and the variable costs of raw materials, production, transportation, and inventory) from the total revenue. They used the Lagrangian relaxation method to solve the problem. There are also some relevant research works in the fuzzy environment of production/distribution planning problems of SCs (Selim et al., 2008; Liang and Cheng, 2009; Bilgen, 2010; Liang, 2011). In the most prominent works, Liang (2011) presented an application of fuzzy sets to PDP decisions in SCs.

Different parts of supply chains have recently been considered simultaneously to make the model more realistic (Sabri and Beamon, 2000; Chen and Lee, 2004; Wang, 2009). Chan et al. (2005) and Altiparmak et al. (2006) developed a multi-objective network structure consisting of manufacturers and CZs with no shortages. Their objective functions were to minimise the costs and the delivery time while balancing the capacities. They used an analytic hierarchy process (AHP) to obtain objectives' weights and utilised a GA to solve the problem. Jolai et al. (2011) first proposed a linear multi-objective multi-product multi-level multi-period PD model and then changed it to a single-objective model via fuzzy goal programming. They solved the problem via three meta-heuristic algorithms [GA, particle swarm optimisation (PSO), and an improved hybrid GA]. Javanshir et al. (2012) presented a bi-objective multi-level SCN with multiple products, single resource, and deterministic cost and demand. The objective functions of their model were the minimisation of the total costs of SC as well as minimisation of delays in delivery of products to customers. They proposed a MILP model and solved it using two meta-heuristic algorithms namely multi-objective PSO (MOPSO) and NSGA-II. Pishvaee and Razmi (2012) proposed a multi-objective fuzzy mathematical programming model for an environmental SCND by considering two objectives of minimising the total cost and minimisation of the total environmental impact, where an interactive fuzzy solution approach was employed to solve the problem. Liu and Papageorgiou (2013) addressed a capacity planning PD problem, in which a universal SC was considered with three objectives of costs, response, and service level. They solved the problem by utilising the ε-constraints and the lexicographic mini-max methods. Cárdenas-Barrón and Treviño-Garza (2014) presented a mathematical model for optimising a three-echelon supply chain network (SCN). Their model is an integer linear programming (ILP) model. In order to solve it, they developed five algorithms; four of them are based on a PSO method and the other is a GA. Sarrafha et al. (2014) proposed an integrated PDPP for a multi-echelon SCN by minimising the total costs and transfer time. A multi-objective evolutionary approach with two Pareto-base meta-heuristic algorithms called multi-objective simulated annealing (MOSA) and NSGA-II is presented to solve the problem. Sarrafha et al. (2015) designed a multi-periodic structure for an SCND by considering a flow shop scheduling model in factory level in order to obtain makespan. A bi-objective MINLP was presented by minimising the total SC cost and the average tardiness of the products to DCs. A new Pareto-based algorithm called multi-objective biogeography-based optimisation (MOBBO) algorithm with tuned parameters is presented to solve it, and compared with MOSA and NSGA-II algorithms. More recently, Ghaderi et al. (2016) reviewed, analysed, and classified some published works on the biomass supply chain network design (SCND) problem. In their paper, the modelling approaches, the decisions, the uncertainties, the solution methodologies, sustainability, model features, entities, data, and regions of the case studies were taken into account. In addition, Lemmens et al. (2016) published a review paper on the works published in integrated vaccine SCND. One of the aims in their work was to see whether the decisions at strategic, tactical and operational levels are able to address vaccine supply chain key issues such as limited shelf life, cold chain distribution and accessing remote areas. Cárdenas-Barrón and Treviño-Garza (2016) presented the correct values in Table 7 of Cárdenas-Barrón and Treviño Garza (2014). Nobil et al. (2017) identified some inconsistencies in the research works of Shankar et al. (2013) and provided some observations and corrections to the shortcomings of location and allocation decisions for multi-echelon SCND. Emdadian et al. (2018) proposed five variants of the differential evolution (DE) algorithms to solve a multi-echelon SCN optimisation problem. The objective in their work was to find the optimal alignment of procurement, production, and distribution while aiming towards maximising profit.

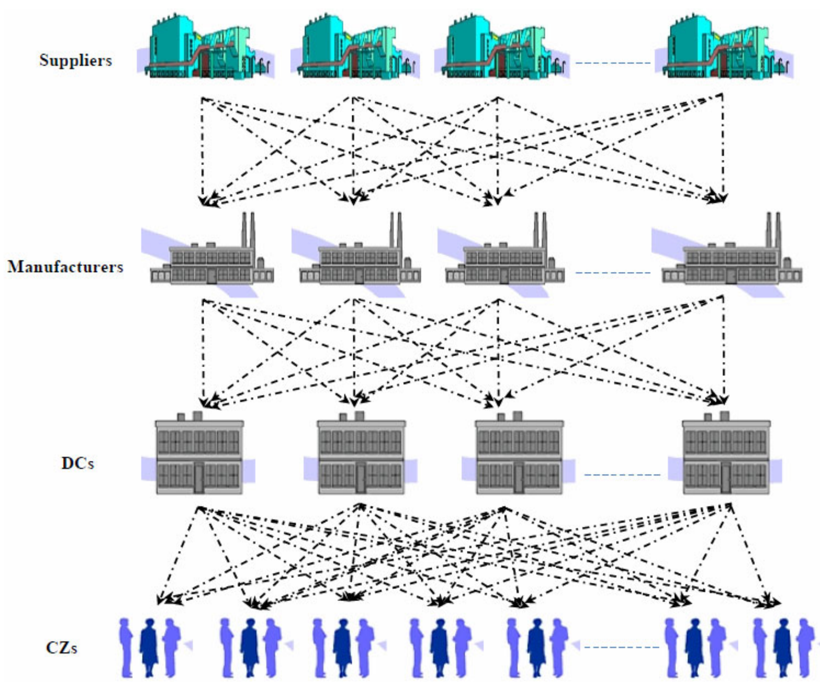
Notably, in the literature of the integrated PDP in SCND, fewer papers have considered the shortage. In addition, those that have considered shortages, only focus on one of its types, i.e., either backorder or lost sale. Therefore, in this paper, a multi-objective model of an integrated PDP in a multi-echelon SCN is presented in which in addition to minimising the total costs of the chain, responsiveness and service level are also considered. In this model, in order to improve the responsiveness, the delivery time of products and in order to increase the service level, shortage quantities are optimised, where shortage includes both the backorder and the lost sale cases. In addition, considering the location and allocation in a supply chain for the production and distribution problem is another contribution. A novel parameter-tuned Pareto-based algorithm called multi-objective vibration damping optimisation (MOVDO) is also developed to solve the problem.

The rest of the paper is organised as follows: the problem is defined in Section 2 that also contains assumptions, indices, parameters, decision variables, objective functions, and constraints. In Section 3, the proposed solving methodologies will be presented. Section 4 describes the computational experiment and tuning the parameters of the algorithms, and introduces a number of metrics to analyse the results. Finally, Section 5 presents the conclusion.

2 Problem definition

Consider an integrated SCN with several suppliers, manufacturers, DCs in potential centres, and CZs in fixed locations, where location decisions are made at the beginning of the time horizon. In the first stage, raw materials are transferred from potential suppliers to production centres or manufacturers. In the second stage, raw materials are transformed into final products. In the next stage, the products are shipped from manufacturers to their assigned DCs. Finally, final products disseminate to CZs by DCs. In case of not being able to fulfil customers' demands, shortage in its two forms of backorder and lost sales can occur in a period, where a certain percentage is considered backorder and is fulfilled in the next period, and the rest treated lost sales. Furthermore, the allocation between different levels of SC in order to obtain a suitable quantity of procurement, production, distribution, transportation, and inventory is investigated in this research.

Figure 1 A four-echelon SCN (see online version for colours)



The proposed SCND problem is formulated as a multi-objective MILP model, consisting of minimisation of the total SC costs, minimising the transfer time of products to customers, and minimisation of the shortage quantities of products. One of the objectives of this model is a function of time, while the other two correspond to cost and shortage quantity. It should be noted that the last two objectives contradict each other. In other words, on the one hand, distributors aim to minimise their shortages. On the other hand, minimising shortage in DCs may lead to an increase in the total SC cost. The scheme of the SCN under consideration is illustrated in Figure 1. Besides, the assumptions, notations, and mathematical formulation are presented in the following subsections.

