
Multi-objective hazardous materials routing and scheduling for balancing safety and travel time

Kamran S. Moghaddam

Department of Marketing and Supply Chain Management,
College of Business,
Clayton State University,
Morrow, GA 30260, USA
Email: KamranMoghaddam@clayton.edu

Abstract: This research develops a new multi-objective multi-period optimisation model to find optimal links and routes to maintain a balance between safe and fast distribution of hazmats between origins and destinations through the transport network. The transport network includes multiple origins and destinations along with multiple hazmat classes to better mimic the challenges faced by practitioners. We consider unknown probabilities for hazmat incidents along with a game-theoretic demon approach and formulate a link-based routing and scheduling hazmat shipment problem. The objective functions are defined to minimise both the probability of population exposure affected by hazmat transport risks and the total transportation time in the distribution network. This paper also proposes a solution method based on an integrated Monte Carlo simulation and fuzzy goal programming to obtain Pareto-optimal solutions. A numerical example is provided to evaluate the effectiveness of the developed mathematical model and the solution method in obtaining Pareto-optimal solutions.

Keywords: transportation; vehicle routing; game theory; hazardous materials; multi-objective optimisation; fuzzy goal programming.

Reference to this paper should be made as follows: Moghaddam, K.S. (2021) 'Multi-objective hazardous materials routing and scheduling for balancing safety and travel time', *Int. J. Applied Management Science*, Vol. 13, No. 1, pp.69–94.

Biographical notes: Kamran S. Moghaddam is an Associate Professor of Supply Chain Management at the Clayton State University. Over the past 18 years, he has been active in the field of industrial engineering and operations management in academic and industry environments. He has conducted variety of projects such as multi-reservoir systems operations management, transportation and logistics systems design and improvement, optimal maintenance scheduling in manufacturing systems, and reverse logistics systems design and evaluation. He is a Licensed Professional Engineer (PE) and a Certified Quality Engineer (CQE) by the American Society for Quality.

1 Introduction

With the rapid development of global economy, the demand for hazardous materials is continuously increasing in recent years. Hazardous materials are defined and regulated in the USA primarily by laws and regulations administered by the US Environmental Protection Agency (EPA), the US Occupational Safety and Health Administration (OSHA), the US Department of Transportation (DOT), and the US Nuclear Regulatory Commission (NRC). Each has its own definition of a 'hazardous material' (IHHM, 2018). For example, OSHA's definition includes any substance or chemical, which is a 'health hazard' or 'physical hazard'. EPA incorporates the OSHA definition, and adds any item or chemical, which can cause harm to people, plants, or animals when released by spilling, leaking, pumping, pouring, emitting, emptying, discharging, injecting, escaping, leaching, dumping or disposing into the environment (Reese, 2017). Depending on their physical, chemical, and nuclear properties hazardous materials have been classified into nine different categories as explosives, gases, flammable and combustible liquids, flammable solids, oxidisers and organic peroxides, toxic materials and infectious substances, radioactive materials, corrosive materials, and miscellaneous dangerous products (Keller, 2017).

In most developed countries, significant amount of hazardous materials shipments is transported by commercial carriers through highway and interstate networks of which 90% are long distance travels. Commercial carriers in the US transport over 3 billion tons of hazardous materials each year. Such hazmat shipments can be highly risky and highly sensitive and if improperly handled, labelled, or packaged could result in the loss of life, property damage, and harm to national security interests (US Government Accountability Office, 2014). The most common hazardous materials for transportation include cooking gas, fuel oil, and chemicals such as ethyl alcohol, and gasoline (Verter and Kara, 2008). Because of highly flammable, corrosive, explosive, poisonous or radioactive nature of dangerous goods, the carriers of hazmats, when involved in a road accident, may lead to disastrous consequences such as fire, explosion, spillage, and leakage, resulting in a large number of fatalities and injuries besides property loss and environmental pollution (Yang et al., 2010).

Compared to the number of shipments and the amount of hazmat transported through the transport networks, the number of hazmat incidents, fatalities, injuries and economic loss compared to the flow of hazmats are diminutive. A study of available data on hazmat transportation accident statistics in North America reveals that accident probabilities are extremely small, usually estimated at 10^{-6} per mile (Harwood et al., 1993). As such, hazardous material transportation planning has been categorised as low-probability-high-consequence (LPHC) problem. Although the number of hazmat transportation incidents is a very small fraction compared to the total number of shipments, due to nature of hazmats there will be tremendous societal costs as well as environmental impacts in a case of an incident. This makes hazardous materials transportation, routing, and scheduling operations are very important and sensitive logistical decisions.

In general, a dispatcher can adopt routing or scheduling strategies to avoid or reduce the disastrous consequences of hazmat incidents in transportation networks. Routing strategy tries to split the flow of hazmat shipments on several routes between suppliers and consumers points to minimise severe consequences in a single route in the event of an incident. On the other hand, scheduling strategy seeks to distribute the flow of hazmat

shipments over multiple periods to avoid simultaneous accumulation of incidents in a single route. In this research, routing and scheduling strategies are concurrently considered into our modelling approach.

Hazmat incidents are low probability events. On the other hand, there is always development and expansion of existing roads, which makes the historical data unreliable due to changes in condition of the roads. Besides, usually some new roads are constructed over time and added to the transportation network where there is no sufficient historical data on them. As such, we encounter a transportation network in which incidents probabilities are unknown or uncertain. Therefore, unlike the majority of the traditional literature this research focuses on unknown incidents probabilities. In order to cope with the routing-scheduling problem with unknown incident probabilities, Szeto et al. (2017) assume that the dispatcher is risk averse. Therefore, instead of focusing on probability of accidents we consider the worst-case scenario, which converts a probabilistic problem to a deterministic version. This worst-case scenario may be criticised by practitioners because:

- 1 it searches the safest routes and does not consider travel distance; therefore, sometimes a dispatcher may travel a long distance from origin to destination that can be costly
- 2 not all dispatchers are fully risk-averse; how a risk-seeking or risk-neutral company can use such a model.

In order to address these issues, this research aims at focusing on:

- 1 unknown probabilities
- 2 while providing a decision-making platform for a wider range of companies in terms of their risk attitude.

In other words, in addition to risk-averse companies, we also consider risk seeking and risk neutral companies. We define two objectives for the model:

- 1 minimisation of total number of affected people
- 2 minimisation of total transport time.

If we have a model with just a single-objective (i.e., the risk objective function), then we are dealing with a risk-averse decision maker. On the other hand, when we solve the model with the other objective function (distance) then the decision maker is risk seeking. Any linear combination of these two objective functions will give us a Pareto solution, which is neither risk averse, nor risk seeking.

The original single-objective dispatcher problem assumes that the dispatcher is risk-averse. In other word, he seeks strategies based on worst-case scenario. In multi-objective context, it is equivalent that he is looking for solutions far away from the worst possible solution (nadir solution). We chose fuzzy goal programming to mimic the risk-averse behaviour of the dispatcher in finding non-dominated solutions. Other goal programming methods try to minimise the gap between candidate solutions and the best possible solution (ideal solution) whereas fuzzy goal programming tries to maximise the gap between candidate solutions and the worst possible solution.

