A cost allocation model based on the combination of data envelopment analysis and Shannon entropy (case study: branches of the central post of Isfahan)

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Abstract: The method used for allocation of costs in this study has several stages. In stage 1, various types of data envelopment analysis (DEA) models were used to assess the efficiency of decision-making units (DMUs), including constant return to scale and variable return to scale with a variety of input and output types in radial and non-radial states. In stage 2, Shannon entropy method was used to combine the obtained efficiencies for each decision making unit. In stage 3, the allocation of costs was done based on the combined efficiency number for each unit obtained in phase 2. The proposed cost allocation model was then implemented in an example and its fairness was compared with similar methods using the Gini coefficient method. Finally, the proposed model was investigated in Central Post of Isfahan.

Keywords: cost allocation; data envelopment analysis; DEA; Shannon entropy; Gini coefficient method.

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1 Introduction

Data envelopment analysis (DEA) technique was first proposed by Charnes et al. in 1978. Since then, it has become a popular empirical method for measuring relative efficiency of a set of homogeneous and comparable decision-making units (DMUs) (see Jahanshahloo et al., 2011, 2012, 2013; Maddahi et al., 2014). In recent years, an important application of DEA is to solve the problem of resource allocation among a set of DMUs in an equitable way. The first study on fair allocation of costs was done by Cook and Kress (1999), and was based on two principles:

1 the principle of invariance

2 the minimum input Pareto principle.

Based on this idea, they decided to consider a common share of the total input in the allocation of fixed costs to each DMU, and they started with a single input – single output example; however, they could not use this method directly for DMUs. Subsequently, Jahanshahloo et al. (2004) showed with a numerical example that Cook and Kress method did not work for the minimum input Pareto principle and provided a similar formula for achieving cost allocation without solving complex linear programming problems. Cook and Zhu (C2005) developed Cook and Kress's model in 2005, and presented a method for this allocation in their paper titled 'Cost allocation among DEA DMUs'. Lin (2011) proved in 2011 that with a new limitation added to Cook and Zhu's model, this model will no longer be able to provide a feasible solution. In recent years, many studies have been conducted on cost allocation using DEA (Ding et al., 2017; Yang and Zhang, 2015; Lotfi et al., 2013; Fang, 2013; Li and Cui, 2008). The aim of this study is to provide a model for cost allocation based on a combination of DEA and Shannon entropy. The rest of this article is organised as follows. In Section 2, research method and the proposed model for fair allocation of costs are discussed. In Section 3, the proposed method is compared with other methods through an illustrative example. Finally, in Section 4, the proposed method is applied to the central post of Isfahan and the results are discussed.

2 Method

The issue that is discussed in this paper as fixed cost allocation is the costs used to build a common platform for the subunits of an organisation. It has been recognised that different subunits of the same organisation are homogeneous and comparable, because they consume the same kind of inputs and generate the same kind of outputs via a similar production process. For example, television advertising to attract customers that a central bank can do and all its branches will benefit from its advantages. The question that arises in this context is 'how the cost should be allocated between the units (bank branches), which is the best and fairest method?' In our opinion, the purpose for which the cost is spent directly affects the efficiency of the DMUs of the organisation. Therefore, the cost assigned to each DMU should be proportional to its efficiency. Thus, the method presented in this paper is based on the efficiency score obtained from DEA models.

In using DEA models, two important factors should be considered regarding efficiency:

1 the type of return to scale (RTS) of the production set (see Banker and Thrall, 1992; Banker et al., 1996; Tone, 1996; Golany and Yu, 1997)

2 the nature of the model used, i.e. whether it is input-oriented or output-oriented.

The appropriate model is determined based on the focus, access, and control capability of the DMU on inputs and outputs (see Paradi et al., 2018). To solve these two basic problems, several articles have been presented, the main idea of which is to aggregate the efficiency number from different DEA models using various methods, including multi-criteria decision-making methods (see Soleimani-Damaneh and Zarepisheh, 2009; Xie et al., 2014). In order to solve these two problems in this study, the method used by Soleimani-Damaneh and Zarepisheh (2009) in combining various efficiencies in DEA was used in the first step of the proposed model. For this purpose, assume there are n DMUs DMU_j, (j = 1, 2, …, n), each using m inputs as vector $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ to generate s outputs as vector $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$, and the goal is to allocate a fair amount of R units of cost among these DMUs. The proposed model has the following steps:

Step 1 Efficiency calculation

In this section, famous DEA models that are used to calculate the efficiency of DMUs are first introduced. These models include:

1 CCR

This model was introduced by Charnes et al. in 1978 and was named using the initial letters of their names. The mathematical form of this model is as follows:

$$
\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} > y_{ro} \quad r = 1, \ldots, s \\
& \lambda_j \ge 0 \qquad\qquad j = 1, \ldots, n
\end{aligned}
$$

2 BCC

This model was presented by Banker et al. (1984) and, like the CCR model, the initial letters of the authors' names were used as its name. This model can be applied in both input-oriented and output-oriented forms defined mathematically as follows.

