Maximisation of energy and exergy efficiencies for a sustainable thermoelectric cooling system by applying genetic algorithm

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Abstract: The efficient thermal management of thermoelectric cooler (TEC) as a sustainable cooling technology is important. The energy loss, electrical power requirement, and irreversibility of the TEC need to be minimised. Through this work, authors separately optimised TEC energy efficiency (η_I) and exergy efficiency (η_{II}) considering thermoelectric elements geometry and electric current by using the genetic algorithm (GA). The effects of electrical contact resistance and thermal resistance are considered in the mathematical model. Unlike previously reported works, the authors have used junction temperatures different from surface temperatures at the respective cold and hot sides of TEC. This study reveals that maximum energy and exergy efficiencies are obtainable at the same values of electric current, length, and cross-sectional area of thermoelectric elements. At cold surface temperature (T_c) of 20°C, the maximum energy efficiency of 4.11 and the maximum exergy efficiency of 0.0715 are obtained. GA result is validated through ANSYS® finite-element simulation.

Keywords: thermoelectric cooler; TEC; energy efficiency; exergy efficiency; genetic algorithm; optimisation; finite-element simulation.

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1 Introduction

At present, more than three-fourths of refrigeration systems work on vapour compression and use refrigerants. These systems are one of the most important factors of stratospheric ozone depletion. The adverse environmental impacts of chloro-fluoro-carbons (CFCs), hydro-chloro-fluoro-carbons (HCFCs), and hydro-fluoro-carbons (HFCs) used in vapour compression-based cooling systems have forced the energy researchers to strongly explore renewable alternatives. A promising eco-friendly replacement for the cooling systems using refrigerants is thermoelectric cooler (TEC). A TEC is a solid-state and practically silent device. TEC is powered by the Peltier effect and achieved by supplying electric current through n-type and p-type thermoelectric (TE) material. TEC does not have moving components. TECs are significant because of the need for steady, small size and environmental sustainability for many applications. TECs are prominently used for cooling requirements of small volumes. These are preferred in various high technology applications such as electronics, space, medicine, telecommunications, and others. TECs are best suited to areas that are not accessible to compressors.

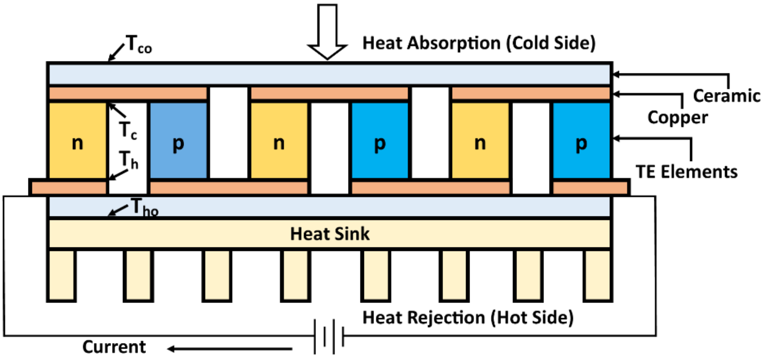
Notwithstanding, currently, TECs have low rate of cooling and energy conversion efficiency compared to other traditional cooling technologies that impeded their wide-spread usage (Hermes and Barbosa, 2012). Various models and hypotheses have been presented in the literature on this problem, and several solutions have been suggested (Abramzon, 2007; Astrain et al., 2003; Cheng and Lin, 2005; Fabián-Mijangos et al., 2017; Ferreira-Teixeira and Pereira, 2018; Ghoshal et al., 2002; Giri and Nain, 2019; Huang et al., 2013; Jeong, 2014; Lee, 2013; Lee and Kim, 2007; Manikandan et al., 2016; Miner et al., 1999; Nain et al., 2010; Pan et al., 2007; Seifert and Pluschke, 2014; Shen et al., 2020; Thiébaut et al., 2017; Völklein et al., 1999; Yamanashi, 1996; Yu and Wang, 2009; Zhang, 2010). A significant number of papers are concerned with the design and simulation of TE cooling or heat pumping systems. However, it is important to note that most of these optimisation attempts were primarily made to improve rate of cooling or coefficient of performance (COP) of TEC. Energy has two faces: exergy and anergy. The expression ‘exergy’ derives from the Greek terms ex and ergon, representing from and work (Dincer and Rosen, 2013). Exergy represents available part of energy. Anergy shows dispersed part of energy source or lost potential. Very few articles appear to deal with the exergy aspects of the TE cooling or heat pumping devices, two of such articles which are prominent (Kaushik et al., 2015; Nami et al., 2017). Kaushik et al. (2015) performed exergy analysis of reversible, endoreversible, exoreversible, and irreversible heat pump. Analytical expressions of thermodynamic models were derived. Exergy output for the different temperature difference between hot and cold sides was observed. It was found that exergy output increases at a higher temperature difference. The increment in contact resistance reduces exergy output. Nami et al. (2017) reported a comparison of single-stage and two-stage TE devices for COP and exergetic efficiencies. It was established that a higher current is needed in a single-stage device for maximum values of energy efficiency and exergy efficiency. The research on exergy is focused on the quality of energy used to assess the energy process in compliance with ideal thermodynamic equality (Bejan, 2002; Dincer and Kanoglu, 2017). The exergy study is used to recognise exergy losses. It is used to understand irreversible energy conversion losses in the design of the system. To provide effective efficiency and meet several design requirements, computer-aided design, optimisation, and analysis have been important parts for the design of TE cooling systems.