In this research, we use a link-based modelling approach, which has been proved to be more effective than traditional route-based algorithms in reducing computational

efforts (Szeto et al., 2017). Using link-based approach and game theoretic concepts, the problem is formulated a mixed-integer linear programming model. If the model was single-objective (either with loss or transport time objectives), it could be solved using exact solution techniques such as branch-and-bound on commercial software products for large-scale problems. Since we are dealing with a multi-objective model, the Monte Carlo simulation is used to solve the model for different randomly generated set of weights. In other words, Monte Carlo simulation helps us generate random weights for membership functions to find non-dominated solutions (that should be uniformly extended over the objective function space) of the multi-objective model. If we do not use Monte Carlo model we have to assign user-specified weights to the membership functions and we will not be able to capture the entire Pareto-optimal spectrum.

The remainder of this paper is organised as follows: Literature review is presented in Section 2. The detailed problem description is provided in Section 3 and it is mathematically formulated in Section 4. In Section 5, the solution approach is presented and discussed. Computational results including presentation of a sample case study, practicality of the optimal solutions, and validation of the model and solution algorithm are presented in Section 6. Section 7 concludes the paper by highlighting the contributions and provides direction for future research.

2 Literature review

Transportation, logistics, and distribution of hazardous materials have been studied extensively in operation research in last 20 years. In this section, we examine the relevant literature to our study and categorise the body of knowledge into single-objective and multi-objective models to better understand the current trends and be able to position the contributions of our research compared to existing literature.

2.1 Single-objective models

An early study of Batta and Chiu (1988) addressed the problem of finding an optimal path that minimises the weighted sum of lengths over which hazardous material shipment is within a threshold distance of population centres. Gopalan et al. (1990) developed and analysed an integer programming model to generate an equitable set of routes for hazardous material shipments. The objective was to determine a set of routes to minimise the total risk of travel and spread the risk equitably among the zones of the geographical region covered by the transportation network. Erkut and Verter (1998) performed an empirical analysis on the US road network and showed that different risk models usually select different ‘optimal’ paths for a hazmat shipment between a given origin-destination pair. Their study examined that the optimal path for one model could perform very poorly under another model. Other significant early studies in the area of routing and scheduling of hazmat shipments can be found in List et al. (1991), Klein (1991), Jin et al. (1996) and Nozick et al. (1997).

One of the groundbreaking studies in the area of hazmat logistics was found in Bell (2000) in which a two-player non-cooperative game is envisaged between on the one hand the network user seeking a path to minimise the expected trip cost and on the other hand an ‘evil entity’ choosing link performance scenarios to maximise the expected trip cost. The applications of the classis two-player non-cooperative game are also found in

Bell and Cassir (2002), Bell (2003, 2004, 2006, 2007), Szeto (2013) and Szeto et al. (2017). However, all of these studies used single-objective optimisation models to formulate the two-player non-cooperative games.

Akgün et al. (2007) examined the effects of weather systems on hazmat routing by analysing the effects of a weather system on a vehicle traversing to characterise the time-dependent attributes of a link due to movement of the weather systems. In addition, it was demonstrated that the heuristic algorithms can provide near-optimal solutions for large-scale practical problems. Carotenuto et al. (2007) dealt with the generation of minimal risk paths for the road transportation of hazardous materials between an origin-destination pair of a given regional area by formulating the problem mathematically and proposing two heuristic algorithms. Erkut and Alp (2007) considered an integrated routing and scheduling problem in hazardous materials transportation where accident rates, population exposure, and link durations on the network vary with time of day and tried to minimise risk in form of accident probability multiplied by exposure subject to a constraint on the total duration of the trip. The authors also proposed pseudo-polynomial dynamic programming algorithms for different versions of their model and examined the computational effectiveness of the algorithms on a realistic example network.

Verter and Kara (2008) provided a path-based formulation for this network design problem by which alternative solutions can be generated by varying the routing options included in the model for each shipment. Each solution corresponds to a certain compromise between the two parties in terms of transport risk and economic viability. Dadkar et al. (2010) developed a game-theoretic model of the interactions among government agencies, shippers/carriers and terrorists as a framework for the analysis. Reilly et al. (2012) developed a three-player game of the interactions among a government agency, a carrier, and a terrorist along with an effective solution procedure for the game. Hu et al. (2017) considered a time-dependent hazardous materials vehicle routing problem in a two-echelon supply chain system to determine the departure time and the optimal route with a minimum risk value for hazardous materials transportation.

The conditional value-at-risk (CVaR) models are known to be flexible and suitable for hazmat transportation that can be solved efficiently. For instance, Toumazis and Kwon (2013) proposed a new method for mitigating risk in routing hazardous materials based on CVaR measure on time-dependent networks. Their research extended the previous studies by considering CVaR for hazmat transportation in the case where accident probabilities and accident consequences are time-dependent. Kang et al. (2014) extended the value-at-risk (VaR) framework to apply the VaR concept to a more realistic multi-trip multi-hazmat type framework, which determines routes that minimise the global VaR value while satisfying equity constraints.

2.2 Multi-objective models

Huang et al. (2004) considered safety, costs and, security criteria and used GIS to quantify the factors on each link in the network that contribute to the evaluation criteria for a possible route. Chang et al. (2005) developed a method for finding non-dominated routes for multiple routing objectives in networks in which the routing attributes are uncertain and the probability distributions that describe those attributes vary by time of day.

Zografos and Androutsopoulos (2004) and Androutsopoulos and Zografos (2010) developed bi-criterion routing and scheduling problem to minimise the total cost and risk as time-dependent attributes. The authors formulated a routing and scheduling model to determine the non-dominated time-dependent paths for servicing a given and fixed sequence of customers (intermediate stops) within specified time windows. They also constructed a special-purpose dynamic programming algorithm to determine the k -shortest time-dependent paths on different numerical examples. In another study, Androutsopoulos and Zografos (2012) expanded their previous work by applying weighted-sum method to decompose the bi-objective vehicle routing and scheduling problem to a series of single-objective instances of the problem where a route-building heuristic algorithm is presented for addressing each of the constituent single-objective problems.

In the recent study of Asgari et al. (2017), a model for obnoxious waste location-routing problem considering various types of wastes and several treatment technologies is proposed. They formulated a multi-objective location-routing model with three objective functions minimising the treatment and disposal facility undesirability, different costs related to the problem, and eventually the risk associated with transportation of untreated materials. The researchers also developed an effective memetic algorithm in which a tabu search algorithm performs the local search tested on a real-life case study. The most recent study in the area of hazmat logistics is found in Kumar et al. (2018) in which an integrated fleet mix and routing decision for hazmat transportation is developed to minimise the overall costs for long-haul shipments. The formulated nonlinear model with integer variables for the number and type of trucks, and the route choices is solved via a genetic algorithm.

