Gold price forecasting with a neuro-fuzzy-based inference system

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Abstract: Following the importance of gold in the global economy and the high interest that has attracted recently, the objective of this paper is twofold: to predict the price of gold by using the Adaptive Neuro-Fuzzy Inference System (ANFIS) and compare its forecasting accuracy with various time-series forecasting methods and the ‘Buy and Hold’ (B&H) strategy. The results show that the ANFIS’s accuracy is far superior to the performance of all compared methods and therefore ANFIS demonstrates the potential of neuro fuzzy-based modelling for predicting the gold’s price.

Keywords: gold price forecasting; neuro-fuzzy system; ANFIS; B&H strategy; time-series forecasting methods; financial management.

1 Introduction

Gold is a precious commodity, globally accepted as a potential currency. Gold is used as a hedge against inflation and variations of the US dollar apart from uses in industrial fields such as the jewelry industry. What is more, gold is a ‘safe haven’ investment for investors due to its high liquidity. In fact, gold is the most influential metal because of its
multifunctions such as storage of value, reserve for money insurance and anti-inflation shelter. Gold affects not only economic development but also the social aspect of a country at the aggregate level.

Gold has been traded in the market since 1967, and the price has increased with rapid fluctuations. Historically, gold has been considered a ‘frontier-less currency’ that may be traded at any time and under virtually all circumstances. Gold proved to be the most effective way to collect cash during the stock exchange crash in 1987, and once again in 1997 and 1998 during the Asian crisis. In both Gulf wars, the demand for gold increased significantly, and hence, maintaining a small proportion of a portfolio in gold could be invaluable when cash is essential. Gold’s importance lies in reflecting investors’ expectations, marking the trends and expectations of growth and decline of global economies.

Nowadays, the price of gold is higher than its historical trend. In 2008 and early 2009, most metal prices dropped, and the global economy was in recession. The price and production behaviour of gold differ from most other mineral commodities. In the 2008 financial crisis, the price of gold increased by 6%, while many key mineral prices fell, and other equities dropped by around 40%. This was not a surprise to many, because, given the turmoil in the markets and the fear for the banking system, gold should be attracting safe-haven buying. In the current environment, upward pressure on the price of gold is likely being driven by the weakness of the US dollar, higher oil prices and geopolitical tensions. What is more, it is commonly believed that the increasing geopolitical risk combined with the increasing economic uncertainty (weakness of the US dollar) should continue to provide incentive to invest in gold markets.

In the short-term, there are two main reasons gold prices increase dramatically. Firstly, in a period when global financial markets crash and the global economy is in recession, investors are less trusting of financial markets as reliable investments. Consequently, investors switch to speculation or to any market that does not have heavy liability or unpredictability, such as the gold market. Secondly, the devaluation of the US dollar versus other currencies, and international inflation with high oil prices are reasons big companies hedge gold against fluctuations in the US dollar and inflation. This means that gold trading will offset the potential movement of real value in the short-term market against US dollar oscillations and inflation. In the long-term, there are three major reasons for increasing gold prices. Firstly, mine production has gradually declined in recent years due to increased mining costs, decreased exploration and difficulty finding new deposits. Secondly, institutional and retail investors have rational expectations when markets are uncertain, and therefore, gold is kept in institutional and retail investment portfolios as it is more liquid or marketable in unstable financial markets. Thirdly, investing in gold is becoming easier via gold Exchange Traded Funds (ETFs) compared to other finance markets.

Due to gold’s importance and its increasing price, the development of a forecasting model that reflects the structure of the gold market is necessary. The contribution in predicting the gold price in a dynamic way can be important for investors, economists, politicians or any agent who sees gold as an indicator of the future performance of the world economy. Many studies have been conducted to predict trends in the price of gold. Forecasting the next day’s price is difficult as there is no graphic or method to follow to predict the future of the price of metal. Therefore, a model that can reflect the structure of the gold market and forecast the movement of the price of gold is necessary.
In this paper, the Adaptive Neuro-Fuzzy Inference System (ANFIS) is performed for the first time for predicting the next-day gold price. At the second stage, the ANFIS’s forecasting accuracy is compared with various time-series forecasting methods (AR, ARMA and neural network) or the ‘Buy and Hold’ (B&H) strategy. The rest of this paper follows the following structure: Section 2 presents a literature review, followed in Section 3 by the presentation of the main methodological tools employed in the analysis. Section 4 presents the simulation results, and Section 5 concludes the paper.

2 Literature review

Many authors have worked to develop a forecasting model that reflects the structure of the gold market. Lipschitz and Otani (1977) were among the pioneers who tried to forecast the price of gold. They developed a quarterly econometric model and contended that gold prices adjust when investors absorb the existing excess supply since the gold stocks are large relative to annual flows.