2.1 Assumptions

The following assumptions are considered to formulate the problem at hand:

- an SCND problem with several suppliers, manufacturers, DCs, and CZs is investigated
- decisions are made for the flow of raw materials and products in different time periods
- there is a limited capacity for the transportation system
- all types of raw materials can be supplied by suppliers in all periods
- all manufacturers are able to produce all types of products in all periods
- shortage in form of backorder and lost sale is allowed
- not all the potential centres to supply raw materials, to manufacture products, and to distribute items, are to be established.

2.2 Indices

s an index used for a supplier ($s = 1, 2, \ldots, S$)

p an index used for a manufacturer ($p = 1, 2, \ldots, P$)

d an index used for a DC ($d = 1, 2, \ldots, D$)

c an index used for a CZ ($c = 1, 2, \ldots, C$)

i index of a product ($i = 1, 2, \ldots, I$)

m index of a raw material ($m = 1, 2, \ldots, M$)

t index of a time period ($t = 1, 2, \ldots, T$).

2.3 Parameters

NS number of potential supply centres that are to be established

NP number of potential production centres that are to be established

ND number of potential DCs that are to be established

FS_s fixed cost of establishing the supply centre s

FP_p fixed cost of establishing a plant for the manufacturer p

FD_d fixed cost of establishing a DC d

DE_{ci}^t demand of product i by customer zone (CZ) c at the beginning of the period t

CTM_{spm}^t transportation and supply cost of raw material m from supplier s to the manufacturer p in period t

CP_{pi}^t production cost of product i for manufacturer p in period t

CSE_{pi}^{t} production preparation cost of product i for manufacturer p in period t

CH_{pm}^{t} inventory holding cost of raw material m for manufacturer p in period t

CH_{pi}^{t} inventory holding cost of the product i for manufacturer p in period t

CT_{pdi}^{t} transportation and purchase cost of product i from manufacturer p to DC d in period t

CH_{di}^{t} inventory holding cost of product i at DC d in period t

CT_{dci}^{t} transportation and purchase cost of product i from DC d to CZ c in period t

CPT_{sp}^{t} transportation capacity of raw materials from supplier s to manufacturer p in period t

CPT_{pd}^{t} transportation capacity of products from manufacturer p to DC d in period t

CPT_{dc}^{t} transportation capacity of products from DC d to CZ c in period t

CPL_{pi}^{t} minimum production capacity of product i at manufacturer p in period t

CPU_{pi}^{t} maximum production capacity of product i at manufacturer p in period t

CPD_{pm}^{t} holding capacity of raw material m at manufacturer p in period t

CPD_{pi}^{t} holding capacity of product i at manufacturer p in period t

CPD_{di}^{t} holding capacity of product i at DC d in period t

TSM_{spm}^{t} transfer time of raw material m from supplier s to manufacturer p in period t

TP_{pi}^{t} production and holding time of product i at manufacturer p in period t

TT_{pdi}^{t} transfer time of product i from manufacturer p to DC d in period t

TT_{dci}^{t} transfer time of product i from DC d to CZ c in period t

α_{mi} quantity of raw material m consumed in product i

β percentage of lost demand.

2.4 Decision variables

QSM_{spm}^{t} quantity of raw material m supplied from supplier s to manufacturer p in period t

QP_{pi}^{t} production quantity of product i at manufacturer p in period t

QS_{pdi}^{t} number of product i shipped from manufacturer p to DC d in period t

QS_{dci}^{t} number of product i shipped from DC d to CZ c in period t

I_{pm}^{t} inventory of raw material m at manufacturer p in period t

I_{pi}^{t} inventory of product i at manufacturer p in period t

I_{di}^{t} inventory of product i at DC d in period t

QLS_{ci}^{t} shortage quantity of product i for CZ c in period t

BL_{ci}^{t} backorder level of product i for CZ c at the end of period t

$$W_{pi}^{t}:\begin{cases}1 & \text{if manufacturer } p \text{ procudes product } i \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$X_{sp}^{t}:\begin{cases}1 & \text{if supplier } s \text{ is assigned to manufacturer } p \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$X_{pd}^{t}:\begin{cases}1 & \text{if manufacturer } p \text{ is assigned to DC } d \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$X_{dc}^{t}:\begin{cases}1 & \text{if DC } d \text{ is assigned to CZ } c \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$YS_{s}:\begin{cases}1 & \text{if supplier } s \text{ is established in a potential location}\\ 0 & \text{otherwise}\end{cases}$$

$$YP_{p}:\begin{cases}1 & \text{if manufacturer } p \text{ is established in a potential location}\\ 0 & \text{otherwise}\end{cases}$$

$$YD_{d}:\begin{cases}1 & \text{if DC } d \text{ is established in a potential location}\\ 0 & \text{otherwise}\end{cases}.$$

2.5 Objective functions

The first objective function of the proposed model given in equation (1) minimises the total cost of the SCN. This includes the fixed costs of establishing supply, production and DCs, transportation and supply cost of raw materials from suppliers to manufacturers, production preparation and production costs of products made by manufacturers, inventory holding of raw materials and products stored by manufacturers, transportation and purchase cost of products from manufacturers to DCs, inventory holding of products stored at DCs, and transportation and purchase cost of products from DCs to CZs.

$$\begin{aligned}
Min\ Z_1 = &\sum_{s=1}^{S}\left(FS_S \times YS_s\right)+\sum_{p=1}^{P}\left(FP_p \times YP_p\right)+\sum_{d=1}^{D}\left(FD_d \times YD_d\right)\\
&+\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{t=1}^{T}\left(CTM_{spm}^{t} \times QSM_{spm}^{t}\right)+\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CSE_{pi}^{t} \times W_{pi}^{t}\right)\\
&+\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CP_{pi}^{t} \times QP_{pi}^{t}\right)+\sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{t=1}^{T}\left(CH_{pm}^{t} \times I_{pm}^{t}\right)\\
&+\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CH_{pi}^{t} \times I_{pi}^{t}\right)+\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CT_{pdi}^{t} \times QS_{pdi}^{t}\right)\\
&+\sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CH_{di}^{t} \times I_{di}^{t}\right)+\sum_{d=1}^{D}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(CT_{dci}^{t} \times QS_{dci}^{t}\right)
\end{aligned} \tag{1}$$

The second objective function shown in equation (2) minimises the transfer time of products for CZs. This includes the transportation time of raw materials from suppliers to manufacturers, production and holding time of products by manufacturers, transportation time of products from manufacturers to DCs, and transportation time of products from DCs to CZs.

$$\begin{aligned}
Min\ Z_2 = &\sum_{s=1}^{S}\sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{t=1}^{T}\left(TSM_{spm}^{t} \times QSM_{spm}^{t}\right)+\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(TP_{pi}^{t} \times QP_{pi}^{t}\right)\\
&+\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(TT_{pdi}^{t} \times QS_{pdi}^{t}\right)+\sum_{d=1}^{D}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(TT_{dci}^{t} \times QS_{dci}^{t}\right)
\end{aligned} \tag{2}$$

The third objective function is given in equation (3) minimises the backorder level and the lost demand for products.

$$Min\ Z_3 = \sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(BL_{ci}^{t}\right)+\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{t=1}^{T}\left(\beta \times QLS_{ci}^{t}\right) \tag{3}$$

2.6 Constraints

Constraints (4), (5), and (6) ensure the number of supply, production, and DCs that are to be established among the potential centres.

$$1 \le \sum_{s=1}^{S} YS_s \le NS \tag{4}$$

$$1 \le \sum_{p=1}^{P} YP_p \le NP \tag{5}$$

$$1 \le \sum_{d=1}^{D} YD_d \le ND \tag{6}$$

Constraint (7) indicates that a link between the established supplier s and manufacturer p may exist in period t. Constraint (8) shows that the established manufacturer p can be served by more than one suppliers in the period t.

$$X_{sp}^{t} \leq YS_{s} \qquad \forall s, p, t \tag{7}$$

$$\sum_{s=1}^{S} X_{sp}^{t} \geq YP_{p} \qquad \forall p, t \tag{8}$$

Inequality (9) shows that a link between the established manufacturer p and DC d may exist in period t. Inequality (10) indicates that the established DC d can be served by more than one manufacturer in the period t.

$$X_{pd}^{t} \leq YP_{p} \qquad \forall p, d, t \tag{9}$$

$$\sum_{p=1}^{P} X_{pd}^{t} \geq YD_{d} \qquad \forall d, t \tag{10}$$

Constraint (11) means that a link between an established DC d and CZ c may exist in period t. Inequality (12) ensures that more than one DC can supply a CZ c in each period.

