Local search-based dynamically adapted bat algorithm in image enhancement domain

Krishna Gopal Dhal*

Department of Computer Science and Application,
Midnapore College (Autonomous),
Paschim Medinipur, West Bengal, India
Email: krishnacse42@gmail.com
*Corresponding author

Sanjoy Das

Department of Engineering and Technological Studies,
University of Kalyani,
Kalyani, Nadia, India
Email: dassanjoy0810@hotmail.com

Abstract: Bat algorithm (BA) is a new metaheuristic optimisation algorithm, which has already proved its supreme performance on many optimisation fields. However, it is possible to increase its efficiency when solving complex optimisation problems. This study concentrates on improving the efficiency of BA by incorporating different types of local search strategies and novel self-adaption strategies of parameters such as loudness, pulse rate and frequency. Comparative analysis of three different proposed local search strategies has been performed to find the best one. The proposed modified BAs with local search strategies are employed to solve five popular image enhancement models. Experimental results prove that self-adaption of parameters enhances the capability of standard BA. But the addition of efficient local search technique with self-adaption increases the effectiveness of the standard BA to a great extent.

Keywords: image enhancement; bat algorithm; self-adaptive; local search; chaos; optimisation.

Reference to this paper should be made as follows: Dhal, K.G. and Das, S. (2020) 'Local search-based dynamically adapted bat algorithm in image enhancement domain', *Int. J. Computing Science and Mathematics*, Vol. 11, No. 1, pp.1–28.

Biographical notes: Krishna Gopal Dhal completed his BTech and MTech from the Kalyani Government Engineering College, Nadia, West Bengal, India. Currently, he is working as Assistant Professor in Department of Computer Science and Application, Midnapore College (Autonomous). His research interests are image processing and nature-inspired optimisation algorithms.

Sanjoy Das completed his BE from the Regional Engineering College, Durgapur, ME from the Bengal Engineering College, Deemed University and Howrah, PhD from the Bengal Engineering and Science University, Shibpur.

Currently, he is working as Associate Professor in Department of Engineering and Technological Studies, University of Kalyani. His research interests are tribology and optimisation techniques.

1 Introduction

Recently, several real time problems have been formulated as optimisation problem such as image enhancement (Gorai and Ghosh, 2009; Dhal et al., 2016; Dhal and Das, 2017), image retrieval (Liang et al., 2016), classification (Li et al., 2016; Geng et al., 2016), resource allocation and utilisation in flash and SSD (Tai et al., 2015; Yang et al., 2016a; 201b), big data processing (Wang et al., 2017b; Gao et al., 2017; Bhimani et al., 2017) and so on. In this study, image enhancement techniques (Gonzalez and Woods, 2002) have been formulated as optimisation problems and solved by nature inspired optimisation algorithms. Genetic algorithm (GA), particle swarm optimisation (PSO), differential evolution (DE), Cuckoo search (CS), artificial bee colony (ABC), firefly algorithm (FA) are some nature inspired metaheuristic algorithms which were effectively used in image enhancement field (Gorai and Ghosh, 2009, 2011; Munteanu and Rosa, 2001; Pal et al., 1994; Hashemi et al., 2010; Coelho et al., 2009; Dhal et al., 2015a, 2015b, 2015c, 2015d, 2017b, 2017a; Shanmugavadivu et al., 2014; Braik et al., 2007; Quraishi et al., 2012; Dhal and Das, 2016, 2015; Quraishi et al., 2013). PSO gave better results than GA in the image enhancement domain by maximising employed entropy-based objective function. ABC, CS, FA outperformed the GA in terms of better convergence rate when those algorithms maximised or minimised the used objective functions (Gorai and Ghosh, 2009, 2011; Yang and Deb, 2009; Yang, 2010c, 2010b; Dhal et al., 2015e). CS and FA also outperformed PSO by considering the ability to maximise or minimise some benchmark mathematical functions (Yang, 2010c, 2010b). The performance of these algorithms can be increased by some mechanism such as use of different random number generators to modify the solutions (Coelho et al., 2009; Dhal et al., 2015b, 2015c, 2015d), use of different inertia weight to get a better convergence rate (Dhal et al., 2015d; Bansal et al., 2011), use of communication between solutions (Caponetto et al., 2003) and so on. Mutation factor and crossover rate had been modified by chaotic sequence of traditional DE algorithm and experimental result showed that modified DE was far better than traditional DE in image enhancement field with faster convergence rate and also maintained a good diversity property (Coelho et al., 2009). Lévy flight with chaotic step length had been used to generate new solutions and one population diversity measurement technique had been used as a safe guard from premature convergence in Dhal et al. (2015b, 2015c) to modify FA and DE respectively, which outperformed the traditional FA and DE in the image enhancement domain by maximising the employed objective function with faster convergence rate. A newly proposed metaheuristic algorithm called bat algorithm (BA) which is developed based on the echolocation behaviour of bats, already outperformed GA and PSO (Yang, 2010a, 2013), also gave better results than CS algorithm in image enhancement field (Dhal et al., 2015d). Some modifications were also reported on BA in literature to improve its performance (Fister et al., 2014, 2013; Yilmaz et al., 2014; Liu et al., 2012; Gandomi and

Yang, 2014). Fister proposed a new BA having capability of a self-adaption of control parameters that used DE as local search heuristics and their BA returned better results than traditional BA. The modified BA presented by Yilmaz et al. (2014) in which pulse rate and loudness modification rules of BA had been changed and modified BA gave better results. In Fister et al. (2013), BA was hybridised with DE where DE was applied as local optimiser and this modification also increased the efficiency of traditional BA. In Liu et al. (2012), Doppler Effect had been used to formulate new equation for frequency modification. In standard BA, velocity and location of the bats crucially depend on frequency. Hence, frequency modification also increases the efficiency of the BA. Gandomi and Yang (2014) proposed one chaotic BA variant where pulse rate, loudness and frequency were replaced by different chaotic sequences. Experimental results showed the supremacy of modified BA over standard one to some great extent when only the pulse rate was replaced by sinusoidal chaotic map. Therefore, modification and hybridisation carry out important roles to enhance the effectiveness of BA. BA had also been effectively employed in the multi-objective optimisation domain (Yang, 2011). In this study, the efficiency of BA has been enhanced by using three proposed local search strategies (LSS) which are discussed in Section 3. Self-adaption rules for loudness (A), pulse rate (r) and frequency (f) have been proposed based on individual performance which is discussed in Sub-Section 3.2.4. Application of proposed self-adaption rules for parameters and LSS significantly increases the ability of traditional BA. Formulation of image enhancement models as optimisation problems has been discussed in Section 2. The experimental study carries out in Section 4 and it proves that the proposed BA outperforms the standard BA and some other existing modified BAs.

2 Enhancement models for experiments

Five enhancement models have been taken into consideration for experiments which are discussed below:

2.1 1st enhancement model (EM_1)

In the first enhancement model, one parameterised contrast stretching function has been used (Gorai and Ghosh, 2009, 2011):

$$g(i, j) = \frac{(D \times k)}{(\sigma(i, j) + b)} \times [\{f(i, j) - c \times m(i, j)\}] + m(i, j)^a \tag{1}$$

where $g(i, j)$ and $f(i, j)$ is the gray level intensity of pixels in the output and input images respectively. $m(i, j)$ and $\sigma(i, j)$ are the mean and standard deviation of the image, computed using [3 × 3] window. D is the global mean. In equation (1), c, k, b are the associated three parameters to obtain a large variation in resultant image. Range of three parameters is same as Gorai and Ghosh (2009). $c \in [0, 1]$, $b \in [0, 0.5]$, $k \in [0.5, 1.5]$, $a \in [0, 1.5]$. Initially k and c have been taken as 1 and b as 0. Parameter c always has been takes as a fractional value so that a fraction of the mean always subtracted from the pixel's gray level intensity value.

2.1.1 Objective function for EM_1 (OBJ1)

Combination of entropy and edge information is considered as objective function which is given below (Dhal et al., 2015d):

$$OBJ1(z) = \log_e^{(E(I_e))} \times \left(n_{edgels(I_e)} / (M \times N) \right) \times H(z) \tag{2}$$

*OBJ*1 is the fitness value of enhanced image, $E(I_e)$ is the sum of pixel intensities of Sobel edge image, i.e., I_e. $n_{edgels(I_e)}$ is the number of edge-pixels whose intensity value is above a threshold in the Sobel edge image. Based on the histogram, entropy value $H(z)$ is calculated on the enhanced image. M, N are the number of rows and columns of the image respectively.

2.2 2nd enhancement model (EM_2)

In the 2nd enhancement model, one parameterised contrast stretching function has been used (Dhal et al., 2015b):

$$g(i, j) = [\{f(i, j) - c \times m(i, j)\} / \max - m(i, j)] \times (M \times k) / (\sigma(i, j) + b) \tag{3}$$

where $g(i, j)$ and $f(i, j)$ are the gray level intensity of pixels in the output and input images and max is the maximum gray scale value; $m(i, j)$ and $\sigma(i, j)$ are the mean and standard deviations of the image, computed using [3 × 3] window; M is the number of pixels in horizontal line. In equation (3), c, k, b are the associated three parameters to obtain a large variation in resultant image. Ranges of three parameters are same as Gorai and Ghosh (2009). $c \in [0, 1]$, $b \in [0, 0.5]$, $k \in [0.5, 1.5]$. Initially k and c have been taken as 1 and b as 0. Parameter c always has been takes as a fractional value so that a fraction of the mean always subtracted from the pixel's gray level intensity value.

2.2.1 Objective function for EM_2 (OBJ2)

Combination of entropy, contrast and energy is considered as objective function which is given as equation (4) (Dhal et al., 2017a):

$$OBJ2(I_{en}) = \log\left\{ \left[(I_{con}) \times \exp(H_{en}) \right] / I_{ER} \right\} \tag{4}$$

where *OBJ*2(.) is the fitness function. I_{en} is the enhanced image, H_{en} is the entropy of the enhanced image, I_{con}, I_{ER} are the contrast and energy of image computed from the co-occurrence matrix. exp is the exponential operator.

