
New similarity measures for Pythagorean fuzzy sets with applications

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Abstract: The concept of Pythagorean fuzzy sets (PFSs) is pertinent in finding reliable solution to decision-making problems, because of its unique nature of indeterminacy. Pythagorean fuzzy set is characterised by membership degree, non-membership degree, and indeterminate degree in such a way that the sum of the square of each of the parameters is one. The objective of this paper is to present some new similarity measures for PFSs by incorporating the conventional parameters that describe PFSs, with applications to some real-life decision-making problems. Furthermore, an illustrative example is used to establish the applicability and validity of the proposed similarity measures and compare the results with the existing comparable similarity measures to show the effectiveness of the proposed similarity measures. While analysing the reliability of the proposed similarity measures in comparison to analogous similarity measures for PFSs in literature, we discover that the proposed similarity measures, especially, s_4 yields the most reasonable measure. Finally, we apply s_4 to decision-making problems such as career placement, medical diagnosis, and electioneering process. Additional applications of these new similarity measures could be exploited in decision making of real-life problems embedded with uncertainty such as in multi-criteria decision-making (MCDM) and multi-attribute decision-making (MADM), respectively.

Keywords: fuzzy set; intuitionistic fuzzy set; similarity measure; Pythagorean fuzzy set.

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1 Introduction

Decision making is a universal process in the life of human beings, which can be described as the final outcome of some mental and reasoning processes that lead to the selection of the best alternative. In many situations, it is difficult for decision makers to precisely express a preference regarding relevant alternatives under several criteria, especially when relying on inaccurate, uncertain, or incomplete information (Zhang, 2016). Multi-criteria decision-making (MCDM) is defined as the process that involves the analysis of a finite set of alternatives and ranking them in terms of how credible they are to decision makers when all the criteria are considered simultaneously (Garg and Rani, 2019a). To this end, the theory of fuzzy sets was introduced by Zadeh (1965) to address multi-criteria decision-making (MCDM) problems within uncertainty. Fuzzy set is characterised by a membership function, μ which takes value from a crisp set to a unit interval $I = [0, 1]$. With the massive imprecise and vague information in the real world, diverse extensions of fuzzy set have been developed by some researchers. The notion of intuitionistic fuzzy sets (IFSs) was proposed by Atanassov (1983, 1986, 1989, 2012) as a generalised framework of fuzzy sets. Sequel to the introduction of IFSs, a lot of attentions have been paid on developing measures for IFSs, as a way to apply them to solving many decision-making problems. As a result, some measures were proposed (see Szmidt, 2014; Ye, 2011; Liang and Shi, 2003; Boran and Akay, 2014). Some applications of IFSs have been carried out in medical diagnosis (see Davvaz and Sadrabadi, 2016; De et al., 2001; Ejegwa and Modom, 2015; Szmidt and Kacprzyk, 2001, 2004; Ejegwa and Onasanya, 2019), career determination (Ejegwa et al., 2014a; Ejegwa and Onyike, 2019), selection process (Ejegwa, 2015), and other multi-criteria decision making problems (Ejegwa et al., 2014b, 2014c, 2016; Garg and Singh, 2018, 2018a, 2018b), among other, some using measures for IFSs.

The idea of Pythagorean fuzzy sets (PFSs) proposed by Yager (2013a, 2013b) is a new tool to deal with vagueness considering the membership grade, μ and non-membership grade, ν in such a way that the sum of the square of each of the membership grade and the non-membership grade is less than or equal to one, unlike in IFSs. Honestly speaking, the origin of Pythagorean fuzzy sets emanated from intuitionistic fuzzy sets of second type (IFSST) introduced in Atanassov (1989, 1999) as generalised IFSs. As a generalised set, PFS has close relationship with IFS. The concept of PFSs can be used to characterise uncertain information more sufficiently and accurately than IFSs. Garg (2017b) presented an improved score function for the ranking order of interval-valued Pythagorean fuzzy sets (IVPFSs). Based on it, a Pythagorean fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) method by taking the preferences of the experts in the form of interval-valued Pythagorean fuzzy decision matrices was discussed. In fact, the theory of PFSs has been extensively studied, as shown in Beliakov and James (2014), Dick et al. (2016), Garg (2018a, 2018b, 2018d, 2018e, 2019b), Gou et al. (2016), He et al. (2016), Liang and Xu (2017), Mohagheghi et al. (2017), Peng and Yang (2015) and Peng and Selvachandran (2017).

In connection to the applications of PFSs in real-life situations, Rahman et al. (2017) worked on some geometric aggregation operators on interval-valued PFSs (IVPFSs) and applied same to group decision-making problem. Perez-Dominguez et al. (2018) presented a multiobjective optimisation on the basis of ratio analysis (MOORA) under PFS setting and applied it to MCDM problem. Rahman et al. (2018b) proposed some approaches to multi-attribute group decision making based on induced interval-valued

Pythagorean fuzzy Einstein aggregation operator. In a nutshell, the idea of Pythagorean fuzzy sets has attracted great attentions of many scholars, and the concept has been applied to several application areas viz; multi-criteria decision making (MCDM), multi-attribute decision making (MADM), aggregation operators, information measures, etc. For details, see Ejegwa (2019a, 2019b, Ejegwa and Awolola (2019), Garg (2018a, 2018b, 2018d, 2018e, 2019b), Liang and Xu (2017), Mohagheghi et al. (2017), Gao and Wei (2018), Rahman et al. (2018a), Rahman and Abdullah (2018), Khan et al. (2018a, 2018b), Garg (2016a, 2016b, 2016c, 2017a, 2018c), Du et al. (2017), Hadi-Venchen and Mirjaberi (2014), Yager (2014, 2016), Yager and Abbasov (2013), Zhang and Xu (2014) and Zhang (2016).

