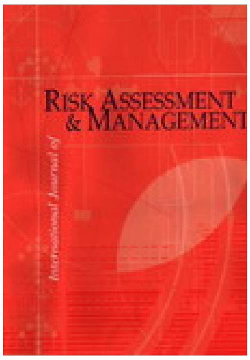

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Scenario-based stochastic model for supplier selection and order allocation under disruption risk and quantity discount

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Abstract: In this paper, we develop two stochastic mixed integers linear programming (SMILP) models for supplier selection under disruption risk considering different capacity, failure probability, uncertain demand and quantity discounts. The suppliers are assumed domestic suppliers and global suppliers. The obtained combinatorial stochastic optimisation problem is formulated as a mixed integer program with conditional value-at-risk technique (CVaR). Numerical examples and computational results are presented. The proposed models can optimise the present problem through an estimated value at risk (VaR) and minimised CVaR simultaneously. The computational results reveal that the proposed models allow the decision maker to make an efficient selection of suppliers under disruption risk. Results also show that the decisions are not univocal because they depend on the risk proneness of the decision maker.

Keywords: selection supplier; disruption risk; stochastic mixed integer linear programming; total quantity discount; VaR; value-at-risk technique; CVaR; conditional value-at-risk technique; neutral risk; aversion risk.

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1 Introduction

Speedily developing technologies and a competitive market have resulted in numerous renovations in the existing supply chain. Modern supply chains have become complex networks that includes different actors such as suppliers, manufacturers and consumers distributed everywhere. This mutation has created different sets of risk and uncertainty in whole supply chains. Moreover, all actors become more vulnerable to disruption, which is caused by unexpected events such as earthquakes, hurricanes, economic crises, labour strikes and terrorist attacks. Among those unexpected events, it is worth mentioning the Kobe earthquake in 1995, which disrupted some supply chains that relied on liquid crystal displays. Hurricane Katrina in 2005 disrupted the supply chains of companies that were dependent on the port of New Orleans (Xu and Nozick, 2009). In order to build a robust supply chain, companies required careful attention to their supply disruption. “Supply chain disruptions are unplanned and unanticipated events that disrupt the normal flow of goods and materials within a supply chain”, as it was defined by Kleindorfer and Saad (2005). Generally speaking, most supply chain disruptions can be broadly classified into three categories, namely supply-related, demand-related, and miscellaneous risks (Oke and Gopalakrishnana, 2009). In fact, supply disruption happens when suppliers are unable to fulfil their requirements. This set of disruption may potentially affect the flow of a product or a service that the supply chain offers to their customers (Li et al., 2010).

This paper addresses the development of two stochastic mixed integer linear programs for selection supplier under supply chain risk management. In this model, the value-at-risk (VaR) and the conditional value-at-risk (CVaR) techniques are applied. These two techniques will be integrated in order to estimate and optimise the expected cost in the worst case. In fact, several previous works looked at either to maximise the expected profit or minimise the expected costs (Hammami et al., 2014), Zhang and Chen (2013) Meena and Sarmah (2013). However, the mean or expected values are good decision-making tools, but they are not enough in presence of risk aversion of the decision maker. The contribution of this work is minimising expected cost and expected cost in worst case simultaneously.

The remainder of this paper is organised as follows: At first, the related literature is reviewed. Section 2, presents the two stochastic mixed integer linear programs by considering both techniques of risk such as value-at-risk (VaR) and conditional value-at-risk (CVaR). Section 3 describes the different data. Results and discussion are subsequently presented in Section 4. Finally, concluding remarks and possible future extensions will be drawn in Section 5.

2 Literature review

There exists several literatures related to supplier selection under disruption risk. Considering the relevance, we review the background in the field. First, literature review has been devoted to supplier selection under disruption risk then, the selection supplier problem under risk and discount criteria will be treated.

2.1 Related literature to supplier selection under disruption risk

Nowadays, the supply chain has become very expanding globally. Thus, firms are allocating their business to foreign or global suppliers. This mutation has, in turn, created and increased different sets of risk in the supply chain. For this reason, different researches treated the notions of supplier selection and order allocation by considering disruption risk. The objective of this section is to present an overview of the literature for the present issue. In this context, we consider two sets of researches: qualitative and quantitative researches.