Input and output oriented BCC models:

$$
\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad j = 1, \ldots, n \\
& \lambda_j \ge 0, \qquad j = 1, 2, \ldots, n
\end{aligned}
$$

$$
\begin{aligned}
\max \quad & \varphi \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi y_{ro} \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad j = 1, \ldots, n \\
& \lambda_j \ge 0
\end{aligned}
$$

The CCR model can also be input- or output-oriented for the calculation of efficiency; however, it is proved that the efficiency number in the input-oriented model is the inverse of the efficiency number generated in the output-oriented model, and in fact, there is no difference in these two model types in calculating efficiency. Therefore, only the input-oriented (or output-oriented) model is used to calculate the efficiency. While in the BCC model, these two values (the input-/output-oriented efficiency number) are not inversely related, so both cases are used to calculate efficiency. In other words, the input-oriented efficiency number is different from the output-oriented efficiency number in BCC.

3 Non-radial models

Non-radial models can be designed as input- or output-oriented, or with no specific orientation. In the latter case, both inputs and outputs can be considered simultaneously in the calculation of efficiency. The main difference between these models and the radial models is how a unit is checked for efficiency. In radial models, this is done by checking if the system will still be able to produce if inputs (or outputs) of the DMU are reduced or increased at a similar ratio. In the answer is positive, the unit is considered efficient; otherwise, it is regarded as inefficient. In non-radial models, inputs (or outputs) are not equally reduced (or increased), and each input and output can be evaluated at a different ratio. However, similar to radial models, return-to-scale in non-radial models can be considered either fixed or variable. Some non-radial models used in this study are as follows:

The MJ1 model is an input-oriented non-radial model with variable return-to-scale.

$$\begin{aligned} \min \quad & \frac{1}{m}\sum_{i=1}^{m}\theta_i \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} = \theta_i x_{io} \quad i = 1, \dots, m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} \quad r = 1, \dots, s \\ & \sum_{j=1}^{n}\lambda_j = 1 \qquad j = 1, \dots, n \\ & \lambda_j \ge 0 \end{aligned}$$

The MJ2 model is an output-oriented model with variable return-to-scale.

$$\begin{aligned} \max \quad & \frac{1}{s}\sum_{r=1}^{s}\varphi_r \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io} \quad i = 1, \dots, m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} = \varphi_r y_{ro} \quad r = 1, \dots, s \\ & \sum_{j=1}^{n}\lambda_j = 1 \qquad j = 1, \dots, n \\ & \lambda_j \ge 0 \end{aligned}$$

The MJ3 model is an input-oriented model with fixed return-to-scale.

$$\begin{aligned} \min \quad & \frac{1}{m}\sum_{i=1}^{m}\theta_i \\ \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} = \theta_i x_{io} \quad i = 1, \dots, m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} \quad r = 1, \dots, s \\ & \lambda_j \ge 0 \qquad j = 1, \dots, n \end{aligned}$$

The MJ4 model is an output-oriented model with fixed return-to-scale.

$$
\begin{aligned}
\max \quad & \frac{1}{s}\sum_{r=1}^{s} \varphi_r \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \quad i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = \varphi_r y_{ro} \qquad r = 1,\ldots,s \\
& \lambda_j \ge 0 \qquad\qquad\quad j = 1,\ldots,n
\end{aligned}
$$

In the first step of the proposed method, the efficiency of n independent and comparable DMUs is evaluated using the above models, i.e., CCR model, input- and output-oriented BCC models, non-radial models MJ1, MJ2, MJ3, and MJ4 (henceforth called the seven models).