Many researchers suggest that Box-Jenkins’s Auto-Regressive Integrated Moving Average (ARIMA) model is the most accurate forecasting model and wins over other models; Pravit Khaemasunun (2007) developed two forecasting models: Multiple-Regressive and Auto-Regressive Integrated Moving Average to forecast the Thai gold price. He proved that the ARIMA (1, 1, 1) is the most suitable model, while the Multiple-Regression gives many factors that affect the price of gold (such as Australian Dollars, Japanese Yen, US dollars, Canadian Dollars, EU pounds, Oil prices and gold future prices). He also proved that the simultaneous use of the Multiple-Regression and ARIMA (1, 1, 1) models increases the accuracy of forecasting the Thai gold price in the short term. The idea that the ARIMA can predict the price of gold is also accepted in many countries. In Australia, for example, Selvanathan (1991) compared the forecasted London daily gold prices from the Economic Research Centre with those from the ARIMA model, and proved that the ARIMA model is very low cost and effective enough to predict the price of gold.

Mui and Chu (1993) employed a combined time series forecasting technique and a composite time series forecasting technique to predict the price of gold. They applied three weighting methods (the traditional equal weight [EW] method, the variance-covariance matrix [VC] method, the odds matrix [OM] method) in the combined forecasting modelling. Comparing the forecasts generated by univariate analysis using the ARIMA model, bivariate analysis with combined forecasts and multivariate analysis for composite forecasts, the authors concluded that the combined forecasting models outperform the univariate ARIMA model and the bivariate transfer function models, while the composite forecasting model is superior.

Ismail et al. (2009) proposed the Multiple Linear Regression (MLR) model for predicting the future price of gold. The MLR model is based on economic factors that influence the price of gold such as inflation, currency price movements and others. Two models were considered. The first model considered all possible independent variables, and the second model considered only four independent variables (Commodity Research Bureau future index [CRB] lagged one, USD/Euro Foreign Exchange Rate [EUROUSD] lagged one, Inflation rate [INF] lagged two and Money Supply [M1] lagged two). In terms of prediction, the second model achieved a higher level of predictive accuracy than the first model.
The frequent, alternating rise and fall, as well as the range of the daily closing prices during a period, can significantly increase the difficulty of prediction (Hsu, 2011). The Artificial Neural Network (ANN) offers researchers the possibility of making predictions on the future prices of data, based on pre-existing series. The ANN uses the process of learning, during which the network is trained on the existing data series, and then, based on the training the network has received, estimates future prices. Several distinguishing features of ANNs make them valuable and attractive in forecasting. First, ANNs are non-linear data-driven. They are capable of performing non-linear modelling without prior knowledge of the relationships between the input and output variables. Thus, ANNs are more general and flexible modelling tools for forecasting. The non-parametric ANN model may be preferred over traditional parametric statistical models in situations where the input data does not meet the assumptions required by the parametric model, or when large outliers are evident in the dataset. Second, ANNs are universal functional approximators. A network can approximate any continuous function to any desired accuracy. ANNs have more general and flexible functional forms than the traditional statistical methods can effectively deal with. Third, ANNs can generalise. After learning the data presented to them, ANNs can often correctly infer the unseen part of a population even if the sample data contain noisy information. As forecasting is performed via predicting future behaviour (the unseen part) from examples of past behaviour, this application area is ideal for neural networks, at least in principle. These unique features make ANNs valuable for solving many practical forecasting problems.

Any time series forecasting model assumes that there is an underlying process from which data are generated and the future value of a time series is solely determined by the past and current observations. Neural networks capture the underlying pattern or autocorrelation structure within a time series even when the underlying law governing the system is unknown or too complex to describe.