Similarity measure for PFSs is a function that shows how comparable two or more PFSs are to each other. In fact, it is a dual concept of distance measure for PFSs. Similarity measures for PFSs have gained much attentions for their wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. Many measures of similarity between PFSs have been proposed and researched in recent years, in short, from different perspectives. In Li and Zeng (2018), some distance/dissimilarity measures for PFSs and Pythagorean fuzzy numbers, which take into account four parameters, were proposed. It is observed that, the four parameters are not the conventional features of PFSs. Also, in Peng (2018), a new similarity measure and a new dissimilarity measure for Pythagorean fuzzy set were introduced by incorporating four parameters more than the three traditional components of PFSs. The notions of similarity and dissimilarity of PFSs as extension of the work in Li and Zeng (2018) were introduced in Zeng et al. (2018) by incorporating five parameters, and applied to multi-criteria decision-making (MCDM) problems. Howbeit, the five parameters captured are not the traditional components of PFSs. In Wei and Wei (2018), some similarity measures between PFSs based on the cosine function were proposed by considering the degree of membership, degree of non-membership and degree of hesitation, and applied to pattern recognition and medical diagnosis. The notion of dissimilarity measure for PFSs studied in Zhang and Xu (2014) incorporated only the three traditional parameters of PFSs, notwithstanding, the measure failed the metric distance conditions. A similarity measure for PFSs based on the combination of cosine similarity measure and Euclidean distance measure featuring only membership and non-membership degrees were introduced in Mohd and Abdullah (2018). Of recent, some dissimilarity and similarity measures for PFSs which satisfied the metric distance conditions were introduced in Ejegwa (2018) by incorporating the three conventional parameters of PFSs.

As a result of the survey of some similarity measures for PFSs (Peng, 2018; Zeng et al., 2018; Wei and Wei, 2018; Mohd and Abdullah, 2018); especially, those similarity measures (Ejegwa, 2018) that incorporated the three conventional parameters of PFSs, the need to propose new similarity measures for PFSs with more reasonable, reliable, and efficient output, is undeniable. Thus, the motivation of this work. This paper explores some new similarity measures for PFSs. By taking into account the three parameters characterisation of PFSs (viz; membership degree, non-membership degree and indeterminate degree), we propose some new similarity measures for PFSs with applications to decision-making problems. The paper is organised by presenting some basic notions of PFSs in Section 2. In Section 3, we reiterate some similarity measures for PFSs studied in Ejegwa (2018) with some numerical examples. Also in Section 3,

some new similarity measures for PFSs are proposed with numerical examples. Section 4 discusses some applications of the most accurate of the similarity measures for PFSs to decision-making problems. Finally, Section 5 summarises the resulted outcomes of the paper with direction for future studies.

2 Basic notions of Pythagorean fuzzy sets

We recall some basic notions of fuzzy sets, IFSs and PFSs to be used in the sequel.

Definition 2.1 (Zadeh, 1965): Let X be a non-empty set. A fuzzy set A of X is characterised by a membership function

$$\mu_A : X \rightarrow [0, 1].$$

That is,

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \notin X \\ (0, 1), & \text{if } x \text{ is partly in } X \end{cases}$$

Alternatively, a fuzzy set A of X is an object having the form

$$A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \} \text{ or } A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle \mid x \in X \right\},$$

where the function

$$\mu_A(x) : X \rightarrow [0, 1]$$

defines the degree of membership of the element, $x \in X$.

Definition 2.2 (Atanassov, 1983, 1986): Let a non-empty set X be fixed. An IFS A of X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

or

$$A = \left\{ \left\langle \frac{\mu_A(x), \nu_A(x)}{x} \right\rangle \mid x \in X \right\},$$

where the functions

$$\mu_A(x) : X \rightarrow [0, 1] \text{ and } \nu_A(x) : X \rightarrow [0, 1]$$

define the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For each A in X ,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is the intuitionistic fuzzy set index or hesitation margin of x in X . The hesitation margin $\pi_A(x)$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A(x) \in [0, 1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus,

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1.$$

Definition 2.3 (Yager, 2013a, 2013b): Let X be a universal set. Then a Pythagorean fuzzy set A which is a set of ordered pairs over X , is defined by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

or

$$A = \left\{ \left\langle \frac{\mu_A(x), \nu_A(x)}{x} \right\rangle \mid x \in X \right\},$$

where the functions

$$\mu_A(x) : X \rightarrow [0, 1] \text{ and } \nu_A(x) : X \rightarrow [0, 1]$$

define the degree of membership and the degree of non-membership, respectively of the element $x \in X$ to A , which is a subset of X , and for every $x \in X$,

$$0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1.$$

Supposing $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]}$ and $\pi_A(x) \in [0, 1]$. In what follows, $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$.

We denote the set of all PFSs over X by $PFS(X)$.

Example 2.1: Let $A \in PFS(X)$. Suppose $\mu_A(x) = 0.70$ and $\nu_A(x) = 0.50$ for $X = \{x\}$. Clearly, $0.70 + 0.50 \not\leq 1$, but $0.70^2 + 0.50^2 \leq 1$. Thus $\pi_A(x) = 0.5099$, and hence $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$.