Formerly, several methods have been used in the literature review. Among these approaches, we mention the decision tree, the game theory, and AHP. In this context, Berger and Zeng (2006) used the decision-tree approach to help a buying firm to determine the optimal size of its supply base in presence of risks. They assumed two states of suppliers: either all suppliers are unavailable to satisfy the buying firm's demand or all suppliers are available to satisfy the buying firm's demand. Ruiz-Torres and Mahmoodi (2006) used the same approach, but they considered other sets such as the cost of maintaining suppliers, the purchased quantity and the loss per unit not delivered. Ruiz-Torres and Mahmoodi (2007) criticised the above hypotheses and proposed a more realistic model of a decision-making process. They consider the risks' independence of individual supplier failures. In this case, they assumed when the probability of failure for each supplier is equal, as well as, the case where the probability of failure for each of the suppliers is not equal. Wu et al. (2006) used AHP approach to calculate the weight of some sets of risk. They proposed a classification of supply chain risks, which can be internal/external controllable, internal/external partially controllable and internal/external uncontrollable. Chan and Kumar (2007) combined a fuzzy logic with AHP in order to identify and discuss some of the important and critical decision criteria. They included risk factors to develop an efficient system of global supplier selection. Vinodh et al. (2011) used a fuzzy ANP approach for selecting the best supplier. Hsieh et al. (2014) used game theory approach to model the supply chain, which is composed by a set of manufacturers and a common retail. They considered uncertain demand and sensitive selling prices. Yin and Nishi (2014) used the game theory to model an asymmetric information among the suppliers and the manufacturer. Model includes uncertain demand.

With respect to the second group, different papers have been used to dealing with this problem. In fact, in the proposed approach, there are many different techniques, such as:

the linear program (LP), the mixed integer linear program (MILP), the multi-objective program, and so on. Based on this, Chen et al. (2012) used the linear program to model a periodic review of the inventory system with two suppliers. The first is an unreliable regular supplier and the second is a reliable one. The latter is characterised by a limited capacity, a higher unit purchasing cost and a fixed order cost compared to the unreliable supplier. Ravindran et al. (2010) used the goal programming by incorporating VaR and miss the target (MttR). VaR is used to estimate the supplier's risk. MttR is used to model the frequent events. Sawik (2011) developed a mixed integer linear program to model an operational risk. The objective is to mitigate the impact of delay's risk of the total cost. Fang et al. (2016) developed a quantitative approach to model both the supply chain operation risk and the disruption risk for both selection supplier and order allocation problems. Hosseininasab and Ahmadi (2015) developed a multi objectives program for a supplier portfolio problem. The objective is to maximise the expected value and development of suppliers and minimise their correlated risk simultaneously. Hamdi et al. (2016), developed two stochastic mixed integer linear programs to model supply chain with three levels, which are composed of: a set of suppliers, a set of customers and a central purchasing. In the first model, the objective is to maximise an expected profit in which the decision maker is a neutral risk and they did not consider losses criteria, whereas the objective of the second model is to maximise an expected profit subject to a fixed threshold of loss. However, while there is abundant literature on the various problems of supplier selection and order allocation, significantly fewer studies have taken the supplier selection under disruption risk into account (for comprehensive surveys, see Hamdi et al. (2015) on supplier selection under supply chain risk management and Ivanov et al. (2017) on supply chain disruption and recovery policies).

The decision related to the supplier selection problem is usually about which supplier is to be selected. How many suppliers are there? And how much quantity can be ordered? Thus, the problem becomes more complicated when the potential supplier offers some discounts to promote more orders. In this paper, the focus is on these criteria and their combination with the disruption risk.