2.3 Contributions of this research

After reviewing and analysing the relevant literature in order to determine the exciting gaps in the body of knowledge, the contribution of this research can be summarised as follows:

- 1 A new multi-objective network optimisation model is formulated to minimise the total expected loss (based on risk of population exposure) and the total transport time for hazmat logistics and distribution problem with multiple hazmat classes and multiple departure times.
- 2 Because of time-varying nature of the population exposure and transport time the expected loss and transport time are treated as multi-period attributes in the problem. In our modelling approach, travel time is known and deterministic but it depends on the choice of departure time. Therefore, we face a multi-period situation in which travel time on a link in each period may change. In addition, the number of people affected in the event of an incident on each link is known but it also depends on the choice of departure time. The number of periods required depends on major changes in population density over the time horizon. For example, in urban areas, it can be divided into three periods: morning peak hours, off-peak hours and evening peak hours. Splitting the day to more periods may not introduce us safer routes but increases computational efforts drastically.

- 3 A new hybrid algorithm based on Monte Carlo simulation and fuzzy goal programming is developed to solve the multi-objective model and obtain the Pareto-optimal solutions. The computational performance of the proposed algorithm is also evaluated using hyper-volume metric.

3 Problem description

We consider a network $G = [N, A]$ which consists of a set of nodes N with $|N| = n$ and a set of undirected links A with $|A| = m$. The links correspond to highway segments and the nodes to highway intersections. The network G has multiple origin and destination points identified by the sets O and D . In addition to the nodes and links in the base network, we define different departure times and different hazmat classes for the flow of the hazmat throughout the network links. Based on this network, a space-time expanded network (STEN) (or an expanded network) $G' = [N', A']$ can be constructed to capture T departure time choices of the dispatch problem, where N' and A' are the set of nodes and directed links of the space-time expanded network respectively.

The proposed integrated routing and scheduling problem involves three definitions: loss, expected loss, and total expected loss. Loss (expected loss) is defined as the number (expected number) of people affected in the event of accidents. Expected loss can be defined on the link, route, and network levels. The expected loss on a link (route) is the expected number of people affected in the events of accidents on that link (or route). The sum of the expected loss on all links or routes gives the total expected loss in the network. For each link ij , $loss_{ijt}$ is measured by the population inside a circle of given impact radius, centred at any point on link ij , in the event of an accident from node i to node j at departure time t . The impact radius would depend on the hazardous material under consideration, and can vary from several feet to several miles. This is the most popular measure of hazardous materials transport risk in the literature (Erkut and Verter, 1998) and corresponds to the definition of transport risk for hazardous material proposed by the US Department of Transportation (1994).

It is obvious that for low probability high consequence (LPHC) events such as link failures, there is little historical and reliable data available. As such, the decision maker cannot base his estimates to measure the likelihood of an incident. In addition, any historical data that may exist is likely to be out of date, in the sense that probabilities based on them may not reflect the current situation in making decisions. Over the time it would take for sufficient data to accumulate, there may have been significant technological progress with regard to vehicle or infrastructure safety. As a result, a risk-averse dispatcher may prefer to base choices on pessimistic assumptions about link incident probabilities. Link failure is basically rare, so as previously mentioned there is likely to be insufficient data from which to estimate failure probabilities. Hence q_{ijt} , the conditional probability of an incident on link ij during departure time t given that an incident occurs, are unknown. However, the dispatcher is risk-averse so we assume he plans on the basis of one link failure $\left(\sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T q_{ijt} = 1\right)$. Multiple link failures are excluded from consideration in this study as they are being too unlikely. However, they can be considered in a case of an extensive natural disaster such as earthquake, flood, or hurricane. We are now interested in determining the worst case scenarios for q_{ijt} ,

where $ij \in A$, $t \in T$. It is proven in the literature that the risk-averse dispatcher will in general not wish to use one route but rather a mix of routes. Apart from equity of exposure considerations addressed in other work, which are of course important too, exposure of hazmat to the population can be reduced by using a mix of routes. In addition to the total expected loss, the dispatcher is interested to find the shortest paths in the distribution network that can start from any of the origins and end to any of the destinations. So the dispatcher's problem involves in finding the safest links and routes with regard to risk of population exposure and simultaneously determining the shortest paths within the distribution network.

We define the problem as an extension of classic network optimisation problems. In this case, each link in the network depending on its type (interstate highway or state roads) and time of day (morning peak hours, off-peak hours, and evening peak hours) is associated with a transport time and unknown probability of a incident (e.g., traffic accidents, inclement weather, etc.). Here the transport time and risk are considered as multi-period parameters so they may vary for the links during different time of day. The outcome of this model would be an interesting decision-making trade-off for the hazmat company's dispatcher when he is planning to choose routes, departure times, and intermediate stopping locations for the outgoing hazmat-carrying trucks (containers and tankers). According to the preceding discussion, it is concluded that the following assumptions are necessary to develop the time-space expanded network.

- Assumption 1 The network has multiple origins and destinations.
- Assumption 2 Unlike previous hazmat research papers that consider shipment distribution between each OD pair (which is similar trip distribution in conventional transportation systems), we consider supply (in origins) and demand (in destinations) are known and deterministic for different hazmat classes at OD pairs. In fact, these data resembles trip attraction and trip generation in classic transportation systems. The total supply and total demand are not necessarily equal for different hazmat classes which may result to the surplus of supply in the supply points because of their capacity.
- Assumption 3 The departure time of the dispatcher is deterministic and the departure time choice set is known.
- Assumption 4 Travel time is known and deterministic but it depends on the choice of departure time. Therefore, we face a multi-period model in which travel time on a link in each period may change.
- Assumption 5 The number of people affected in the event of an incident on each link is known but it also depends on the choice of departure time.
- Assumption 6 Probability of using a link at specific departure time is unknown.
- Assumption 7 Probability of incident in a link at specific departure time is unknown.
- Assumption 8 The dispatcher is both risk-averse with regard to hazmat transport risks and optimal-oriented with regard to travel time when selecting links and departure times.

Assumption 9 Simultaneous hazmat incidents will not happen. The reason is that hazmat incident is a low-probability event. Naturally, multiple link failures can be unlikely (Bell, 2007).

4 Problem formulation

4.1 List of notations

In this section, we introduce the notation used in the paper and based on the description of the problem and its assumptions. Then, we formulate the problem mathematically.

4.1.1 Sets

N	Set of nodes in the network G .
A	Set of links in the network G .
O	Set of origin nodes in the network G .
D	Set of destination nodes in the network G .
T	Set of departure times.
H	Set of hazmat classes.
K	Set of objective functions.

4.1.2 Indices

i	Index for node i in the network $G, i = 1, \dots, N$.
j	Index for node j in the network $G, j = 1, \dots, N$.
t	Index for departure times, $t = 1, \dots, T$.
h	Index for hazmat classes, $h = 1, \dots, H$.
k	Index for objective functions, $k = 1, \dots, K$.