Table 1 explains the difference between Pythagorean fuzzy sets and intuitionistic fuzzy sets (Ejzegwa, 2018).

Table 1 Intuitionistic fuzzy sets and Pythagorean fuzzy sets

| <i>Intuitionistic fuzzy sets</i> | <i>Pythagorean fuzzy sets</i> |
|----------------------------------|--|
| $\mu + \nu \leq 1$ | $\mu + \nu \leq 1$ or $\mu + \nu \geq 1$ |
| $0 \leq \mu + \nu \leq 1$ | $0 \leq \mu^2 + \nu^2 \leq 1$ |
| $\pi = 1 - (\mu + \nu)$ | $\pi = \sqrt{1 - [\mu^2 + \nu^2]}$ |
| $\mu + \nu + \pi = 1$ | $\mu^2 + \nu^2 + \pi^2 = 1$ |

Definition 2.4 (Yager, 2013a): Let $A, B \in PFS(X)$. Then $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \forall x \in X$.

Definition 2.5 (Yager, 2013a, 2013b): Let $A, B \in PFS(X)$. Then the following are defined thus:

- 1 $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle | x \in X\}$.
- 2 $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle | x \in X\}$.
- 3 $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \in X\}$.
- 4 $A \oplus B = \{\langle x, \sqrt{(\mu_A(x))^2 + (\mu_B(x))^2 - (\mu_A(x))^2(\mu_B(x))^2}, \nu_A(x)\nu_B(x) \rangle | x \in X\}$.
- 5 $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \sqrt{(\nu_A(x))^2 + (\nu_B(x))^2 - (\nu_A(x))^2(\nu_B(x))^2} \rangle | x \in X\}$.

Remark 2.1: (Ejegwa, 2018): Let $A, B, C \in PFS(X)$. By Definition 2.5, the following properties hold:

- 1 Complementary property:

$$(A^c)^c = A$$

- 2 Idempotent property:

$$A \cap A = A$$

$$A \cup A = A$$

$$A \oplus A \neq A$$

$$A \otimes A \neq A$$

- 3 Commutative property:

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

$$A \oplus B = B \oplus A$$

$$A \otimes B = B \otimes A$$

- 4 Associative property:

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

$$A \otimes (B \otimes C) = (A \otimes B) \otimes C$$

5 Distributive property:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

$$A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$$

$$A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$$

$$A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$$

6 De Morgan property:

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \oplus B)^c = A^c \otimes B^c$$

$$(A \otimes B)^c = A^c \oplus B^c.$$

Definition 2.6: Let $A \in PFS(X)$. Then the level/ground set of A is defined by

$$A_* = \{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 1 \forall x\}.$$

Certainly, A_* is a subset of X .

3 Similarity measures for Pythagorean fuzzy sets

Similarity measure (SM) for PFSs is a dual concept of distance measure for PFSs (Ejegwa, 2018). Firstly, we recall the axiomatic definition of similarity for Pythagorean fuzzy sets.

Definition 3.1(Ejegwa, 2018): Let X be non-empty set and $A, B, C \in PFS(X)$. The similarity measure s between A and B is a function $s : PFS \times PFS \rightarrow [0, 1]$ satisfies

1 $0 \leq s(A, B) \leq 1$ (boundedness)

2 $s(A, B) = 1$ iff $A = B$ (separability)

3 $s(A, B) = s(B, A)$ (symmetric)

4 $s(A, C) + s(B, C) \geq s(A, B)$ (triangle inequality).

Proposition 3.1 (Ejegwa, 2018): Let $A, B, C \in PFS(X)$. Suppose $A \subseteq B \subseteq C$, then

1 $d(A, C) \geq d(A, B)$ and $d(A, C) \geq d(B, C)$

2 $s(A, C) \leq s(A, B)$ and $s(A, C) \leq s(B, C)$.

Now, we state a new result on similarity measure for PFSs.

Proposition 3.2: If $A, B, C \in PFS(X)$, such that $A \subseteq B \subseteq C$, then

$$s(A, C) \leq \min[s(A, B), s(B, C)].$$

Proof: Let $A, B, C \in PFS(X)$. Assume that $A \subseteq B \subseteq C$, then by Proposition 3.1 we have $s(A, B) \geq s(A, C)$ and $s(B, C) \geq s(A, C)$. Hence

$$s(A, C) \leq \min[s(A, B), s(B, C)].$$

□

By incorporating the three parameters of PFSs, the following similarity measures for PFSs were proposed in Ejegwa (2018). Let $A, B \in PFS(X)$ such that $X = \{x_1, \dots, x_n\}$, then

$$\begin{aligned} s_1(A, B) &= 1 - \frac{1}{2n} \sum_{i=1}^n [|\mu_A(x_i) - \mu_B(x_i)| \\ &\quad + |\nu_A(x_i) - \nu_B(x_i)| \\ &\quad + |\pi_A(x_i) - \pi_B(x_i)|], \\ s_2(A, B) &= 1 - \left(\frac{1}{2n} \sum_{i=1}^n [(\mu_A(x_i) - \mu_B(x_i))^2 \right. \\ &\quad + (\nu_A(x_i) - \nu_B(x_i))^2 \\ &\quad \left. + (\pi_A(x_i) - \pi_B(x_i))^2] \right)^{\frac{1}{2}}, \\ s_3(A, B) &= 1 - \frac{1}{2n} \sum_{i=1}^n [|(\mu_A(x_i))^2 - (\mu_B(x_i))^2| \\ &\quad + |(\nu_A(x_i))^2 - (\nu_B(x_i))^2| \\ &\quad + |(\pi_A(x_i))^2 - (\pi_B(x_i))^2|] \end{aligned}$$

3.1 Some new similarity measures for PFSs

We propose some new similarity measures for Pythagorean fuzzy sets, and exemplify the measures to determine their compliant to Definition 3.1.