Due to their unique ability to approximate arbitrary non-linear functions and their ability to mimic classical regression models, neural network models have been examined in forecasting future gold trends. The use of neural networks is not new in forecasting the price of gold, stock market indexes and individual assets (McCann and Kalman, 1994; Tsibouris and Zeidenberg, 1995; Chen and Leung, 2005). Many researchers have used non-linear techniques, have compared them with traditional linear regression analysis and concluded in favour of precious metals being non-linear in nature. Dunis and Nathani (2007) forecasted gold and silver daily returns with advanced regression analysis using various linear and non-linear models. Their prime aim was to find out which quantitative model does best in forecasting the returns of these two metals in order to implement a daily quantitative trading strategy. The authors used ARMA models as a linear benchmark for comparison purposes with non-linear models such as Nearest Neighbours, Multilayer Perceptron (MLP) and Higher Order Neural Networks (HONNs). The results showed that the non-linear models like MLPs and HONNs outperformed the linear ARMA models. In addition, the authors confirmed the presence of non-linearities in gold and silver prices as they found that non-linear models can be effectively used for generating excess returns in these markets. Parisi, Parisi and Diaz (2008) analysed recursive and rolling neural network models to forecast one-step-ahead sign variations in the price of gold. The results showed that the rolling ward networks exceed the recursive ward networks and feed forward networks in forecasting the price of gold sign variation. Furthermore, the rolling ward net exceeded the ARIMA model supporting the use of neural networks with a dynamic framework to forecast the price of gold. What is more,
the neural network forecasting systems in the gold market proved to be capable of generating returns far above those of typical regression models (Brauner et al., 1997). Shafiee and Topal (2010) applied a modified econometric version of the long-term trend reverting jump and dip diffusion model for forecasting gold prices. The authors used two forecasting statistical errors (RMSE, MAE) to compare forecasts across this model and the ARIMA model. The comparison showed that predicting the price of gold using this model that includes the jump and dip components as parameters is an improvement over the ARIMA model.

Neural networks equipped with genetic algorithms have the advantage of simulating non-linear models when little a priori knowledge of the structure of problem domains exists. Studies suggest that this system provides better predictions when compared with traditional econometric models. For example, Mirmirani and Li (2004) used the back-propagation neural networks with genetic algorithms to predict the movement of the price of gold and proved that there is a short-term time dependence in movements of the price of gold. What is more, using a genetic algorithm to find the optimal net architecture, rather than relying on the modeler in network selection, minimises the possibility of finding a solution that is actually sub-optimal. More recently, Zhou, Lai, and Yen (2012) proposed an improved Empirical Mode Decomposition (EMD) meta-learning rate-based model for forecasting the price of gold. The experiment results showed that this model offers good prediction results and running time.

Neural networks are being used in prediction and classification, areas where regression models and other related statistical techniques have traditionally been used. Some of the commonly used traditional statistical techniques applied for predicting and classifying are multiple regression, discriminant analysis and logistic regression. These methods are being widely used in various applications due to the established methodology. However, neural networks are being used as an alternative to these traditional techniques and have been gaining popularity in recent years. This has led to many studies comparing traditional statistical techniques with neural networks in various applications. Paliwal and Kumar (2009) carried out a comprehensive review of articles that involve a comparative study of feed forward neural networks and statistical techniques used for prediction and classification problems in various application areas. The authors concluded that the neural networks in most cases outperformed or at least performed as well as other methods.

Many studies using neural networks have also been conducted to forecast the price of oil. Yu, Wang and Lai (2008) indicated that an Empirical Mode Decomposition (EMD) based on a neural network ensemble learning paradigm can be used as a very promising methodology for predicting the price of world crude oil. He et al. (2009) investigated the multi-scale heteroscedasticity behaviour of crude oil risks across different time horizons proposing a multi-scale non-linear ensemble system to account for multi-scale and autocorrelation data characteristics. Two emerging techniques including wavelet analysis and neural network were integrated in the proposed system. Experiment results suggested that the proposed wavelet decomposed non-linear ensemble value at risk (WDNEVaR) model is superior to more traditional approaches as it provides Value at Risk (VaR) estimates at higher reliability and accuracy. Mingming and Jinliang (2012) constructed a Multiple Wavelet Recurrent Neural Network (MWRNN) simulation model, in which trends and random components of crude oil and gold prices were considered. The simulation results showed that the model has high prediction accuracy. Azadeh et al. (2012) presented a flexible algorithm based on an Artificial Neural Network (ANN) and
Fuzzy Regression (FR) to cope with optimum long-term oil price forecasting in noisy, uncertain and complex environments. The researchers concluded that the selected ANN models considerably outperform the FR models in terms of Mean Absolute Percentage Error (MAPE) and the flexible ANN-FLR algorithm, as a comprehensive framework can be an ideal alternative for oil price forecasting in most cases.

Wang, Dong and Sun (2010) developed a feed-forward-type network based on a genetic algorithm for predicting saturates of sour vacuum gas oil. The study showed that the genetic algorithm found the optimal networks architecture and parameters of the back-propagation algorithm compared with other forecasting models and improved the prediction accuracy. Movagharnejad et al. (2011) introduced a neural network for forecasting the prices of different commercial oils from the Persian Gulf region. The designed neural network predicted the differences in the oil prices with an average error of 8.82% for testing and 7.24% for training data, and the researchers claimed that the method could promote the forecasting power of existing models to predict the price of any commercial oil instead of an average or benchmark value.

