A queuing-based mathematical model for the congested network design problem under demand uncertainty

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Abstract: In this study, we propose a queuing-based mathematical model for analysing the congestion phenomenon in a transportation network. For this purpose, we present a mathematical model for network design with the objective of the cost minimisation and optimal demand satisfaction under demand uncertainty situation. The queuing theory usage helps to better model the congestion item caused by customers' referrals to the facilities of the transportation network. Moreover, a numerical example is solved to demonstrate the validity of the proposed model. To the best of the authors' knowledge, this is the first study which looks at the congested transportation network problem considering demand uncertainty from a queuing theory perspective.

Keywords: queuing theory; congested networks; transportation; mathematical modelling; demand uncertainty; systems; optimisation.

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1 Introduction

In the recent years, network design problems (NDP) with their various applications have attracted much interest of researchers in computer science, operations research, engineering and other related fields. In general, the NDP investigates the design and development communication paths for transferring demand among a set of nodes (Yildiz et al., 2018; Farahani et al., 2013; Yaghini and Akhavan, 2012). Design of transport networks (road, air, and rail), communication networks (telecom and computer networks) and energy transmission systems (electrical networks, transmission networks for fuel, gas, water, etc.) are various applications of network design models. Many scholars have studied the problem of transportation NDP (e.g., Ukkusuri et al., 2007; Wieberneit, 2008; Bianco et al., 2009; Zhu et al., 2014; Miandoabchi et al., 2015; Wang et al., 2015; Tan et al., 2016; Chainas, 2017).

It is assumed in the traditional NDP that there are a set of nodes encompasses a set of facility points and a series of non-hub nodes to connect the network points. The aim of the problem is to determine the nodes which transfer the demand between each origin-destination (O-D) pair and minimise both total setup and total flow routing costs (Wang et al., 2015; Alumur et al., 2016). In addition, there are plenty of studies in the NDP literature in which scholars have developed different mathematical models for optimal allocating customers to facilities. The combination of facility location and NDP is an example of the mentioned problems. For this kind of problems, in addition to the determination of nodes and optimal routes to facilities, the allocation of customers to the facilities is also considered (Elkady and Abdelsalam, 2016).

Gollowitzer et al. (2012) studied a combined facility location and NDP simultaneously and presented a mathematical integer programming model considering the capacity constraints on the network nodes as well as facilities. In another study, Contreras et al. (2012) presented a mathematical model for the facility location and NDP. The objective function of the model minimised the maximum cost of the service received by the customer. In most models of NDP, two types of received service costs by costumers can be minimised that are defined in most network design models and thus, the objective function is usually considered as minimising the total costs including the fixed costs of network nodes construction and variable costs of demand flow.

In a general classification, NDP is divided into the discrete and continuous problems. In the discrete environment, the decision variables are discrete and in relation to the hub and the non-hub nodes. The models proposed in the studies like Wang and Pardalos

(2017), Fontaine and Minner (2017) and Wang et al. (2015) include discrete NDP in order to optimise the network design.

Changing the demand volume is influenced by factors such as economic growth, changing of social patterns and population growth or changes in other parameters of the problem resulting in decreased performance of the available network. Under continuous conditions, NDP seeks to optimise by changes in the network structure and by modification of the network performance. It is assumed that in the continuous NDP, a network of nodes and arcs are predetermined and the capacity of each arc is determined. Decision variables of these problems are discussed in relation to the changes in arc capacity, pricing, and the construction of new nodes. Some studies such as Hua et al. (2011), Boland et al. (2017) and Han et al. (2016) have addressed the continuous network design models and have utilised some algorithms for solving the investigated problems.

In addition to these categories, NDP is divided into two categories according to the assumption of certainty or uncertainty. In many of the previous models, all input information is assumed to be certain and static. However, the uncertainty in real situations such as demand uncertainty reduces the effectiveness of the earlier presented models. Thus, the models based on the assumption of uncertainty in demand, have grown substantially during the past decade. Some studies have considered demand uncertainty in formulating the studied transportation NDP (i.e., Chen et al., 2007; Ukkusuri et al., 2007; Hua et al., 2011; Ukkusuri and Patil, 2009; Mudchanatongsuk et al., 2008; Tsao et al., 2016; Pishvaee et al., 2014).