Step 2 Combining efficiencies with shannon entropy method

After obtaining the efficiency of each DMU, the efficiency numbers are combined with the Shannon Entropy method through the following steps. For each DMU, there is an efficiency number, hereinafter called 'combined efficiency' [the main idea of this method has been adapted from Soleimani-Damaneh and Zarepisheh's (2009) paper]. The steps of this combination using Shannon Entropy method are as follows:

Stage 1 Creating Shannon entropy matrix.

First, the Shannon entropy matrix (this matrix is actually the decision matrix in the multi-criteria decision-making method), in which the rows are DMUs, and the columns show the efficiencies derived from the seven-dimensional model, is created. If the efficiency of the j^{th} model for DMU_i is indicated by e_{ij}, the above matrix will be like A] [e_{ij}]. If, in general, there are L models to calculate the efficiency and n DMUs, matrix A becomes a $n \times L$ matrix.

It should be noted that the efficiency of inefficient units is greater than one in the output-oriented models, and smaller than one in the input-oriented models. Since the proposed method combines these efficiencies, the concept of efficiency should be considered similarly in the input- and output-oriented models. Therefore, for all output-oriented models in the seven models, i.e., the output-oriented BCC models, MJ2, and MJ4, the inverse of the optimal value of the objective function is considered as the efficiency number of these models in the Shannon entropy matrix.

Stage 2 Normalising the matrix.

The above matrix is normalised as follows: each element of each column is divided by the sum of the elements of that column and each normalised element is represented by P_{ij}. In other words, $P_{ij} = \dfrac{e_{ij}}{\sum_{i=1}^{n} e_{ij}} \quad \forall i, j.$

Stage 3 Calculating the entropy.

At this stage, the entropy of E_j is calculated using the values in P_{ij} via the following formula. k is a constant value and helps keeping the value of E_j between 0 and 1.

$$E_j = -k \sum_i P_{ij} Ln P_{ij} \quad j = 1, 2, \ldots, n, k = \frac{1}{Ln\ L} \tag{1}$$

In equation (1), L is the number of elements in each column of the Shannon entropy matrix, and in fact the number of models, which is 7 in this study.

Stage 4 Calculating d_j (deviation).

d_j values are calculated using the following formula, which states how much useful information is given to the DMU by each indicator. Therefore, the role of the indicator in decision making should be reduced by the same amount.

$$d_j = 1 - E_j, \quad j = 1, 2, \ldots, n$$

Stage 5 Calculating the weights w_j.

In this stage, the weights are calculated using the following equation:

$$w_j = \frac{d_j}{\sum_{j=1}^{L} d_j}, \ j = 1, 2, \ldots, L.$$

Stage 6 Obtaining combined efficiency (τ_i)

At this stage, the 'combined efficiency' for each DMU_i is obtained using the weights calculated for each model in the previous stage:

$$\tau_i = \sum_{j=1}^{L} w_j e_{ij}, \quad i = 1, 2, \ldots, n$$

Step 3 The allocation

In this step, using the combined efficiency calculated in the previous step, the cost is fairly allocated to the DMUs. For this purpose, the combined efficiency of each unit is considered as the basis of allocation. Assuming that the purpose is to allocate a cost of value R to n DMUs with combined efficiency, then the following can be stated: if we base fairness on the idea that the more a DMU is efficient, the more cost must be allocated to it, then the allocation must be a coefficient of the efficiency of each unit; thus, we define the cost allocated to $DMUi$, $i = 1, 2, \ldots, n$ equal to $R_i = \frac{\tau_i}{\sum_{i=1}^{n} \tau_i} R.$ If cost allocation is done as above, the total allocations will be equal to the total cost (R). The proposed allocation in this section is encouraging and can be used in cases such as allocation of funds to organisations. This method can also be used if the goal is to allocate advertising costs to the homogeneous subdivisions of an organisation, as an organisation that has become more productive has benefited more from advertising and therefore needs to go under more costs. In any case, if the senior management of the organisation, for whatever reason, defines fairness as ‘a more efficient unit must endure less costs’, then the allocation can be done as follows: $R_i = \dfrac{\frac{1}{\tau_i}}{\sum_{i=1}^{n} \frac{1}{\tau_i}}, \; i = 1, 2, \ldots, n.$

What is used in the following sections is the same as the previous one.

3 Example

In this section, the proposed model is illustrated using an example from Yang and Zhang’s (2015) paper, and then it is compared with similar methods. Consider Table 1. The rows in this table represent 12 DMUs that use three inputs to generate two outputs. Inputs are displayed in the first three columns and outputs are displayed in the last two columns.