The development and applications of neural networks are not limited to a specific application area. Neural networks have received considerable attention in various fields of research such as statistical science, engineering, computer science, artificial intelligence, energy consumption, solar radiation, rate of heat transfer, accounting and finance, health and medicine, manufacturing, marketing and many others. The ANN is a well-known tool for solving complex, non-linear biological systems, and it can give reasonable solutions even in extreme cases or in the event of technological faults.

In the finance and economics fields, many price behaviours have a memory and may be modelled more accurately with techniques other than the traditional linear statistical methods. Some emerging non-linear methods, in particular neural networks, are being increasingly applied in finance and economics since neural networks have more general functional forms than those that can be effectively dealt with by well-developed statistical methods. The application involves the interaction of many diverse variables that are highly correlated, frequently assumed to be non-linear, unclearly related, and too complex to be described by a mathematical model. Huang et al. (2007) presented many studies using neural networks in finance and economics for predicting foreign exchange rates, stock market index and economic growth. The authors found mixed comparison results for forecasting performance between neural networks and other models because of the difference in data, forecasting horizons, types of neural network models and so on. The results supported the hypothesis that the prediction performance of neural networks is improved by integrating it with other technologies, and the combination of non-linear forecasting with neural networks provides encouraging results.

Some researchers integrate neural networks and other techniques such as genetic techniques, wavelet analysis, fuzzy inference, pattern recognition and traditional time series models for finance and economic forecasting. The advantages of genetic techniques include adaptivity and robustness, which avoid neural networks to get stuck at a local optimum. Roy and Cosset (1990) used artificial neural networks and logistic regression models in predicting country risk ratings using economic and political indicators. The artificial neural network models had lower mean absolute error for predictions of country risk ratings and were more sensitive to changes in risk indicators than were their logistic counterparts. Duliba (1991) compared artificial neural network models with four types of regression models in predicting the financial performance of a single group of transportation companies. She found that for this decision, the artificial
neural network model outperformed the random effects regression model but not the fixed-effects model. Wilson and Sharda (1994) presented a rigorous experimental design methodology to test ANNs’ effectiveness for predicting firm bankruptcy compared to the classical multivariate discriminant analysis. The study showed that neural networks perform significantly better than discriminant analysis in predicting firm bankruptcies. This problem of predicting business failure was also considered by Boritz and Kennedy (1995), who compared neural networks with methods such as discriminant analysis, logit and probit. The authors demonstrated that neural network performance is sensitive to the choice of the variables selected, and hence a number of replications may have to be carried out to obtain a reliable measure of model performance.


Applying neural networks in marketing is relatively new but is becoming popular because of the ability to capture non-linear relationships between the variables. Numerous applications of NN models in the marketing discipline are available. Chiang, Zhang and Zhou (2006) used neural networks in predicting and explaining patronage behaviour toward web and traditional stores and compared the results with logistic regression. Authors have provided statistical evidence that neural networks significantly outperform logistic regression models for most of the surveyed products in terms of predicting power.

Neural networks are also being applied to an increasing range of medical problems. For example, Ture et al. (2005) compared the performance of classification techniques to predict the risk of essential hypertension disease and concluded that the neural networks procedures performed better than other techniques.

Recently, neural networks have also been used by many authors for different decision-making problems. For predicting the final prices of online auction items, Xuefeng et al. (2006) used back propagation neural network models and compared the results with those obtained from multiple regression for predicting continuous variable and logistic regression in a discrete variable in predicting the final prices of online auction items. The authors concluded that for predicting the final price, the neural network performed better than traditional statistical methods, and the results may help sellers in optimising the selling price of their items and auction attributes. Sharda and Delen (2006) used neural networks in predicting the success of a movie at the box office before its theatrical release. The authors compared the results with other models (i.e., logistic regression, discriminant analysis and classification and regression trees), using exactly the same experimental conditions. Comparison results showed that the neural networks do a much better job in prediction in this setting. Nikolopoulos et al. (2007) compared the accuracy of multiple linear regression in forecasting the impact of different TV programs on a Greek TV audience by a simple bivariate regression model, three different types of an artificial neural network and three forms of nearest neighbour analysis. Much higher accuracy was obtained from forecasts based on a simple bivariate regression model, a simple nearest neighbour procedure and from two types of artificial neural networks used in the study.