Let $A, B \in PFS(X)$ such that $X = \{x_1, \dots, x_n\}$. By incorporating the three parameters of PFSs, we propose the following new similarity measures for PFSs:

$$\begin{aligned} s_4(A, B) &= 1 - \frac{1}{4n} \sum_{i=1}^n [|\mu_A(x_i) - \mu_B(x_i)| \\ &\quad + ||\mu_A(x_i) - \nu_A(x_i)| - |\mu_B(x_i) - \nu_B(x_i)|| \\ &\quad + ||\mu_A(x_i) - \pi_A(x_i)| - |\mu_B(x_i) - \pi_B(x_i)||] \end{aligned}$$

$$s_5(A, B) = 1 - \frac{1}{4n} \sum_{i=1}^n [|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| + 2 \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)|\}]$$

$$s_6(A, B) = 1 - \left(\frac{1}{4n} \sum_{i=1}^n [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2 + 2 \max\{(\mu_A(x_i) - \mu_B(x_i))^2, (\nu_A(x_i) - \nu_B(x_i))^2, (\pi_A(x_i) - \pi_B(x_i))^2\}] \right)^{\frac{1}{2}}$$

where

$$\pi_A(x_i) = \sqrt{1 - [(\mu_A(x_i))^2 + (\nu_A(x_i))^2]}$$

and

$$\pi_B(x_i) = \sqrt{1 - [(\mu_B(x_i))^2 + (\nu_B(x_i))^2]}.$$

3.2 Numerical examples

We now verify whether these similarity measures satisfy the conditions in Definition 3.1 by using the examples in Ejegwa (2018).

Example 3.1 Let $A, B, C \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$. Suppose

$$A = \left\{ \left\langle \frac{0.6, 0.2}{x_1} \right\rangle, \left\langle \frac{0.4, 0.6}{x_2} \right\rangle, \left\langle \frac{0.5, 0.3}{x_3} \right\rangle \right\},$$

$$B = \left\{ \left\langle \frac{0.8, 0.1}{x_1} \right\rangle, \left\langle \frac{0.7, 0.3}{x_2} \right\rangle, \left\langle \frac{0.6, 0.1}{x_3} \right\rangle \right\}$$

and

$$C = \left\{ \left\langle \frac{0.9, 0.2}{x_1} \right\rangle, \left\langle \frac{0.8, 0.2}{x_2} \right\rangle, \left\langle \frac{0.7, 0.3}{x_3} \right\rangle \right\}.$$

Calculating the similarity using the proposed similarity measures above, we have

$$s_4(A, B) = 1 - \frac{1}{12} \sum_{i=1}^3 [|0.6 - 0.8| + ||0.6 - 0.2| - |0.8 - 0.1|| + ||0.6 - 0.7746| - |0.8 - 0.5916|| + |0.4 - 0.7| + ||0.4 - 0.6| - |0.7 - 0.3|| + ||0.4 - 0.6928| - |0.7 - 0.6481|| + |0.5 - 0.6| + ||0.5 - 0.3| - |0.6 - 0.1|| + ||0.5 - 0.8124| - |0.6 - 0.7937||] = 0.8505$$

$$\begin{aligned}
 s_5(A, B) &= 1 - \frac{1}{12} \sum_{i=1}^3 [|0.6 - 0.8| + |0.2 - 0.1| + |0.7746 - 0.5916| \\
 &\quad + 2 \max\{ |0.6 - 0.8|, |0.2 - 0.1|, |0.7746 - 0.5916| \} \\
 &\quad + |0.4 - 0.7| + |0.6 - 0.3| + |0.6928 - 0.6481| \\
 &\quad + 2 \max\{ |0.4 - 0.7|, |0.6 - 0.3|, |0.6928 - 0.6481| \} \\
 &\quad + |0.5 - 0.6| + |0.3 - 0.1| + |0.8124 - 0.7937| \\
 &\quad + 2 \max\{ |0.5 - 0.6|, |0.3 - 0.1|, |0.8124 - 0.7937| \}] \\
 &= 0.7628
 \end{aligned}$$

$$\begin{aligned}
 s_6(A, B) &= 1 - \left(\frac{1}{12} \sum_{i=1}^3 [(0.6 - 0.8)^2 + (0.2 - 0.1)^2 + (0.7746 - 0.5916)^2 \right. \\
 &\quad + 2 \max\{ (0.6 - 0.8)^2, (0.2 - 0.1)^2, (0.7746 - 0.5916)^2 \} \\
 &\quad + (0.4 - 0.7)^2 + (0.6 - 0.3)^2 + (0.6928 - 0.6481)^2 \\
 &\quad + 2 \max\{ (0.4 - 0.7)^2, (0.6 - 0.3)^2, (0.6928 - 0.6481)^2 \} \\
 &\quad + (0.5 - 0.6)^2 + (0.3 - 0.1)^2 + (0.8124 - 0.7937)^2 \\
 &\quad \left. + 2 \max\{ (0.5 - 0.6)^2, (0.3 - 0.1)^2, (0.8124 - 0.7937)^2 \} \right)^{\frac{1}{2}} \\
 &= 0.7662
 \end{aligned}$$