This work is a focussed effort on optimisation of TEC to achieve maximum energy and exergy efficiencies. The TEC geometry and current can change the efficiencies, and optimum values can be obtained under given circumstances. In this article, the authors attempted to present an optimisation method for the TEC design for improved results by considering three of the main parameters: namely electric current, length of TE elements, and cross-sectional area of TE elements. Thereafter, a three-dimensional TEC model is developed, and finite-element simulation is carried out following the optimised value of exergy efficiency to validate genetic algorithm (GA) results.

2 System description

A TEC consists of several couples made out of n-type and p-type TE elements, as shown in Figure 1. These TE elements are tied together electrically by using highly conductive copper tabs. These elements are connected thermally parallel. This arrangement is sandwiched between two ceramic layers.

Figure 1 Schematic diagram of a TEC (see online version for colours)



A TE cooling system is configured to transfer thermal energy from the source at temperature T_{co} to a sink at a higher temperature T_{ho}. The basic units of a TEC are thermocouples that possess temperatures T_c and T_h at cold and hot junctions respectively. Unlike previous works in the TEC system, the authors have used a model that appreciates the difference between T_{co} and T_c at the cold side and the difference between T_{ho} and T_h at the hot side. The material properties and heat balance equations are fundamentally established for thermocouple portions of TEC. In the present work, the authors have modelled TEC realistically considering copper and ceramic effects with cold and hot side temperatures of T_{co} and T_{ho} for GA-based optimisation and finite-element simulations.

Electric power is provided as input to the TEC system, and the Peltier effect is at work according to the direction of electric current.

TECs are inherently irreversible because the flow of heat and current is necessary during operation. These irreversibilities are the reason for the requirement of the performance optimisation of TECs.

3 Theoretical model of an irreversible TEC system

The reversible cycle concept as explained by Sadi Carnot is well established and significant. Thermodynamically, a practical TEC can be modelled as an irreversible reversed heat engine. An irreversible TEC possesses both types of the irreversibilities, i.e., internal and external. The internal irreversibility is generated as TEC operates between cold and hot junctions of n-type and p-type TE elements at temperatures T_c and T_h, respectively. The external irreversibility is generated for temperature gap $T_{co} - T_c$ carrying heat absorption rate, Q_c at the cold side and for temperature gap $T_h - T_{ho}$ carrying heat rejection rate, Q_h at the hot side of the TEC system. In the present work, unlike previous papers, the authors have used temperature T_c different from T_{co} and temperature T_h different from T_{ho}.

At the cold side of the TEC, the heat energy balance may be expressed as:

$$Q_c = Q_{Peltier} - Q_{conduction} - Q_{Joule} \tag{1}$$

At the hot side of the TEC, the heat energy balance may be expressed as:

$$Q_h = Q_{Peltier} - Q_{conduction} + Q_{Joule} \tag{2}$$

The Peltier heat ($Q_{Peltier}$) at the cold and hot sides corresponds to $I\alpha T_c$ and $I\alpha T_h$ respectively. The irreversible heat transfer due to conduction ($Q_{conduction}$) at the cold side and hot side corresponds to $K(T_h - T_c)$. The irreversible Joule heat generation (Q_{Joule}) corresponds to ½ I^2R. In these expressions, I refers to supplied electric current (A), α is the Seebeck coefficient (V/K) and R represents the electrical resistance. T_c and T_h are the cold side and hot side temperatures (K) of n-type and p-type elements. If N represents the total number of TE couples used in the TEC, the heat energy balance equations become:

$$Q_c = 2N\left[I\alpha T_c - K(T_h - T_c) - \frac{1}{2}I^2(R)\right] \tag{3}$$

$$Q_h = 2N\left[I\alpha T_c - K(T_h - T_c) + \frac{1}{2}I^2(R)\right] \tag{4}$$

If k and r_c represent the thermal conductivity (W/mK) and the electrical contact resistance (Ωm^2), respectively. L and A represent the length (m) and cross-sectional area (m^2) of n-type and p-type TE elements, respectively. Equations (3) and (4) can be rearranged as:

$$Q_c = 2N\left[I\alpha T_c - \frac{kA(T_h - T_c)}{L} - \frac{1}{2}I^2\left(\frac{\rho L}{A} + 2\frac{r_c}{A}\right)\right] \tag{5}$$

$$Q_h = 2N\left[I\alpha T_h - \frac{kA(T_h - T_c)}{L} + \frac{1}{2}I^2\left(\frac{\rho L}{A} + 2\frac{r_c}{A}\right)\right] \tag{6}$$

The choice of the TE element's material directly affects TEC performance. Bismuth telluride (Bi_2Te_3) is the premium material used in TECs. The temperature-dependent properties of Bismuth telluride used in this study are calculated from the below-mentioned expressions as specified (Fraisse et al., 2013).

$$\alpha = \left(22{,}224 + 930.6\, T_{ave} - 0.9905 T_{ave}^2\right) \times 10^{-9} \tag{7}$$

$$\rho = \left(5{,}112 + 163.4\, T_{ave} + 0.6279 T_{ave}^2\right) \times 10^{-10} \tag{8}$$

$$k = \left(62{,}605 - 277.7\, T_{ave} + 0.4131 T_{ave}^2\right) \times 10^{-4} \tag{9}$$

where $Average\ Temperature, T_{ave} = 0.5\left(T_c + T_h\right)$

The irreversible heat flow rate with the heat source or TEC cold surface temperature (T_{co}) to the cold side temperature of n-type and p-type elements (T_c) is given by:

$$Q_c = U_c A_c \left(T_{co} - T_c\right) \tag{10}$$

Similarly, the irreversible heat flow rate with the hot side temperature of n-type and p-type elements (T_h) to the heat sink or TEC hot surface temperature (T_{ho}) is given by:

$$Q_h = U_h A_h \left(T_h - T_{ho}\right) \tag{11}$$

In the above expressions, U_c and U_h represent the overall heat transfer coefficients (W/m^2K) while A_c and A_h represent heat transfer surface areas (m^2) at the cold and hot surfaces of TEC.

Thermal conductance is reciprocal thermal resistance. The authors have used a thermal resistance model. They have incorporated thermal resistances of ceramic substrates, copper conductors, and heat sink at the hot side for realistic TEC model consideration. By using thermal resistance models, equations (10) and (11) are restructured as:

$$T_h = Q_h \left(R_{hs} + R_{cr} + R_{cu}\right) + T_o \tag{12}$$

$$T_c = T_{co} - Q_c \left(R_{cr} + R_{cu}\right) \tag{13}$$

where R_{hs}, R_{cr} and R_{cu} are the thermal resistances (°C/W) of the ceramic substrates, copper conductors and heat sink.

From the 'available work' or 'exergy' point of view, it is well established by the researchers that the exergy input to the TE cooling system is equal to the input electric power (P), and it is 100% exergy. The concept of exergy needs the first law of thermodynamics as well as the second law of thermodynamics. In the TE cooling system, the temperature of the cold side of TEC (T_{co}) is less than the environment temperature (T_o). The temperature difference ($T_o - T_{co}$) leads to the absorption of heat from the refrigerated space.