$$X_{dc}^{t} \leq YD_{d} \qquad \forall d, c, t \tag{11}$$

$$\sum_{d=1}^{D} X_{dc}^{t} \geq 1 \qquad \forall c, t \tag{12}$$

Constraints (13), (14), and (15) show the limited transportation capacities for raw materials and products to be shipped in all the levels of the SC in the period t.

$$\sum_{m=1}^{M} QSM_{spm}^{t} \leq CPT_{sp}^{t} \times X_{sp}^{t} \qquad \forall s, p, t \tag{13}$$

$$\sum_{i=1}^{I} QS_{pdi}^{t} \leq CPT_{pd}^{t} \times X_{pd}^{t} \qquad \forall p, d, t \tag{14}$$

$$\sum_{i=1}^{I} QS_{dci}^{t} \leq CPT_{dc}^{t} \times X_{dc}^{t} \qquad \forall d, c, t \tag{15}$$

Constraint (16) ensures that the quantity of a product shipped by all DCs to a CZ in period t cannot exceed the customer demand.

$$\sum_{d=1}^{D} QS_{dci}^{t} \leq DE_{ci}^{t} \qquad \forall c, i, t \tag{16}$$

Constraint (17) ensures a production capacity for manufacturer p who produces product i in period t.

$$CPL_{pi}^{t} \times W_{pi}^{t} \leq QP_{pi}^{t} \leq CPU_{pi}^{t} \times W_{pi}^{t} \qquad \forall p, i, t \tag{17}$$

Constraint (18) shows that the quantity of raw material m shipped from all suppliers to manufacturer p plus the inventory level of raw material m in period t is limited to its corresponding inventory capacity.

$$\sum_{s=1}^{S} QSM_{spm}^{t} + I_{pm}^{t} \leq CPD_{pm}^{t} \qquad \forall p, m, t \tag{18}$$

Constraint (19) states that the production quantity alongside the inventory of product i in period t is limited to the storage capacity of manufacturer p.

$$QP_{pi}^{t} + I_{pi}^{t} \leq CPD_{pi}^{t} \qquad \forall p, i, t \tag{19}$$

Constraint (20) states that the number of product i shipped from all manufacturers to DC d along with the inventory of product i in period t is limited to the storage capacity of DC d in the period t.

$$\sum_{p=1}^{P} QS_{pdi}^{t} + I_{di}^{t} \leq CPD_{di}^{t} \qquad \forall d, i, t \tag{20}$$

Manufacturers' balancing equations for raw materials and products are given in constraints (21) and (22). For example, constraint (21) indicates that the inventory of raw material m for manufacturer p in period t is equal to the inventory of raw material m in the previous period plus the procurement quantity of raw material m from all suppliers to manufacturer p in period t minus the quantity of raw material m consumed to produce product i in period t.

$$I_{pm}^{t} = I_{pm}^{t-1} + \sum_{s=1}^{S} QSM_{spm}^{t} - \sum_{i=1}^{I} \alpha_{mi} \times QP_{pi}^{t} \qquad \forall p, m, t \tag{21}$$

$$I_{pi}^{t} = I_{pi}^{t-1} + QP_{pi}^{t} - \sum_{d=1}^{D} QS_{pdi}^{t} \qquad \forall p, i, t \tag{22}$$

Constraint (23) indicates the balance equation for product i in DCs. In other words, the product i inventory of DC d in period t is equivalent to product i inventory of DC d in the previous period plus the quantity of product i shipped from manufactures to DC d in period t minus the number of product i shipped from DC d to CZs in the period t.

$$I_{di}^{t} = I_{di}^{t-1} + \sum_{p=1}^{P} QS_{pdi}^{t} - \sum_{c=1}^{C} QS_{dci}^{t} \qquad \forall d, i, t \tag{23}$$

Constraint (24) shows that the shortage quantity of a product for CZ c in period t is equal to the demand of product i by CZ c in period t minus the number of product i shipped from DCs to CZ c in the period t.

$$QLS_{ci}^{t} = DE_{ci}^{t} - \sum_{d=1}^{D} QS_{dci}^{t} \qquad \forall c, i, t \tag{24}$$

Constraint (25) indicates that the backorder quantity of product i for CZ c in period t is equal to the backorder level in the previous period plus the backorder demand of product

i for CZ c in the period t. Moreover, equation (26) ensures that the backorders of all CZs have to be fulfilled in the final period.

$$BL_{ci}^{t} = BL_{ci}^{t-1} + \left((1-\beta)\times QLS_{ci}^{t}\right) \qquad \forall c, i, t \tag{25}$$

$$\sum_{c=1}^{C} BL_{ci}^{T} = 0 \qquad \forall i \tag{26}$$

Finally, constraints (27) and (28) ensure the non-negativity and binary states of all variables.

$$QSM_{spm}^{t}, QP_{pi}^{t}, I_{pm}^{t}, I_{pi}^{t}, I_{di}^{t}, QS_{pdi}^{t}, QS_{dci}^{t}, QLS_{ci}^{t}, BL_{ci}^{t} \geq 0 \quad \forall s, p, d, c, m, i, t \tag{27}$$

$$YS_s, YP_p, YD_d, X_{sp}^{t}, X_{pd}^{t}, X_{dc}^{t}, W_{pi}^{t} \in \{0, 1\} \qquad \forall s, p, d, i, t \tag{28}$$

Note that the initial states of the inventories and the backorders are:

$$I_{pm}^{0}, I_{pi}^{0}, I_{di}^{0}, BL_{ci}^{0} = 0 \qquad \forall p, d, c, m, i \tag{29}$$

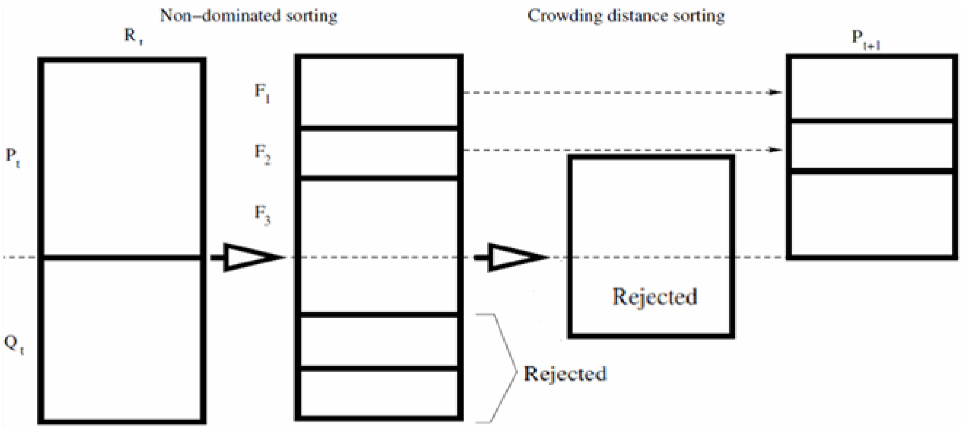
3 Solving methodology

In this section, a novel multi-objective algorithm, namely the MOVDO algorithm, is developed to solve the complicated problem given in Section 2. Besides, since there is no benchmark available in the literature, two commonly employed multi-objective optimisation algorithms, namely NSGA-II and NRGA, are also used to validate the solution obtained and to evaluate the performance of the proposed algorithm. In what comes next, the required basic structures used in NSGA-II and NRGA algorithm are first discussed. Then, the MOVDO is developed.

3.1 The basic structure of NSGA-II

One of the most popular algorithms that work according to the domination concept (Deb, 2001) is NSGA-II (Deb et al., 2002). The multi-objective evolution process of NSGA-II begins by generating an initial population P_t. In the next step, using genetic operators including selection, crossover, and mutation, new population Q_t is generated. In this step, a combination of P_t and Q_t creates R_t. Now, fast non-dominated sorting (FNDS) is used to sort solutions of R_t in several fronts. Finally, to create the population of the next iteration P_{t+1}, the binary tournament selection (BTS) is employed. To do so, as long as the capacity of P_{t+1} is not exceeded, the fronts are added to P_{t+1} according to increasing order of their ranks. However, when P_{t+1} has fewer members compared to the determined population size, some solutions must be added to reach the determined size. Here, the front members are first sorted on the base of the decreasing order of their crowding distances (CDs), and then the members of the next iteration are chosen from the top of the front. A scheme of the evolution process is shown in Figure 2 (Deb, 2001).