Similarly, we obtain

$$\begin{aligned}
 s_4(A, C) &= 0.7952, \quad s_5(A, C) = 0.6706, \quad s_6(A, C) = 0.6654, \\
 s_4(B, C) &= 0.8976, \quad s_5(B, C) = 0.8216, \quad s_6(B, C) = 0.8309.
 \end{aligned}$$

Remark 3.1: From Example 3.1, we observe that

- 1) $s_i(A, B), s_i(A, C), s_i(B, C) \in [0, 1] \forall i.$
- 2) $s_i(A, B) = s_i(A, C) = s_i(B, C) = 1$ iff $A = B = C \forall i.$
- 3) $s_i(A, B) = s_i(B, A), s_i(A, C) = s_i(C, A)$ and $s_i(B, C) = s_i(C, B) \forall i.$
- 4) $s_i(A, C) + s_i(B, C) \geq s_i(A, B) \forall i,$

where $i = 1, 2, 3, 4, 5, 6.$

Clearly, conditions 1–4 of Definition 3.1 hold for all the similarity measures.

Table 2 contains all the values of the existing and the proposed similarity measures using Example 3.1 via the three parameters approach.

Table 2 Numerical Outputs of Example 3.1

| <i>SM</i> | s_1 | s_2 | s_3 | s_4 | s_5 | s_6 |
|-----------|--------|--------|--------|--------|--------|--------|
| $s(A, B)$ | 0.7589 | 0.7706 | 0.7600 | 0.8505 | 0.7628 | 0.7662 |
| $s(A, C)$ | 0.6702 | 0.6726 | 0.6100 | 0.7952 | 0.6706 | 0.6654 |
| $s(B, C)$ | 0.8113 | 0.8368 | 0.8133 | 0.8976 | 0.8216 | 0.8309 |

Howbeit, we consider a case where the level/ground sets of the PFSs are not equal, to determine whether the similarities will satisfy the aforesaid conditions.

Example 3.2: Let $A, B, C \in PFS(X)$ for $X = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose

$$A = \left\{ \left\langle \frac{0.6, 0.4}{x_1} \right\rangle, \left\langle \frac{0.5, 0.7}{x_2} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\},$$

$$B = \left\{ \left\langle \frac{0.7, 0.3}{x_1} \right\rangle, \left\langle \frac{0.4, 0.7}{x_3} \right\rangle, \left\langle \frac{0.9, 0.2}{x_4} \right\rangle \right\}$$

and

$$C = \left\{ \left\langle \frac{0.6, 0.4}{x_2} \right\rangle, \left\langle \frac{0.7, 0.3}{x_3} \right\rangle, \left\langle \frac{0.5, 0.4}{x_4} \right\rangle \right\}.$$

These PFSs could be rewritten thus:

$$A = \left\{ \left\langle \frac{0.6, 0.4}{x_1} \right\rangle, \left\langle \frac{0.5, 0.7}{x_2} \right\rangle, \left\langle \frac{0.8, 0.4}{x_3} \right\rangle, \left\langle \frac{0.0, 1.0}{x_4} \right\rangle, \left\langle \frac{0.7, 0.2}{x_5} \right\rangle \right\},$$

$$B = \left\{ \left\langle \frac{0.7, 0.3}{x_1} \right\rangle, \left\langle \frac{0.0, 1.0}{x_2} \right\rangle, \left\langle \frac{0.4, 0.7}{x_3} \right\rangle, \left\langle \frac{0.9, 0.2}{x_4} \right\rangle, \left\langle \frac{0.0, 1.0}{x_5} \right\rangle \right\}$$

and

$$C = \left\{ \left\langle \frac{0.0, 1.0}{x_1} \right\rangle, \left\langle \frac{0.6, 0.4}{x_2} \right\rangle, \left\langle \frac{0.7, 0.3}{x_3} \right\rangle, \left\langle \frac{0.5, 0.4}{x_4} \right\rangle, \left\langle \frac{0.0, 1.0}{x_5} \right\rangle \right\}.$$

We rewrite the PFSs for easily calculations. Using the proposed similarity measures, we obtain the following similarities;

$$s_4(A, B) = 0.5634, s_5(A, B) = 0.4054, s_6(A, B) = 0.3855.$$

Similarly, we get

$$s_4(A, C) = 0.5867, s_5(A, C) = 0.3873, s_6(A, C) = 0.3846,$$

and

$$s_4(B, C) = 0.6142, s_5(B, C) = 0.4968, s_6(B, C) = 0.4625.$$

Remark 3.2: Although the level/ground sets of the PFSs considered here are not equal, we get observations that coincide with Remark 3.1.

Table 3 contains all the values of the existing and the proposed similarity measures using Example 3.2 via the three parameters approach.

Table 3 Numerical outputs of Example 3.2

| <i>SM</i> | s_1 | s_2 | s_3 | s_4 | s_5 | s_6 |
|-----------|--------|--------|--------|--------|--------|--------|
| $s(A, B)$ | 0.3328 | 0.3602 | 0.4220 | 0.5634 | 0.4054 | 0.3855 |
| $s(A, C)$ | 0.3070 | 0.3524 | 0.3620 | 0.5867 | 0.3873 | 0.3846 |
| $s(B, C)$ | 0.4322 | 0.4345 | 0.4580 | 0.6142 | 0.4968 | 0.4625 |

3.3 Discussion

From Tables 2 and 3, it follows that the new similarity measures, s_4 , s_5 and s_6 satisfy the conditions of Definition 3.1 and hence, they are appropriate similarity measures for PFSs. Notwithstanding, the existing similarity measures, s_1 , s_2 and s_3 are certified similarity measures for PFSs (Ejegwa, 2018).