Table 1 Previous studies versus the current research

Research	*Studied problem*	*Proposed model/approach*
Marcotte (1986)	NDP with congestion effects	Bi-level programming and game theory
Verhoef (2002)	Congested transportation network considering the elastic demand	Second-best optimisation model
Nagurney (2000)	Congested urban transportation network with fixed travel demand and emission	Traffic network equilibrium model
Osorio and Bierlaire (2009)	Congested urban networks considering a fixed-time traffic signal	Queueing network framework applying simulation tool
Szeto et al. (2011)	The risk – aversive stochastic transit assignment problem considering travel time, waiting time, capacity, and congestion	Nonlinear complementarity problem formulation
Abouee-Mehrizi et al. (2011)	Congested network with balking	A location-pricing-queuing model
Poudel et al. (2017)	Biomass co-firing supply chain network design considering the impact of congestion	The two-stage stochastic programming model
Current study	Congested transportation NDP considering demand uncertainty	A queuing-based mathematical model

While many studies have intended to consider demand uncertainty as a major assumption, however, the role of congestion caused by this factor has not been properly studied in the literature. The occurrence of congestion in the nodes leads to block the demand transmission paths which reduces the network performance. In the present study, a mathematical model for transportation NDP is presented considering congestion occurrence in the network arcs. For this purpose, it is assumed that the number of facilities located to provide services to customers and the demand volume in each node are both stochastic variables. It is also assumed that a set of potential nodes are intended to connect the network nodes and the capacity of each potential node is predetermined. In order to better highlight the research gap that this study aims to fill, Table 1 summarises the recent relevant studies on the NDP considering the congestion effect. It can be found out from Table 1 that there is no study in the literature in which a congested NDP is formulated by queuing theory fundamentals by taking demand uncertainty into account.

The organisation of the paper is as follows: the next section presents the problem statement. Then, Section 3 introduces the proposed mathematical model and the related assumptions. In Section 4, the proposed model is evaluated. For this purpose, a numerical example is solved by GAMS software. The final section, Section 5, is about the conclusion and several recommendations for future studies.

2 Problem definition

Considering the objective of this study, this paper presents a queuing-based mathematical model for modelling the congested transportation networks. The study assumes that there is a predetermined network consisted of nodes and potential arcs and the customers located in some of the network nodes. The times among the occurrences of successive demands in each of the nodes follow the exponential distribution with a specified rate. Each potential arc has a certain and limited capacity and the average travel time in each arc is specified. There is a number of facilities located in some nodes of the network to provide service to customers. Each customer will be assigned to one of the facilities and demand satisfaction is possible only through one path. The cost of arc establishment is determined and the variable cost of transporting demand from each customer to the corresponding facility depends on the route length.

The purpose of the model is selecting the network arcs, allocating customers to facilities and determining the route of each customer to the corresponding facility while total fixed and variable costs are minimised. Because of the demand uncertainty and limitation of arc capacity, congestion phenomenon would be unavoidable and as a result, the blocking possibilities in arcs require to be considered. Thus, in the proposed mathematical model, we specify an upper limit for the probability of blocking of each arc in the network.

3 Mathematical model

A set of indices, variables, and parameters of the mathematical model are as follows:

3.1 *Set of indices*

V set of network nodes (n and m: network point's index)

A set of potential arcs (n, m): interface arc between pairs of nodes n and m)

$I \in V$ set of customers (i: costumer index)

$J \in V$ set of facilities (j: facility index).

3.2 Parameters

d_i demand rate (number of requests per unit of time) for customer i

μ_{nm} rate of traverse of arc (n, m) (unlike average travel time)

C_{nm} fixed cost of the arc construction (n, m)

t_{nm} time traverse of arc (n, m)

v_{nm} arc capacity

α upper limit on the probability of blocking for each network arc.

3.3 Decision variables

X_{ij} if the costumer i allocated to facilitate j^{th}, it is equal to 1 and otherwise is equal to 0

y_{nm} if the arc (n, m) is enabled it is equal to 1 and otherwise is equal to 0

z_{nm}^{ij} if the arc (n, m) is on the route of customer i to facilitate j it is equal to 1 and otherwise is equal to 0

P_{nm} limiting probability of blocking of arc (n, m)

λ_{nm} total arrival rate of demand to arc (n, m)

ρ_{nm} productivity coefficient in the arc (n, m). [The ratio of the arrival rate of request to arc to traverse rate of arc (n, m)]

In this section, before introducing the mathematical model, firstly, limiting probabilities for blocking of each arc in the network is calculated. In this study, queuing systems principles are used in each arc to calculating the limiting probabilities. Each arc will be considered as a multi-service facility with capacity limitation. For this purpose, it is assumed that service to any customer immediately begins upon his/her arrival to the arc and the duration of servicing to each customer is equal to the duration of traverse of the arc. Since each customer has its own path, the service provider is servicing to a number of customers in the arc and the number of service providers in each arc will be limited to the capacity of the arc. Thus, each arc follows a queuing system M/D/C/C (Poisson inputs, times of definite servicing and C service providers in each facility and capacity limits equal to the number of service providers) and so the analytical results of this queuing system can be used to calculate the limiting probabilities of blocking of each arc. Equations (1) and (2) calculate the mentioned limiting probabilities (Gnedenko and Kovalenko, 1989).