2.3 3rd enhancement model (EM_3)

One parameterised variant of histogram equalisation (HE) has been considered as enhancement method in EM_3 (Shanmugavadivu et al., 2014).

The idea behind the proposed method is as follows:

1 Histogram of the image has been separated based on the mean of the image.

2 Formulate the four upper and lower weighing constraints to modify the lower and upper histogram.

3 Optimise the constraints by optimisation algorithms.

4 Apply HE method over lower and upper histogram independently.

5 Then unite the both to get an enhanced image.

2.3.1 Objective function for EM_3 (OBJ3)

Entropy of the image has been considered as objective function (Shanmugavadivu et al., 2014). Shannon defined the entropy as a function of probability of occurrence of states of the system (Dhal and Das, 2015). Principally, an image can be thought as information resource. The source is described by probability vector which is nothing but gray level histogram. Let probability vector $P = (p_0, p_1, \ldots, p_i, \ldots, p_{L-1})$. For an 8 bit image the value L is 256. p_i is the probability of occurrence of i^{th} gray level in an image, for $0 \le i \le 255$. Basically, p_i is nothing but the ratio between numbers of occurrence of a specific gray level to the total number of pixels in that image. Entropy value reveals the information content in the image. If the distribution of the intensities is uniform, then we can say that histogram is equalised and the entropy of the image is more. Entropy can be defined as (Dhal and Das, 2015):

$$Entropy(H_e) = \sum_{i=0}^{n} p_i \log_2 p_i \tag{5}$$

2.4 4th enhancement model (EM_4)

Here another parameterised variant of HE has been considered as an enhancement method in EM_4 (Dhal and Das, 2016; Dhal et al., 2017b).

The idea behind the proposed method is as follows:

1 Histogram of the image has been separated based on the threshold level, which maximises the sum of the entropy (Dhal and Das, 2016).

2 Formulate the four upper and lower weighing constraints to modify the lower and upper histogram.

3 Optimise the constraints by optimisation algorithms.

4 Apply HE method over lower and upper histogram independently.

5 Then unite the both to get an enhanced image.

2.4.1 Objective function for EM_4 (OBJ4)

Peak-signal to noise ratio (PSNR) has been employed as objective function (Dhal and Das, 2016). Mean squared error (MSE) is defined as:

$$MSE(f, G) = \frac{\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} [f(i, j) - G(i, j)]^2}{M \times N} \tag{6}$$

where f and G are the input and output images respectively. M and N are the number of rows and columns of the input images.

The PSNR is calculated as follows:

$$PSNR(f, G) = 10\log_{10}\left(\frac{(L-1)^2}{MSE(f, G)}\right) \quad (7)$$

where L is the number of discrete gray level. For 8 bit image it is 256.

2.5 5th enhancement model (EM_5)

Enhancement method is same as the method described in EM_5 (Dhal and Das, 2016; Dhal et al., 2017b). Kapur's entropy-based (Dhal and Das, 2016; Dhal et al., 2017b) method has been used to segment the input histogram.

2.5.1 Objective function for EM_5 (OBJ5)

The objective function for EM_5 is defined as (Dhal et al., 2017b):

$$OBJ5(I_{en}) = \{\exp(FD) + \exp(QILV)\} \quad (8)$$

where $OBJ5(.)$ is the objective function. FD and $QILV$ represent the fractal dimension (Dhal et al., 2017b) and quality index based on local variance (Dhal et al., 2017b) of the corresponding image respectively. exp is the exponential operator.

All the above enhancement models have been formulated as maximisation problem as it is proved that if any of the above stated objective function is maximised then the quality of the enhanced image always better than the input image for some specific application (Gorai and Ghosh, 2009, 2011; Munteanu and Rosa, 2001; Pal et al., 1994; Hashemi et al., 2010; Coelho et al., 2009; Dhal et al., 2015a, 2015b, 2015c, 2015d, 2017b, 2017a; Shanmugavadivu et al., 2014; Braik et al., 2007; Quraishi et al., 2012; Dhal and Das, 2016, 2015; Quraishi et al., 2013). Therefore, the general maximisation problem is formulated as:

$$\{v_0, v_1, ..., v_c\} = \arg\left[\max_{v_0, v_1, ..., v_c}\{OBJ_i(v_0, v_1, ..., v_c)\}\right] \quad (9)$$

c is the number of parameters associated with corresponding objective function OBJ_i and $i = 1, 2, \ldots 5$.

Image enhancement is formulated as optimisation problem for the following reasons:

a Parameters of the transformation functions are associated with diverse ranges and fixed values of the parameters unable to produce proper enhanced images for different kinds of images.

b Manual selection of the parameters does not impart full automation power to the enhancement method.

c Efficient nature inspired optimisation algorithms could produce proper enhanced images within reasonable time.

3 BA and its variants

BA, originally presented by Xin-She Yang under inspiration of echolocation behaviour of bats (Yang, 2010a, 2013) is one of the most efficient and rigid metaheuristic algorithm for solving computational problems.

3.1 Idealise the behaviour of bats

There are three ideal rules which are assumed in the development of the algorithm.

1 All bats use echolocation to sense distance and they also 'know' the difference between food/prey and background barriers in some magical way.

2 Bats fly randomly with velocity V_i at position X_i with a fixed frequency $f_{\min}$, varying wavelength λ and loudness A to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission $r \in [0, 1]$, depending on the proximity of their target.

3 Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive) A_0 to a minimum constant value $A_{\min}$ (Yang, 2010a).

4 Pulse rate increase and loudness decrease when the bat going to reach to its prey.

5 In general, the frequency f in a range $[f_{\min}, f_{\max}]$ corresponds to a range of wavelengths $[\lambda_{\min}, \lambda_{\max}]$ (Yang, 2010a).

3.1.1 Bat algorithm

The pseudo-code of BA is as follows:

1 Objective function has been taken.

2 Initialise the population of bats, $X = \{X_i | i = 1, 2, 3, \ldots, n\}$ by using uniform distribution, where n is the number of bats or population size and X_i is the i^{th} bat.

3 Initialise the velocity (V), loudness (A), β_1 and pulse rate (r) to each bat with the help of the uniform distribution.

4 Global search step: Generate new solutions by adjusting frequency using equations those are given below:

$$
\begin{aligned}
f_i &= f_{\min} + \left(f_{\max} - f_{\min}\right) \times \beta_1 \\
V_i^t &= V_i^{t-1} + \left(X_i^t - X_*\right) \times f_i \\
X_i^t &= X_i^{t-1} + V_i^t
\end{aligned}
\tag{10}
$$

Here f_i controls the pace and range of the movement of every bat.

5 For selection of best solutions probability theory is used. According to objective function fit_i of X_i, probability value p_i is defined as

$$
p_i = fit_i \,/\, Max_{\forall i}\left(fit_i\right); 0 < p_i \leq 1 \tag{11}
$$

where fit_i is the fitness of the i^{th} enhanced image and if p_i is greater than some threshold ($t_h = 0.6$) then the corresponding solution is included to best solution.

6 Local search step: Select corresponding best solution X_i and create new solution using equation as:

Otherwise, remains same

$$\begin{array}{l} \text{if}\left(rand > r_i\right) X_{new_i} = X_{old_i} + \varepsilon \times A_{avg} \\ \text{Otherwise, remains same} \end{array} \tag{12}$$

where A_{avg} is the average loudness of all bats and $\varepsilon \in [-1, 1]$, $rand \in [0, 1]$ is taken from uniform distribution.

7 Simulated annealing step for maintaining population diversity.

$$\begin{array}{l} \text{if}\left(rand > A_i \ \& \ fit\left(X_i\right) < fit\left(X_*\right)\right) \\ \text{accept, solutions} \end{array}$$

As pulse rate increases and loudness decreases when a bat is going to reach to its prey, So, increase r_i using

$$r_i^{t+1} = r_i^o \times (1 - \exp(\alpha t)) \tag{13}$$

where $\alpha = 0.3$, t is the current iteration number. r_i^o is the initial pulse rate of i^{th} bat.

Decrease A_i using

$$A_i^{t+1} = A_i^t \times \gamma; \text{where } \gamma = 0.9 \tag{14}$$

8 Rank the bats and find the current best X_*.

9 Repeat steps 4 to 8 until stopping criterion.

10 Post process and visualise the result.

BA is a composition of global search, local search and simulated annealing steps. At first, global search step has been done followed by a random local search on those solutions which have been found at global search step. The simulated annealing step helps the algorithm to maintain population diversity. Mature convergence to the actual global best position in less time is a crucial matter in swarm-based algorithms which can be significantly controlled by the proper parameter setting strategies and employed local search techniques. For that reason, this study mainly concentrates to develop effective LSS and self-adaption techniques of the parameters on which local search depend. In traditional BA, the simulated annealing search and local search crucially depend on loudness (A) and pulse rate (r). That's why novel dynamic updation rules of those parameters are also proposed based on individual's performances, which are explained below:

3.2 Dynamic adaption of parameters

Loudness (A), pulse rate (r)) and frequency (f_i) are the significant parameters of BA. It is proved that proper modification of the updated rules of those parameters greatly assists to enhance the ability of the traditional BA (Fister et al., 2014; Yilmaz et al., 2014; Gandomi and Yang, 2014). Three different rules for parameters modification were reported which were able to enhance the capability of simple BA (Fister et al., 2014; Yilmaz et al., 2014; Gandomi and Yang, 2014). In this study, four different *dynamically adapted bat algorithms* (DABAs) have been according to the existing as well as proposed parameters adjustment mechanisms which are explained below:

3.2.1 1st DABA (DABA1)

Fister et al. (2014) proposed one hybrid self-adaptive BA where DE was used as local search heuristic and self-adaption of r and A had been done by the following rules (Fister et al., 2014):

$$\begin{aligned} A^{t+1} &= A^t_{lb} + rand_0\left(A^t_{ub} - A^t_{lb}\right) && \text{if } rand_1 < \tau_1 \\ &= A^t && \text{Otherwise} \end{aligned} \tag{15}$$

$$\begin{aligned} r^{t+1} &= r^t_{lb} + rand_2\left(r^t_{ub} - r^t_{lb}\right) && \text{if } rand_3 < \tau_2 \\ &= r^t && \text{Otherwise} \end{aligned} \tag{16}$$

where A^t_{ub}, r^t_{ub} and A^t_{lb}, r^t_{lb} are the upper and lower bound of the loudness and pulse rate respectively. $rand_0$, $rand_1$, $rand_2$ and $rand_3$ are the random numbers. $\tau_1 = 0.1$ and $\tau_2 = 0.1$ are the predefined constants.