Figure 2 Evolution process of NSGA-II



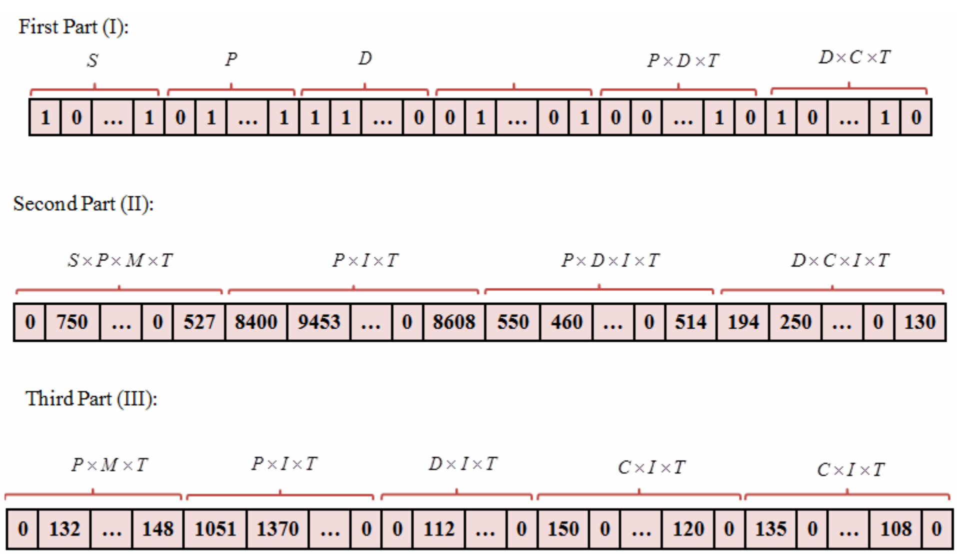
Source: Deb (2001)

In the rest of this sub-section, the structure of GA is briefly explained.

3.1.1 Solution representation and evaluation

A chromosome in GA consists of three parts. The first part represents potential sites such as supply, production, and DCs that are either open or close. They are coded as binary numbers. Besides, the relationship between the sites in each period is given as a binary number in this part. The quantity of procurement, production, and distribution of raw materials and products are decisions that should be determined in the second part. The third part shows the inventory of the manufacturers and DCs in each period along with the shortage quantities of the products for CZs.

Figure 3 Chromosome representation (see online version for colours)



Since decisions are made for S suppliers, P manufacturers, D DCs, and C CZs for I products and M raw materials in T periods, the full length of the chromosome is the sum of the lengths of these three sections. This length for the first section is $[(S + P + D) + (S \times P \times T) + (P \times D \times T) + (D \times C \times T)]$ genes, for the second section is $[(S \times P \times M \times T) + (P \times I \times T) + (P \times D \times I \times T) + (D \times C \times I \times T)]$ genes, and for the third sections is $[(P \times M \times T) + (P \times I \times T) + (D \times I \times T) + (D \times C \times I \times T) + (D \times C \times I \times T)]$ genes. The structure of a chromosome is shown in Figure 3.

In the first part of the solution, while the number of centres that are established is less than a predetermined number, a centre is selected randomly and its corresponding gene is set equal to 1. Conversely, if the number of established centres is more than the predetermined number, a centre is selected randomly and its corresponding gene is set equal to 0. Similarly, the relationship between the two centres together is set when both centres are already established. This helps in to produce feasible solutions.

A similar approach is taken for the second part. In the third part, one must assure that the inventory and the shortage balance constraints are satisfied. To do so, the approach proposed in Sarrafha et al. (2015) is used. Interested readers are referred to Sarrafha et al. (2015) for more details.

3.1.2 Crossover operation

A uniform crossover operator is used in this paper to create new offspring. The uniform crossover operator has often been employed in situations the chromosomes with proper characteristics (genes) are spread over the population (Bate and Jones, 2008). In this operation that takes place with probability p_c, some genes swap within the chromosome of the parents to produce offspring. Figure 4 illustrates this operator. Note that the feasibility of the generated solutions must be confirmed when this operation takes place.

Figure 4 A sample of the crossover process (see online version for colours)

Random Number	**2**	**1**	**1**	**…**	**1**	**2**

Parent 1	**8400**	**9255**	**8870**	**…**	**0**	**9319**
Parent 2	**9830**	**0**	**8336**	**…**	**8970**	**8274**
Offspring 1	**9830**	**9551**	**8870**	**…**	**0**	**8274**
Offspring 2	**8400**	**0**	**8336**	**…**	**8970**	**9319**

3.1.3 Mutation operation

In this paper, the mask operation is utilised for mutated solutions. Here, uniform random numbers in [0, 1] are first generated to have a vector with a proper dimension. The dimension of this vector is equal to the number of the variables in a chromosome. If the generated random number corresponding to a gene is smaller than a certain value, the mutation is applied to the gene to mutate it. Figure 5 depicts this operation. Once again the feasibility of the solutions must be checked when this operation takes place.

Figure 5 A sample of the mutation process (see online version for colours)

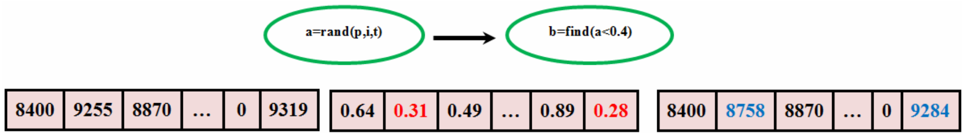
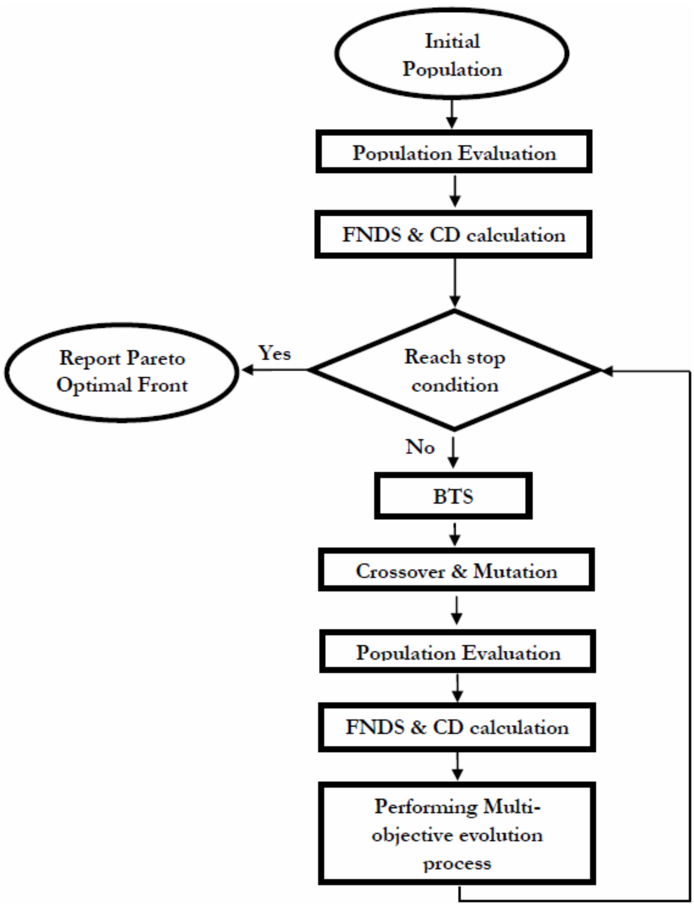


Figure 6 illustrates the different steps of the developed NSGA-II, schematically.

Figure 6 The flowchart of NSGA-II



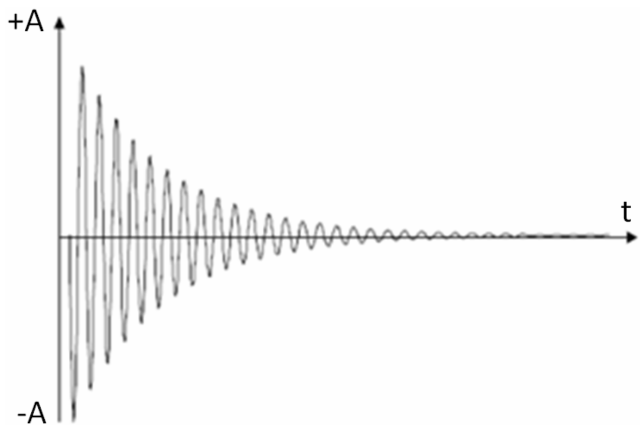
3.2 The NRGA

The other algorithm used for the validation of the results obtained using the MOVDO algorithm is an NSGA-II-mimicked algorithm called NRGA (Bate and Jones, 2008; Al Jaddan et al., 2009). NRGA differs from NSGA-II only in the selection methodology to generate P_{t+1}. In NRGA, instead of BTS, the roulette wheel selection (RWS) method is employed, where tiers of rank-based RWS are used. One tier for the front selection based on FNDSs and one tier for selecting solutions from the front based on CDs.