2.2 Related literature to supplier selection in context of quantity discount

Quantity discount and pricing discount are common and effective strategies for the suppliers to promote their products. A quantity discount is based on the quantity of promoting items, which promote the buyer to order large quantities of a given items. There are numerous researches that address the selection supplier problem and its various extensions. Order quantity or lot sizing decisions can be largely influenced by alternative supplier pricing schemes. In prior research, the most common pricing schemes that have been assumed are: the constant price, the all units of price discount, and the incremental unit of price discount. So, the motivation to integrate the quantity discount from the fact that they encourage the buyer to supply a large quantity and win operating advantages (economy of scale). In fact, by the integration of the quantity discount, both supplier and buyer can realise higher overall profits. Among works, which include the present criteria, we mention, Burke et al. (2008a) investigated the problem of central organisation for a major office product distributor. In which, the purchasing organisation must source a quantity of a particular resale item from a set of capacitated suppliers. In their work, they considered each supplier offered an incremental quantity, discount in purchase price structure but they neglected risk criteria. Wang and Yang (2009), mentioned that the problem of selection supplier must take into account numerous heterogeneous criteria. They considered that the present problem became very complicated when quantity discounts are considered at the same time. In this context, they combined AHP and fuzzy compromise programming. The net price with a variable quantity discount rate, quality and delivery are the most important factors in evaluating alternative suppliers. Burke et al. (2008b) considered that the integration of quantity discounts often complicates the order allocation problem under multiple sourcing. They studied different sets of discounts such as: linear pricing discount, incremental units pricing discount and all unit pricing discounts in order allocation models, whereas they neglected risk of disruption. Hammami et al. (2014) studied supplier selection problem by considering uncertain fluctuations of currency exchange rates and price discounts. They assumed that the suppliers are located worldwide, and the price is offered in suppliers' local currencies. Zhang and Chen (2013) developed a supplier selection and a procurement decision model. They considered uncertain demand, quantity discounts and fixed selection costs. Suppliers offer quantity discounts based on the ordering quantity and a fixed cost. Meena and Sarmah (2013) investigated the order allocation problem of a manufacturer/buyer among multiple suppliers under risk of supply disruption. They developed mixed integer non-linear programming by considering different capacities, failure probability and quantity discount for each supplier. Genetic algorithm approach is used to solve it. Mansini et al. (2012) developed an integer programming model to study a procurement setting in which suppliers offer total quantity discount and transportation costs, which are based on truckload shipping rates. Choudhary and Shankar (2011) studied a multi period purchasing problem in which a buyer procures a single product from a single supplier considering economies of scale in purchasing and transaction costs along with supply chain disruption. More recently, Choudhary and Shankar (2013) studied a procurement setting in which a buyer needs to purchase a single product from a set of suppliers over finite discrete time periods to satisfy service level requirements. The suppliers offer all-unit quantity discounts.

In this work, we developed two mathematical models by considering different criteria. In order to represent a realistic situation, each supplier is considered to have different capacity, different interval on total quantity discount and different probability of disruption. Also, we consider two sets of disruption such as local and global disruption. In addition, we integrated two techniques of computing risk such as VaR and CVaR to control cost in the worst case. Therefore, by considering all these settings together the problem has become more realistic.

2.3 Quantification of disruption risk

Most recently, some works start to model supply chain disruption of supplier or demand disruption by integrating quantitative techniques. This way makes the problem more realistic with reasonable manner to moderate, estimate and cope with disruption successfully. In fact, quantified risk of channel member plays an important role for the decision maker. Also, decision preferences are generally assumed to be risk-neutral or risk-averse in many research fields such as finance, economics and so on.

Risk-averse is defined when the decision maker prefers a lower return with a known risk rather than a higher return with an unknown risk. As for the risk-neutral, the decision maker is an adventurous person. He usually seeks a higher return and does not give importance to the risk level. In supply chain researches, the majority suppose that the decision maker is neutral-risk whereas few papers presuppose that the decision maker's behaviour is averse-risk. VaR and CVaR techniques are among the techniques used to model the aversion-risk of the decision maker preference. Here, we briefly define and compare the two present techniques.

Value at risk (VaR) can be used to compute risk and control risk management. Value at risk measures the risk of probable loss of cost. VaR is defined for a confidence level $\alpha \in (0, 1]$.

$$VaR_\alpha(x) = \inf\{u \mid \Pr\{L(x,\zeta) \leq u\} \geq \alpha\} \tag{1}$$

x is a fixed variable, ζ is a random variables, $L(x, \zeta)$ is the loss function and α is a confidence level. This technique allows us to quantify and compare plausible losses attached to each portfolio or position without considering its composition.