The first law of thermodynamics can be applied to obtain:

$$Exergy\ Input = Input\ Electric\ Power(P) = Q_h - Q_c \tag{14}$$

Applying the second law of thermodynamics for irreversible TEC, we can write:

$$\frac{Q_c}{T_{co}} - \frac{Q_h}{T_{ho}} < 0 \tag{15}$$

The second law of thermodynamics can also be applied to obtain rate of entropy generation as:

$$Rate\ of\ entropy\ Generation,\ S_{gen} = \left(\frac{Q_h}{T_{ho}} - \frac{Q_c}{T_{co}} \right) > 0 \tag{16}$$

If we combine the first law and second law of thermodynamics used for conservation of energy and non-conservation of entropy, exergy balance for a thermodynamic system can be obtained as:

$$Exergy\ Input = Exergy\ Output + Exergy\ Losses \tag{17}$$

$$Exergy\ Input = Exergy\ Output + (Exergy\ Waste\ Emission + Exergy\ Destruction) \tag{18}$$

The exergy output refers to the thermal exergy deposited at the cold side of the TEC. This exergy transfer accompanying heat transfer Q_c can be calculated by (Bejan, 2016; Çengel et al., 2019; Moran et al., 2018):

$$Exergy\ Output = Q_c \left(\frac{T_o}{T_{co}} - 1 \right) \tag{19}$$

where T_o represents a typical environment or outside temperature.

Exergy waste emission or exergy lost represents the exergy rejected at the hot side of the TEC (If present) and can be written as (Manikandan et al., 2016):

$$Exergy\ Lost = Q_h \left(1 - \frac{T_o}{T_{ho}} \right) \tag{20}$$

Substituting equations (14), (19), and (20) into equation (18), irreversibility or exergy destruction in the process can be obtained as:

$$Irreversibility\ or\ Exergy\ Destruction = T_o \left(\frac{Q_h}{T_{ho}} - \frac{Q_c}{T_{co}} \right) = T_o S_{gen} \tag{21}$$

First law efficiency or *energy efficiency* of a TEC is obtained as:

$$\begin{aligned} First\ Law\ Efficiency\ or\ Energy\ Efficiency(\eta_I) &= \frac{Energy\ Output}{Energy\ Input} \\ &= \frac{Q_c}{P} = \frac{Q_c}{Q_h - Q_c} \end{aligned} \tag{22}$$

The *second law efficiency* or *exergy efficiency* is a measure of approach to reversibility or ideality. A key goal of exergy analysis is to identify this significant efficiency and actual values of exergy losses with the identification of possible reasons (Dincer, 2016).

$$Second\ Law\ Efficiency\ or\ Exergy\ Efficiency(\eta_{II}) = \frac{Exergy\ Output}{Exergy\ Input} \tag{23}$$

Alternatively, this can be expressed as:

$$\eta_{II} = 1 - \frac{Exergy\ Losses}{Exergy\ Input} \tag{24}$$

The purpose of the TEC is to absorb heat from the space to be cooled. Thus, it helps in increasing exergy output while this heat absorption takes place. The exergy efficiency of a TEC can be understood as the ratio of minimum exergy or work requirement to the actual exergy input. For simplicity, the thermal reservoir at hot surface temperature T_{ho} is set to be the typical environment. Hence, T_{ho} is the outside temperature (T_o) for the TEC.

Hence

$$Exergy\ Output = Q_c\left(\frac{T_{ho}}{T_{co}} - 1\right) \tag{25}$$

Second law efficiency or exergy efficiency of an irreversible TEC is obtained as:

$$\eta_{II} = \frac{Q_c\left(\frac{T_{ho}}{T_{co}} - 1\right)}{P} = 1 - \frac{T_{ho}S_{gen}}{P} \tag{26}$$

This establishes a relationship between first law efficiency (η_I) and second law efficiency (η_{II}) of the TEC:

$$\eta_{II} = \frac{\eta_I}{\frac{1}{\left(\frac{T_{ho}}{T_{co}} - 1\right)}} \tag{27}$$

The COP of the TEC is the ratio of useful energy to the required energy and is equal to energy efficiency as described in equation (22). According to the reversed Carnot cycle phenomenon, $\frac{1}{\left(\frac{T_{ho}}{T_{co}} - 1\right)}$ represents the Carnot or reversible COP, the highest theoretical COP of TEC working within temperature bounds T_{co} and T_{ho}. Hence, the second law efficiency of TEC can also be expressed as:

$$\eta_{II} = \frac{Actual\ COP}{Reversible\ COP}\ or\ \frac{Actual\ COP}{Maximum\ COP} \tag{28}$$

It is clear that Second law efficiency indicates the irreversibilities associated with the TE cooling system and can vary from 0 to 1.