$$\rho_{nm} = \frac{\lambda_{nm}}{\mu_{nm}} \quad \forall (n, m) \in A \tag{1}$$

$$\rho_{nm} = \frac{\rho_{nm} v_{nm} \frac{1}{v_{nm}!}}{\sum_{l=0}^{v_{nm}} \rho^l \frac{1}{l!}} \quad \forall (n, m) \in A \tag{2}$$

According to the results of the analysis system M/D/C/C, the mathematical model is presented as follows:

$$Min\ w = \sum_{(n,m)\in A} c_{nm} y_{nm} + \sum_{(n,m)\in A} t_{nm} \lambda_{nm} \tag{3}$$

St:

$$\lambda_{nm} = \sum_i \sum_j d_i z_{nm}^{ij} \quad \forall (n, m) \in A \tag{4}$$

$$\rho_{nm} = \frac{\lambda_{nm}}{\mu_{nm}} \quad (n, m) \in A \tag{5}$$

$$p_{nm} = \frac{\rho_{nm} v_{nm} \frac{1}{v_{nm}!}}{\sum_{l=0}^{v_{nm}} \rho^l \frac{1}{l!}} \quad \forall (n, m) \in A \tag{6}$$

$$\sum_j x_{ij} = 1 \quad \forall i \in I \tag{7}$$

$$\sum_{n|(i,n)\in A} Z_{in}^{ij} = x_{ij} \quad \forall i, j \in Vi \neq j \tag{8}$$

$$\sum_{n|(i,j)\in A} Z_{nj}^{ij} = x_{ij} \quad \forall i, j \in Vi \neq j \tag{9}$$

$$\sum_{n|(n,m)\in A} Z_{nm}^{ij} - \sum_{n|(m,n)\in A} Z_{mn}^{ij} = 0 \quad \forall i, j, m \in Vi \neq j, m \neq i, j \tag{10}$$

$$Z_{nm}^{ij} \leq y_{nm} \quad \forall i, j \in Vi \neq j, (n, m) \in A \tag{11}$$

$$Z_{nm}^{ij} \leq x_{ij} \quad \forall i, j \in Vi \neq j, (n, m) \in A \tag{12}$$

$$p_{nm} \leq \alpha \quad \forall (n, m) \in A \tag{13}$$

$$x_{ij}, Z_{nm}^{ij}, y_{nm} \in \{0, 1\} \quad \forall i, j \in V, A(n, m) \in A \tag{14}$$

$$p_{nm}, \rho_{nm}, \lambda_{nm} \geq 0 \quad \forall (n, m) \in A \tag{15}$$

The objective function [equation (3)] minimises the total cost of arc network establishment and transferring costs of demand from costumers to facilities. Equation (4) calculates total customer arrival rate for each arc. The ratio of the arrival rate of customers to each arc to traverse rate is calculated via equation (5) and the probability of blocking each arc is measured by equation (6). Equation (7) ensures that each customer is assigned to only one facility. Equations (8) to (10) establish demand flow in network nodes. Equation (11) ensures that each arc can only place among the network nodes for demand transfer provided that the arc is previously established. Equation (12) ensures that the route determination variables only can be activated for each customer with the facility assigned to it. Equation (13) limits the probability of the blocking each network arc to a certain upper limit. Equations (14) and (15) show the characteristics of the model variables.

3.4 Assumptions of the model

The following assumptions in the problem modelling have been considered:

- coordinates of the network nodes and thus, the distance between each pair of nodes of the network is pre-determined
- customers are deployed in network nodes and the interval time between consecutive demands in any of the demand nodes for receiving the services follows an exponential distribution with the specified rate
- numbers of facilities are deployed in a number of network nodes
- set of the potential arcs between each pair of nodes of the network is known and the capacity of each arc is limited and its amount is pre-specified
- each customer is assigned to a facility and access to that facility will be possible only through one path
- the probability of blocking of an arc of the network is limited to a specified upper limit
- fixed costs of network arc establishments are predefined and variable costs of demand transferring from customer to the corresponding facility depend on the route length.

4 An illustrative example

In this section, a numerical example is solved in order to evaluate the effectiveness of the proposed model. In this example, an asymmetric network with ten points (customers) is considered. The distances matrix between each pair of nodes of the network and demand values at each node are given in Table 2. It is also assumed that two facilities were deployed to provide service in 1 and 2 nodes. Potential arcs among pairs of network nodes and the capacity of each arc are determined and Table 3 shows the potential capacity of each arc. In this example, it is assumed that the fixed costs of construction and the variable cost of each arc transfer demand are equal and the cost of each arc values are provided in Table 4. The upper limit of the probability of blocking of each arc was considered as 0.1. This model, for the solved instance, has been resolved by optimiser software GAMS, BARON solver.