There are a variety of models to estimate VaR. A common model includes variance-covariance, the historical data and Monte Carlo simulation. However, this technique presents some limitations. It does not give any information about extreme losses and does not capture the scenarios exceeding VaR. To overcome the disadvantages, conditional VaR has been introduced. CVaR aims to compute the excess of VaR. It is characterised by stronger mathematical properties rather than VaR, including the sub-additive, homogeneity, and invariance properties. CVaR focuses on the tail of the cost distribution. CVaR is the expected value of loss exceeding α – VaR. It can be expressed by the following formula:

$$\begin{aligned} CVaR_\alpha(x) &= \phi_\alpha(x) = E[L(x,\zeta) \geq VaR_\alpha(x)] \\ &= \frac{1}{1-\alpha}\int_{L(x,y)\geq VaR_\alpha(x)} L(x,y)f(y)\mathrm{d}y \end{aligned} \tag{2}$$

$F(y)$ is the density function of ζ and VaR$\alpha(x)$ which is defined by (1)

$$F_\alpha(x,u) = u + \frac{1}{1-\alpha}E[[L(x,\zeta)-u]^+] \tag{3}$$

Rockafeller and Uryasev (2000, 2002) proved that the CVaR can be decreased by minimising the auxiliary function (3).

The present work proposes a new stochastic mixed integer programming approach for supplier selection and order allocation problem by considering disruption risk quantity discount and including quantitative technique in order to estimate the expected cost in worst cases. Taking into consideration all these settings together has made the problem more realistic.

3 Research gaps and our contributions

This paper studies the problem of the suppliers' selection and order allocation in three level supply chain (suppliers, central purchasing office, customers) in the presence of failure disruption. It is noted that, in the relevant literature (Meena and Sarmah, 2013; Bohner and Minner, 2017), the decisions regarding the suppliers' selection problem are mainly related to which suppliers to select and how much to order for each of them at each period and on the horizon planning. They concentrate only on sole objective either to maximise expected profit or to minimise expected costs. However, the mean or expected values are good decision-making tools, but they are not sufficient in the presence of risk aversion of the decision maker. In addition, the origin of many supply chain risks stem from the supplier selection problem. Nonetheless, the literature lacks adequate integrated approaches for the quantification of risk variants. In fact, risk quantification is an imperative and a vital task to estimate and optimise the value of disruption risk.

The literature of supplier selection lacks a multi-methodological perspective for the mathematical program and quantitative techniques. This study attempts to bring together the two axes by proposing a unified analytical framework for the supplier selection problem under disruption risk. In fact, supplier selection models generally assume that decision maker is a neutral risk, and therefore prefer to find opportunities for optimising expected cost or profit. Risk-averse decision-makers on the other hand, seek to minimise risk and cost simultaneously and consider the risk criteria implications of their decisions.

Thus, in this work, we develop two stochastic MILP, which are based on scenario analysis. In the first one, the decision maker is presented as a neutral risk who seeks to minimise the expected cost. In the second one, it is considered as an aversion risk who seeks to find a compromise between minimising the expected cost and minimising the expected cost in worst case. These development models are based on integrating quantitative techniques such as VaR and CVaR. The two techniques allow to estimate the expected cost in worst cases. Both programs are based on scenarios analyses. Some settings follow a uniform distribution, such as demand, purchasing price and the probability of disruption. Sensitivity analysis is performed to determine the influence of the preference of decision maker vs. risk.

4 Problem description and formulation

In this section, we present the different settings and the related assumptions under which the problem is solved. Formally, we consider a supply chain with three levels: a set of suppliers, a central purchasing organisation and a set of customers.

4.1 Supplier

Let $i = \{1 \ldots m\}$ a set of suppliers located worldwide. Each supplier is limited to different constraints such as the capacity of production and the minimum of price. Also, each supplier is subject to a local disruption, which can be affected by: a problem of quality, a lack of materials, or late delivery. In this case, none of customers' request can be satisfying. The set of disruption affects only the sole supplier which is dependent to the other sets.