4 Optimisation model formulation and implementation of GA

In this study, two objectives are optimised separately. The first optimisation problem's objective is the first law efficiency (η_I), which is of significance to obtain higher energy efficiency in a restricted space. The second optimisation problem's objective is the second law efficiency (η_{II}), which is important to achieve a higher exergy efficiency. After modelling, we select the design variables to achieve optimum engineering goals. The geometry, material and operating conditions play a vital role in designing a TEC with higher performances. The restriction of space in designing TEC is a major constraint. At present, TECs are widely used in the high technology fields such as electronics, telecommunications, space, and others. Space restrictions are very prevalent in these fields and hence there are practical limits to the size of the TEC. Therefore, the space constraints of the TE elements are very relevant assumptions to consider. This study introduces a new approach by selecting three design variables, i.e., length of n-type and p-type TE elements, the cross-sectional area of n-type and p-type TE elements and the input electric current. For simplifying the complexity of calculation, some assumptions are made for this study.

- Heat transfer is assumed to take place along the length of TE elements.
- The TE elements have the same cross-section and length.
- Thomson effect is not considered.
- Steady-state condition is prevailing.

4.1 Optimisation of energy efficiency or first law efficiency (η_I)

The first single-objective optimisation problem for maximisation of the energy efficiency or first law efficiency (η_I) of the TEC is formulated mathematically as:

$$\begin{cases} & \text{Maximise } \eta_I \\ \text{Subject to} & I_{\min} \le I \le I_{\max} \\ & L_{\min} \le L \le L_{\max} \\ & A_{\min} \le A \le A_{\max} \end{cases} \tag{29}$$

where ($I_{\min}$, $I_{\max}$), ($L_{\min}$, $L_{\max}$) and ($A_{\min}$, $A_{\max}$) are the bounds of design variables I, L and A, respectively. The bounds of the design variables of this optimisation study are listed in Table 1.

Besides, the total number of TE couples (N) depends on the cross-sectional area of TE elements and packaging density. Its value can be evaluated using this equation:

$$N = \frac{Available\ area(S)\ of\ TEC \times packaging\ density(PD)}{2 \times A} \tag{30}$$

The detailed specifications of the TE cooling system considered for this work are presented in Table 2.

Table 1 Lower and upper bounds of design variables

Variable	*Description*	*Lower bound*	*Upper bound*
I	Input electric current, A	0.1	3.0
L	Length of n-type and p-type TE elements, mm	1.0	2.0
A	Cross-sectional area of n-type and p-type TE elements, mm^2	1.0	2.0

Table 2 Specifications of the TE cooling system

Description	*Parameter*	*Value*
Available C.S. area of TEC	S	15 mm × 15 mm
Packaging density	PD	80%
Ambient temperature	T_o	25°C or 298.15 K
Cold surface temperature	T_{co}	20°C or 293.15 K
Electrical contact resistance	r_c	$1 \times 10^{-8}\ \Omega\ m^2$
Thermal resistance of heat sink	R_{hs}	0.10 0C/W
Ceramic thermal conductivity	k_{cr}	35.3 W/m°C
Copper thermal conductivity	k_{cu}	386 W/m-°C

The lower bound and upper bound of the number of TE couples is the dependent variable and become 45 and 90, respectively for the present work. The cold side and hot side ceramic substrates are 0.2 mm in thickness. The cold side and hot side copper tabs are considered 90% of the available cross-sectional area of TEC and 0.1 mm in thickness. The parameter values as described in Table 1 and Table 2 are based on information provided by TEC manufacturers on technical catalogues.

4.2 Optimisation of exergy efficiency or second law efficiency (η_{II})

The second single-objective optimisation problem for maximisation of the exergy efficiency or second law efficiency (η_{II}) of the TEC is formulated mathematically as:

$$\begin{cases} & Maximise\ \eta_{II} \\ Subject\ to & I_{\min} \le I \le I_{\max} \\ & L_{\min} \le L \le L_{\max} \\ & A_{\min} \le A \le A_{\max} \end{cases} \tag{31}$$

The bounds of design variables and specifications of TE cooling system resulting in variations of output are identical to previous optimisation problem of maximisation of energy efficiency as discussed in Section 4.1.

4.3 Implementation of GA

In complex engineering applications, the implementation of stochastic algorithms has become popular and common. The stochastic algorithm, like GA attempts to perform a global search within the search space of design variables. Because of the promising potential researchers are using stochastic methods to evaluate, predict, and optimise different systems.