Forecasting stock prices is one of the most challenging tasks in investment/financial decision-making since stock prices/indices are inherently noisy and non-stationary. Many
successful applications have shown that ANNs can be useful techniques for stock price forecasting due to their ability to capture subtle functional relationships among the empirical data even though the underlying relationships are unknown or hard to describe. Atsalakis and Valavanis (2009) surveyed more than 100 related published articles that focus on neural and neuro-fuzzy techniques derived and applied to forecast stock markets. Experiments demonstrated that soft computing techniques outperform conventional models in most cases. These techniques return better results than trading systems and higher forecasting accuracy. For example, Lin, Yang and Song (2009) used a novel recurrent neural network-Echo State Network (ESN) to predict the next closing price in stock markets. The experiments demonstrated that an ESN is an effective model for predicting time series with short-term memory and performs better than other conventional neural networks in most cases. Lu (2010) proposed an integrated Independent Component Analysis (ICA)-based noising scheme with neural network for predicting stock prices. Experimental results showed that the proposed model can produce the lowest prediction error and the highest prediction accuracy and outperformed the Wavelet-back propagation neural network (BPN), the single BPN and random walk models. Kara, Boyacioglu and Baykan (2011) developed two classification techniques, ANNs and Support Vector Machines (SVM), and compared their performances in predicting the direction of movement in the daily Istanbul Stock Exchange (ISE) National 100 Index. The results showed that the average performance of the ANN model (75.74%) was significantly better than that of the SVM model (71.52%). Chang et al. (2012), in contrast with previous studies that tried to optimise the controlling parameters of ANNs using local search algorithms, combined global search algorithms with a partially connected neural network to forecast stock price index trends. The Evolving Partially Connected Neural Networks (EPCNNs) model was performed and returned an accurate prediction of the stock price index for most of the data proving that this model is a very promising tool in forecasting financial time series data. Artificial neural networks were also used to predict the future prices of fuelwood (Koutroumanidis et al., 2009), electricity prices (Amjady and Keynia, 2010; Shafie-khah et al., 2011), carbon prices (Zhu and Wei, 2012) and other prices. The common conclusion of these studies is that neural networks are generally a very promising methodology in the field of forecasting.

In this research, the ANFIS forecasting the next-day gold price is examined for the first time. The ANFIS has been performed in many other cases such as analysing the economic development of the Greek non-metallic sector. Atsalakis, Ucenic and Skiadas (2005) predicted the manufacturing index using the ANFIS and compared it with various time-series forecasting methods such as the AR and ARMA models. The model displayed a high degree of prediction of the correct trend. In addition, Atsalakis et al. (2009) used the proposed model (ANFIS) to control and forecast the next-day stock price trends for various stocks. The results showed that the ANFIS performed very well in returning results superior to the ‘Buy and Hold’ strategy and several other reported methods in terms of percentage of prediction accuracy.

3 The proposed methodology and other forecasting models

At the first stage, well-known forecasting models are employed for forecasting the next-day price of gold. In this section, the theoretical background of these models is discussed.
a. Neural Network (NN). An ANN or a NN is a computational method used to model data. Neural networks try to imitate the architecture of the human brain. Three elements are particularly important in any model of artificial neural networks: the structure of the nodes, the topology of the network and the learning algorithm used to find the weights of the network. The ANN is composed by many nodes that operate in parallel and communicate with each other through connecting weights. In terms of topology structures, neural networks can be divided into two types: feed-forward networks and recurrent networks. Two types of learning algorithms are used in neural networks: supervised and unsupervised learning. The ANN can realise complex non-linear mapping by using the learning algorithm to adjust the connection weights based on its distributed structure. The ANN can discover patterns adaptively from the data and is capable of learning complex relationships from many individual examples (or experiences). In addition, they have fault and noise tolerance, high robustness and generalisation capability. These are very efficient in modelling non-linear problems.

The main advantages of the neural networks are their ability to learn by examples – in other words, to create knowledge from past data – and to recognise patterns and adapt themselves to cope with changing environments. However, neural networks have been criticised mainly due to the high computational power required, which limits the number of input variables that can be used. The main disadvantage of a neural network is the lack of information regarding the impact of every input on the output; therefore, they are commonly called a ‘black box’ as no information is given other than the output (Atsalakis et al., 2011).