In this study, depends on only modified rules of r and A as per equations (15) and (16), *DABA*1 has been developed.

3.2.2 2nd DABA (DABA2)

Yilmaz et al. increased the efficiency of traditional BA by modifying local search technique and the rules of r and A (Yilmaz et al., 2014). The modified rules for r and A are given below:

$$\begin{aligned} A^{t+1}_{ij} &= \alpha A^t_{ij} && \text{if } rand_j < r_{ij} \\ &= A^t_{ij} && \text{Otherwise} \end{aligned} \tag{17}$$

$$\begin{aligned} r^{t+1}_{ij} &= r^0_i\left(1 - e^{-\gamma t}\right) && \text{if } rand_j < r_{ij} \\ &= r^t_{ij} && \text{Otherwise} \end{aligned} \tag{18}$$

If the solution of the problem is d dimensional then r^t_{ij} or A^t_{ij} are the pulse rate and loudness of the j^{th} dimension of the i^{th} solution respectively. α and γ are the predefined constants. $rand_j$ is the random number associated with j^{th} dimension. *DABA*2 is built by considering only the modified rule of r and A as per equations (17) and (18).

3.2.3 3rd DABA (DABA3)

Gandomi et al. used 13 different chaotic maps to update r and A (Gandomi and Yang, 2014). But experimental study showed that the efficiency of standard BA increased when only pulse rate (r) of standard *BA* was replaced by sinusoidal map-based chaotic generator. In this study, this adaptive *BA* is called *DABA*3 where r is replaced by sinusoidal map.

3.2.4 Proposed modified rule

All the above discussed modifications of *r*, *A* and *f* purely depend on the random number generators such as chaotic sequence and uniform distribution. But in this study, modifications of these parameters have been done, based on the performance of the each individual. In order to do that of each individual has been measured which is discussed below.

a Evolutionary speed factor (ESF)

$$Evolutionary\ speed\ \ factor\left(ESF_i^t = \left| \frac{Fit_{gbest} - Fit_i^t}{\max\left(Fit_{gbest}, Fit_i^t\right)} \right| \right) \tag{19}$$

where Fit_{gbest} is the fitness value of global best solution up to generation number *t*. Fit_i^t is the fitness value of i^{th} individual at generation number *t*. It can be obtained that $0 \le ESF \le 1$. From the above equation it is clear that *ESF* strictly depends on the speed of the solution means how fast the solution reach to the global best solution. If the corresponding *ESF* value is small, then it is a good solution and the opposite is also true.

b Self-adaption of pulse rate (*r*) and loudness (*A*)

Pulse rate and loudness are initialised by logistic equation. Pulse rate is used to control the intensification and loudness is used for simulating annealing step. As the iteration number increases the pulse rate increase and loudness decrease but these traditional updating rules of both parameters distorted the main characteristics of any metaheuristic algorithm, which is when the number of generation is increased, then intensification is done around the solutions and takes the best solutions for next generation. But the basic BA shows the opposite characteristics due to the conditions of the intensification and simulated annealing parts. For this reason a new updation technique for pulse rate and loudness has been taken into account in this study as given below:

$$\begin{aligned} r_i^{t+1} &= r_i^o \times (1 - \exp(-\alpha t)) && \text{if } rand < ESF_i^t \\ &= r_i^t && \text{Otherwise} \end{aligned} \tag{20}$$

$$\begin{aligned} A_i^{t+1} &= A_i^t \times \gamma && \text{if } rand < ESF_i^t \\ &= A_i^t && \text{Otherwise} \end{aligned} \tag{21}$$

r_i^t is increased if X_i^t is a bad solution and is remained same if it is a good solution. A_i^t decreases if X_i^t is a bad solution. The modification of pulse rate and loudness of each individual is greatly affected by ESF value of the corresponding solution. If ESF_i^t an approach to 1, then it is a bad solution, otherwise it is a good solution. It is a quite good idea that if a solution is good then increases the probability of intensification around the corresponding solution and takes it for next generation.

But if a solution is not good then increases the chance of diversification around that solution to reduce its chance to go in the next generation. These facts are clearly supported by the conditions of the pulse rate and loudness modification.

c Updation of frequency (f_i)

Frequency plays a key role in BA. It controls the rate of convergent of a solution to the global best solution. If it changes randomly for all solutions then it can affect the convergent rate of BA. At first the frequency has been given to all bats randomly using logistic chaotic generator within the range [0, 1].The range of f_i must be small for a good solution because it needs intensification and reduction for bad solution as it desires diversification. As *ESF* value determines whether a solution is good or bad the modification is done depending upon that value as defined below:

$$f_i^{t+1} = \min\left(f_i^t, ESF_i^t\right) \tag{22}$$

ESF always takes small value for best solutions as described before. This modification always correct because if the solution is among the best solutions then the change in magnitude of that solution is small. So intensification is done around the best solution which is the target of any metaheuristic algorithm.

d Creation of initial population

The initial population is usually created randomly. The equation of standard method is given below:

$$x_i = low + (up - low) \times \partial \tag{23}$$

x_i is the i^{th} individual. *up* and *low* are the upper and lower bound of the search space of objective function. ∂ is the random variable belongs to [0,1].

If the initial population carries a great variance then it helps to restrict the premature convergence of the algorithm. Average population diversity is good when ∂ is generated using logistic equation which is one well known chaotic sequence generator (Bansal et al., 2011). In this study, logistic equation-based initial population has been used in proposed modified BA to generate better variance-based population.

e Proposed DABA

Proposed DABA is developed based on the proposed dynamically adaption rules for *r*, *A* and *f* as per equations (20), (21) and (22) respectively.

3.3 Proposed LSS

LSS are beneficial when large jumps move the solution into a poor position and finding the global optimum is a step by step fine tuning procedure. In this study three LSS have been developed based on chaotic sequence, k-neighbourhood concept and ESF which are as follows:

a Chaotic sequence

Recently, chaotic sequence has been incorporated with nature inspired algorithms to enhance their capability (Dhal et al., 2015b, 2017a; Dhal and Das, 2015; Leandro and Viviana, 2009; Sheikholeslami and Kaveh, 2013; Coelho and Mariani, 2008). Chaotic sequences are used in metaheuristic algorithms for three purposes:

- To generate random numbers.
- To generate inertia weight.
- To perform the local search.

Chaotic local search is helpful for finding the promising solution (Jordehi, 2014; Choi and Lee, 1998). Chaotic inertia weight helpful to maintain the balance between exploitation and exploration (Bansal et al., 2011). In this study, the chaotic sequence-based inertia weight, local and global search have been successfully applied. There are several chaotic generators like logistic map, tent map, gauss map, sinusoidal iterator, lozi map, Chua's oscillator, etc.(Wang et al., 2011). Among those logistic equation is used in this paper as it carries greater variance (Dhal et al., 2017a; Choi and Lee, 1998). The equation of logistic map is given below:

$$L_{m+1} = aL_m(1 - L_m) \tag{24}$$

a is a control parameter and 0< $a \leq 4$, L_m is the chaotic value at m^{th} iteration. The behaviour of the system is mostly depends on the variation of a. Value of a is set to 4 and L_0 does not belong to {0, 0.25, 0.5, 0.75, 1} otherwise the logistic equation does not show chaotic behaviour (Coelho et al., 2009).

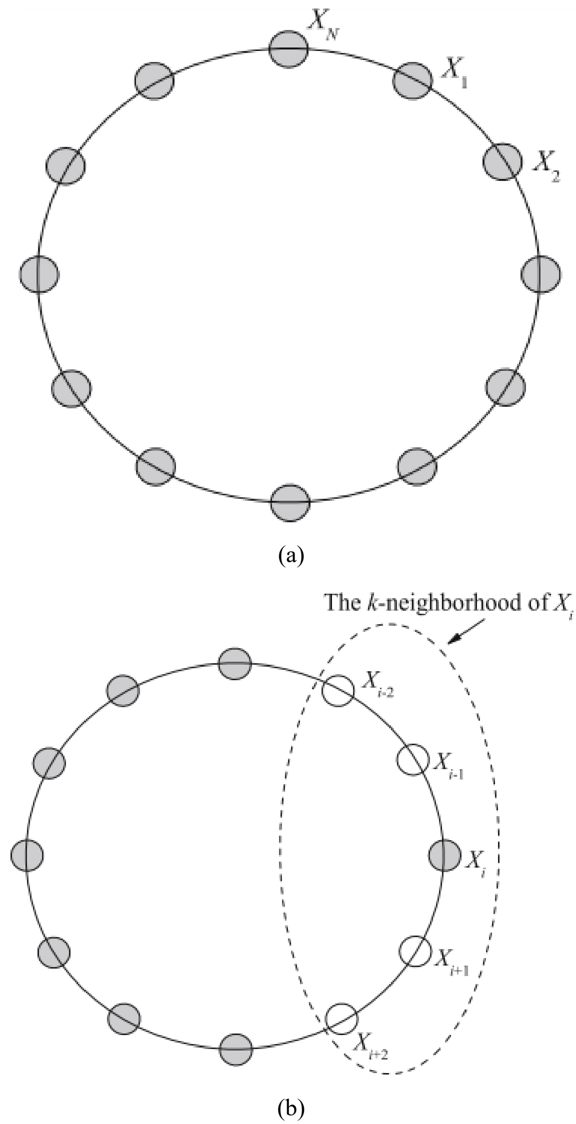
b Topology structure among individuals:

Topology structures among individuals affect the efficiency of the algorithms. Kennedy (1999) proposed four topology structures which are circle, wheel, star and random. It has been reported that PSO with small neighbourhood perform better for complex problems (Wang et al., 2011). But with large neighbourhood it can be better for multi-modal problems because large neighbourhood increases the chance of escaping from local optima (Das et al., 2009). In this study circle topology has been used as it is successfully already employed for DE (Das et al., 2009), PSO (Wang et al., 2011) and FA (Wang et al., 2017a). Assume that all N individuals are organised in a circle topology according to their indices. For example, X_N and X_2 are two immediate neighbour of X_1. Figure 1 represents the circle topology with 12 individuals. Based on circle topology k-neighbourhood concept has been proposed by Das et al. which is discussed below:

c K-neighbourhood concept

Das et al. (2009) proposed k-neighbourhood concept to balance between exploration and exploitation. Balance between exploration and exploitation plays a significant role in any swarm-based algorithm. Exploration indicates the capability of global search or to explore the every region of the feasible search space. On the other hand exploitation means the ability of local search which accelerates the algorithm to converge into a near optimal solution. For each individual, X_i, its k-neighbourhood consisting of $2k + 1$ individuals, i.e., $X_{i-k}, \ldots, X_i, \ldots, X_{i+k}$ where k is an integer $0 \leq k \leq \frac{n-1}{2}$. Figure 1 represents the k-neighbourhood for X_i where $k = 2$. The concept of k-neighbourhood is successfully applied to increase the efficiency of DE, PSO and FA. Neighbourhood-based mutation operator has been used by Das et al. to increase the efficiency of the traditional DE algorithm (Das et al., 2009). Wang et al. (2011) proposed two global and one LSS with the help of k-neighbourhood concept to increase the efficiency of the PSO algorithm. The same strategy has been employed for FA by Wang et al. (2017a).

Figure 1 (a) Circle topology (b) K-neighbourhood of i^{th} individual and k = 2



3.3.1 1st local search strategy (LSS1)

$$X_i = w.X_i + \left(X_{pbest} - X_i\right)\theta + \left(X_p - X_q\right)\theta \tag{25}$$

This local search strategy has been developed based on the previous best position, i.e., X_{pbest} of the corresponding solution and the k-neighbourhood concept. Guidance by the previous best position of the solution helps to increase the local search ability (Wang et al., 2011). Here, X_p and X_q are the k-neighbourhood of the X_i solution, p, $q \in [i - k, i + k]$ and $p \neq q \neq i$, $\theta = \frac{2}{\sqrt{5}+1}$. It is reported that this k-neighbourhood-based searching strategy is significantly helpful to perform local search (Wang et al., 2011; Das et al., 2009; Wang et al., 2017a). w is serving as inertia weight which is generated by the chaotic sequence (logistic equation). w helps to balance between exploration and exploitation (Bansal et al., 2011).

3.3.2 2nd local search strategy (LSS2)

$$X_i = X_i + K(t)\frac{(\max - \min)}{t_{\max}};\text{where } K(t) = \left(K_i + \frac{\left(K_f - K_i\right).^t}{t_{\max}}\right)L_t \tag{26}$$

K_i and K_f are two exploration parameters to control the chaos generated value L_t.t and $t_{\max}$ represent the current and maximum generation respectively. max and min represent the upper and lower bound of the decision variable. Chaotic search with lower values of exploration parameters carries out the vital role as local search in stochastic optimisation algorithms (Dhal et al., 2015b; Choi and Lee, 1998; Bansal et al., 2011).

Here, $K_i = 15$, $K_f = 1$ and $t_{\max} = 200$ have been taken.

3.3.3 3rd local search strategy (LSS3)

$$X_i = X_{cm} + \left(X_p - X_q\right).ESF_i^t + ESF_i^t.L_t;\text{where } X_{cm} = \frac{\sum_{i=1}^{n}\frac{X_i}{fit_i}}{\sum_{i=1}^{n}\frac{1}{fit_i}} \tag{27}$$

X_{cm} is employed in local search strategy as it is beneficial to maintain the balance between exploration and exploitation (Choi and Lee, 1998). It is also clear that ESF_i^t performs the main criteria of any metaheuristic algorithm that the step size be decreased or increased depending upon whether the solution is good or bad very well. Hence, ESF_i^t may be called also as *fitness-based step-size*.

3.4 Proposed modified bat algorithms (PMBAs)

Depending on proposed LSS and proposed dynamic parameter adjustment mechanisms, three modified BA have been proposed which are explained below:

3.4.1 1st PMBAs (PMBA1)

In PMBA1, three modifications have been done on standard BA which are:

a Initial population has been created with the help of logistic equation as per equation (8).

b LSS1 is employed to replace standard local search of traditional BA in equation (25).

c Proposed updation rules of r, A and f have been used.

3.4.2 2nd PMBAs (PMBA2)

PMBA2 is same as PMBA1 but LSS2 is employed to replace standard local search of traditional BA as per equation (26).

3.4.3 3rd PMBAs (PMBA3)

PMBA3 is same as PMBA1 but LSS3 is employed to replace standard local search of traditional BA as per equation (27).

PMBAs have been proposed depending on proposed LSS and novel dynamically adapted rules of r, A and f. The general flowchart for the PMBAs is given as Figure 4.

3.5 Stopping condition

Find the optimal stopping condition is a challenging matter. It has been chosen experimentally. Its stopping conditions are:

1 When the fitness value of the global best solution does not change for continuous 10 iterations for a specific image

2 But the maximum limit of iterations number is 200 to find the optimal parameters.

4 Experimental results

The experiment has been performed over 100 benchmark images with MATLAB R2009b with Windows-7 OS, x32-based PC, Intel(R) Pentium (R)-CPU, 2.20 GHz with 2 GB RAM with five enhancement models. Each optimisation algorithm has been run for 20 times for each case and the gbest solution corresponding to the best run among 20 is recorded. The effect of proposed dynamic adaptive modified rules for pulse rate(r), loudness (A) and frequency (f) has been explained in next sub-section.

4.1 Analysis the performance of DABAs

DABA proposed in this study by modifying the existing updation rule for r, A and f. The proposed DABA is compared with other existing variants of DABAs which are discussed in Section 3.2. Table 3 reveals that the average objective function maximisation ability over 100 images of standard BA always increases when the proposed updation rules of r, A and f have been employed in standard BA. The existing modified rules are also very effective, but the proposed one outperforms those significantly. Table 4 elucidates that the proposed DABA not only better in terms of objective function maximisation ability, but also has the better convergence rate within the less computational time. Friedman ranking is also done in Tables 3 and 4 based on performance and execution time respectively and proposed DABA gets the best rank in both.

Table 1 Features of the different enhancement models

Model name	*Employed enhancement method*	*No. of parameters to be optimised*	*Objective function formulation*	*References*
EM1	Parameterised contrast stretching function	4	Combination of entropy and edge information	Gorai and Ghosh (2009, 2011), Dhal et al. (2015d)
EM2	Parameterised contrast stretching function	3	Combination of entropy, contrast and energy	Dhal et al. (2015b, 2017a)
EM3	Parameterised variant of histogram equalisation	4 and all within range [0, 1]	Entropy	Shanmugavadivu et al. (2014)
EM4	Parameterised variant of histogram equalisation	4 and all within range [0, 1]	Peak-signal to noise ratio (PSNR)	Dhal and Das (2016)
EM5	Parameterised variant of histogram equalisation	4 and all within range [0, 1]	Combination of FD and QILV	Dhal et al. (2017)

Table 2 BA variants used for the comparison

Variants	*Year*	*Reference*
Standard BA	2010	Yang (2010a)
Hybrid Self-adaptive BA (HSABA)	2014	Fister et al. (2014)
Modified BA (MBA)	2014	Yilmaz et al. (2014)
Chaotic BA-IV (CBA-IV)	2014	Gandomi and Yang (2014)
Hybrid BA (HBA)	2013	Fister et al. (2013)
Proposed MBAs	……………………	……………………..

Table 3 Average objective functions of DABAs for all EMs over 100 images and the Friedman ranking based on it

Objective function	*DABA1*	*DABA2*	*DABA3*	*Proposed DABA*
OBJ1	1.6455 (2)	1.6462 (4)	1.6451 (3)	*1.7105 (1)*
OBJ2	18.7985 (4)	18.8102 (2)	18.8094 (3)	*18.8521 (1)*
OBJ3	7.5123 (2)	7.5098 (3)	7.5014 (4)	*7.5312 (1)*
OBJ4	27.65 (2)	27.51 (4)	27.61 (3)	*27.68 (1)*
OBJ5	8.6212 (2)	8.6122 (3)	8.6000(4)	*8.6314 (1)*
Friedman avg. rank	2.4	3.2	3.4	*1*

Table 4 Average Execution time of DABAs over EMs with 100 images and Friedman ranking based on it

	DABA1	*DABA2*	*DABA3*	*Proposed DABA*
Avg. execution time in sec.	49.98	53.13	51.93	*49.51*
Friedman rank	2	4	3	*1*

4.2 Analysis of the PMBAs

Comparison between PMBAs and existing variants of BAs has been performed based on their performance of different objective function maximisation ability, computational time and consistency which are explained as follows:

4.2.1 Analysis based on objective function maximisation ability

Proposed three PMBAs have been compared with other five exiting variants of BA which are enlisted in Table 2. The PMBAs are developed based on three strategies:

a Logistic equation-based initial population.

b Proposed dynamically adaptive modified rules for r, A and f as per equations (20), (21) and (22) respectively.

c Proposed three different local search techniques as per equations (25), (26) and (27) respectively.