Because each supplier is either 'on' or 'off', the total number of different possible scenarios is equal to 2^m. Let $s = \{1 \ldots 2^m\}$ the set of potentials' scenarios. Let π_i the probability of local disruption for the supplier *i*. Let $RS_{is} = 1$ if the supplier is subject to local disruption; otherwise, it's 0. The probability of local disruption is expressed by:

$$P_s = \prod_{i \notin I_s} (1-\pi_i) \prod_{i \in I_s} \pi_i \tag{4}$$

Local and global supplier disruption affects all suppliers simultaneously among the disruption. We can cite: economics crisis and a strike of the transport sector. In fact, the probability of this event's realisation is weak but its consequence is very high.

Let π_i^* the probability of global disruption

$$P_s = \begin{cases} (1-\pi_i^*)p_s & \text{if } \mathrm{I}_s \neq 0 \\ \pi_i^* + (1-\pi_i^*)\prod_{i \in I} \pi_i & \text{if } \mathrm{I}_s = 0 \end{cases} \tag{5}$$

4.2 Purchasing central

The purchasing central office is an intermediary link between the suppliers and the customers. The decision maker of the central receives the product *h* order from the customer *j*. First, he must select the number of suppliers. Let $Y_{ih} = 1$ if the supplier i is selected to deliver the product *h*, 0 otherwise. If the supplier *i* is selected, a fixed cost transaction CT_{ih} is incurred. In a second stage, the decision maker determines the QO_{ijh} to order the product *h* from the supplier *i* for the customer *j*. The decision maker can order more of DN_{jh} quantity requested by the customer. In case of undelivered part, the customers do not pay the purchasing price but the fixed cost is lost.

4.3 Customer

We consider a set $J = \{1 \ldots n\}$ of n customers with a different order of the product h. If a part of the customer *j*'s demand of the product *h* is unmet, a shortage cost CS_{jh} must be paid. Base purchasing price CP_{ih} of the product *h* is fixed by the supplier *i*. If the delivered quantity exceeds the ordered quantity, an inventory cost must be paid.

4.4 Formulation

We modelled the problem of selection supplier in presence of disruption risk. We developed two stochastic mixed integer linear programs. In the first model, we aim to determine the problem of selection supplier in context of minimisation of expected cost. Whereas, in the second model, we aim to select a supplier under disruption risk by calculating VaR and minimising CVaR. As consideration, suppliers' potentials offer different sets of discounts: All quantity discounts, an incremental discount, and a total volume discount. In this model, we consider the discount on total quantity. The present model enriches the newsvendor problem under disruption risk by integrating VaR and CVaR, while considering multiple products and quantity discount simultaneously.

In this section, we developed two stochastic mixed integer programs for three levels of supply chain. The specific notations of these models are the following.

4.5 Index

i Supplier

j Customer

h Product

s Scenario

k Interval of discount.

4.6 Scenario-independent settings

CS_{jh}: Shortage cost for customer *j* for product *h*

TD_{ih}: Interval of quantity discount offered by the supplier *i* to the product *h*

CP_{ih}: Purchasing price fixed by supplier *i* for product *h*

CT_{ih}: Fixed transaction cost for supplier *i* for product *h*

DN_{jh}: Normal demand of customer *j* of the product *h*

CA_{ih} Purchasing cost of supplier *i* for product *h*

PC_{ih} Capacity of supplier *i* for product *h*

CTR_{ih} Transportation cost of product *h* from supplier *i*

Scenario-dependent settings

RS_{is} Binary setting equals to 1 if supplier is in disruption; otherwise 0

PS_s Probability of disruption in the scenarios

Scenario-independent variables

QO_{ijh} Order quantity of the product *h* for the supplier *i* requested by the customer *j*

Y_{ih} Binary variable equals to 1 if the supplier *i* is selected to deliver the product *h*; otherwise it's 0

KD_{ihk} Quantity of the product *h* belongs to the interval of discount *k* of the supplier *i*

U_{ihk} Binary variable equals to 1 if the quantity of product *h* belongs to the interval of discount *k* of the supplier i;0 otherwise

Scenario-dependent variables

QL_{sjh} Delivered quantity of the product *h* to the customer *j* in scenario s

QS_{sjh} Shortage quantity of the product *h* for the customer *j* in scenario s

QD_{sjh} Requested quantity in excessive of the product *h* requested by the customer *j* in scenario s

Model 1: Minimise the expected cost: E(X)