3.3 The MOVDO

MOVDO is a multi-objective optimisation algorithm with a mechanism based on a mechanical vibration called damping process (DP) (Mehdizadeh and Tavakkoli Moghaddam, 2009). His algorithm was originally implemented to solve a parallel machine scheduling problem. It has been also used in other areas such as a capacitated location-allocation problem (Mehdizade et al., 2010, 2011; Mousavi et al., 2013) and scheduling (Aliabadi et al., 2011). Figure 7 presents a sense of this process. According to this figure, as variation amplitude gets wider during the process, a more controlled area for the responses become available. In this figure, +A and –A represents positive and negative variations in the horizontal axis, respectively. By passing the time *t*, the range of this amplitude is decreased, leading to a less probability of finding new responses.

Figure 7 Neutralising performance of DP during the time



Source: Mehdizadeh and Tavakkoli-Moghaddam (2009)

Single objective VDO algorithm copies a similar process to obtain the optimum or a near optimum solution of an optimisation problem. In fact, in this case, the DP is equivalent to an optimisation problem. Accordingly, the response of the DP is equal to problem solution; zero damping is equal to the best-obtained value, and variation amplitude of DP works like a parameter controller. Besides, neighbour responses represent neighbour solutions. Interested readers are referred to Mehdizadeh and Tavakkoli Moghaddam (2009) for more details.

Figure 8 MOVDO selection and elitism schemes

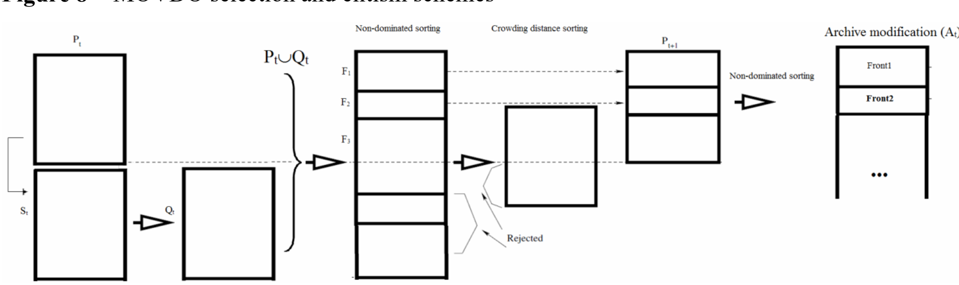


Figure 9 The pseudo code of the MOVDO

```
Parameter setting: Popsize, A, itr, num.struct, Dampcof, Archsize, Stdev
Initialization: Generate initial solutions
Evaluation: Evaluate initial solution
Perform non-dominated sorting and calculate ranks
Calculate crowding distance (CD)
Sort population according to ranks and CDs
P_t=population
For it=1:itr
    For i=1:popsize
        For j=1:num.struct
            S_t(i) = perform neighborhood structure on the solution i of the population P_t(i)
        End
    End
    Perform non-dominated sorting and calculate ranks (S_t)
    Calculate crowding distance (CD) (S_t)
    Sort population according to ranks and CDs (S_t)
    For i= 1:popsize
        If dominates ( S_t(i) , P_t(i) )
            Q_t(i) = S_t(i)
        Else
            p = 1 - e^(-A^2 / 2Stdev^2)
            If rand < P
                Q_t(i) = S_t(i)
            End
        End
    End
    R_t = P_t ∪ Q_t
    Perform non-dominated sorting and calculate ranks (R_t)
    Calculate crowding distance (CD) (R_t)
    Sort population according to ranks and CDs (R_t)
    P_t = choose popsize number of the solution of the R_t
    A_t = A_t ∪ P_t
    Perform non-dominated sorting and calculate ranks (A_t)
    Calculate crowding distance (CD) (A_t)
    If size of A_t > Archsize
    A_t = select frontmax number of the solution
    Else
  Update A ( A = A_0 e^(-γt/2) )
End
```

While the chromosome structure in the multi-objective version of the VDO algorithm is similar to the one explained in NSGA-II, the existing operators of the VDO are combined with Pareto-based multi-objective concepts and operators in the NSGA-II. A scheme of these multi-objective operations is illustrated in Figure 8. In other iterations of the algorithm, a similar process is followed. The comprehensive pseudo code of the MOVDO is shown in Figure 9. Interested readers are referred to Sarrafha et al. (2015) for more details.

4 Computational results

Some test problems are generated and solved in this section using the abovementioned three solution algorithms in order to not only validate the results obtained using MOVDO, but also to compare the performances of the algorithms in terms of some popular multi-objective performance measures. Note that the parameters of all solution algorithms are also tuned in this section using the Taguchi approach in order to find better quality solutions.

4.1 Test problems

Table 1 presents the generated scenario of the input data used for experimentation.

Table 1 Input values and scenarios

Parameter	*Range*	*Parameter*	*Range*
DE_{ci}^{t}	U (3,000, 4,000)	CT_{pdi}^{t}	U (90, 100)
CTM_{spm}^{t}	U (10, 20)	CH_{di}^{t}	U (10, 15)
CSE_{pi}^{t}	U (5, 10)	CT_{dci}^{t}	U (120, 130)
CP_{pi}^{t}	U (20, 30)	NS	$Ceil(Unifrnd(0.7S))$
CH_{pm}^{t}	U (5, 10)	FS_s	U (1,000, 1,500)
CH_{pi}^{t}	U (10, 15)	FP_p	U (1,000, 1,500)
NP	$Ceil(Unifrnd(0.7P))$	FD_d	U (800, 1,000)
TSM_{spm}^{t}	U (24, 48)	TT_{dci}^{t}	U (24, 48)
TP_{pi}^{t}	U (8, 16)	α_{mi}	$Rand\ (m, i)$
TT_{pdi}^{t}	U (48, 96)	β	$Unifrnd$ (0, 0.2)
ND	$Ceil(Unifrnd(0.7D))$	CPT_{sp}^{t}	U (5,000, 8,000)
CPD_{di}^{t}	U (9,500, 10,000)	CPT_{dc}^{t}	U (2,000, 3,000)
$(CPL_{pi}^{t}, CPU_{pi}^{t})$	U (8,000, 10,000)	CPD_{pm}^{t}	U (14,500, 15,000)
CPD_{pi}^{t}	U (9,500, 10,000)	CPT_{pd}^{t}	U (2,000, 3,000)

In this table, U is a uniformly distributed pseudorandom integer, *Ceil* is round toward positive infinity, *Unifrnd* is a continuous uniform random number, and *Rand* is a uniformly distributed pseudorandom number. In addition, Table 2 contains 10 test problems, in which the first row shows the number of suppliers (S), the number of manufacturers (P), the number of DCs (D), and the number of CZs (C). In addition, there are five types of products, ten types of raw materials, and six periods.

Table 2 Generated test problems information

Test problem	*S*	*P*	*D*	*C*
1	5	6	6	4
2	8	8	9	6
3	12	11	11	9
4	16	18	16	15
5	17	20	20	15
6	22	20	20	20
7	25	22	21	22
8	28	23	22	25
9	30	25	25	25
10	35	30	30	28

4.2 Multi-objective performance measures

The two main features of a Pareto-based multi-objective optimisation algorithm are:

1 convergence to the Pareto optimal front

2 good diversity within the set of solutions of the Pareto front.

Various performance measures are introduced in the literature to assess these features. Among them, the diversity (*DI*) metric proposed by Zitzler (1999) is used for evaluating the spread of the front. The spacing (*SP*) measure developed by Schott (1995) measures the distribution of the solutions in a front. The mean ideal distance (*MID*) employed in by Karimi et al. (2010) is applied to measure the closeness of the solutions of a Pareto front to an ideal point (usually (0, 0)). Moreover, the number of solutions (*NOS*) is used to count the number of non-dominated solutions in the final Pareto front.