Moreso, s_4 is the most reasonable of the similarity measures discussed, because it provides the greatest similarity when compare to the existing similarity measures for PFSs that capture the three main parameters of PFSs. Clearly,

$$s_4(A, B) > s_i(A, B),$$

$$s_4(A, C) > s_i(A, C)$$

and

$$s_4(B, C) > s_i(B, C) \quad \forall i = 1, 2, 3, 5, 6.$$

Therefore, we adopt s_4 for application to electoral process, career placement and disease diagnosis, respectively.

4 Applicative examples

In this section, we provide some applications of Pythagoren fuzzy sets using s_4 in the areas of electoral process, career placement and disease diagnosis, respectively.

4.1 Application in electoral process

Suppose there are five provinces in a certain nation with electoral qualification requirements for a position P represented by PFS

$$\tilde{A} = \left\{ \frac{\langle 0.7, 0.2 \rangle}{x_1}, \frac{\langle 0.8, 0.1 \rangle}{x_2}, \frac{\langle 0.7, 0.1 \rangle}{x_3}, \frac{\langle 0.9, 0.0 \rangle}{x_4}, \frac{\langle 0.8, 0.2 \rangle}{x_5} \right\},$$

where the five provinces are represented by

$$X = \{x_1(A), x_2(B), x_3(C), x_4(D), x_5(E)\}.$$

Assume that four candidates C_1, C_2, C_3 and C_4 represented by PFSs $\tilde{B}_1, \tilde{B}_2, \tilde{B}_3$ and \tilde{B}_4 , respectively are vying for position P . After the voting process, the four candidates gathered the following votes in Pythagorean fuzzy values as shown below:

$$\tilde{B}_1 = \left\{ \frac{\langle 0.5, 0.4 \rangle}{x_1}, \frac{\langle 0.8, 0.2 \rangle}{x_2}, \frac{\langle 0.4, 0.3 \rangle}{x_3}, \frac{\langle 0.6, 0.2 \rangle}{x_4}, \frac{\langle 0.7, 0.2 \rangle}{x_5} \right\},$$

$$\begin{aligned}\tilde{B}_2 &= \left\{ \frac{\langle 0.8, 0.2 \rangle}{x_1}, \frac{\langle 0.6, 0.3 \rangle}{x_2}, \frac{\langle 0.6, 0.1 \rangle}{x_3}, \frac{\langle 0.3, 0.5 \rangle}{x_4}, \frac{\langle 0.8, 0.1 \rangle}{x_5} \right\}, \\ \tilde{B}_3 &= \left\{ \frac{\langle 0.6, 0.3 \rangle}{x_1}, \frac{\langle 0.7, 0.3 \rangle}{x_2}, \frac{\langle 0.7, 0.2 \rangle}{x_3}, \frac{\langle 0.2, 0.6 \rangle}{x_4}, \frac{\langle 0.8, 0.2 \rangle}{x_5} \right\}, \\ \tilde{B}_4 &= \left\{ \frac{\langle 0.7, 0.1 \rangle}{x_1}, \frac{\langle 0.8, 0.1 \rangle}{x_2}, \frac{\langle 0.5, 0.4 \rangle}{x_3}, \frac{\langle 0.8, 0.2 \rangle}{x_4}, \frac{\langle 0.5, 0.3 \rangle}{x_5} \right\}.\end{aligned}$$

The collated result is determined by showing which of the candidates \tilde{B}_i ($i = 1, \dots, 4$) has the greatest similarity value with respect to \tilde{A} .

The Pythagorean fuzzy values are gotten thus: let $F(X)$ be the number of voters that voted for, $A(X)$ be the number of voters that voted against, and $U(X)$ be the number of voters that remained undecided or cast invalid votes in X . From the knowledge of PFS, we have

$$\mu(x) = \frac{F(X)}{X} \text{ and } \nu(x) = \frac{A(X)}{X},$$

implying that

$$F(X) = \mu(x)X \text{ and } A(X) = \nu(x)X.$$

Then

$$\begin{aligned}\pi(x) &= \sqrt{1 - \left[\left(\frac{F(X)}{X} \right)^2 + \left(\frac{A(X)}{X} \right)^2 \right]} = \sqrt{1 - \left[\frac{F(X)^2 + A(X)^2}{X^2} \right]} \\ &= \frac{\sqrt{X^2 - F(X)^2 - A(X)^2}}{X}.\end{aligned}$$

Thus,

$$U(X) = \sqrt{X^2 - F(X)^2 - A(X)^2} \text{ and } \pi(x) = \frac{U(X)}{X}.$$

That is,

$$X = \sqrt{F(X)^2 + A(X)^2 + U(X)^2}.$$

Now, we calculate the similarity between \tilde{A} and \tilde{B}_i for ($i = 1, \dots, 4$) as follows:

$$\begin{aligned}s_4(\tilde{A}, \tilde{B}_1) &= 1 - \frac{1}{20} \sum_{i=1}^5 [|\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}_1}(x_i)| \\ &\quad + |\mu_{\tilde{A}}(x_i) - \nu_{\tilde{A}}(x_i)| - |\mu_{\tilde{B}_1}(x_i) - \nu_{\tilde{B}_1}(x_i)| \\ &\quad + ||\mu_{\tilde{A}}(x_i) - \pi_{\tilde{A}}(x_i)| - |\mu_{\tilde{B}_1}(x_i) - \pi_{\tilde{B}_1}(x_i)||] \\ &= 0.8126.\end{aligned}$$

Similarly,

$$s_4(\tilde{A}, \tilde{B}_2) = 0.8526, \quad s_4(\tilde{A}, \tilde{B}_3) = 0.9049, \quad s_4(\tilde{A}, \tilde{B}_4) = 0.8762.$$

Table 4 Electoral results

| s_4 | C_1 | C_2 | C_3 | C_4 |
|-------|--------|--------|--------|--------|
| P | 0.8126 | 0.8526 | 0.9049 | 0.8762 |

From the results in Table 4, candidate C_3 will be declared winner of position P because $s_4(\tilde{A}, \tilde{B}_3)$ is the greatest of the similarities.