Table 5 Values of average objective functions of PMBAs over 100 images and the Friedman ranking based on it

Objective function	*PMBA1*	*PMBA2*	*PMBA3*	*HSABA*	*MBA*	*CBA-IV*	*HBA*	*Standard BA*
OBJ1	1.6972 (2)	1.6911 (3)	*1.6975 (1)*	1.6783 (4)	1.6531 (6)	1.6451 (7)	1.6781 (5)	1.6077 (8)
OBJ2	19.6870 (2)	19.6788 (4)	*19.6872 (1)*	19.6812 (3)	18.8911 (6)	18.8094 (7)	19.6154 (5)	18.5992 (8)
OBJ3	*7.6012 (1)*	7.5657 (2)	7.5613 (4)	7.5621 (3)	7.5338 (6)	7.5014 (7)	7.5583 (5)	7.4501 (8)
OBJ4	28.82 (2)	*28.85 (1)*	28.54 (4)	28.56 (3)	27.95 (6)	27.61 (7)	28.49 (5)	27.12 (8)
OBJ5	*8.7201 (1)*	8.7000 (2)	8.6989 (3)	8.6872 (4)	8.6441 (6)	8.6000 (7)	8.6865 (5)	8.4915 (8)
Friedman avg. rank	*1.6*	2.4	2.6	3.4	6	7	5	8

The experiment has been performed over 100 images with five test enhancement models. Table 5 represents the average objective function over 100 images for each employed algorithm in the case of each enhancement model. From Table 5, it is clear that PMBAs are better than other BA variants in terms of objective function maximisation ability. The fitness of gbest solution of each algorithm has been recorded in Table 5 for each enhancement model. It can be easily verified that all proposed LSS significantly assist the standard BA to increase its efficiency. By comparing Tables 3 and 5, it can be easily concluded that proposed LSSs significantly effective as three PMBAs notably outperform the proposed DABA. In HSABA, when DE is used for local search in standard BA then also the efficiency increases. Therefore, proper LSS considerably increase the efficiency of BA. Friedman ranking is also done for each enhancement model and then average ranking is also computed in Table 5 and the PMBA1 gets the best rank and another two PMBAs are also better than others according to the average Friedman ranking.

4.2.2 Analysis based on computational time

Table 6 represents the average computational time over five enhancement models for each employed optimisation algorithm over 100 images. It can be seen that proposed PMBAs are also better in terms of computational time. When Table 4 and 6 are compared then it can be verified that the proposed local search always reduces the execution time as the proposed DABA ha the longer execution time than all PMBAs. DE is also used as an efficient local optimiser, but it increases the execution time as HSABA has the longer execution time than DABA1. Friedman ranking is also done based on execution time and PMBA1 gets best rank as it has the lowest execution time. Therefore, it can be concluded that proposed LSSs are beneficial to reduce the execution time.

Table 6 Average execution time of PMBAs over all EMs with 100 images and Friedman ranking based on it

	PMBA1	*PMBA2*	*PMBA3*	*HSABA*	*MBA*	*CBA-IV*	*HBA*	*Standard BA*
Avg. execution time in sec.	*43.91*	45.01	47.55	54.34	51.12	51.93	53.09	60.14
Friedman rank	*1*	2	3	7	4	5	6	8

4.2.3 Analysis based on consistency

Consistency of the BA and its variants has been measured by maximising the proposed objective function iteratively. Each variant has been run 20 times for each image and the values of the maximum and minimum objective function have been recorded for that image over that 20 runs. Now the consistency has been taken as the difference between the maximum and minimum objective values of that corresponding ten runs. Therefore, the less difference represents better consistency. The same methodology has been used for 100 images. Then the average difference (AD) over 100 images has been computed by the equation (28):

$$\begin{gathered}\text{Average Difference } (AD) = \frac{1}{p}\sum\nolimits_{i=1}^{p} \text{Diff}\left(X_i\right) \\ \text{Diff}\left(X_i\right) = MaxObj\left(X_i\right) - MinObj\left(X_i\right);\, p = 100\end{gathered} \tag{28}$$

where $MaxObj(X_i)$ and $MinObj(X_i)$ are the maximum and minimum objective function values over 20 iterations for i^{th} image respectively and Diff(X_i) is the difference between these two.

Table 7 represents the AD values of each optimisation algorithm for each EM. It can be seen that PMBAs are also very consistent as per AD. According to the Friedman average ranking, PMBA1gets the best rank. Average values of the objective functions corresponding to the 20 runs have been computed in Table 8 and it is quite clear that PMBAs are also better than other employed algorithms for maximising the objective function averagely. So, the average consistencies of the PMBAs are significantly better than others. According to the Friedman average ranking, PMBA1here also gets the best rank. Therefore, consistency is also increased due to the modification of traditional BA. Table 10 represents the study about the parameters of the PMBAs.

Table 7 Consistency based on AD over 100 images and Friedman ranking based on it

Objective function	*PMBA1*	*PMBA2*	*PMBA3*	*HSABA*	*MBA*	*CBA-IV*	*HBA*	*Standard BA*
AD for OBJ1	*0.0871 (1)*	0.0905 (4)	0.0896 (3)	0.0879 (2)	0.2221 (6)	0.2421 (7)	0.1031 (5)	0.3941 (8)
AD for OBJ2	*0.0783 (1)*	0.0813 (3)	0.0834 (4)	0.0798 (2)	0.2551 (7)	0.2491 (6)	0.1241 (5)	0.3886 (8)
AD for OBJ3	0.0501 (2)	*0.0441 (1)*	0.0710 (3)	0.0723 (4)	0.0911 (6)	0.0985 (7)	0.0849 (5)	0.1120 (8)
AD for OBJ4	*0.12 (1)*	0.16 (3)	0.14 (2)	0.16 (3)	0.32 (6)	0.39 (7)	0.23 (5)	0.42 (8)
AD for OBJ5	0.0597 (2)	0.0613 (3)	*0.0588 (1)*	0.0655 (4)	0.1911 (6)	0.1956 (7)	0.0915 (5)	0.3419 (8)
Friedman avg. rank	*1.4*	2.8	2.6	3	6.2	6.8	5	8

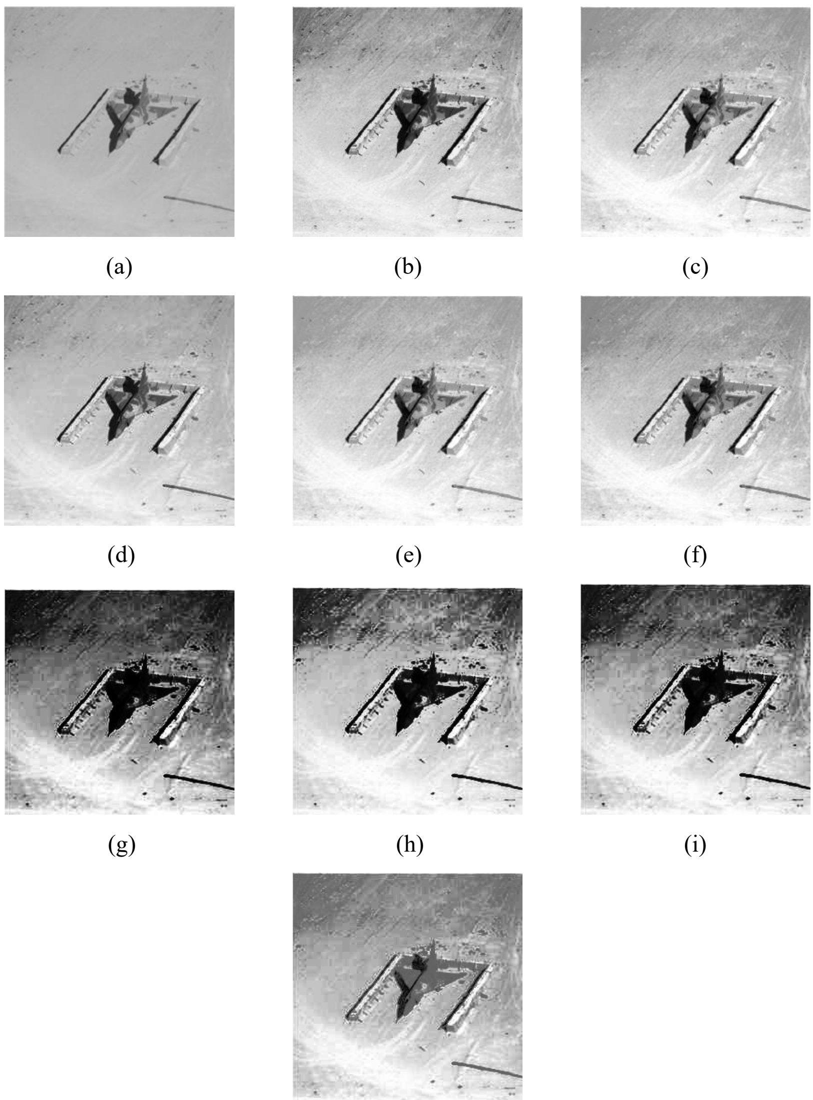
Table 8 Consistency based on average objective function maximisation ability over 20 runs with 100 images and Friedman ranking based on it

Avg. objective function	*PMBA1*	*PMBA2*	*PMBA3*	*HSABA*	*MBA*	*CBA-IV*	*HBA*	*Standard BA*
Avg. OBJ1	*1.6885 (1)*	1.6861 (3)	1.6880 (2)	1.6730 (4)	1.6441 (6)	1.6352 (7)	1.6681 (5)	1.5783 (8)
Avg. OBJ2	*19.6455 (1)*	19.6291 (4)	19.6450 (2)	19.6387 (3)	18.8051 (6)	18.7581 (7)	19.5871 (5)	18.4981 (8)
Avg. OBJ3	*7.5500 (1)*	7.5020 (4)	7.5110 (3)	7.5205 (2)	7.4851 (6)	7.4501 (7)	7.4988 (5)	7.3015 (8)
Avg. OBJ4	28.32 (2)	*28.50 (1)*	28.25 (3)	28.21 (4)	27.40 (6)	27.35 (7)	27.89 (5)	26.98 (8)
Avg. OBJ5	8.6695 (2)	*8.6712 (1)*	8.6595 (3)	8.6425 (4)	8.5550 (6)	8.5331 (7)	8.6411 (5)	8.3955 (8)
Friedman avg. rank	*1.4*	2.6	2.6	3.4	6	7	5	8

4.3 Comparison with different standard techniques

Result of best PMBA, i.e., PMBA1 with each enhancement model have been compared with other state of art methods such as traditional HE, brightness preserving bi-histogram equalisation (BBHE) (Chen and Ramli, 2004; Kim, 1997), dualistic sub-image histogram equalisation (DSIHE) (Kim, 1997), minimum mean brightness error bi-histogram equalisation (MMBEBHE) (Chen and Ramli, 2003). From Figure 2, it can be visually verified that all EMs with PMBA1 outperforms all state of art methods significantly. The enhancement method of EM_3 is the parameterised version of BBHE. But the enhancement efficiency of EM_3 is far better than simple BBHE. So, formulation of image enhancement as optimisation problem is effective for different kind of image enhancement with different perspective as proper enhancement crucially depends on the parameters of the employed enhancement method, objective function and optimisation algorithm.