ANNs have emerged as a powerful statistical modelling technique. They provide an attractive alternative tool for researchers and practitioners. ANNs can detect the underlying functional relationships within a set of data and perform tasks such as pattern recognition, classification, evaluation, modelling, prediction and control. Neural networks are particularly well suited to finding accurate solutions in an environment characterised by complex, noisy, irrelevant or partial information.

b. Fuzzy logic. Fuzzy logic was firstly proposed by Zadeh (1965) as an alternative way to express data. In fuzzy logic, data can belong in more than one category in contrast with the classical reasoning that supports something to be either true or false. What is more, classical reasoning can be seen as a subset of fuzzy logic. Among others, the most important advantage of fuzzy logic is the use of verbal variables that are easily understood by humans. Fuzzy Inference Systems incorporate human knowledge and experience by using fuzzy inference rules represented in ‘If-Then’ statements. They can be used in many problems where information does not have to be precise as it gives a means of representing uncertainty.

c. Neuro-fuzzy system. The need to find a solution to the disadvantages of neural networks and fuzzy logic while maintaining the advantages from both theories has led to Neuro-Fuzzy systems. A Neuro-Fuzzy system uses NNs to extract rules and membership functions from input-output data to be used in a Fuzzy Inference System. The parameters of a Neuro-Fuzzy system are determined by an adaptive algorithm from the existing data. The advantage of the Neuro-Fuzzy architecture is that knowledge is created from the data and the results are easily interpreted with the use of fuzzy verbal variables. Therefore, by using this approach the black box behaviour of NNs and the problems of finding suitable membership values for fuzzy-
logic systems are avoided. The biggest advantage is that they can use the neural networks’ learning capability and avoid the rule-matching time of an inference engine in the traditional fuzzy logic system.

d. Adaptive Neuro-Fuzzy Inference System (ANFIS). One of the most important Neuro-Fuzzy systems is the Adaptive Neuro-Fuzzy Inference System (Jang, 1993). The ANFIS architecture uses a Sugeno-type fuzzy system. Assuming that the FIS under consideration has two inputs and one output, the rule base contains two fuzzy if-then rules of the Takagi and Sugeno type as follows:

Rule 1: If x is A1 and y is B1, then f1 = p1 · x + q1 · y + r1
Rule 2: If x is A2 and y is B2, then f2 = p2 · x + q2 · y + r2

- The ANFIS architecture consists of five layers as described below:

Layer 1. Every node in this layer is a square node with a node function.

\[ O_i^1 = \mu_{A_i}(x) \]

where \( x \) is the input to node \( i \) and \( A_i \) is the linguistic label (small, large, etc.) associated with this node function. In other words, \( O_i^1 \) is the membership function of \( A_i \) and specifies the degree to which the given \( x \) satisfies the quantifier \( A_i \). Parameters in this layer are referred to as premise parameters.

Layer 2. Every node in this layer is a circle node labelled \( n \), which multiplies the incoming signal and sends the product out.

Layer 3. Every node in this layer is a circle node labelled \( N \). The \( i \)th node calculates the ratio of the ith rules firing strength to the sum of all rules’ firing strengths:

\[ \overline{W}_i = \frac{w_i}{w_1 + w_2} \quad i = 1, 2 \]

For convenience, the output of this layer will be called normalised firing strengths.

Layer 4. Every node \( i \) in this layer is a square node with a node function:

\[ O_i^4 = \overline{W}_i * f_i = \overline{W}_i (p_i * x + q_i * y + r_i) \]

where \( w_i \) is the output of layer 3 and \( p_i, q_i, r_i \) is the parameter set. Parameters in this layer will be referred to as consequent parameters.

Layer 5. The single node in this layer is a circle node labelled \( \Sigma \) that computes the overall output as the summation of all incoming signals:

\[ O_i^5 = \sum_i \overline{W}_i * f_i = \frac{\sum_i w_i * f_i}{\sum_i w_i} \]

Training of the system requires establishing the parameters of the membership functions and the parameters of the output functions of the Sugeno-type fuzzy system \( p_i, q_i, r_i \). The latter are found in levels 4 and 5 of the ANFIS. Jang proposed a hybrid method to optimise the ANFIS by using two phases: a forward pass and a backward pass. The forward pass uses the Least Square method to optimise the consequent parameters.
(\(p_i, q_i, r_i\)) in levels 4 and 5, while the backward pass uses a gradient descent algorithm such as the back propagation algorithm to optimise the premise parameters of the membership functions used as inputs in levels 1 to 3.

e. Autoregressive (AR) models. An AR model is a type of random process often used to model and predict various types of natural phenomena. It is one of a group of linear prediction formulas that attempt to predict an output of a system based on the previous outputs. AR models are used in time series analysis to describe stationary time series. These models represent time series that are generated by passing the white noise through a recursive linear filter. The output of such a filter at the moment \(t\) is a weighted sum of \(m\) previous values of the filter output. The integer parameter \(m\) is called the order of the AR model.