4.1.3 Decision variables

x_{ijth}	Flow of hazmat class h from node i to node j at departure time $t, i \in N, j \in N, h \in H, t \in T$.
p_{ijt}	Probability of link ij selected for shipment at departure time $t, i \in N, j \in N, t \in T$.
q_{ijt}	Conditional probability of an incident on link ij during departure time t given that an incident occurs, $i \in N, j \in N, t \in T$.

4.1.4 Parameters

- $loss_{ijt}$ Loss, or population exposure, is defined as the number of people impacted in the event of an accident from node i to node j at departure time t , $i \in N$, $j \in N$, $t \in T$.
- $transport_{ijt}$ Transport time from node i to node j at departure time t , $i \in N$, $j \in N$, $t \in T$.
- $supply_{ih}$ Supply for hazmat class h at origin node i , $i \in O$.
- $demand_{jh}$ Demand for hazmat class h at destination node j , $j \in D$.
- μ_k Membership function of the objective function k , $k \in K$.
- f_k^{\min} The lowest possible value of the objective function k , $k \in K$.
- f_k^{\max} The largest possible value of the objective function k , $k \in K$.

4.2 Mathematical models

4.2.1 Multi-objective minimax optimisation model for risk-averse dispatcher

Based on the network representation, assumptions, and definitions, the multi-objective routing and scheduling problem for hazmat shipment is modelled as a two-person non-cooperative zero-sum game and formulated as a link-based *minimax* problem over the transport network as follows:

$$Min_p \left(Max_q \left(\sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T p_{ijt} \cdot q_{ijt} \cdot loss_{ijt} \right) \right) \quad (1)$$

$$Min f_2 = \sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T \sum_{h=1}^H transport_{ijt} \cdot x_{ijth} \quad (2)$$

subject to:

$$\sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T q_{ijt} = 1 \quad (3)$$

$$\sum_{j=1, j \neq O}^N \sum_{t=1}^T p_{ijt} = 1 \quad \forall i \in O \quad (4)$$

$$\sum_{i=1, i \neq O}^N \sum_{t=1}^T p_{ijt} = 1 \quad \forall j \in D \quad (5)$$

$$\sum_{i=1, i \neq k}^N \sum_{t=1}^T p_{ikt} - \sum_{j=1}^N \sum_{t=1}^T p_{kjt} = 0 \quad \forall k \notin O, k \notin D \quad (6)$$

$$\sum_{j=1, j \neq O}^N \sum_{t=1}^T x_{ijth} \leq supply_{ih} \quad \forall i \in O, h \in H \quad (7)$$

$$\sum_{i=1, i \notin D}^N x_{ijth} \geq \text{demand}_{jh} \quad \forall j \in D, h \in H \quad (8)$$

$$\sum_{i=1, i \neq k}^N \sum_{t=1}^T x_{ikth} - \sum_{j=1, j \neq k}^N \sum_{t=1}^T x_{kjth} = 0 \quad \forall k \notin O, k \notin D, h \in H \quad (9)$$

$$\sum_{h=1}^H x_{ijth} \leq M \cdot p_{ijt} \quad \forall i \in N, j \in N, t \in T \quad (10)$$

$$0 \leq p_{ijt} \leq 1 \quad \forall i \in N, j \in N, t \in T \quad (11)$$

$$0 \leq q_{ijt} \leq 1 \quad \forall i \in N, j \in N, t \in T \quad (12)$$

$$x_{ijth} \geq 0 \text{ and integers} \quad \forall i \in N, j \in N, t \in T, h \in H \quad (13)$$

M is a very large number

The above optimisation model is a multi-objective model with minimisation of transport risk associated with unknown probability incidents and links selection and the minimisation of transport time as formulated in equations (1) and (2). Constraint (3) indicates that there is only one link failure $\left(\sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T q_{ijt} = 1\right)$. The risk-averse dispatcher in general will not wish to use one route but rather a mix of links so generally $\left(\sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T p_{ijt} \geq 1\right)$. Constraints (4) and (5) impose the requirement of using a mixed strategy for link selection probabilities at origin and destination points. Constraint (6) makes sure the inbound and outbound probabilities of link selection at intermediate nodes are equal to each other. Constraint (7) restricts the total amount of transported hazmat from each origin point for each hazmat class to be less than the supply amount of hazmat classes available at each origin point. Constraint (8) requires the total amount of delivered hazmat to each destination point for each hazmat class is greater than the demand of hazmat class at the destination points. The group of constraint (9) makes sure that the inbound and outbound flow transported hazmat for intermediate nodes are equal. To make sure that a link is available to transport hazmat at specific departure times whenever the link is included in a mixed strategy (its probability of selection is greater than zero), constraint (10) uses a very large number, M , to ensure the equivalent if-then-constraint is met. In the above optimisation model, constraints (11) and (12) impose the requirement of probability of link selection, p_{ijt} , and the conditional probability of link failure, q_{ijt} , to be in the range of $[0, 1]$. Because our modelling approach is based on routing and scheduling of trucks (containers and tankers), we restricted the decision variables to be general integer using constraint (13).

4.2.2 Equivalent multi-objective optimisation model for risk-averse dispatcher

Minimax problems in the literature have been found to be difficult to solve as their models are bi-level optimisation problems and are in general non-convex and nonlinear. Although the proposed problem in this research is bi-level in nature, it has the following useful properties:

- 1 the proposed problem has a bi-linear objective function with respect to total expected risk
- 2 the constraints are linear and can be separated into two groups according to decision variables because link selection, p_{ijt} , and conditional link selection, q_{ijt} , probabilities do not simultaneously appear in the constraints
- 3 each element of a solution vector of the link selection probabilities can take any value between zero and one inclusively, and the sum of all the elements equals to one, because the dispatcher chooses a mixed strategy of his/her own game.

The above multi-objective *minimax* problem can be converted to the following multi-objective mixed-integer optimisation model by defining a new free variable V to be minimised in the objective function (14). In addition, a new set of constraint (16) is now included in the original set of constraints to ensure that the free variable V is greater than the maximum possible expected loss of population exposure in each link at specified departure times. For more details of the conversion game theory mixed strategy problems with two-person zero-sum game assumptions to equivalent linear programming models, see Section 15.5 of Hillier and Lieberman (2014).