Figure 2 Aircraft image, (a) original (b) result of EM_1 (c) result of EM_2 (d) result of EM_3 (e) result of EM_4 (f) result of EM_5 (g) result of HE (h) result of BBHE (i) result of DSIHE (j) result of MMBEBHE



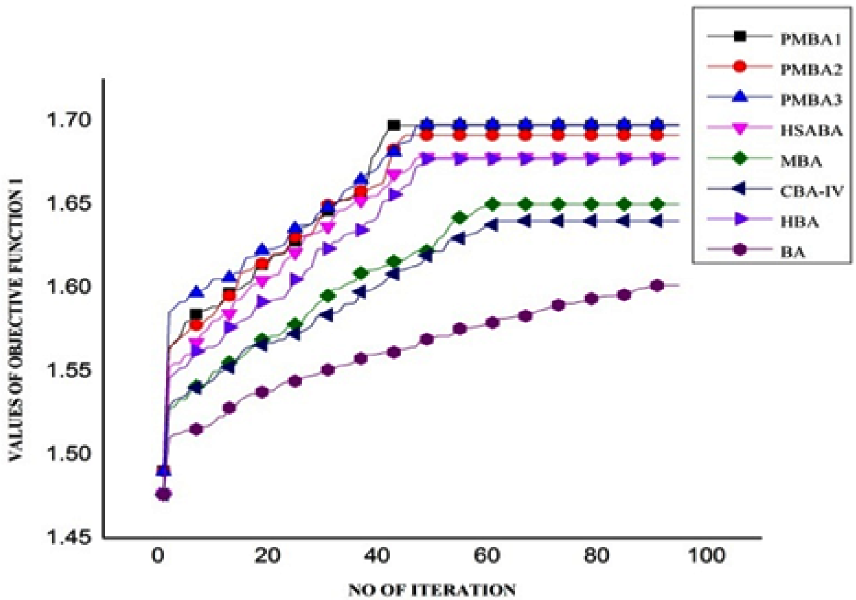
(a) (b) (c)

(d) (e) (f)

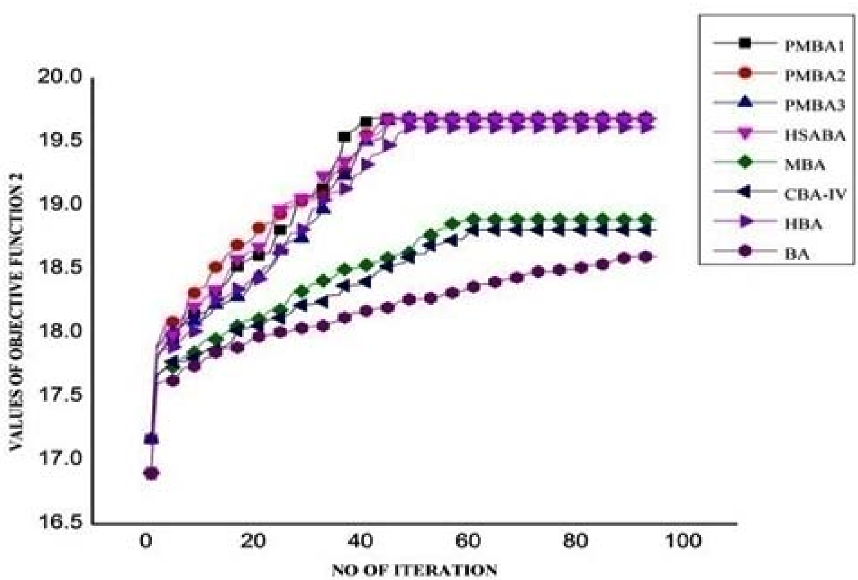
(g) (h) (i)

(j)

Figure 3 Convergence rate of each algorithm, (a) for objective function 1 (b) for objective function 2 (c) for objective function 3 (d) for objective function 4 (e) for objective function 5 (see online version for colours)

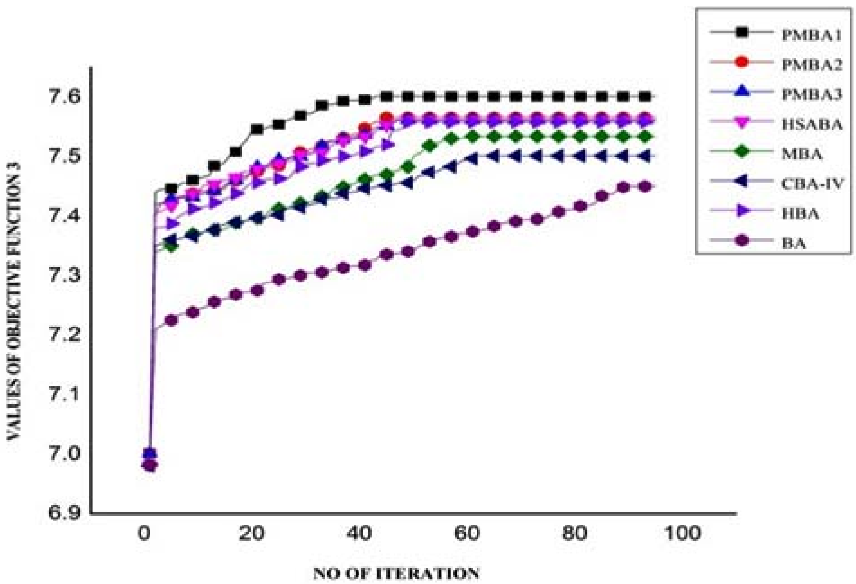


(a)

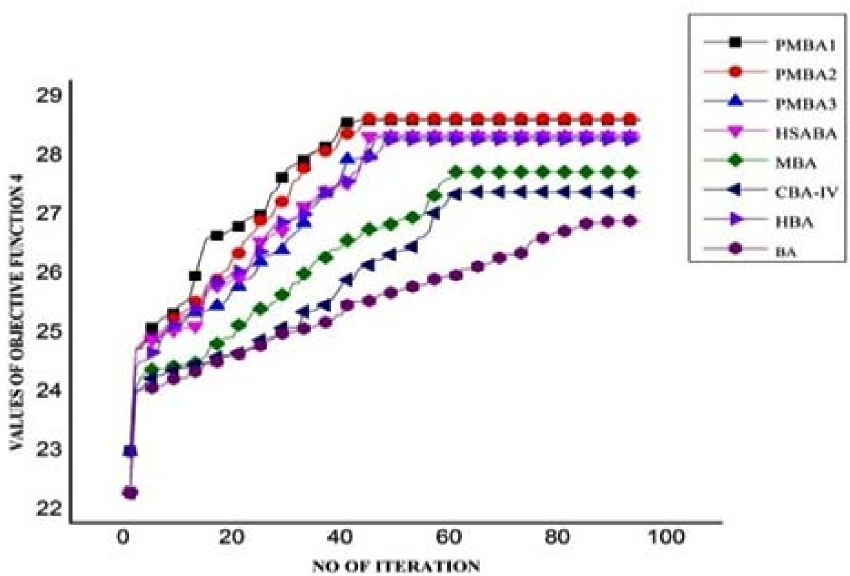


(b)

Figure 3 Convergence rate of each algorithm, (a) for objective function 1 (b) for objective function 2 (c) for objective function 3 (d) for objective function 4 (e) for objective function 5 (continued) (see online version for colours)

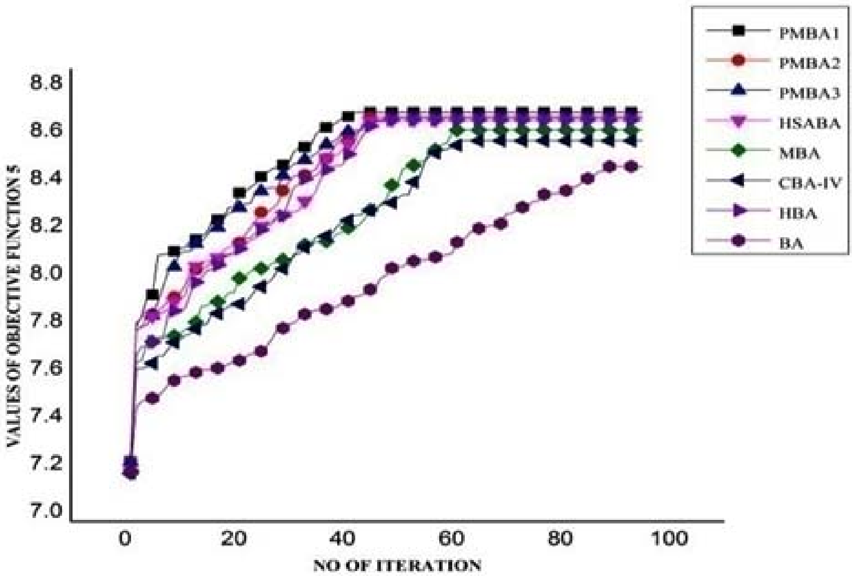


(c)



(d)

Figure 3 Convergence rate of each algorithm, (a) for objective function 1 (b) for objective function 2 (c) for objective function 3 (d) for objective function 4 (e) for objective function 5 (continued) (see online version for colours)



(e)