The AR model of a random process \(y(t)\) in discrete time \(t\) is defined by the following expression:

\[
y(t) = \sum_{i=1}^{m} \alpha(i) y(t-i) + \varepsilon(t)
\]

where \(\alpha_1, \alpha_2, ... \alpha_m\) are the coefficients of the recursive filter, \(m\) is the order of the model and \(\varepsilon(t)\) are output uncorrelated errors.

f. Autoregressive Moving Average (ARMA) models. In statistical and signal processing, ARMA models, sometimes called Box-Jenkins models, are typically applied to autocorrelated time series data. ARMA is the model for predicting time series data. The model uses no other independent variables, but the prediction comes out based on the historical price of gold. ARMA models are particularly useful when information is limited to a single stationary series or when there is no relevant economic theory. This univariate time series model combines a moving average process with a linear difference equation. The model is then explained purely in terms of its own past values. Given a time series of data \(x_t\), the ARMA model is a tool for understanding and perhaps predicting future values in this series. The model consists of two parts, an AR part and an MA part. The model is usually then referred to as the ARMA \((p,q)\) model where \(p\) is the order of the autoregressive part and \(q\) is the order of the moving average part. This model contains AR\((p)\) and MA\((q)\) models:

\[
x_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i x_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}
\]

where \(\varphi_1, \varphi_2, ... \varphi_p\) and \(\theta_1, \theta_2, \theta_q\) are the parameters of the model AR\((p)\) and MA\((q)\), respectively, \(c\) is a constant and \(\varepsilon\) is white noise. The constant term is omitted by many authors for simplicity.

The proposed methodology. For the empirical analysis, we use historical/past daily gold prices as inputs in the forecasting system. The forecasting was carried out using an Adaptive Neural Network with the Fuzzy Inference System (ANFIS). The ANFIS model is a Sugeno first-order model with one input \(x(\cdot-1)\) and one output \(y(\cdot)\) that corresponds to the next day’s price. The data are 1681 daily gold price observations from 31/7/2002 to 2/4/2009. As inputs, the model receives the 1587 observations from 31/7/2002 to 14/11/2008, while the remaining 94 observations from 15/11/2008 to 3/4/2009 are used as testing data. In Figure 1, a graphic representation of the overall data (1681 observations) is shown, and in Figure 2, a scatter plot of the training data that displays the relationship between the input and the output of the system appears.
The remaining datasets (94 observations) are the testing data used to evaluate the suggested model.

Different types of membership functions were examined:

- Generalised bell-shaped (Gbellmf)
- Triangular-shaped (trimf)
- Gaussian-shaped (gaussmf)
- Gauss-2-shaped (gauss2mf)
- Trapezoidal-shaped (trapmf)
In addition, different numbers of membership functions – two, three, four and five – were examined, creating two to five rules, respectively.

Each rule is of the form:

If \( y(k-1) \) is very small, then \( y(k) = f = p_1 \cdot y(k-1) + r_1 \)

where \((p_1, r_1)\) is a parameter set with the values calculated and optimised during the learning phase. The parameters of the first part (premise) of the rule are optimised using the Error Back Propagation gradient descent method while the parameters of the second part (consequent) are optimised using the Least Square error method.

The initial membership functions and the final one after training are presented in Figure 3.

**Figure 3** The membership functions before (a) and after (b) the training of the ANFIS controller (see online version for colours)

After many trial-and-error attempts, it is determined that using 400 epochs is the best number of epochs as using more did not reduce the RMSE any further. The evolution of the RMSE and the step size (0.01) according to the number of epochs are shown in Figure 4.

**Figure 4** The evolution of the RMSE and the step size according to the number of epochs (see online version for colours)
4 Results and discussion

This paper uses the ANFIS to predict the price of gold. Different types and numbers of membership functions are examined. The performance of the ANFIS for all performed combinations of parameters is evaluated according to statistical errors such as Mean Square Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). Table 1 presents the forecasting accuracy results of the ANFIS model for all types and numbers of membership functions. The RMSE has been chosen as the accuracy measure as it is the most commonly used type of error. As can be observed in Table 1, the smallest error corresponds for Gauss-2-shaped and five membership functions and is equal to 2.0154. Therefore, this combination of parameters can lead to more accurate prediction by the ANFIS model for forecasting the next-day price of gold.