$$\text{Min } f_1 = V \quad (14)$$

$$\text{Min } f_2 = \sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T \sum_{h=1}^H \text{transport}_{ijt} \cdot x_{ijth} \quad (15)$$

subject to:

$$p_{ijt} \cdot \text{loss}_{ijt} - V \leq 0 \quad \forall i \in N, j \in N, t \in T \quad (16)$$

$$\sum_{j=1, j \in O}^N \sum_{t=1}^T p_{ijt} = 1 \quad \forall i \in O \quad (17)$$

$$\sum_{i=1, i \in D}^N \sum_{t=1}^T p_{ijt} = 1 \quad \forall j \in D \quad (18)$$

$$\sum_{i=1, i \neq k}^N \sum_{t=1}^T p_{ikt} - \sum_{j=1, j \neq k}^N \sum_{t=1}^T p_{kjt} = 0 \quad \forall k \in O, k \in D \quad (19)$$

$$\sum_{j=1, j \in O}^N \sum_{t=1}^T x_{ijth} \leq \text{supply}_{ih} \quad \forall i \in O, h \in H \quad (20)$$

$$\sum_{i=1, i \in D}^N \sum_{t=1}^T x_{ijth} \geq \text{demand}_{jh} \quad \forall j \in D, h \in H \quad (21)$$

$$\sum_{i=1, i \in D}^N \sum_{t=1}^T x_{ijth} - \sum_{j=1, j \neq k}^N \sum_{t=1}^T x_{kijth} = 0 \quad \forall k \in O, k \in D, h \in H \quad (22)$$

$$\sum_{h=1}^H x_{ijth} \leq M \cdot p_{ijt} \quad \forall i \in N, j \in N, t \in T \quad (23)$$

$$0 \leq p_{ijt} \leq 1 \quad \forall i \in N, j \in N, t \in T \quad (24)$$

$$x_{ijth} \geq 0 \text{ and integer} \quad \forall i \in N, j \in N, t \in T, h \in H \quad (25)$$

$$V \text{ is a free unbounded variable} \quad (26)$$

$$M \text{ is a very large number}$$

Complexity of the optimisation model can be expressed as a function of problem size affected by the model parameters and network setting. The above proposed model contains $I \times J \times T \times (1 + H) + 1$ decision variables from which $I \times J \times T \times H$ are general integer and one variable is a free unbounded variable. The model has $3 \times I \times J \times T + I \times (1 + H)$ linear constraints and two linear objective functions.

5 Solution approach

The classic methods to solve multi-objective optimisation problems are based on preference-based approach in which a relative predetermined vector of weights is used to combine multiple objectives into a single-objective function. Other methods such as ε -constraint method reformulate the multi-objective optimisation problems by just keeping one of the objectives while placing the others into the set of constraints and then restricting them by user-specified values. Goal programming methods try to find the optimal solutions that attain a predefined target values for one or more objectives by minimising deviations from these target values (Masud and Ravindran, 2008). All these traditional methods then employ a point-by-point deterministic optimisation approach by finding single Pareto-optimal solution at each trial. In the past three decades, numerous multi-objective evolutionary algorithms have been developed and tested as trustable and efficient solution methods to solve multi-objective models. However, these algorithms are best known to their capability of obtaining good or near optimal solutions and attainment of the exact optimal solution(s) is never guaranteed. Since multi-objective optimisation problems have equally important Pareto-optimal solutions, an ideal solution approach would be to find multiple Pareto-optimal solutions at once and let the decision maker choose the desired solution based on other higher-level information. The optimal solutions obtained by the ideal approach will be then independent from the user's predefined parameters. An effective multi-objective solution procedure should successfully perform three following conflicting tasks (Zitzler et al., 2000; Deb, 2001):

- 1 The obtained non-dominated solutions should be close enough to the true Pareto-optimal front. Ideally, the non-dominated solutions should be a subset of the Pareto-optimal set.
- 2 The obtained non-dominated solutions should be uniformly distributed over of the Pareto-optimal front in order to provide the decision maker a true insight of existing trade-offs between objectives.

- 3 The obtained non-dominated solutions should capture the whole spectrum of the Pareto-optimal front. This requires investigating non-dominated solutions at the extreme ends of the objective functions space.

Because the developed model in this research has two objectives, fuzzy goal programming (FGP) is considered as the solution approach. The main difference between regular goal programming method and fuzzy goal programming is that regular goal programming tries to minimise the sum of the deviations from the optimal (i.e., best) solutions with regard to the objective functions. Whereas fuzzy goal programming tries to maximise the sum of the membership functions (i.e., distance) from the nadir (i.e., worst) solutions of the objective functions. This solution method is chosen because of risk averse attitude of the dispatcher as he is making decisions to avoid the worst-case outcomes (nadir solutions).

The major drawback of the standard fuzzy goal programming model is that it can obtain only one non-dominated solution which is highly dependent to the decision maker’s choice of the weights of the membership functions. Integrating the standard point-by-point approach with a randomly generated preferences/weights independent from the decision maker can provide the entire Pareto-optimal front in a simulation run. To rectify this dependability and in order to obtain the true Pareto-optimal front the following hybrid Monte Carlo simulation model is also developed in which randomly generated and normalised weights for membership functions are used in fuzzy goal programming sub-model during each simulation replication.

5.1 *Fuzzy membership functions for the objective functions*

In a multi-objective optimisation problem, all objective functions may not be simultaneously optimised under the system constraints so the decision maker may define a tolerance limit such as a membership function $\mu_k(f_k)$ for the k^{th} fuzzy objectives. The fuzzy goal programming employs the nadir solutions (the worst possible solutions of each objective function) as undesired targets and maximises the lower bounds on the membership functions. In this research, we formulate linear membership functions which have a continuously increasing property for maximisation objective functions and continuously decreasing trend for minimisation objective functions.

For maximisation type objective function:

$$\mu_k(f_k) = \begin{cases} 1, & \text{if } f_k \geq f_k^{\max} \\ \frac{f_k - f_k^{\min}}{f_k^{\max} - f_k^{\min}}, & \text{if } f_k^{\min} \leq f_k \leq f_k^{\max} \\ 0, & \text{if } f_k \leq f_k^{\min} \end{cases} \quad \forall k \in K \quad (27)$$

For minimisation type objective function:

$$\mu_k(f_k) = \begin{cases} 1, & \text{if } f_k \leq f_k^{\min} \\ \frac{f_k^{\max} - f_k}{f_k^{\max} - f_k^{\min}}, & \text{if } f_k^{\min} \leq f_k \leq f_k^{\max} \\ 0, & \text{if } f_k \geq f_k^{\max} \end{cases} \quad \forall k \in K \quad (28)$$

In the above equations, f_k^{\max} and f_k^{\min} are the best and the worst possible solutions (also known as *ideal* and *nadir* solutions) of the maximisation type objective function k that can be obtained by incorporating only one of the objectives while ignoring all other objectives subject to the set of functional constraints. For the minimisation type objective functions the definitions are adjusted with respect to *ideal* and *nadir* solutions.