GA, which is used for this study, uses a population of individuals to search the design space. It is based on natural genetics. A population represents a set of solutions. Each individual of a population represents a candidate solution. The initial population is processed for fitness evaluation. Each individual of the population is assigned a fitness score based on the objective function. GA performs its evolution by undergoing processes inspired by evolutionary biology. New generations are created using selection, crossover, and mutation of solutions. In each generation, the individuals are evaluated for fitness scores. Individuals with the highest fitness are chosen to produce offspring for the new generation. GA tries the search process in whole design space and has the ability to concentrate on global optimum or near-global optimum location. After completion of certain generations, GA population finally converges towards the optimum solution.

In the literature, many coding schemes are available for GA such as binary-coded GA, gray coded GA, integer genes, and real-coded GA (RGA). RGA is more popular than others because of its inherent potential to directly provide numeric value solutions with no need to code and decode the solutions. Many real-world applications have shown the advantage of RGA compared to others. The simulated binary crossover (SBX) operator performs excellently in optimisation problems with continuous search space. The real-variable GA is implemented in this paper as created by Deb and Agrawal (1995). GA parameters settings used for this work are given in Table 3.

Table 3 GA parameters

Parameter	*Value*
Population size	50
Generations limit	1,000
Crossover probability	0.80
Mutation probability	0.25

The GA solution procedure adopted for this study has the major steps as described below:

Major steps used for GA optimisation process

Input: TEC dataset and GA parameters

Output: Optimum solution as per the objective

Begin

a Initialise the population.

b Guess the values of T_h and T_c.

c Compute the values of TE material properties and Q_c and Q_h using equations (5)–(9).

d Compute new values of T_h and T_c and replace old values with new values.

e Repeat the process until the difference in new and old values of T_h and T_c becomes negligible.

f Accept the individual (solution) of the population, if it satisfies the conditions at step-e mentioned above.

g Evaluate the individual of the population for fitness values.

h Take the next individual of the current population and repeat the process discussed above.

i After the entire population is evaluated, a new population is formed by using selection, crossover, and mutation operators of GA.

j The entire process is repeated for a stipulated number of maximum generations.

k The important statistics of generations are reported.

5 Results and discussion

In two single-objective optimisation problems, the energy efficiency and the exergy efficiency of the irreversible TEC are maximised using GA. GA-based optimisations are coded in C language. In this study, the GA source code developed by Deb (2001) was employed. Multiple independent runs were performed for each optimisation problem. Table 4 contains the best optimisation results after 1,000 generations for multiple runs of energy efficiency maximisation and exergy efficiency maximisation. These best results are repeatedly found over various runs.

The optimal values of design variables are identical for both the optimisation problems as per simulation results. The optimum input current to obtain maximum energy and exergy efficiency are with the same values of 0.28A. This finding of results is a reliable match as reported by researchers (Kaushik et al., 2015; Manikandan et al., 2016). This study found that besides input current, the length and cross-sectional area of TE elements to obtain maximum energy and exergy efficiencies are also the same. The overall findings of this study are superior to previous approaches because the previous studies reported similar values of one variable while other parameters were kept constant to obtain peak values of η_I and η_{II}. In the optimised solutions, the length of TE element is found as 2.0 mm which is the upper limit of the range. The other design variables (I and A) are unique and well between the bounds of given ranges. Depending on the optimal design values of three different variables, the corresponding temperature T_h is obtained as 25.11°C, and T_c is found as 19.97°C. The corresponding hot surface temperature (T_{ho}) is estimated as 25.09°C. The rate of cooling (Q_c) and the rate of heat rejection (Q_h) are achieved as 0.746 W and 0.927 W, respectively. Exergy parameters for optimised exergy efficiency are given in Table 5.

The heat rejection increases the rate of entropy generation (S_{gen}) and exergy destruction in the TEC system. The work provided to the TEC system includes exergy transfer towards the cold side and loss of available work due to the irreversibility of the system. Hence, the required work exceeds the threshold value. S_{gen} is minimised for maximum exergy efficiency. The exergy efficiency or second law efficiency is maximised and achieved as 7.15%.