The model displayed a high degree of correct prediction. The comparison between the actual values and the ANFIS predicted values is presented in the following figure (Figure 5):

<table>
<thead>
<tr>
<th>Number of MFs</th>
<th>gbellmf</th>
<th>trimf</th>
<th>gaussmf</th>
<th>gauss2mf</th>
<th>trapmf</th>
</tr>
</thead>
<tbody>
<tr>
<td>mf_no = 2</td>
<td>2.0626</td>
<td>2.0637</td>
<td>2.0599</td>
<td>2.0623</td>
<td>2.0600</td>
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<tr>
<td>mf_no = 3</td>
<td>2.0705</td>
<td>2.1200</td>
<td>2.0334</td>
<td>2.0699</td>
<td>2.0575</td>
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<tr>
<td>mf_no = 4</td>
<td>2.0390</td>
<td>2.2017</td>
<td>2.0633</td>
<td>2.0536</td>
<td>2.0677</td>
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<tr>
<td>mf_no = 5</td>
<td>2.0953</td>
<td>2.8602</td>
<td>2.0441</td>
<td>2.0154</td>
<td>2.0342</td>
</tr>
</tbody>
</table>

Figure 5 Actual values and ANFIS predicted values (see online version for colours)

In the following figures (Figures 6, 7 and 8), the comparisons between the actual values and the predicted values from models AR, ARMA and NN are depicted, respectively.
Figure 6  Actual values and AR predicted values (see online version for colours)

Figure 7  Actual values and ARMA predicted values (see online version for colours)

Figure 8  Actual values and NN predicted values (see online version for colours)
Gold price forecasting with a neuro-fuzzy-based inference system

The results derived from the ANFIS application are compared with those obtained from the use of the neural network and traditional methods such as AR and ARMA. The comparison of these models is presented in Table 2 and is carried out using the four types of errors (MSE, RMSE, MAE, MAPE). The results are defined below:

| Table 2 | Evaluation of the models’ performance (ANFIS, NN, AR, ARMA) according to statistical performance measures |
|------------------------|------------------------|------------------------|------------------------|
| Errors | ANFIS | AR | ARMA | Neural network |
| MSE | 4.0619 | 7.5951 | 7.6070 | 4.0671 |
| RMSE | 2.0154 | 2.7559 | 2.7581 | 2.0167 |
| MAE | 1.5532 | 2.0894 | 2.0913 | 1.5395 |
| MAPE | 3.7064 | 4.9241 | 4.9289 | 3.6690 |

As can be observed in Table 2, the ANFIS ranked first as it gives the smallest RMSE followed by NN and then by the AR and ARMA models.

For further evaluation of the ANFIS performance, it is also examined according to gold’s earnings. Thus, a comparison with the ‘Buy and Hold’ strategy is performed. According to this strategy, the investor invests an amount of money in gold and holds the investment until the end of the simulation horizon. However, using the proposed model, the investor buys gold when there is a predicted up-trend and sells when there is a predicted down-trend for the next day. For example, an indicative amount of 10,000 € is allocated as an initial investment. The rate of return (ROR) over the out-of-sample forecasts is equal to 410.72% after 94 days according to the ANFIS while the ROR that the B&H strategy returns is 213% over the same period. Consequently, the ANFIS outperforms by far, giving almost twice the returns of the B&H strategy (197.72% more returns). In Table 3 is presented the ROR that the ANFIS and the B&H strategy returns.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Comparison of the ROR with the B&amp;H strategy</th>
</tr>
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<tbody>
<tr>
<td>Method</td>
<td>Rate of return</td>
</tr>
<tr>
<td>Proposed ANFIS</td>
<td>410.72%</td>
</tr>
<tr>
<td>B&amp;H strategy</td>
<td>213%</td>
</tr>
<tr>
<td>Performance difference</td>
<td>197.72%</td>
</tr>
</tbody>
</table>

5 Conclusions

An ANFIS was developed to forecast the next-day price of gold. The ANFIS model is a Sugeno first-order model with one input \( y(k−1) \) and one output \( y(k) \). As inputs, the model receives the past gold prices while as output it gives the next-day price of gold. Different types and numbers of membership functions are examined. The ANFIS performance for all examined combinations of parameters is evaluated according to statistical errors such as the MSE, RMSE, MAE and MAPE. Among the above, the RMSE is chosen as the accuracy measure of the models’ performance. The smallest RMSE corresponds to the combination of gauss-2-shaped and five membership functions, meaning that this combination of parameters gives the most accurate prediction of the ANFIS.
The results derived from the ANFIS application are compared with those obtained from the use of the NN, AR and ARMA models. The proposed system has performed much better than the other models (AR, ARMA, NN) in terms of the RMSE. For further evaluation, the ANFIS performance is also examined according to the gold trading earnings. Thus, a comparison with the 'Buy and Hold' strategy is performed. The results demonstrate clearly that the proposed system (ROR = 410.72%) outperforms the B&H strategy (ROR = 213%) by far based on the returns. Reported case studies do not consider commission and tax liabilities meaning that the actual returns will be less than those reported.

In conclusion, the ANFIS model clearly demonstrates the potential of neuro-fuzzy-based modelling for predicting the price of gold.

References


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