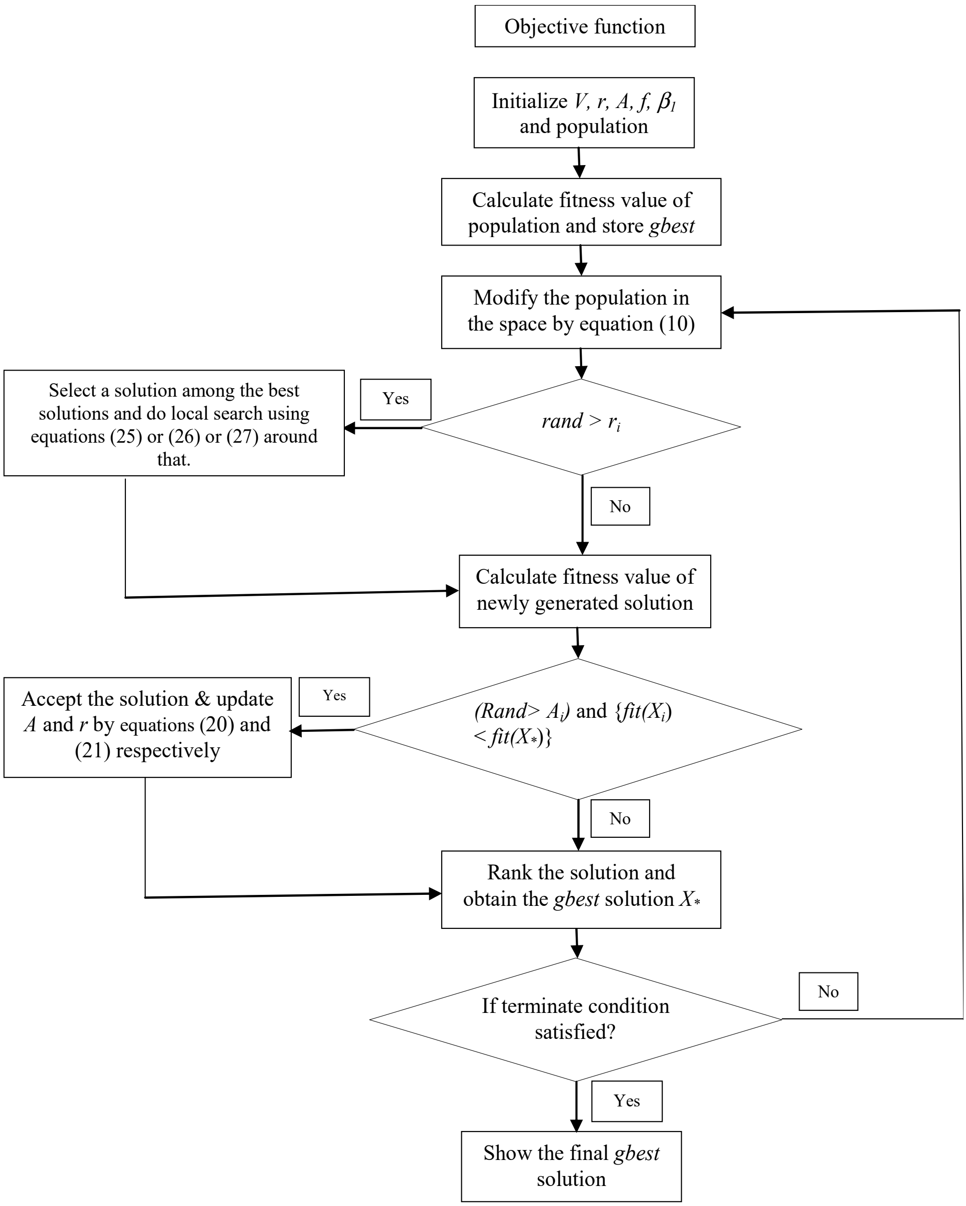
4.4 Effect of logistic equation-based initial population

It is proved that the logistic equation-based initial population always carries greater variance (Dhal et al., 2017a). Table 9 and Figure 3 reveal that logistic equation-based population also carries the global best solution with greater fitness value than uniform distribution based population. The initial starting points of all PMBAs are always above of the other existing BA variants in the corresponding graphs. Table 8 lists the average fitness values of the gbest solution over 100 images of the initial population generated by uniform distribution and logistic equation.

Table 9 Effect of initial population generated by different random number generators

Image	*Enhancement model*	*Avg. fitness of gbest of initial population generated by*	
		Uniform distribution (BA)	*Logistic equation (PMBAs)*
Over 100 images	EM1	1.4763	1.4903
	EM2	16.8971	17.1667
	EM3	6.9811	7.0012
	EM4	22.5201	23.2210
	EM5	7.2010	7.2512

Figure 4 Flowchart of the PMBAs



4.5 Graphical analysis of convergence rate

Convergence rate of the BA and its variants corresponding to Figure 2 has been given as Figure 3. It can be easily verified that convergence rates of PMBAs, HSABA and HBA are far better than other BA variants.

Table 10 Study of parameters for PMBAs

Parameter	*BA*	*PMBAs*
r	$r \in [0, 1]$, uniform distribution	$r \in [0, 1]$, logistic equation
A	$A \in [0, 1]$, uniform distribution	$A \in [0, 1]$, logistic equation
β_1	$\beta_1 \in [0, 1]$, uniform distribution	$\beta_1 \in [0, 1]$, logistic equation
α	0.3	0.3
γ	0.9	0.9
t_h	0.6	0.6
Population size (n)	30	30
∂ (initial population)	$\partial \in [0, 1]$, uniform distribution	$\partial \in [0, 1]$, logistic equation
$[f_{\min}, f_{\max}]$	[0, 1], uniform distribution	[0, 1], logistic distribution
w	………………..	w [0, 1], logistic equation
$t_{\max}$	200	200
θ	……………………	$\dfrac{2}{\sqrt{5}+1}$
K_i, K_f	……………………..	$K_i = 15$, $K_f = 1$

5 Conclusions

This study proposed improved BA variants called PMBAs which employ two strategies:

1 dynamically parameter adjustment method

2 different LSS.

The first strategy intends to change the rules to modify the pulse rate (r), loudness (A) and frequency (f) to accelerate the convergence rate and to reduce the computational time. The second one focuses on enhancing the ability of local search. Five popular image enhancement models have been used to measure the effectiveness of the PMBAs and proposed DABA. In all enhancement models, the proper enhancement of the input image significantly depends on the fine tuning of the parameters associated with these enhancement methods. Experimental results prove that PMBAs give better performance than some existing modified BA in terms of maximisation of the objective functions, convergence rate and computational time. This study also demonstrates that proposed dynamic parameter adjustment method significantly enhances the capability of BA within less computational time and outperforms the existing dynamic parameter adjustment techniques. Application of proposed LSS with the proposed DABA again increases the efficacy within less computational time. In future, these strategies may help other metaheuristic algorithms to increase their effectiveness in different single as well as multi-objective optimisation fields and the proposed image enhancement models can be applied to enhance the different kinds of images. In the same way some global search techniques can also be developed to facilitate the optimisation algorithms.

References

Bansal, J.C., Singh, P.K., Saraswat, M., Verma, A., Jadon, S.S. and Abraham, A. (2011) 'Inertia weight strategies in particle swarm optimization', *Third World Congress on Nature and Biologically Inspired Computing*, pp.640–647.

Bhimani, J., Yang, Z., Leeser, M. and Mi, N. (2017) 'Accelerating big data applications using lightweight virtualization framework on enterprise cloud, *21st IEEE High Performance Extreme Computing Conference*.

Braik, M., Sheta, A. and Ayesh, A. (2007) 'Image enhancement using particle swarm optimization', *Proceedings of the World Congress on Engineering*.

Caponetto, R., Fortuna, L., Fazzino, S. and Xibilia, M.G. (2003) 'Chaotic sequences to improve the performance of evolutionary algorithms', *IEEE Transaction on Evolutionary Computation*, Vol. 7, No. 3, pp.289–304.

Chen, S.D. and Ramli, A.R. (2003) 'Minimum mean brightness error bi-histogram equalization in contrast enhancement', *IEEE Transaction on Consumer Electronics*, Vol. 49, No. 4, pp.1310–1319.

Chen, S.D. and Ramli, A.R. (2004) 'Preserving brightness in histogram equalization based contrast enhancement techniques', *Digital Signal Processing*, Vol. 14, No. 5, pp.413–428.

Choi, C. and Lee, J.J. (1998) 'Chaotic local search algorithm', *Artif. Life Robotics*, Vol. 2, No. 1, pp.41–47.

Coelho, L.D.S. and Mariani, V.C. (2008) 'Use of chaotic sequences in a biologically inspired algorithm for engineering design optimization', *Expert Systems with Applications*, Vol. 34, No. 3, pp.1905–1913.

Coelho, L.D.S., Sauer, J.G. and Rudek, M. (2009) 'Differential evolution optimization combined with chaotic sequences for image contrast enhancement', *Chaos, Solitons and Fractals*, Vol. 42, pp.522–529.

Das, S., Abraham, A., Chakraborty, U.K. and Konar, A. (2009) 'Differential evolution using a neighborhood-based mutation operator', *IEEE Trans. on Evolutionary Computation*, Vol. 13, No. 3, pp.526–553.

Dhal, K.G, Quraishi, I. and Das, S A. (2015a) 'Chaotic Lévy flight approach in bat and firefly algorithm for gray level image enhancement', *I.J. Image, Graphics and Signal Processing* Vol. 7, No. 7, pp.69–76, MECS Publishers, ISSN: 2074-9074.

Dhal, K.G, Quraishi, I.M. and Das, S. (2015b) 'Development of firefly algorithm via chaotic sequence and population diversity to enhance the image contrast', *Natural Computing*, Vol. 15, No. 2, pp.307–318.

Dhal, K.G, Quraishi, I.M. and Das, S. (2015c) 'Performance enhancement of differential evolution by incorporating lévy flight and chaotic sequence for the cases of satellite images', *Int. J. of Applied Methaeuristic Computing*, Vol. 6, pp.69–81.