Table 4 Results of GA-based optimisation for maximisation of energy and exergy efficiencies

Case	η_I	η_{II}	*Optimal values of design variables*			
			I	*L*	*A*	*N (Dependent)*
Optimisation of energy efficiency	4.116	0.0715	0.28 A	2.0 mm	1.956 mm^2	45
Optimisation of exergy efficiency	4.116	0.0715	0.28 A	2.0 mm	1.956 mm^2	45

Table 5 Exergy parameters for maximised exergy efficiency (η_{II})

Maximised exergy efficiency	*Exergy input*	*Exergy output*	*Exergy destructed*	*Entropy generation rate*
7.15%	0.1874 W	0.0134 W	0.1740 W	0.5835×10^{-3} W/K

6 Finite-element simulation with ANSYS®

The finite-element simulation is an effective computational method for approximate solutions to a number of complex real-world engineering problems with boundary conditions. It has become a key software for the analysis of real-world problems in the engineering domain. The authors have used finite-element simulation to validate GA result. Since the maximum energy and exergy efficiency are obtained at the same amount of I, L and A, any of the two maximisation results can be tested. The authors have selected maximum exergy efficiency to be verified as it can be the preferred basis of TEC design because of environmental concerns. The ANSYS® is a multi-purpose analysis tool to generate the finite-element model and perform simulation for a wide range of engineering areas. ANSYS® is capable of providing coupled solutions to electric and thermal fields that are required for TEC simulation. Hence, the authors have used ANSYS® for finite-element simulation for this work. There are three major steps involved in the finite-element analysis of the TE cooling system using ANSYS®:

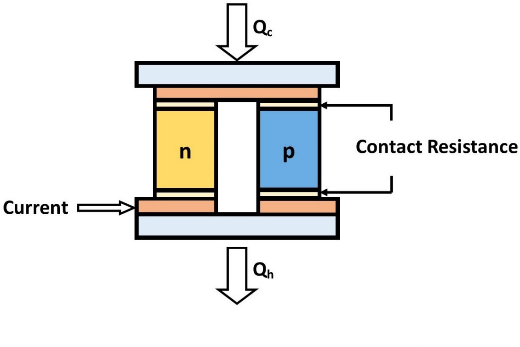
a finite-element model generation

b finding solution after specifying boundary conditions

c reviewing plots and results.

A nonlinear three-dimensional finite-element model has been set up. This finite-element model consists of a TEC structure with one pair of n-type and p-type TE elements following GA result. The authors have used a new approach for taking consideration of electrical contact resistances in both the junctions. A new geometric part at each end of each TE element is added to incorporate the effect of electrical contact resistances. These new parts are termed as 'contact resistance', and the complete TEC structure for finite-element simulation is shown in Figure 2. Each 'contact resistance' part has material properties pertinent to thermo-electric behaviour of electrical contact resistance.

Figure 2 TEC structure for finite-element simulation (see online version for colours)



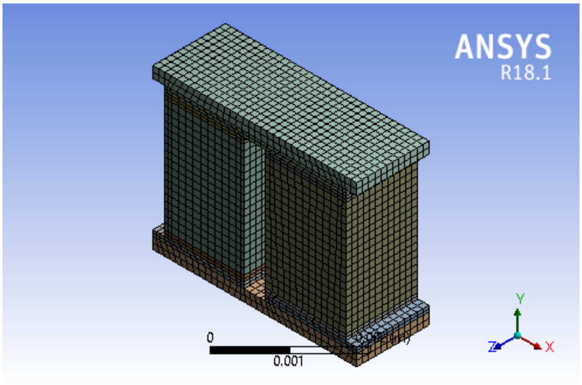
The input parameters for finite-element simulations to validate GA result of maximum exergy efficiency of TEC are given in Table 6. Except for those for which the boundary conditions are specified in Table 6, a small convection loss of 0.000001 W/mK is added to all surfaces of the adiabatic heat transfer models from the exposed surfaces of the TE cooling system.

Table 6 Input parameters for finite-element simulations to validate GA result of maximum exergy efficiency

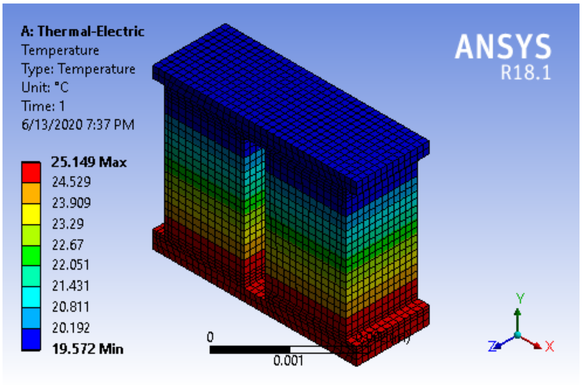
Description	*Parameter*	*Value per pair of TE elements*
Rate of cooling	Q_c	0.0165 W
Current	I	0.28 A
Hot side temperature of TEC	T_{ho}	25.09°C

The steady-state finite-element TEC model for validating GA results has TE elements of length and cross-sectional area as reported in Table 4. The material properties of TE elements are temperature-dependent and computed using T_c and T_h values. The GA predictions for maximum exergy efficiency were tested. The mesh generated for this simulation, electric voltage distribution, and temperature distribution are shown in Figure 3.