5.2 Algorithm: hybrid Monte Carlo simulation and fuzzy goal programming

Begin

Step 1 Calculate the best and the worst possible solutions (also known as *ideal* and *nadir* solutions) of the objective function k by incorporating only one of the objectives while ignoring all other objectives subject to the set of functional constraints;

Step 2

Current replication = 1;

While (current replication <= designated number of replications)

Step 2.1 Read the parameters of the optimisation model;

Step 2.2 Calculate the linear membership function $\mu_k(f_k)$ for each fuzzy objective function k ;

Step 2.3 Generate normalised random weights for the membership functions $\mu_k(f_k)$;

$$w_k = \text{rand}(0, 1) \quad \forall k \in K \quad (29)$$

$$w'_k = \frac{w_k}{\sum_{k \in K} w_k} \quad \forall k \in K \quad (30)$$

Step 2.4 Solve the fuzzy goal programming sub-model;

$$\text{Max} \sum_{k \in K} w'_k \lambda_k \quad (31)$$

Subject to:

$$\lambda_k \leq \mu_k(f_k) \quad \forall k \in K \quad (32)$$

the set of constraints (16)-(26)

Step 2.5 Current replication = Current replication + 1;

End while

End

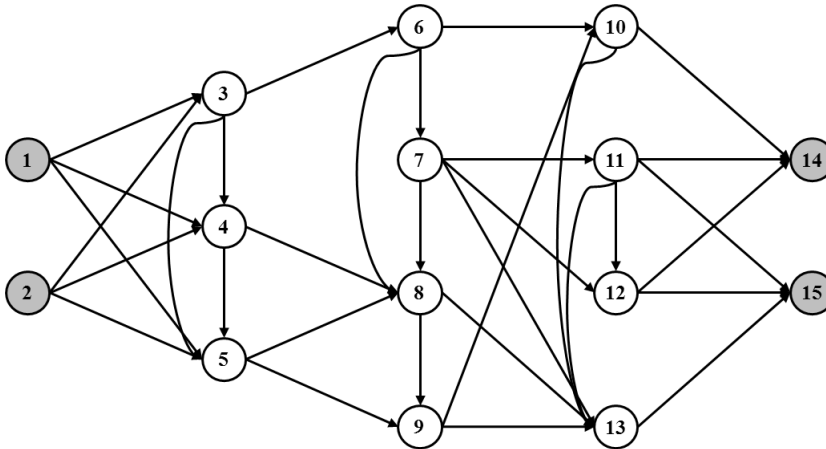
6 Numerical example

6.1 Network and data setting

In order to demonstrate an application of the developed model and the solution method, a transportation network consisting of 15 nodes and 33 transport links was created as illustrated in Figure 1. It is assumed that the network has two origins (nodes 1 and 2) and two destinations (nodes 14 and 15); these nodes are highlighted in grey. In addition,

two different departure times and three classes of hazardous materials are considered to fully capture the complex aspects of the dispatcher’s problem.

Figure 1 An illustration of the network for the numerical study



To illustrate the time-varying nature of the population and transport time, we consider two periods with different impacted populations around the roads as shown in Table 1. We assumed that during the first period, the majority of population stay at home generating fewer work-related trips. As such, the estimated transport time for each link is shorter. For the second period, since the majority of populations are at work or taking work-related trips, the impacted population is assumed to be less but on the other side, transport time is longer compared to the first period. We also estimated supply and demand for three classes of hazmat as the number of standard tankers and containers (to make the transport time measureable) at origin and destination nodes respectively as provided in Table 2. Visual Basic.Net programming environment is used to develop the simulation model in which LINGO optimisation software (LINGO 17.00, unlimited version) is utilised to solve the fuzzy goal programming sub-model in the algorithm all to be run on a laptop computer equipped with Intel® Core™ i7-6600U CPU @ 2.60 GHz and 16.00 GB RAM.

Table 1 Impacted population and transport time for the network links during two periods of a day

Link	Impacted population		Transport time (hours)	
	$t = 1$	$t = 2$	$t = 1$	$t = 2$
1, 3	38,479	26,185	2.12	2.59
1, 4	28,713	20,248	5.88	7.33
1, 5	32,046	23,483	5.56	8.04
2, 3	34,407	25,633	3.92	4.90
2, 4	11,489	7,132	3.06	4.12
2, 5	24,967	18,992	5.30	6.39
3, 4	47,883	34,026	4.78	6.40
3, 5	5,876	3,607	5.70	7.55

Table 1 Impacted population and transport time for the network links during two periods of a day (continued)

<i>Link</i>	<i>Impacted population</i>		<i>Transport time (hours)</i>	
	<i>t = 1</i>	<i>t = 2</i>	<i>t = 1</i>	<i>t = 2</i>
3, 6	21,045	14,239	2.06	2.55
4, 5	38,774	26,533	3.14	4.16
4, 8	47,681	36,962	8.94	11.34
5, 8	37,536	27,116	4.62	5.54
5, 9	9,977	7,304	4.96	6.46
6, 7	43,759	27,796	5.08	6.72
6, 8	34,716	23,201	9.96	12.71
6, 10	42,358	29,850	4.18	5.50
7, 8	10,004	6,596	9.92	13.48
7, 11	35,874	22,027	2.14	3.06
7, 12	39,939	31,947	5.64	7.21
7, 13	31,132	23,455	2.58	3.77
8, 9	22,019	16,690	6.54	9.20
8, 13	30,142	22,721	6.56	8.84
9, 10	20,964	15,264	9.88	13.26
9, 13	10,845	6,570	3.48	5.21
10, 13	49,483	38,948	8.06	10.92
10, 14	40,228	31,458	7.22	10.72
11, 12	40,144	28,554	5.06	6.96
11, 13	7,023	5,247	3.66	4.98
11, 14	45,362	28,918	4.68	5.62
11, 15	45,126	27,315	6.26	7.96
12, 14	22,925	16,054	9.32	12.53
12, 15	16,628	11,483	2.74	3.85
13, 15	28,652	20,420	9.12	11.59

Table 2 Supply and demand for three classes of hazmat

<i>Node</i>	<i>Supply</i>			<i>Demand</i>		
	<i>h = 1</i>	<i>h = 2</i>	<i>h = 3</i>	<i>h = 1</i>	<i>h = 2</i>	<i>h = 3</i>
1	8	12	5			
2	9	7	3			
14				7	11	4
15				8	5	3

6.2 *Computational results*

6.2.1 *Optimal solutions of the objective functions*

Tables 3 and 4 show the optimal (i.e., ideal) solutions with respect to each objective function independently while ignoring the other objective function subject to the functional and operational constraints (16)–(26). It can be observed that through the optimal solution for objective function (1) the lowest possible value for the total expected loss, number of people affected, is 10,033 while the total transport time is 947 hours. In this case, it can be stated that the dispatcher tends to diversify the link selection probabilities as much as possible in order to reduce the negative effect of hazmat spillage due to possible link failures especially during the first departure time. During this period 22 links of the distribution network has non-zero selection probability indicating the possibility of large selection pool. In addition, if a link has zero selection probability during a departure time, it will not be possible to use that link to transport any classes of hazmat [e.g., link (3, 4) for $t = 1$]. During the second departure time, since less population can be impacted overall, the dispatcher's optimal link selection is less diversified as only 12 links with non-zero selection probability are be considered for selection. However, these candidate links have generally higher chance of being selected compared to the first departure time in which there are more candidate links with lower probability of selection. According to this optimal scenario during the first departure time, 14 candidate links are selected to transport different classes of hazmat while seven links are used for the shipments during the second departure time.