4.2 Application in career placement

Suppose three students $S_1, S_2,$ and S_3 represented by PFSs $\tilde{E}_1, \tilde{E}_2,$ and \tilde{E}_3 are vying to study course C in a certain institution, wherein the subject requirements for admission are represented by the set

$$X = \{x_1, x_2, x_3, x_4, x_5\},$$

where $x_1 =$ English language, $x_2 =$ mathematics, $x_3 =$ physics, $x_4 =$ chemistry, and $x_5 =$ biology.

The following Pythagorean fuzzy values are the scores of the students or applicants after they sat for an examination over 100% of multi-choice questions on the listed subjects, within a stipulated time.

$$\begin{aligned} \tilde{E}_1 &= \left\{ \frac{\langle 0.6, 0.3 \rangle}{x_1}, \frac{\langle 0.5, 0.4 \rangle}{x_2}, \frac{\langle 0.6, 0.2 \rangle}{x_3}, \frac{\langle 0.5, 0.3 \rangle}{x_4}, \frac{\langle 0.5, 0.5 \rangle}{x_5} \right\}, \\ \tilde{E}_2 &= \left\{ \frac{\langle 0.5, 0.3 \rangle}{x_1}, \frac{\langle 0.6, 0.2 \rangle}{x_2}, \frac{\langle 0.5, 0.3 \rangle}{x_3}, \frac{\langle 0.4, 0.5 \rangle}{x_4}, \frac{\langle 0.7, 0.2 \rangle}{x_5} \right\}, \\ \tilde{E}_3 &= \left\{ \frac{\langle 0.7, 0.1 \rangle}{x_1}, \frac{\langle 0.6, 0.3 \rangle}{x_2}, \frac{\langle 0.7, 0.1 \rangle}{x_3}, \frac{\langle 0.5, 0.4 \rangle}{x_4}, \frac{\langle 0.4, 0.5 \rangle}{x_5} \right\}. \end{aligned}$$

Assume the institution has a bench-mark for admission into studying C represented by PFS \tilde{D} as

$$\tilde{D} = \left\{ \frac{\langle 0.8, 0.1 \rangle}{x_1}, \frac{\langle 0.7, 0.2 \rangle}{x_2}, \frac{\langle 0.9, 0.0 \rangle}{x_3}, \frac{\langle 0.6, 0.3 \rangle}{x_4}, \frac{\langle 0.8, 0.1 \rangle}{x_5} \right\}.$$

Our task is to find which of the students \tilde{E}_i ($i = 1, \dots, 3$) is suitable to study C from the sense of similarity between \tilde{E}_i ($i = 1, \dots, 3$) and \tilde{D} . Thus,

$$\begin{aligned} s_4(\tilde{D}, \tilde{E}_1) &= 1 - \frac{1}{20} \sum_{i=1}^5 [|\mu_{\tilde{D}}(x_i) - \mu_{\tilde{E}_1}(x_i)| \\ &\quad + ||\mu_{\tilde{D}}(x_i) - \nu_{\tilde{D}}(x_i)| - |\mu_{\tilde{E}_1}(x_i) - \nu_{\tilde{E}_1}(x_i)|| \\ &\quad + ||\mu_{\tilde{D}}(x_i) - \pi_{\tilde{D}}(x_i)| - |\mu_{\tilde{E}_1}(x_i) - \pi_{\tilde{E}_1}(x_i)||] \\ &= 0.8009. \end{aligned}$$

Similarly,

$$s_4(\tilde{D}, \tilde{E}_2) = 0.8182, \quad s_4(\tilde{D}, \tilde{E}_3) = 0.8314.$$

From the results in Table 5, we can see that $s_4(\tilde{D}, \tilde{E}_3) > s_4(\tilde{D}, \tilde{E}_2) > s_4(\tilde{D}, \tilde{E}_1)$. Hence, student S_3 is the most suitable to study course C from the career placement exercise.

Table 5 Qualification results

| s_4 | S_1 | S_2 | S_3 |
|-------|--------|--------|--------|
| C | 0.8009 | 0.8182 | 0.8314 |

4.3 Application in disease diagnosis

Let us consider a set of diseases given as $D = \{D_1, D_2, D_3, D_4, D_5\}$, where $D_1 =$ viral fever, $D_2 =$ malaria fever, $D_3 =$ typhoid fever, $D_4 =$ stomach problem, and $D_5 =$ chest problem; and a set of symptoms is thus:

$$X = \{x_1, x_2, x_3, x_4, x_5\},$$

where $x_1 =$ temperature, $x_2 =$ headache, $x_3 =$ stomach pain, $x_4 =$ cough, and $x_5 =$ chest pain.