Table 3 Parameter levels of the solution algorithms used in experiments

Multi-objective algorithms	*Algorithm parameters*	*Parameters range*	*Low (1)*	*Medium (2)*	*High (3)*
NSGA-II	*nPop* (A)	25–75	25	50	75
	P_c (B)	0.8–0.9	0.8	0.85	0.9
	P_m (C)	0.05–0.15	0.05	0.1	0.15
	nIter (D)	50–100	50	75	100
NRGA	*nPop* (A)	25–75	25	50	75
	P_c (B)	0.8–0.9	0.8	0.85	0.9
	P_m (C)	0.05–0.15	0.05	0.1	0.15
	nIter (D)	50–100	50	75	100
MOVDO	*Ampl0* (A)	50–200	50	125	200
	Dampcof (B)	0.005–0.5	0.005	0.05	0.5
	Stdev σ (C)	20–50	20	35	50
	MaxA (D)	100–200	100	150	200
	nPop (E)	10–20	10	15	20
	MaxIt (F)	100–300	100	200	300
	nMove (G)	5–10	5	8	10

Table 4 NSGA-II and NRGA responses

Run order	*Algorithm parameters*				*Response*					
					NSGA-II			*NRGA*		
	nPop	P_c	P_m	*nIter*	*MID*	*DI*	*R*	*MID*	*DI*	*R*
1	1	1	1	1	566,967,499.99	9,613,238,732.26	0.058	731,734,897.01	13,999,778,022.59	0.052
2	1	2	2	2	981,276,941.64	18,257,768,689.82	0.053	723,226,899.76	13,998,997,960.23	0.051
3	1	3	3	3	345,457,491.76	7,003,717,632.79	0.049	2,587,391,945.80	28,012,197,125.16	0.092
4	2	1	2	3	1,325,483,615.41	28,005,183,143.52	0.047	132,065,167.07	285,423,315.69	0.462
5	2	2	3	1	2,658,018,435.6	21,425,484,232.55	0.124	135,363,266.2	295,768,233.72	0.457
6	2	3	1	2	155,245,959.3	280,024,750.3	0.554	619,833,680.95	21,018,620,503.99	0.029
7	3	1	3	2	437,956,274.91	21,007,079,146.63	0.020	149,607,503.22	279,582,315.4	0.535
8	3	2	1	3	484,365,322.43	26,235,189,058.55	0.018	132,289,925.74	256,465,864.49	0.515
9	3	3	2	1	139,106,336.03	294,927,088.12	0.471	133,917,784.30	299,890,585.06	0.446

Table 5 MOVDO responses

Run order	Algorithm parameters							Response		
	A	*B*	*C*	*D*	*E*	*F*	*G*	*MID*	*DI*	*R*
1	1	1	1	1	1	1	1	287,634,731.65	145,877,145.91	1.97176
2	1	1	1	1	2	2	2	183,161,316.18	202,843,521.20	0.902969
3	1	1	1	1	3	3	3	173,894,258.90	206,272,974.54	0.84303
4	1	2	2	2	1	1	1	194,613,252.93	215,821,876.14	0.901731
5	1	2	2	2	2	2	2	217,347,216.29	217,899,252.67	0.997467
6	1	2	2	2	3	3	3	213,706,998.49	124,061,063.23	1.722595
7	1	3	3	3	1	1	1	229,743,088.94	95,690,423.62	2.400899
8	1	3	3	3	2	2	2	159,123,970.67	188,805,356.31	0.842794
9	1	3	3	3	3	3	3	220,718,006.71	135,908,178.14	1.624023
10	2	1	2	3	1	2	3	158,074,412.03	75,336,752.02	2.098238
11	2	1	2	3	2	3	1	188,241,302.75	166,838,503.82	1.128285
12	2	1	2	3	3	1	2	203,509,486.07	248,840,666.85	0.81783
13	2	2	3	1	1	2	3	279,804,252.34	108,167,954.63	2.586757
14	2	2	3	1	2	3	1	205,164,906.24	234,960,700.51	0.873188
15	2	2	3	1	3	1	2	262,191,781.38	193,993,270.31	1.351551
16	2	3	1	2	1	2	3	169,118,052.13	128,314,609.15	1.317995
17	2	3	1	2	2	3	1	211,144,347.59	162,804,438.57	1.29692
18	2	3	1	2	3	1	2	205,974,990.29	218,102,138.37	0.944397
19	3	1	3	2	1	3	2	285,016,955.39	116,796,845.79	2.44028
20	3	1	3	2	2	1	3	184,546,183.47	193,600,855.19	0.95323
21	3	1	3	2	3	2	1	199,849,449.61	231,866,907.07	0.861915
22	3	2	1	3	1	3	2	219,701,811.11	171,615,368.34	1.280199
23	3	2	1	3	2	1	3	238,941,875.33	187,525,614.71	1.274183
24	3	2	1	3	3	2	1	243,900,533.76	258,196,563.32	0.944631
25	3	3	2	1	1	3	2	286,892,788.22	160,753,146.62	1.784679
26	3	3	2	1	2	1	3	222,606,192.36	195,778,575.85	1.13703
27	3	3	2	1	3	2	1	215,714,805.79	285,510,358.03	0.755541

4.3 Parameter tuning

In order to tune the parameters of the algorithms, the Taguchi method is employed. To do so, a new response is first defined. This response considers the two main features of multi-objective optimisation algorithms. This metric, called *Response* (*R*), is formulated in equation (30). In this equation, *MID* represents the first goal (convergence) and *DI* shows the second goal (diversity).

$$R = \frac{MID}{DI} \tag{30}$$

The values of the algorithm parameters (levels) are shown in Table 3.

Table 4 shows the obtained response values of NSGA-II and NRGA and Table 5 does so for MOVDO.

Figure 10 illustrates the related S/N ratio of the Taguchi method, obtained for all algorithms using the Minitab software. The selected (tuned) parameter levels of the algorithms can be determined according to this figure based on the higher values of the S/N ratio. These selected values are highlighted in Table 3.

4.4 Outputs

Tables 6 and 7 contain the performance measures of the algorithms when they solve the test problems. The statistical analysis of the metrics is summarised in Table 8. These algorithms are programmed by MATLAB language on a PC with 4-GB RAM and 2.4-GHz CPU.

Here, Tables 6 and 7 are separated into two different tables, each showing a distinct set of performance measures. In the second row of these tables, (↑) shows the desired high and (↓) presents the desired low values of the measures. In addition, the averages of the measures are given in the bottom row, based on which the best algorithm is determined and its corresponding value is highlighted. Figure 11 depicts the performances of the algorithms on all measures in Tables 4 and 5. Based on these tables and figures, the following conclusion can be made:

- for *DI* and *R*, NSGA-II has the best performance
- for *MID*, *SP*, and *NOS* MOVDO has the best performance.

The statistical comparison results are given in Table 8, where a one-way analysis of variance test (ANOVA) is employed. The typical test of hypothesis in this table is as follows:

$$\begin{cases} H_0 : \mu_{DI(NSGA-II)} = \mu_{DI(NRGA)} = \mu_{DI(MOVDO)} \\ H_1 : At\ least\ two\ of\ the\ means\ are\ not\ equal \end{cases} \tag{31}$$

The results in Table 8 as well as the box plot in Figure 12 lead one to conclude that:

- for *MID*, *SP*, and *NOS*, NSGA-II and NRGA are dominated by MOVDO
- for *DI*, NSGA-II is the superior algorithm
- for the *R* measure, there are no significant differences among the algorithms.

While the algorithms perform different in terms of different measures, one can take advantage of the popular multi-criteria decision-making method called TOPSIS (Hwang and Yoon, 1981) in order to simultaneously rank all algorithms in terms of all metrics. In this method, alternatives are ranked based on the farthest distance they are from the negative ideal solution and also the shortest distance they are from the positive ideal solution. The decision matrix consists of *j* alternatives and *i* criteria. In our problem, the decision matrix is reported in Table 9.