Figure 3 (a) Mesh generation (b) Temperature distribution (c) Voltage distribution in the finite-element model for maximum exergy efficiency (see online version for colours)

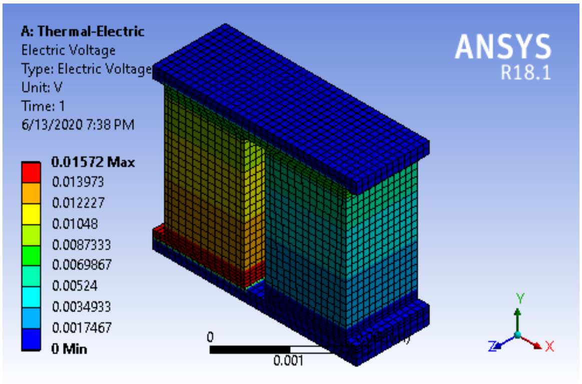


(a)



(b)

Figure 3 (a) Mesh generation (b) Temperature distribution (c) Voltage distribution in the finite-element model for maximum exergy efficiency (continued) (see online version for colours)



(c)

The result of the finite-element simulation is in strong alignment with those predicted by GA. This comparison of the results is shown in Table 7.

Table 7 Comparison of GA and ANSYS® results for maximum exergy efficiency

Description	*Parameter*	*GA*	*ANSYS®*	*Remarks*
Input electric power	P	0.004 W	0.004 W	Value for one pair of TE elements
Rate of heat rejection	Q_h	0.021 W	0.021 W	
Cold surface temperature	T_{co}	20°C	19.61°C	

As can be seen from the comparison presented in Table 7, the values of P and Q_h perfectly match while T_{co} slightly differs which is quite acceptable. The ANSYS® result strongly matches the GA result. The GA optimisation result is verified by the ANSYS® thermal-electric module solutions.

7 Conclusions

The purpose of the first and second laws of thermodynamic analysis of this work is to find maximised energy efficiency (η_I) and exergy efficiency (η_{II}) of a TEC. This study introduces a new approach by selecting three design variables, i.e., length of n-type and p-type TE elements, the cross-sectional area of n-type and p-type TE elements, and the input electric current. The authors have used a thermal resistance model in both analytical and numerical models. The thermal resistances of ceramic substrates, copper conductors, and heat sink at the hot side have been incorporated for realistic TEC model consideration. Unlike previously reported works, the authors have used junction temperatures different from surface temperatures at the respective cold and hot sides of TEC.

Previous studies (Kaushik et al., 2015; Manikandan et al., 2016) indicate that η_I and η_{II} are maximum at the same current. This study has shown that the maximum values of η_I and η_{II} are obtained not only at the same current but the length and cross-sectional area of TE elements are also the same. In previous studies, the identical electric current for maximum η_I and η_{II} were recorded, while other parameters were kept constant. If we consider three performance indicators for TECs, i.e., cooling capacity, energy efficiency (η_I) and exergy efficiency (η_{II}), through this work it is clear that maximum η_{II} automatically assures maximum η_I.

For environmental consideration, the authors recommend the maximisation of exergy efficiency as the basis of TEC design. The improvement in exergy efficiency is a reflection of the thermodynamic improvement of the TE cooling operation and high sustainable score. Exergy analysis includes all losses (irreversibilities) in the parts and completes TE cooling system. Irreversibilities can be minimised to get maximum exergy efficiency, which makes the TEC system more sustainable.

Since it is a numerical simulation, a different technique is required to validate the GA result. Hence, the authors have tried to validate the maximised exergy efficiency solution and tested with the finite-element TEC model. The GA solution is validated through finite-element simulation using ANSYS® thermal-electric module which is well in agreement with each other.

The dimensionless figure of merit (*ZT*) of TE materials plays an important role in characterising the performance of TEC. It is important to develop more efficient TE materials with a focus on maximising *ZT* value. When significant advances in the peak value of the dimensionless figure of merit of TE materials will be found, the exergy efficiency of TEC can be higher than the vapour compression refrigeration systems. Future research on sustainable TEC design should consider different TE materials at different temperatures range.

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