The objective function is to minimise the expected cost which depends on the transaction cost, the purchasing cost, the transportation cost, the inventory cost and the shortage cost, if the order quantity is unmet due to the supplier disruption. Our analysis is based on supplier selection under disruption risk.

$$\begin{aligned} Min\ \mathrm{E(X)} = &\sum_i \sum_h CT_{ih} * Y_{ih} + \sum_i \sum_h \sum_s Ps_s * CA_{ih} * PS_s * (1 - RS_{is}) \\ &+ \sum_i \sum_h \sum_j \sum_s Ps_s * CTR_{ihj} * QO_{ihj} * (1 - RS_{is}) \\ &+ \sum_j \sum_h \sum_s Ps_s * SD_{jh} * QD_{jh} + \sum_j \sum_h \sum_s Ps_s * CS_{jh} * QS_{jh} \end{aligned} \tag{6}$$

Subject to

The supply chain is always subject to a different set of constraints such as: the extreme capacity, the delivery date, the resources' allocation, etc. The different constraints can be modelled as forms of equations or inequalities. In the following section, we will mention the different constraints of our model:

$$QL_{sjh} = \sum_i QO_{ihj} * (1 - RS_{is}) \quad \forall j, \forall s \tag{7}$$

$$\sum_j QO_{ihj} < PC_{ih} * Y_{ih} \qquad \forall i\ \forall h \tag{8}$$

$$QL_{sjh} - DN_{jh} = QD_{sjh} - QS_{sjh} \quad \forall j, s \tag{9}$$

$$QD_{sjh} < DD_{jh} \quad \forall j\ , s \tag{10}$$

$$QA_{ih} = \sum_{\mathrm{j}} QO_{ihj} \qquad \forall i, \forall h \tag{11}$$

$$\sum_k QD_{ikh} = QA_{ih} \quad \forall i, h \tag{12}$$

$$QD_{ih1} \le BD_{ih1} * U_{ih1} \qquad \forall i \forall h \tag{13}$$

$$BD_{ihk-1} * U_{ihk} \le QD_{ihk} \le BD_{ihk} * U_{ihk} \quad \forall i \in I, \forall h \in H, \forall k = 2 ... TD_i \tag{14}$$

$$\sum_k U_{ihk} \ = 1 \quad \forall i\ \forall h \tag{15}$$

Purchasing cost of the product *h* for the supplier i

$$CA_{ih} \ = CP_{ih} * \ \sum_k (QD_{ihk} * (1 - AD_{ihk}) \qquad \forall\ i, \forall h \tag{16}$$

Model 2: Minimise the expected cost in the worst cases: CVaR

$$Min\ \mathrm{CVaR} = VaR + (1 + \alpha)^{-1} * \sum_s PS_s * TS_s \tag{17}$$

Subject to

- Constraints (7) to (16)
- Constraint of risk.

The distribution of tail cost in scenario s is non-negative in which the cost exceeds the value of VaR in scenario s

$$\sum_i \sum_h CT_{ih} * Y_{ih} + \sum_i \sum_h CA_{ih} + \sum_i \sum_h \sum_j \sum_s CTR_{ihj} * QO_{ihj} * (1 - RS_{is}) + \sum_s \sum_j \sum_h SD_{jh} * QD_{sjh} + \sum_s \sum_j \sum_h CS_{jh} * QS_{sjh} - VaR \le TS_s \quad (18)$$

Binary and non-negative variables (19)

$$Y_{ih},\ U_{ihk},\ QO_{ijh},\ KD_{ihk},\ QD_{sjh},\ QS_{sjh},\ QL_{sjh},\ TS_s$$

5 Setting presentation

The two models are based on an analysis scenario. We consider a model with 6 suppliers, 10 customers and 2 products. Hence, we have 64 different scenarios numbered from 0 to 63 in binary numbers (000000 to 111111). For example, the 38th scenario is numbered 100110 in binary number and it corresponds to: suppliers 1, 4 and 5 fail, and suppliers 2, 3 and 6 deliver the products without any problems. Table 1 gives the values' settings for the different suppliers. We mention here some settings are generated uniformly random demand, probability of disruption, purchasing price such as: DN_{jh}: U[100 : 600], CA_{ih}: U[10 : 40], πi: U[0.03 : 0.3], CTR_{ij}: U[5 :40].