Table 6 The performances of the algorithms in terms of *DI*, *MID*, and *R* metrics

Num.	*NSGA-II*			*NRGA*			*MOVDO*		
	DI ↑	*MID* ↓	*R* ↓	*DI* ↑	*MID* ↓	*R* ↓	*DI* ↑	*MID* ↓	*R* ↓
1	15,376,310,165	781,557,066.2	0.05	28,010,509,949	2,409,552,751	0.08	177,014,604.9	223,133,389.32	1.26
2	97,464,617,823	8,256,112,032	0.084	72,916,076,576	5,983,878,355	0.08	190,893,275.1	378,082,600.26	1.98
3	115,495,494.6	250,435,091.3	2.17	4,801,686,368	453,497,526.5	0.09	778,580,116.9	860,526,694.65	1.105
4	8.79793E+11	1.05436E+11	0.119	3.77051E+11	28,097,210,944	0.07	781,865,593.4	1,585,483,308.7	2.027
5	84,404,360,574	3,894,579,672	0.05	107,574,814.8	547,402,902.6	5.08	905,940,312.2	1,918,203,409.9	2.117
6	3.49449E+11	15,934,001,719	0.046	1.69444E+12	68,559,095,835	0.04	1,425,987,857	2,111,679,647.8	1.481
7	1.61143E+12	67,963,292,959	0.04	106,812,215.4	798,402,439.1	7.47	1,886,197,009	2,122,808,845.3	1.125
8	3.16476E+12	1.76126E+11	0.055	1.88542E+12	79,789,755,196	0.042	2,455,191,140	4,358,420,149.9	1.775
9	2.8792E+12	1.21144E+11	0.042	2.13386E+12	1.59632E+11	0.075	2,405,028,381	3,458,199,988.1	1.437
10	3.76066E+12	2.35054E+11	0.062	1.17178E+12	48,685,552,282	0.041	2,438,247,211	4,963,445,950.3	2.035
Av.	*1.28427E+12*	73,483,913,833	*0.27*	7.36849E+11	39,495,639,795	1.31	1,344,494,550	*2,197,998,398*	1.63

Figure 10 Taguchi S/N ratio plots (see online version for colours)

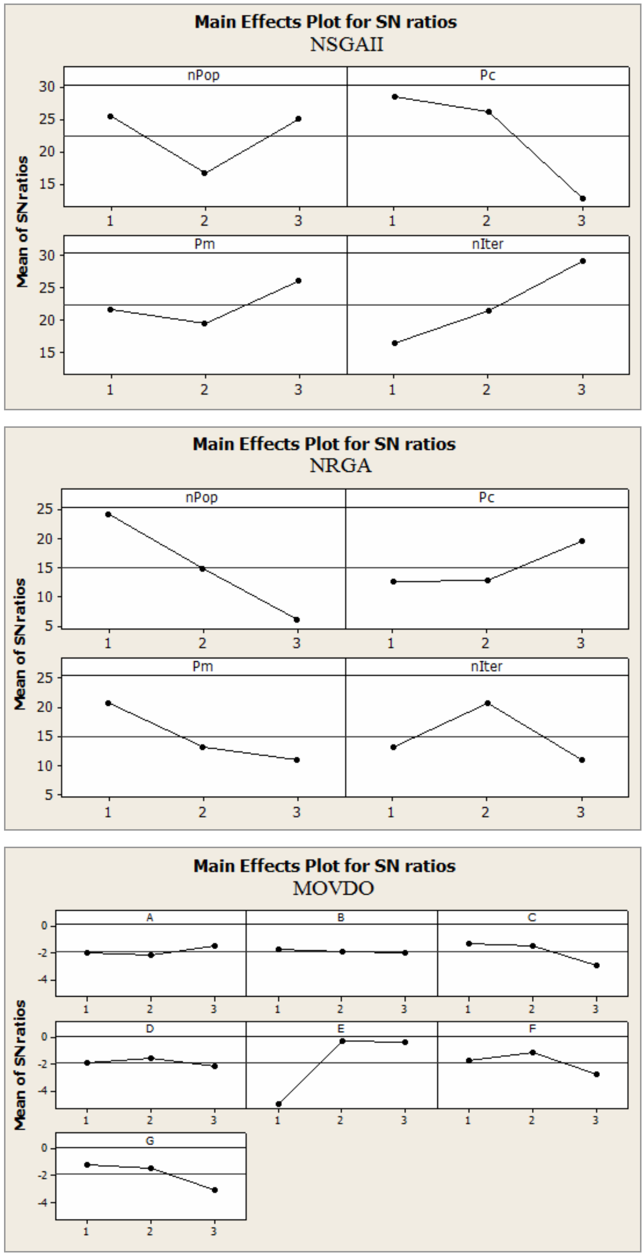


Figure 11 Performances of the solution algorithms (see online version for colours)

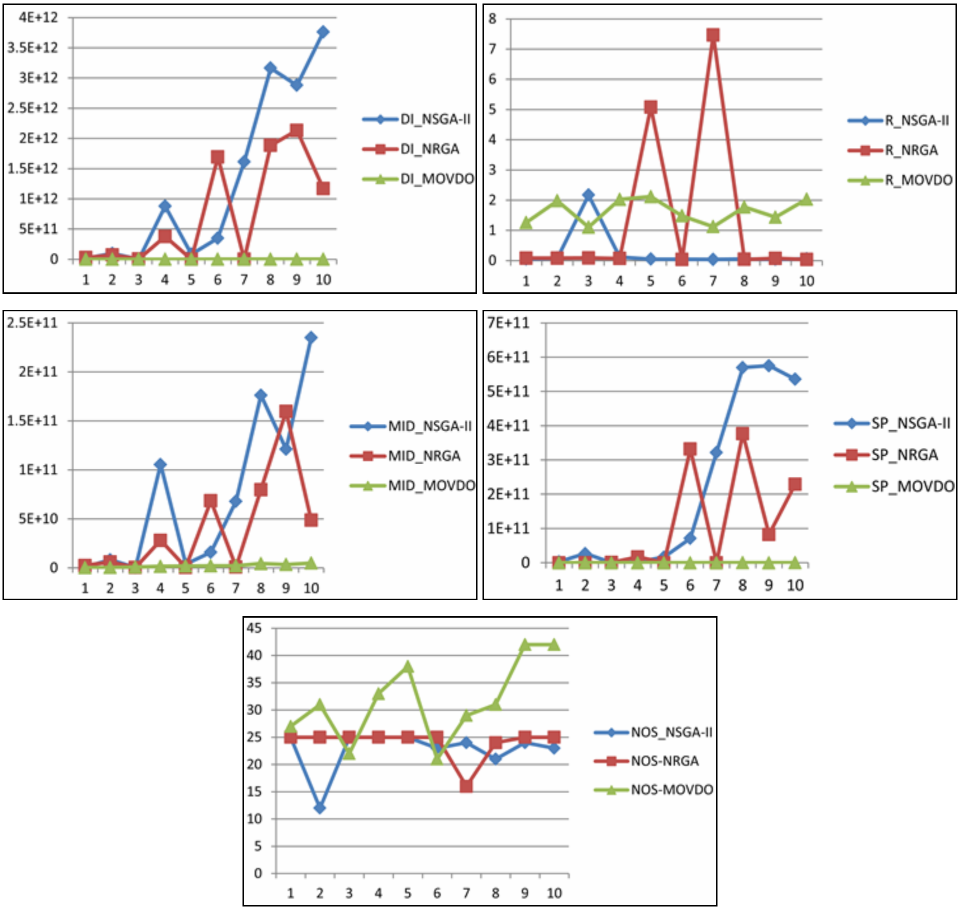


Table 7 The performances of the algorithms in terms of *S* and *NOS* metrics

No.	*NSGA-II*		*NRGA*		*MOVDO*	
	SP ↓	*NOS* ↑	*SP* ↓	*NOS* ↑	*SP* ↓	*NOS* ↑
1	3,007,867,069	25	14,133,971.15	25	5,672,312.1	27
2	26,938,770,139	12	2,179,581.59	25	13,551,821.8	31
3	4,795,683.31	25	937,348,151.2	25	69,911,827.5	22
4	2,617,829,766	25	17,046,291,596	25	44,817,519.1	33
5	16,523,578,375	25	4,007,640.62	25	15,189,670.3	38
6	71,260,276,631	23	3.32032E+11	25	132,007,589.5	21
7	3.21999E+11	24	9,784,816.48	16	127,375,515.5	29
8	5.69873E+11	21	3.7675E+11	24	193,663,739.2	31
9	5.75333E+11	24	82,714,613,634	25	135,351,603.8	42
10	5.35842E+11	23	2.29601E+11	25	350,380,147.2	42
Average	2.1234E+11	22.7	1.03911E+11	24	*108,792,174.6*	*31.6*

Table 8 Statistical comparisons of the solution algorithms (MOVDO vs. NSGA-II and NRGA)

	ANOVA	
	P-value	*Result*
DI	0.025	H_0 is rejected
MID	0.032	H_0 is rejected
R	0.162	H_0 is not rejected
SP	0.036	H_0 is rejected
NOS	0.001	H_0 is rejected

Figure 12 Box plots to compare the algorithms in terms of all metrics (see online version for colours)