Table 1 Data

DMUs	*Input 1*	*Input 2*	*Input 3*	*Output 1*	*Output 2*
DMU1	350	39	9	67	751
DMU 2	298	26	8	73	611
DMU 3	422	31	7	75	584
DMU 4	281	16	9	70	665
DMU 5	301	16	6	75	445
DMU 6	360	29	17	83	1,070
DMU 7	540	18	10	72	457
DMU 8	276	33	5	78	590
DMU 9	323	25	5	75	1,074
DMU 10	444	64	6	74	1,072
DMU11	232	25	5	25	350
DMU12	444	64	6	104	1,199

Also, assume the goal is to allocate a fixed cost of 100 units to DMUs. In Table 2, the last column represents the amount of cost assigned to each DMU using the proposed method. The first four columns show the cost allocated to each unit by similar methods.

To measure the fairness of these methods, Gini coefficient (Gini, 1921) is used. The results of evaluating the methods presented in Table 2 by means of Gini coefficient are shown in Table 3. In Gini method, the more the calculated number is close to zero, the fairer the allocation. As can be seen, in the proposed method, the number obtained by the Gini coefficient method is zero, which means that the allocation is quite fair.

In the following section, the proposed method is used for the central post of Isfahan.

Table 2 Example cost allocation

	Lin's method	*Bizley's method*	*Lee's method (2008)*	*Lee's method (2012)*	*The proposed method*
DMU1	6/86	6/78	7/33	6/38	7/95
DMU2	6/18	7/21	5/6	7/42	8/78
DMU3	7/28	6/83	8/35	6/68	7/53
DMU4	6/09	8/47	8/45	8/83	9/20
DMU5	4/03	7/08	6/9	7/63	9/20
DMU6	12/51	10/06	11/55	9/69	8/7
DMU7	7/2	5/09	7/45	4/27	8/36
DMU8	5/61	7/74	5/83	8/35	9/2
DMU9	5/8	15/11	7/43	15/87	9/2
DMU10	13/62	10/08	11/63	9/75	8/36
DMU11	5/8	1/58	5/69	0/45	4/1
DMU12	18/6	13/97	13/71	14/64	9/20

Table 3 G values

	Lin's method	*Bizley's method*	*Lee's method (2012)*	*Lee's method (2008)*	*The proposed method*
G	0.32	0.35	0.38	0.26	0.0

4 Implementation of the model at the central post of Isfahan

4.1 DMUs and input and output indicators

To implement the proposed model in the central post of Isfahan, the following seven areas were considered as DMUs: Masjed Seyyed, Neshat, Khorasgan, Feyz, Ghadir, Malekhahr, and Laleh. In order to evaluate the efficiency of the above areas using DEA, and to use them in the proposed model, appropriate indicators should be identified and used as inputs and outputs. Many studies were reviewed for this purpose (Borenstein et al., 2004; Doble, 1995; Filippini and Zola, 2005; Iturralde and Quirós, 2008; Horncastle et al., 2006; Register, 1988). Finally, using the Delphi method and collecting the opinion of experts in this industry, the following six indicators were selected: Operating income, sent items, timely delivery, operating costs, total productivity, and traffic. These indicators were then divided into input and output categories for inclusion in the DEA models. According to the definition of each indicator, presented by the Performance Evaluation Unit, inputs are those indicators that if increased, while other factors remain constant, the efficiency of the DMU decreases, and outputs are those indicators that if increased, while other factors remain constant, the efficiency of the DMU also increases. Therefore, the first indicator is the operational cost that refers to the total of current, personnel, and contract costs, and is considered as input according to its nature. It should be noted that due to the unavailability of current and contract costs and that personnel costs cover about 95% of the costs, operational costs were limited to personnel costs in this study. The second indicator is timely delivery, which is called the waiting time from the acceptance to the distribution of the item, and according to its definition it was considered as an input. The third indicator is the operating income derived from the total revenue earned by the post. The fourth indicator is traffic and indicates a unit of acceptance of a consignment in the source. The fifth indicator, sent items, is the number of items that are sent in time to the target exchange centre, and the final indicator is the total productivity. The factors in the latter include capital, energy, and human resources; however, due to the difficulty of obtaining productivity and the fact that it is affected by so many factors, we calculated productivity by dividing the income of each area by the number of employees in that area. These four indicators were considered as outputs according to their nature. The selected indicators (inputs and outputs) used for assessing the seven areas of central post of Isfahan are briefly summarised in Table 4.