Table 1 Supplier settings

		i1	*i2*	*i3*	*i4*	*i5*	*i6*
Capacity of supplier	h1	1100	1200	900	800	1200	800
	h2	918	1277	1129	917	1389	866
Transaction cost	h1	1000	1500	1000	1000	1000	1000
	h2	1100	1000	1200	1200	1000	1200
Discount Interval	h1	120	150	130	120	130	140
	h2	100	120	130	150	130	120
	h1	500	550	600	450	500	600
	h2	500	600	600	560	500	700
	h1	1100	1200	900	800	1200	800
	h2	918	1277	1129	917	1389	866
Rate discount	h1	0%	0%	0%	0%	0%	0%
	h2	0%	0%	0%	0%	0%	0%
	h1	5%	5%	5%	5%	5%	5%
	h2	5%	5%	5%	5%	5%	5%
	h1	10%	10%	10%	10%	10%	10%
	h2	15%	15%	15%	15%	15%	15%

Table 2 gives the parameters' values for the customers. For each of them, the maximum quantity that can be sold at a discounted price is 20% of his demand. The same table presents also the cost of shortage and the inventory cost.

Figure 1 shows the level of product demand for h1 and h2.

Table 2 Customer settings with product

	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*	
	h1	*97*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*
DDj	75	23	72	86	98	88	64	98	97	66	87	78	82	79	77	97
SDj	27	62	23	25	25	23	23	22	21	25	25	23	23	22	24	23
CSj	45	69	61	61	45	61	60	64	63	63	66	68	69	67	68	62

Figure 1 Customer demand for each product

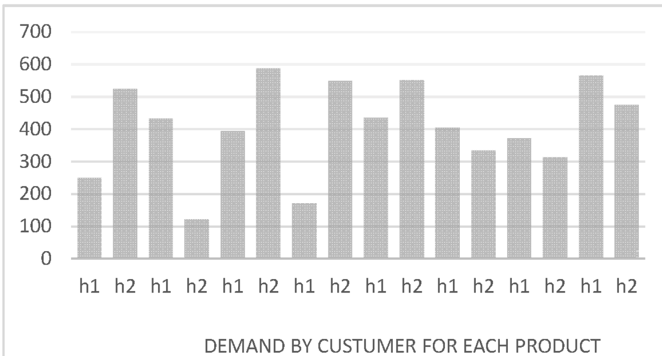
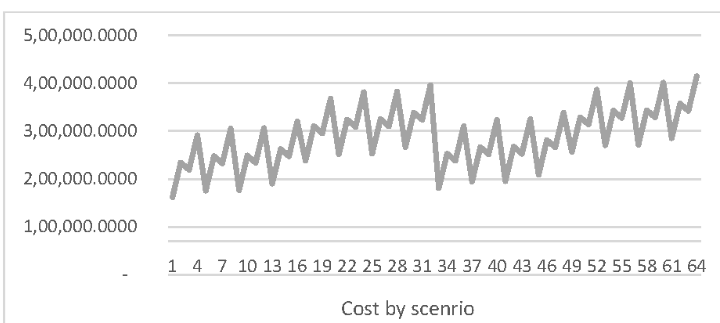


Figure 2 shows the distribution of the cost by scenario. The values of cost comprised between 161901.30 as a minimum value of cost (this value reflects the scenario in which all suppliers can deliver (000000)), and 414 773,000 as a maximum value where all suppliers are in disruption (1111111). Based on scenarios' analysis, the value of the expected cost is equal to 185600, 6. In this case, the selected suppliers for h1 are (111101) and for h2 are (010011). Table 3 presents the quantity's order for the risk neutral decision maker.

Figure 2 Cost by scenario



After calculating the value of the expected cost, we determine the value of VaR in which we fix the confidence level and we give the corresponding value of VaR. Based on this value, we minimise the expected cost in the worst case by determining the value of CVaR.