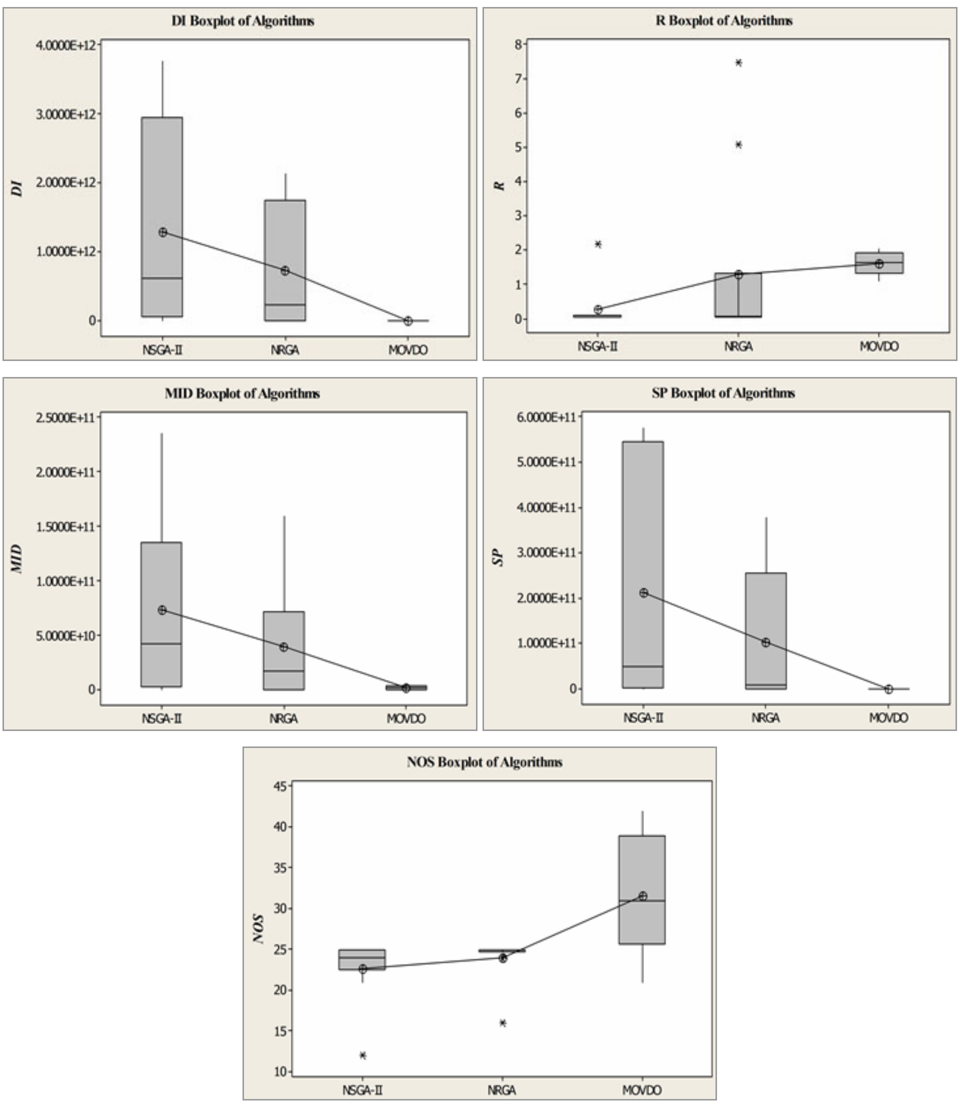


Table 9 The decision matrix

Algorithm	*DI* ↑	*MID* ↓	*R* ↓	*SP* ↓	*NOS* ↑
NSGA-II	1.28427E+12	73,483,913,833	0.27	212,340,000,000	22.7
NRGA	7.36849E+11	39,495,639,795	1.31	103,911,000,000	24
MOVDO	1,344,494,550	2,197,998,398	1.63	108,792,174.6	31.6

The steps involved in TOPSIS are:

Step 1 The normalised decision matrix is obtained using equation (32) with the results shown in Table 10.

$$r_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{m} r_{ij}^2}}, \quad j = 1, 2, ..., m, i = 1, 2, ..., n \tag{32}$$

Step 2 The normalised weight matrix is calculated by equation (33) with an equal weight for all metrics. The outputs are reported in Table 11.

$$v_{ij} = w_i \times r_{ij}, \;\; j = 1, 2, ..., m, \;\; i = 1, 2, ..., n \tag{33}$$

Step 3 In this step, ideal and negative ideal solutions for each index are obtained based on equation (34) and equation (35). Table 11 contains these ideal points, where green cells are associated with ideal solutions and red ones are associated with negative ideal solutions.

$$A^+ = \left\{ \left(\max_j v_{ij} \middle| i \in I \right), \left(\min_j v_{ij} \middle| i \in J \right) \right\} = \left\{ v_1^+, v_2^+, ..., v_n^+ \right\} \tag{34}$$

$$A^- = \left\{ \left(\min_j v_{ij} \middle| i \in I \right), \left(\max_j v_{ij} \middle| i \in J \right) \right\} = \left\{ v_1^-, v_2^-, ..., v_n^- \right\} \tag{35}$$

Step 4 The distances of each option to ideal and non-ideal solutions are calculated by equation (36) and equation (37).

$$d_j^+ = \left\{ \sum_{i=1}^{n} \left(v_{ij} - v_i^+ \right)^2 \right\}^{0.5}, \; j = 1, 2, ..., m \tag{36}$$

$$d_j^- = \left\{ \sum_{i=1}^{n} \left(v_{ij} - v_i^- \right)^2 \right\}^{0.5}, \quad j = 1, 2, ..., m \tag{37}$$

The outputs of these distances are summarised in Table 12.

Step 5 The proportional distance of each option to the ideal solution is calculated based on equation (38). Since $d_j^+ \geq 0$ and $d_j^- \geq 0$, then clearly $CL_j \in [0, 1]$. The final ranking of this method is summarised in Table 13.

$$CL_j = \frac{d_j^-}{d_j^- + d_j^+}, \quad j = 1, 2, ..., m \tag{38}$$

The results obtained in Table 13 show that MOVDO performs the best in terms of all measures simultaneously.

Table 10 The normalised decision matrix

Algorithm	*DI* ↑	*MID* ↓	*R* ↓	*SP* ↓	*NOS* ↑
NSGA-II	0.867374349	0.88052865	0.12805128	0.898216734	0.49655607
NRGA	0.497655417	0.47326062	0.62128583	0.4395526	0.52499321
MOVDO	0.000908049	0.02633774	0.77305031	0.0004602	0.69124106

Table 11 The normalised weight matrix (see online version for colours)

Algorithm	*DI* ↑	*MID* ↓	*R* ↓	*SP* ↓	*NOS* ↑
NSGA-II	0.17347487	0.17610573	0.02561026	0.179643347	0.09931121
NRGA	0.099531083	0.09465212	0.12425717	0.08791052	0.10499864
MOVDO	0.00018161	0.00526755	0.15461006	0.00009204	0.13824821

Table 12 The distances of each option to ideal and non-ideal solutions

Algorithm	d_1^+	d_1^-
NSGA-II	0.250879345	0.216035886
NRGA	0.178902538	0.160852618
MOVDO	0.216035886	0.250879345

Table 13 Final ranking of the proportional distance of each option to the ideal solution

Algorithm	*CL*	*Ranking*
NSGA-II	0.4626876	3
NRGA	0.47343687	2
MOVDO	0.5373124	1

5 Conclusions

The aim of this paper was to study the combination of the procurement, production, and distribution, for a multi-period multi-echelon SCND with multiple products and raw materials, where shortages in a combination of backorder and lost sales were considered. The goal was three-fold; minimising the total cost of the chain, minimising the transfer time of the products to CZs, and minimising the shortage quantities. A multi-objective optimisation model was developed for the problem. Since the problem was NP-Hard, a Pareto-based algorithm called MOVDO was used to find Pareto frontiers. Since there was no benchmark available in the literature the results obtained were validated using two common multi-objective optimisation algorithms called NSGA-II and NRGA, where the parameters of all algorithms were tuned using the Taguchi method. Some multi-objective measures were used for comparison and both types of statistical, called ANOVA, and non-statistical tests were used. While we showed that the algorithms perform different in terms of different measures, the TOPSIS method was implemented to compare the performances of the three solution algorithms in terms of some multi-objectives metrics simultaneously. The results obtained from the TOPSIS method indicated that the MOVDO solution algorithm had the best performance.

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