Table 2 Distance matrix between pairs of network nodes and demand for each node

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	0.00	0.27	0.93	0.40	0.68	0.28	0.55	0.85	0.57	*0.96*
2	0.48	0.00	0.69	0.87	0.50	0.24	0.67	0.83	0.19	*0.69*
3	0.69	0.68	0.00	0.82	0.70	0.19	0.70	0.04	0.22	*0.17*
4	0.86	0.45	0.59	0.00	0.26	0.03	0.20	0.91	0.31	*0.15*
5	0.50	0.90	0.63	0.95	0.00	0.76	0.79	0.13	0.52	*0.18*
6	0.66	0.30	0.68	0.97	0.99	0.00	0.82	0.20	0.88	*0.19*
7	0.49	0.95	0.84	0.58	0.57	0.58	0.00	0.63	0.73	*0.86*
8	0.78	0.80	0.79	0.30	0.75	0.94	0.34	0.00	0.54	*0.51*
9	0.78	0.78	0.62	0.58	0.68	0.74	0.11	0.91	0.00	*0.30*
10	0.23	0.50	0.44	0.56	0.13	0.38	0.62	0.10	0.34	*0.00*
d_i	0.30	0.50	0.73	1.20	0.60	0.18	0.49	0.45	1.50	*0.90*

Table 3 The capacity of the potential network arcs

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1			8	9			4			2
2						5		8		5
3							2	9	2	2
4					2	2	7		3	3
5		13				9	5	3	2	6
6	3			2					8	
7	9					7		1	4	2
8	1	2	4						3	
9				5	2	3	1	4		9
10	7		6		9		2	2		

Table 4 The fixed cost of construction and cost of demand transferring for each potential network arc

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1			0.93	0.40			0.55			*0.96*
2						0.24		0.83		*0.69*
3							0.70	0.04	0.22	*0.17*
4					0.26	0.03	0.20		0.31	*0.15*
5		0.90				0.76	0.79	0.13	0.52	*0.18*
6	0.66			0.97					0.88	
7	0.49					0.58		0.63	0.73	*0.86*
8	0.78	0.80	0.79						0.54	
9				0.58	0.68	0.74	0.11	0.91		*0.30*
10	0.23		0.44		0.13		0.62	0.10		

The results of the numerical examples show that in the optimal solution, costumer of node 8 is assigned to the facility in node 2 and the remained customers are assigned to the facility in node 1. Also arcs (6, 1), (7, 1), (10, 1), (8, 2), (9, 3), (6, 4), (10, 5), (9, 7) and (10, 9) are activated. The optimal routes considering the congestion for the customers in nodes 3 to 10 are 3-9-7-1, 5-10-1, 6-1, 7-1, 8-2, 9-10-1 and 10-1, respectively. The objective function value per the optimal solution is equal to 9.302.

5 Concluding remarks

NDP is considered as one of the most important issues in the planning of the transportation problems. Such problem is addressed about the establishment and development of optimal arc among the network nodes. This paper presents a mathematical model to NDP in transportation with assuming the occurrence of congestion in network arcs. For this purpose, the amount of demand at each node was considered as a random variable with exponential distribution and the specified rate. It also assumes a number of facilities to serve are deployed in some spots of the network and each customer is assigned to one of these facilities to getting service. A set of potential arcs with limited capacity are known in advance.

The restriction in the network capacity and the uncertain nature of the demands at each node cause occurring congestion in the network arcs. In this study, the queuing system M/D/C/C was used to calculate the limiting probabilities for blocking of each arc. The model aimed to restrict the probabilities of blocking the network arc as well as to minimise the total fixed and variable costs. The decision variables in the model as well as determining the optimal arcs will be determined the allocation method of each customer to one of the facilities and the demand transferring route of each customer to the corresponding facility.

In the proposed model, facility location as a parameter of the problem was considered as specified in advance if the decision variables associated with determining the location of the facility also be added to this model. In addition to determining the network arcs, optimal location of facilities should also be specified. Adding of facility locating problem to the NDP will cause to dealing with a more practical model. Therefore, the development of the proposed model with regard to the problem of locating of the facility into NDP was considered as suggestions for future research. Considering the assumption of congestion in the network recovery problem also is considered as appropriate fields of research for future studies. It also offers innovative or meta-heuristic methods to solve the problem are among the suitable research areas for studies. Extending the current presented model to a multi-objective optimisation problem would be another idea for further studies. At last, fuzzifying all the decision variables can be another suggestion for developing the mathematical model presented in this study.

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