Suppose a sample of a patient \mathbf{P} is collected and analysed. Let us represent the patient \mathbf{P} laboratory result as an PFS \tilde{F} given as

$$\tilde{F} = \left\{ \frac{\langle 0.8, 0.1 \rangle}{x_1}, \frac{\langle 0.6, 0.1 \rangle}{x_2}, \frac{\langle 0.2, 0.8 \rangle}{x_3}, \frac{\langle 0.6, 0.1 \rangle}{x_4}, \frac{\langle 0.1, 0.6 \rangle}{x_5} \right\}.$$

And then, let each of the diseases D_i ($i = 1, \dots, 5$) be viewed as PFSs

$$\tilde{G}_1 = \left\{ \frac{\langle 0.4, 0.0 \rangle}{x_1}, \frac{\langle 0.3, 0.5 \rangle}{x_2}, \frac{\langle 0.1, 0.7 \rangle}{x_3}, \frac{\langle 0.4, 0.3 \rangle}{x_4}, \frac{\langle 0.1, 0.7 \rangle}{x_5} \right\},$$

$$\tilde{G}_2 = \left\{ \frac{\langle 0.7, 0.0 \rangle}{x_1}, \frac{\langle 0.2, 0.6 \rangle}{x_2}, \frac{\langle 0.0, 0.9 \rangle}{x_3}, \frac{\langle 0.7, 0.0 \rangle}{x_4}, \frac{\langle 0.1, 0.8 \rangle}{x_5} \right\},$$

$$\tilde{G}_3 = \left\{ \frac{\langle 0.3, 0.3 \rangle}{x_1}, \frac{\langle 0.6, 0.1 \rangle}{x_2}, \frac{\langle 0.2, 0.7 \rangle}{x_3}, \frac{\langle 0.2, 0.6 \rangle}{x_4}, \frac{\langle 0.1, 0.9 \rangle}{x_5} \right\},$$

$$\tilde{G}_4 = \left\{ \frac{\langle 0.1, 0.7 \rangle}{x_1}, \frac{\langle 0.2, 0.4 \rangle}{x_2}, \frac{\langle 0.8, 0.0 \rangle}{x_3}, \frac{\langle 0.2, 0.7 \rangle}{x_4}, \frac{\langle 0.2, 0.7 \rangle}{x_5} \right\},$$

$$\tilde{G}_5 = \left\{ \frac{\langle 0.1, 0.8 \rangle}{x_1}, \frac{\langle 0.0, 0.8 \rangle}{x_2}, \frac{\langle 0.2, 0.8 \rangle}{x_3}, \frac{\langle 0.2, 0.8 \rangle}{x_4}, \frac{\langle 0.8, 0.1 \rangle}{x_5} \right\}.$$

Now, we calculate the similarity between \tilde{F} and \tilde{G}_i for ($i = 1, \dots, 5$) as follows:

$$\begin{aligned} s_4(\tilde{F}, \tilde{G}_1) &= 1 - \frac{1}{20} \sum_{i=1}^5 [|\mu_{\tilde{F}}(x_i) - \mu_{\tilde{G}_1}(x_i)| \\ &\quad + |\mu_{\tilde{F}}(x_i) - \nu_{\tilde{F}}(x_i) - \mu_{\tilde{G}_1}(x_i) - \nu_{\tilde{G}_1}(x_i)| \\ &\quad + |\mu_{\tilde{F}}(x_i) - \pi_{\tilde{F}}(x_i) - \mu_{\tilde{G}_1}(x_i) - \pi_{\tilde{G}_1}(x_i)|] \\ &= 0.8336. \end{aligned}$$

Similarly,

$$s_4(\tilde{F}, \tilde{G}_2) = 0.8686, \quad s_4(\tilde{F}, \tilde{G}_3) = 0.8316, \quad s_4(\tilde{F}, \tilde{G}_4) = 0.7817, \quad s_4(\tilde{F}, \tilde{G}_5) = 0.7827$$

From the results in Table 6, we can infer that patient \mathbf{P} is suffering from malaria fever since $s_4(\tilde{F}, \tilde{G}_2)$ is the greatest of the similarities.

Table 6 Medical diagnostic results

| s_4 | D_1 | D_2 | D_3 | D_4 | D_5 |
|----------|--------|--------|--------|--------|--------|
| P | 0.8336 | 0.8686 | 0.8316 | 0.7817 | 0.7827 |

5 Conclusions

Motivated by the advantages of Pythagorean fuzzy environment having the ability to represent uncertainty more reasonably than IFSSs, in this paper, an attempt has been made to propose some new similarity measures for PFSs. We have introduced some new similarity measures for PFSs which satisfied the properties of similarity measure, by taking into account the traditional parameters of PFSs. We verified the authenticity of the proposed similarity measures with reference to some similarity measures for PFSs that also used the three parameters characterisation of Pythagorean fuzzy sets, as studied in Ejegwa (2018), and found that the proposed similarity measures, especially, s_4 yields better output. To test the applicability of the proposed similarity measures, some real-life problems such as, career placement, electioneering process, and disease diagnosis were considered via s_4 , for reliable output. The similarity measures introduced in this work could be used as viable tools in applying PFSs to decision-making problems. From this paper, it has been concluded that the proposed similarity measures for PFSs can easily handle real-life decision-making problems and hence beneficial for system analysis, decision science, etc. In the future, the proposed measures could be extended to different environments such as linguistic single-valued neutrosophic sets, hesitant Pythagorean fuzzy sets, complex intuitionistic fuzzy sets, among others, as seen in Garg (2019a), Garg and Arora (2019), Garg and Nancy (2019) and Garg and Rani (2019b).

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