The modelling approach enables the dispatcher to choose different departure times in the supply nodes as well as in the intermediate nodes. For example, the total supply for hazmat class 1 is $8 + 9 = 17$ and the total demand is $7 + 8 = 15$. We observe in intermediate node 3 that the total inbound shipment is $6 + 7 = 13$ units from which 11 units are shipped to node 6 during the first departure time and two units are shipped to node 6 during the second departure time (shipping to the same node during two departure times). Intermediate node 6 receives 13 units of shipment from which six units are shipped to node 7 during the first departure time and seven units are shipped to node 10 during the second departure time (shipping to different nodes during two departure times).

The optimal solution with respect to the total transport time results to minimum 648 hours of transport but it affects as twice as the population, 20,154 as shown in Table 4. In this optimal scenario, the transport time can be mostly improved only by 32% while the total expected loss is more than doubled. The link selection probabilities in this situation reveal a different pattern as the optimal solution enforces the dispatcher against diversification. During the first departure time, only 14 candidate links have non-zero selection probabilities making the selection pool more restricted while for the second departure time the dispatcher is even more confined with only eight available links with non-zero selection probability. This pattern of optimal solution indicates that if the dispatcher goal is to minimise the total transport time only, the optimal solution narrows down the dispatcher's choice to few candidate links from the distribution network but these links will have much higher likelihood for selection resulting to a significant increase of impacted population. Note that this optimal scenario uses only eight links during the first departure time and no links during the second departure time to transport

all the hazmat classes through the distribution network to meet the demand specified at nodes 14 and 15.

Table 3 The optimal solution with regard to expected loss ($f_{1,\min} = 10,033$ and $f_2 = 947$ hours)

Link	Link selection probability		Transported hazmat					
	$t = 1$	$t = 2$	$t = 1$			$t = 2$		
			$h = 1$	$h = 2$	$h = 3$	$h = 1$	$h = 2$	$h = 3$
1, 3	0.261	0.383	6	0	5	0	0	0
1, 4	0.043	0.000	2	0	0	0	0	0
1, 5	0.313	0.000	0	11	0	0	0	0
2, 3	0.292	0.058	7	0	2	0	0	0
2, 4	0.167	0.000	0	5	0	0	0	0
2, 5	0.402	0.081	0	0	0	0	0	0
3, 4	0.000	0.000	0	0	0	0	0	0
3, 5	0.000	0.000	0	0	0	0	0	0
3, 6	0.300	0.694	11	0	7	2	0	0
4, 5	0.000	0.000	0	0	0	0	0	0
4, 8	0.210	0.000	2	5	0	0	0	0
5, 8	0.267	0.000	0	0	0	0	0	0
5, 9	0.529	0.000	0	11	0	0	0	0
6, 7	0.229	0.361	6	0	3	0	0	0
6, 8	0.000	0.000	0	0	0	0	0	0
6, 10	0.237	0.167	0	0	1	7	0	3
7, 8	0.000	0.000	0	0	0	0	0	0
7, 11	0.000	0.380	0	0	0	6	0	3
7, 12	0.211	0.000	0	0	0	0	0	0
7, 13	0.000	0.000	0	0	0	0	0	0
8, 9	0.112	0.000	0	0	0	0	0	0
8, 13	0.333	0.033	0	5	0	2	0	0
9, 10	0.239	0.183	0	0	0	0	11	0
9, 13	0.218	0.000	0	0	0	0	0	0
10, 13	0.000	0.258	0	0	0	0	0	0
10, 14	0.249	0.319	0	11	3	7	0	1
11, 12	0.000	0.000	0	0	0	0	0	0
11, 13	0.000	0.000	0	0	0	0	0	0
11, 14	0.221	0.000	0	0	0	0	0	0
11, 15	0.159	0.000	6	0	3	0	0	0
12, 14	0.211	0.000	0	0	0	0	0	0
12, 15	0.000	0.000	0	0	0	0	0	0
13, 15	0.350	0.491	0	5	0	2	0	0

Table 4 The optimal solution with regard to total transport time ($f_1 = 20,154$ and $f_{2,\min} = 648$ hours)

Link	Link selection probability		Transported hazmat					
	$t = 1$	$t = 2$	$t = 1$			$t = 2$		
			$h = 1$	$h = 2$	$h = 3$	$h = 1$	$h = 2$	$h = 3$
1, 3	0.520	0.166	8	12	5	0	0	0
1, 4	0.000	0.000	0	0	0	0	0	0
1, 5	0.000	0.315	0	0	0	0	0	0
2, 3	0.581	0.000	7	4	2	0	0	0
2, 4	0.000	0.419	0	0	0	0	0	0
2, 5	0.000	0.000	0	0	0	0	0	0
3, 4	0.000	0.000	0	0	0	0	0	0
3, 5	0.000	0.000	0	0	0	0	0	0
3, 6	0.633	0.633	15	16	7	0	0	0
4, 5	0.000	0.000	0	0	0	0	0	0
4, 8	0.419	0.000	0	0	0	0	0	0
5, 8	0.245	0.000	0	0	0	0	0	0
5, 9	0.000	0.070	0	0	0	0	0	0
6, 7	0.457	0.443	8	5	3	0	0	0
6, 8	0.000	0.000	0	0	0	0	0	0
6, 10	0.367	0.000	7	11	4	0	0	0
7, 8	0.000	0.000	0	0	0	0	0	0
7, 11	0.399	0.000	0	0	0	0	0	0
7, 12	0.501	0.000	8	5	3	0	0	0
7, 13	0.000	0.000	0	0	0	0	0	0
8, 9	0.000	0.000	0	0	0	0	0	0
8, 13	0.664	0.000	0	0	0	0	0	0
9, 10	0.000	0.000	0	0	0	0	0	0
9, 13	0.070	0.000	0	0	0	0	0	0
10, 13	0.000	0.000	0	0	0	0	0	0
10, 14	0.367	0.000	7	11	4	0	0	0
11, 12	0.399	0.000	0	0	0	0	0	0
11, 13	0.000	0.000	0	0	0	0	0	0
11, 14	0.000	0.000	0	0	0	0	0	0
11, 15	0.000	0.000	0	0	0	0	0	0
12, 14	0.000	0.633	0	0	0	0	0	0
12, 15	0.267	0.000	8	5	3	0	0	0
13, 15	0.000	0.733	0	0	0	0	0	0

6.2.2 Practicality of the trade-offs among the objective functions

Figure 2 illustrates the Pareto-optimal (non-dominated) solutions obtained by the hybrid Monte Carlo simulation and fuzzy goal programming. As can be seen, Pareto frontier is convex in the objective functions space. It means a risk-averse decision maker who considers a single-objective of risk sacrifices travel time significantly less than a risk-seeking decision maker who considers a single-objective of economic objective. It is observed that the algorithm performs well in finding the Pareto-optimal solutions close enough to the ideal solutions and far enough from nadir solutions make them more likely to be close enough to the true Pareto front (task 1). The ideal solutions are depicted at the two corners of the trade-off curve as ($f_{1,\min} = 10,033$ and $f_2 = 947$ hours) and ($f_1 = 20,154$ and $f_{2,\min} = 648$ hours). The algorithm is also capable in capturing Pareto-optimal solutions at the extreme ends of the objective functions space (task 3). On the other hand and with respect to the second task of solving multi-objective optimisation, it can be verified that the algorithm is able to find almost uniformly distributed Pareto-optimal solutions over of the Pareto region. Another conclusion is that the Pareto-optimal solutions are more clustered within the range of 11,000–13,000 for population loss and 660–730 hours of transport time giving more choices of optimal dispatching strategies to the decision maker.