Dhal, K.G., Quraishi, M.I. and Das, S. (2015d) 'Performance analysis of chaotic lévy bat algorithm and chaotic cuckoo search algorithm for gray level image enhancement', *Information Systems Design and Intelligent Applications*., pp.233–244.

Dhal, K.G., Quraishi, Md. I. and Das, S. (2015e) 'Gray level image enhancement using particle swarm optimization with lévy flight: a hybrid approach', *1st Int. Conference on Emerging Trends in Computer Science and Information Technology-2015 (ETCSIT-2015)*, Kalyani Govt. Engg. College, West Bengal, India.

Dhal, K.G. and Das, S. (2016) 'Colour retinal images enhancement using modified histogram equalization methods and firefly algorithm', *International Journal of Biomedical Engineering and Technology*, InderScience Publication, (accepted).

Dhal, K.G., Sen, S., Sarkar, K. and Das, S. (2016) 'Entropy based range optimized brightness preserved histogram-equalization for image contrast enhancement', *Int. Journal of Computer Vision and Image Processing*, Vol. 6, No. 1, pp.59–72, IGI-GLOBAL Publishers.

Dhal, K.G. and Das, S. (2017) 'Combination of histogram segmentation and modification to preserve the original brightness of the images', *Pattern Recognition and Image Analysis*, Vol. 27, No. 2, pp.200–212, Springer.

Dhal, K.G, Quraishi, I. and Das, S. (2017a) 'An improved cuckoo search based optimal ranged brightness preserved histogram equalization and contrast stretching method', *Int. Journal of Swarm Intelligence Research*, Vol. 8, No. 1, pp.1–29, IGI-GLOBAL Publishers.

Dhal, K.G, Sen, M. and Das, S. (2017b) 'Cuckoo search based modified Bi-Histogram Equalization method to enhance the cancerous tissues in mammography images', *Int. Journal of Medical Engineering and Informatics*, InderScience Publication, (accepted).

Dhal, K.G. and Das, S. (2015) 'Diversity conserved chaotic artificial bee colony algorithm based brightness preserved histogram equalization and contrast stretching method', *International Journal of Natural Computing Research*, Vol. 5, No. 4, pp.45–73, IGI-GLOBAL Publishers.

Fister, I.J, Fong, S., Brest, J. and Fister, I (2014) 'A novel hybrid self-adaptive bat algorithm', *The Scientific World Journal*, Vol. 2014, No. 2014, pp.1–12.

Fister., I.J, Fister, D. and Yang, X.S (2013) 'A hybrid bat algorithm', *Skivestnik Elektrotehni*, Vol. 80, Nos. 1/2, pp.1–7.

Gandomi, A.H. and Yang, X.S (2014) 'Chaotic bat algorithm', *Journal of Computational Science*, Vol. 5, No. 2, pp.224–232.

Gao, H., Yang, Z., Bhimani, J., Wang, T., Wang, J., Sheng, B. and Mi, N. (2017) 'AutoPath: harnessing parallel execution paths for efficient resource allocation in multi-stage big data frameworks, *26th International Conference on Computer Communications and*.

Geng, Y, Liang, R.Z, Li, W., Wang, J, Liang, G., Xu, C. and Wang, J.Y (2016) 'Learning convolutional neural network to maximize Pos @Top performance measure', arXiv preprint arXiv: 1609.08417.

Gonzalez, R.C. and Woods, R.E. (2002) *Digital Image Processing*, 2nd ed., Prentice Hall, New York.

Gorai, A. and Ghosh, A. (2009) 'Gray-level image enhancement by particle swarm optimization', in *Proceedings of World Congress on Nature and Biologically Inspired Computing*.

Gorai, A. and Ghosh, A. (2011) 'Hue preserving colour image enhancement by particle swarm optimization', *IEEE*, pp.563–568.

Hashemi, S., Kiani, S., Noroozi, N. and Moghaddam, M.E. (2010) 'An image contrast enhancement method based on genetic algorithm', *Pattern Recognition Letters*, Vol. 31, No. 13, pp.1816–1824.

Jordehi, A.R. (2014) 'A chaotic-based big bang–big crunch algorithm for solving global optimisation problems', *Neural Comput. and Applic.*, Vol. 25, No. 6, pp.1329–1335.

Kennedy, J. (1999) 'Small worlds and mega-minds: effects of neighborhood topology on particle swarm performance', *Proceedings of the 1999 Congress on Evolutionary Computation*.

Kim, Y.T. (1997) 'Contrast enhancement using brightness preserving bi-histogram equalization', *IEEE Trans. Consum. Electron.*, Vol. 43, No. 1, pp.1–8.

Leandro, C.S.D. and Viviana, C.M. (2009) 'A novel particle swarm optimization approach using Henon map and implicit filtering local search for economic load dispatch', *Chaos, Solitons and Fractals*, Vol. 39, No. 2009, pp.510–518.

Li, Q., Zhou, X., Gu, A. et al. (2016) 'Nuclear norm regularized convolutional Max Pos@Top machine', *Neural Computing and Applications* [online] http://www.doi.org/10.1007/s00521-016-2680-2.

Liang, RZ, Shi, L, Wang, H, Meng, J, Wang, J.J.Y, Sun, Q. and Gu, Y. (2016) 'Optimizing top precision performance measure of content based image retrieval by learning similarity function, *23st International Conference on Pattern Recognition (ICPR)*.

Liu, G., Huang, H., Wang, S. and Chen, Z (2012) 'An improved bat algorithm with doppler effect for stochastic optimization', *International Journal of Digital Content Technology and its Applications (JDCTA)*, Vol. 6, No. 21, pp.326–336.

Munteanu, C. and Rosa, A. (2001) 'Evolutionary image enhancement with user behavior modeling', *ACM SIGAPP Applied Computing Review*, Vol. 9, No. 1, pp.8–14.

Pal, S.K., Bhandari, D. and Kundu, M.K. (1994) 'Genetic algorithms for optimal image enhancement', *Pattern Recognition Letters*, Vol. 15, No. 1994, pp.261–271.

Quraishi, I, De, M., Dhal, K.G., Mondal, S. and Das, G. (2013) 'A novel hybrid approach to restore historical degraded documents', *International Conference on Intelligent system and Signal Processing (ISSP*).

Quraishi, I., Dhal, K.G., De, M., Choudhury, J.P. and Pattanayak, K. (2012) 'A novel hybrid approach to enhance low resolution images using particle swarm optimization', *2nd IEEE International Conference on Parallel, Distributed and Grid Computing (PDGC 2012)*.

Roemer, J., Groman, M., Yang, Z., Wang, Y., Tan, C.C. and Mi, N. (2014) 'Improving virtual machine migration via deduplication', *11th IEEE International Conference on Mobile Ad Hoc and Sensor*.

Shanmugavadivu, P., Balasubramanian, K. and Muruganandam, A. (2014) 'Particle swarm optimized bi-histogram equalization for contrast enhancement and brightness preservation of images', *Vis Comput.*, Vol. 30, No. 4, pp.387–399.

Sheikholeslami, R. and Kaveh, A. (2013) 'A survey of chaos embedded meta-heuristic algorithms', *Int. J. Optim. Civil. Eng.*, Vol. 3, No. 4, pp.617–633.

Tai, J., Liu, D., Yang, Z., Zhu, X., Lo, J. and Mi, J. (2015) 'Improving flash resource utilization at minimal management cost in virtualized flash-based storage systems', *IEEE Transactions on Cloud Computing*, Vol. 5, No. 99, pp.537–549.

Wang, H., Cui, Z., Sun, H., Rahnamayan, S. and Yang, X.S. (2017a) 'Randomly attracted firefly algorithm with neighborhood search and dynamic parameter adjustment mechanism', *Soft Comput.*, Vol. 21, No. 18, pp.5325–5339, DOI: 10.1007/s00500-016-2116-z.

Wang, J., Wang, T., Yang, Z., Mao, Y., Mi, N. and Sheng, B. (2017b) 'SEINA: A stealthy and effective internal attack in hadoop systems, *International Conference on Computing, Networking and Communications*.

Wang, H., Wu, Z. and Rahnamayan, S. (2011) 'Particle swarm optimisation with simple and efficient neighbourhood search strategies', *Int. J. Innovative Computing and Applications*, Vol. 3, No. 2, pp.97–104.

Yang, X.S (2010a) 'A new metaheuristic bat-inspired algorithm, in: nature inspired cooperative strategies for optimization (NISCO 2010) in Gonzalez, J.R. et al. (Eds.): *Studies Computational Intelligence*, Vol. 284, pp.65–74, Springer, Springer Berlin.

Yang, X.S. (2010b *Engineering Optimization: An Introduction to Metaheuristic Applications*, Wiley, Hoboken, New Jersey.

Yang, X.S. (2010c)*Nature-Inspired Metaheuristic Algorithms*, 2nd ed., Luniver Press, UK.

Yang, X.S (2011) 'Bat algorithm for multiobjective optimization', *Int. J. Bio-Inspired Computation*, Vol. 3, No. 5, pp.267–274.

Yang, X.S (2013) 'Bat algorithm: literature review and applications', *Int. J. Bio-Inspired Computat*ion, Vol. 5, No. 3, pp.141–149, DOI: 10.1504/IJBIC.2013.055093.

Yang, X.S. and Deb, S. (2009) 'Cuckoo search via lévy flight', in *Proceedings of World Congress on Nature and Biologically Inspired Computing*.

Yang, Z., Awasthi, M., Ghosh, M. and Mi, N. (2016a) 'A fresh perspective on total cost of ownership models for flash storage in datacenters', *8th IEEE International Conference on Cloud Computing Technology and Science*.

Yang, Z., Tai, T., Bhimani, J., Wang, J., Mi, N. and Sheng, B. (2016b) 'GREM: dynamic SSD resource allocation in virtualized storage systems with heterogeneous IO workloads', 35th *IEEE International Performance Computing and Communications Conference*.

Yilmaz, S., Kucuksille, E.U. and Cengiz, Y (2014) 'Modified bat algorithm', *Elektronikair Elektrotechnika*, Vol. 20, No. 3, pp.71–78.