Table 4 Inputs and outputs

Input variables		*Output variables*	
1	Operational costs	1	Operational income
2	Timely delivery	2	Traffic
		3	Total productivity
		4	Sent items

4.2 Data for each indicator

The data used here is real data. The seven DMUs that are the branches of the post of Isfahan can be evaluated with two inputs and four outputs. The information in this regard was extracted from valid documents available in the BSC system of Post of Isfahan, under the supervision of the manager of Performance Evaluation Unit. The data are presented in Table 5.

Table 5 The data

Postal area	*Operational costs*	*Timely delivery*	*Operational income*	*Traffic*	*Sent items*	*Productivity*
	i1	*i2*	*o1*	*o2*	*o3*	*o4*
DMU1	476,611	94.55	1,784,123	31,351,886	94	0.14
DMU2	1,715,800	90.25	5,188,396	69,427,201	93	0.32
DMU3	142,983	95.28	253,484	8,712,959	89	0.04
DMU4	667,256	94.29	1,573,414	54,683,004	95	0.25
DMU5	428,950	95.26	913,222	21,743,611	93	0.10
DMU6	381,289	95.12	348,737	17,920,082	89	0.08
DMU7	190,644	95.79	335,670	14,410,406	91	0.07

In Table 5, each row represents a DMU, and the columns represent the inputs and outputs of each unit. For example, DMU1 represents Masjed Seyyed postal area, and the corresponding row shows the values of indicators in that area, such as the area's revenue, which is equal to 31351886. DMU2, DMU3, DMU4, DMU5, DMU6, and DMU7 represent Neshat, Khorasgan, Feyz, Ghadir, Malek Shahr, Laleh areas, respectively. In addition, in this table, the first input is the operational cost (i1), the second input is timely delivery (i2), the first output is operational income (o1), the second output is traffic (o2), the third output is sent items (o3), and the fourth output is productivity (o4).

4.3 Allocation of costs to postal areas using the proposed model

In this section, the proposed model is used to evaluate and allocate costs to the previously described postal areas. The top management intends to allocate 600,000,000 Tomans to the seven areas as emolument. The goal is to allocate this amount by scientific methods and based on the efficiency of each area, so that the area with the highest efficiency receives the highest possible emolument. The steps of implementing the proposed model in central post of Isfahan are as follows:

Stage 1 Calculating the efficiency of the areas using DEA models

As discussed in the previous section, the efficiency of the postal areas is obtained via the models introduced in Section 2. For convenience, these models are named as follows:

- CCR model: the efficiency obtained by this model is shown with ZCCR.
- Input-oriented BCC model (BCC.I): The efficiency obtained by this model is shown with ZBCCI.
- Output-oriented BCC model (BCC.O): The efficiency obtained by this model is shown with ZBCCO.

Table 6 Efficiency values

	ZCCR	*ZBCC.I*	$\frac{1}{z_{BCC.O}}$	*M(J1)*	*M(J2)*	*M(J3)*	$\frac{1}{M(j4)}$
dmu1	1	1	1	1	1	1	0.98
dmu2	1	1	1	1	1	1	1
dmu3	1	1	1	1	1	1	0.862
dmu4	1	1	1	1	1	1	1
dmu5	0.988	0.994	0.992	0.923	0.757	0.868	0.756
dmu6	0.954	0.993	0.956	0.81	0.524	0.796	0.52
dmu7	1	1	1	1	0.996	1	0.936

The efficiency of the rest of the models is represented by the same naming symbols as M (J1), M (J2), M (J3) and M (J4). The results of evaluating the area using the above models are shown in Table 6. In this table, each row represents a DMU and each column represents the efficiency obtained from one of the models. As explained earlier, in the column relating to the efficiency of the output-oriented models, the inverse of the efficiency number is used. For example, for DMU5, which represents Ghadir postal area, and the efficiency number derived from the output-oriented BCC model, represented by $\frac{1}{z_{BCC.O}}$ is equal to 0.992.

As can be seen in Table 6, the areas whose efficiency is equal to 1 in all models are Feyz and Neshat.

Stage 2 Calculation of combined efficiency of postal areas using Shannon entropy

After the efficiency of each area is calculated, the efficiencies are combined using Shannon entropy, which is carried out through the following steps.

Step 1 Creating Shannon entropy matrix.