Figure 2 Pareto-optimal solutions depicting the trade-off between total expected loss and transport time (see online version for colours)



6.2.3 Performance evaluation of the proposed algorithm

In order to evaluate the performance of the proposed algorithm and to testify how it determines the set of non-dominated solutions, the algorithm is executed over 500 simulation replications. The performance parameters of the algorithm are given in Table 5. As expected, the minimum expected loss was found as 10,033 people (for 947 hours of transport) and minimum transport time is optimised as 648 hours (for 20,154 people). The averages reveal that for different choices of dispatching strategy the overall average for total expected population loss is 11,952 people while the average transport time is 737 hours. It can be concluded that regardless of the decision maker and

his/her choice of dispatching strategy expected number of affected people is about 11,952 and expected transport time is about 737 hours. By examining the standard deviations and coefficient of variations, it can be expressed that population loss varies more than the transport time with respect to different optimal dispatching strategies making it more sensitive than transport time for a specified selected strategy. Computational times (in the form of CPU time) are also examined to ascertain any need for improvement. It is found that the mathematical formulation and computer implementation of the model is not ill-structured as all the solution times are less than a second. However, this needs to be testified against very large-scale problems with hundreds of nodes and thousands of transport links.

Table 5 Statistical output of the simulation model

	<i>Objective function</i>		<i>CPU time (second)</i>
	<i>Expected loss</i>	<i>Transport time</i>	
Min.	10,033	648	0.17
Max.	20,154	947	0.29
Average	11,952	737	0.21
StDev	2,346	80	0.02
CV	0.20	0.11	0.07

In order to quantitatively evaluate the proposed hybrid algorithm, the hyper-volume metric is adopted which calculates an approximation of the hyper-cube volume formed by the non-dominated solutions in the objective functions space (van Veldhuizen, 1999). This metric can measure three necessary tasks in solving multi-objective optimisation problems; closeness to the true Pareto-optimal front, spread of non-dominated solutions, and extent of the generated solutions. A volume, v_i , can be constructed by each non-dominated solution and a selected reference point such as f_{nadir} to form the diagonal corners of the hyper-cube. Then the approximate volume of the hyper-cube can be found as the union of all these Q volumes as presented in equation (33). This performance measure has been used and recommended by many researchers to evaluate Pareto sets obtained by multi-objective evolutionary algorithms (Zitzler et al., 2000; van Veldhuizen and Lamont, 2000). However, the concept can be applied to any type of multi-objective optimisation solution method.

$$HV = volume \left(\bigcup_{i=1}^{|Q|} v_i \right) \quad (33)$$

In order to calculate the hyper-volume metric, a reference point such as the nadir solution should be selected. In this research, the nadir solution f_{nadir} (*expected loss* = 20,154, *transport time* = 947) is selected as the reference point (the worst possible values for each objective function). Because of different order of magnitude of expected loss and transport time, the normalised values of the non-dominated solutions along with normalised nadir point $f_{nadir}^{normalised}$ (*expected loss* = 1, *transport time* = 1) are used to calculate the $HV_{normalised}$ (with maximum possible value of one). Since we only have two objective functions here, the hyper-volume is reduced to two-dimensional area in this research. Then the area formed by the reference point and the non-dominated solutions is to be calculated. To prevent overlapping of the areas, the non-dominated solutions are

sorted so the $f_{1,1}$ represents the first value of the first objective function (expected loss) and $f_{2,1}$ represents the first value of the second objective function (transport time). The rest of the areas are calculated with respect to the adjacent sorted non-dominated solutions using equation (34).

$$HV_{normalised} = \left| \left(f_{1,1}^{normalised} - f_{1,nadir}^{normalised} \right) \times \left(f_{2,1}^{normalised} - f_{2,nadir}^{normalised} \right) \right| + \sum_{i=2}^{|Q|} \left| \left(f_{1,i}^{normalised} - f_{1,i-1}^{normalised} \right) \times \left(f_{2,i}^{normalised} - f_{2,nadir}^{normalised} \right) \right| \quad (34)$$

The calculated $HV_{normalised} = 0.887362$ is very close to one, which proves the capability of the proposed solution method in obtaining diverse and well-distributed solutions while being close enough to the true Pareto-optimal front.

7 Conclusions and future research

This paper studies an integrated routing and scheduling hazmat transportation problem with multiple supply and demand locations and unknown link incident probabilities. A new multi-objective network optimisation model is formulated to minimise the total expected loss (based on risk of population exposure) and the total transport time for hazmat logistics and distribution problem with multiple hazmat classes and multiple departure times. Since the formulated problem has multiple objectives, a new hybrid algorithm to solve the multi-objective model and obtain the Pareto-optimal solutions based on Monte Carlo simulation and fuzzy goal programming is also developed and tested. In order to illustrate the effectiveness of the developed mathematical model and the solution method in obtaining Pareto-optimal solutions, a realistic numerical example consisting multiple supply and demand points, three hazmat classes with two departure times is provided. The most interesting insight for practitioners extracted from the model and its solution technique is that the Pareto frontier is convex in the objective functions space. It means a risk-averse decision maker who considers a single-objective of risk sacrifices travel time significantly less than a risk-seeking decision maker who considers a single-objective of economic objective. Therefore, we recommend even risk neutral decision makers consider the higher weight for risk. This way, while they well observe societal impacts of hazmat incidents, they ensure adverse economic impacts are not drastically increased.

Similar to all research activities, there are limitations and assumptions in this research that can lead an interested reader toward future research directions. First, the transport network in the problem is formed by unidirectional (one-way) links but in reality, the transport networks are formed by a combination of both one-way and two-way links. Of course, it is very likely that the optimal solutions will be eventually based on one-way links but the input can also be in a two-way format. Although probabilistic concepts are employed in this research, the main parameters of the developed model are deterministic. The second area for future research is to capture uncertainty to the input parameters of the model mainly for population exposure that is defined as the number of people impacted in the event of an accident and the transport time through the links of the distribution network. Capturing the uncertainty with regard to the above parameters makes the problem more interesting but more complicated to model and solve. It is

expected that the use of robust optimisation techniques such as fuzzy set theory and algorithms or stochastic programming will be of necessity in such future research.

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