First, the Shannon entropy matrix (the decision matrix) in which the rows represent the postal as DMUs, and in the columns show the efficiency number of each area based on the solved models. Table 6 shows the entropy matrix.

Step 2 Normalising the matrix.

The above matrix is normalised as follows: each element of each column is divided by the sum of the elements of that column. Each normalised element is represented by P_{ij} (Table 7).

Table 7 Normalised decision matrix

	Normalisation						
	ZCCR	*ZBCC.I*	$\frac{1}{z_{BCC.O}}$	*M(J1)*	*M(J2)*	*M(J3)*	$\frac{1}{M(j4)}$
dmu1	0.144051	0.143123	0.14392631	0.1485222	0.15931177	0.15006	0.16187645
dmu2	0.144051	0.143123	0.14392631	0.1485222	0.15931177	0.15006	0.16518005
dmu3	0.144051	0.143123	0.14392631	0.1485222	0.15931177	0.15006	0.1423852
dmu4	0.144051	0.143123	0.14392631	0.1485222	0.15931177	0.15006	0.16518005
dmu5	0.142322	0.142264	0.1427749	0.13708599	0.12059901	0.130252	0.12487611
dmu6	0.137424	0.142121	0.13759355	0.12030299	0.08347937	0.119448	0.08589362
dmu7	0.144051	0.143123	0.14392631	0.1485222	0.15867453	0.15006	0.15460852

Step 3 Calculating E_j, d_j, and w_j.

E_j, d_j, and w_j were calculated according to Section 2, step 2 (the main part of the proposed model). The results are shown in Table 8.

Table 8 Entropy values

	ZCCR	*ZBCC.I*	$\frac{1}{z_{BCC.O}}$	*M(J1)*	*M(J2)*	*M(J3)*	$\frac{1}{M(j4)}$
E_j	0.995	0.997	0.987	0.997	0.942	0.874	0.992
d_j	0.005	0.003	0.013	0.003	0.058	0.126	0.008
w_j	0.023	0.013	0.06	0.013	0.268	0.583	0.037

d_j indicates the extent to which each indicator gives useful information to the decision maker for decision making. Accordingly, the role of that indicator in decision making should be reduced to the same extent.

The final row of Table 8 shows wj values calculated using $W_j = \frac{d_j}{\sum d_j}$.

The next step is to calculate the combined efficiency (τ_i) of each DMU. The results are shown in Table 9. It is observed that DMUs 2 and 4 are the most efficient units, while DMU 6 is the most inefficient.

Table 9 Combined efficiency values

	DMU1	*DMU2*	*DMU3*	*DMU4*	*DMU5*	*DMU6*	*DMU7*
τ_i	0.996	1	0.991	1	0.844	0.726	0.993

Now using $R_i = \frac{\tau_i}{\sum_{i=1}^{n} \tau_i} R$, 600 million Tomans is fairly allocated to the seven units based on the combined efficiency. The results are shown in Table 10.

Table 10 Values assigned to the seven postal areas

	DMU1	*DMU2*	*DMU3*	*DMU4*	*DMU5*	*DMU6*	*DMU7*
$\frac{\tau_i}{\sum_{i=1}^{n} \tau_i}$	0.15206	0.15267	0.15129	0.15267	0.12885	0.11083	0.15160
R_i	91236000	91602000	90774000	91602000	77310000	66498000	90960000

5 Conclusions

A good allocation plan should include justice, impartiality and rationality. For example, the amount of cost received by each DMU should be consistent with its share in the system. In this study, a model based on DEA was presented for allocating costs. In this model, in order to increase the accuracy of the calculation of the efficiency of DMUs, seven different DEA models with different capabilities were used to calculate the efficiency. Then, the above-mentioned efficiencies were combined using Shannon entropy. The combined efficiency of each DMU was obtained and then cost allocation was done based on this number. The proposed method ensures that fixed costs are allocated fairly to all units. The fairness of the proposed model was investigated in the first example of the study using the Gini coefficient method. It is suggested that future research in this area uses dynamic DEA models, since these models examine the performance of DMUs at different time periods and hence their evaluations are more accurate, leading to a fairer allocation. In addition, to combine the efficiency of different models, Shannon entropy can be replaced with other multi-criterion decision making methods, such as AHP hierarchical process analysis. However, this method requires full understanding of the possibility of production in the system being evaluated. In this way, the performance of each of these models in assessing DMU efficiency can be determined.

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