Table 3 Optimal solution for neutral risk decision maker

	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	0	286	0	214	0	0	0	0	0	0	0	0	0	0	0	500	0
2	0	0	146	0	0	588	0	0	0	0	404	0	0	313	0	299	550	1200
3	0	0	0	0	108	0	170	0	0	0	0	0	0	0	322	0	600	0
4	0	0	0	0	0	0	0	0	435	0	0	0	0	0	15	0	450	0
5	0	0	0	121	0	0	0	548	0	542	0	0	0	0	0	176	0	1387
6	0	523	0	0	0	0	0	0	0	9	0	334	372	0	228	0	600	866
Total	0	523	432	121	322	588	170	548	435	551	404	334	372	313	565	475		

A sensitivity analysis is performed to identify and analyse the preference of disruption effect. In this case, six different confidence levels were selected which are: 0.5; 0.75; 0.9; 0.95; 0.99; 0.995, while the remaining parameters stayed unchanged. Hence, for each confidence level α, we determine the value of VaR, CVaR and the expected cost. The value of α, reflects the confidence level of the decision maker vs. to risk. The aversion decision maker gives an importance to the values of the highest cost although their probability of scenario of realisation is very low. While the risk-neutral decision maker focuses only on the highest risk with a high probability. We mention here, the more the confidence level decreases, the more the decision maker is neutral risk and the more the confidence level increases, the more the decision maker is aversion risk.

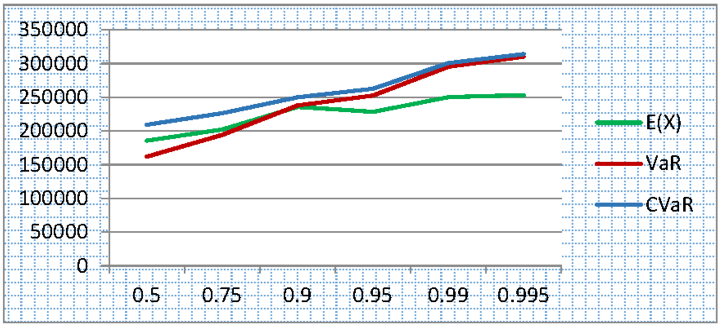
Table 4 shows the different values, such as VaR, CVaR, for each confidence level, the expected cost and the number of selected suppliers for each product. For example, when $\alpha = 0.75$, there is 75% of chance that the value of cost cannot exceed 194213.3. As mentioned previously, the technique of VaR is unable to estimate the extreme value and the worst cases of the expected cost. Thus, we integrated the CVaR technique to minimise the expected cost in the worst case. In this case, there is 75% of the expected cost's value cannot exceed 226,466.3 in the worst case.

Table 4 VaR, CVaR, and $E(X)$ with different confidence levels

Confidence level α		0.5	0.75	0.9	0.95	0.99	0.995
Expected cost		185600.6	202276.5	235709	228454.3	250098.6	253232.5
VaR		161901.3	194213.3	237943.9	252709.8	295374.4	310364.4
CVaR		209300	226466.3	249950.1	262696.3	300861.3	314170.4
Number of selected supplier	h1	5	6	6	6	6	6
	h2	3	4	4	4	5	5

Results shows also that the values of both VaR and CVaR increase with the confidence level. Figure 3 presents the distribution of the different notions with different confidence levels. Also, the number of suppliers increases with the confidence level, which means that the risk of disruption can mitigate by diversification of the suppliers and the order allocation. We notice also that the value of the expected cost is greater than the value of VaR. When $\alpha = 0.5$ and $\alpha = 0.75$, the value of VaR draws near the value of CVaR with the confidence level 0.995.

Figure 3 Distribution of value of $E(X)$, VaR, CVaR (see online version for colours)



Tables 5–9 present the order allocation with the different fixed confidence levels for the local disruption case. Here, we note that the order allocation varied from one confidence level to another that is similar to the number of selected supplier.

Table 5 Order allocation with confidence level $\alpha = 0.75$

	Order quantity QO with α = 0.75																	
	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	105	27	73	0	0	0	95	14	110	404	67	55	63	0	0	500	511
2	0	105	405	24	32	588	0	95	14	110	0	67	55	188	44	24	550	1200
3	0	0	0	0	290	0	170	0	41	0	0	0	55	0	44	0	600	0
4	0	0	0	0	0	0	0	0	351	0	0	0	55	0	44	0	450	0
5	0	105	0	24	72	0	0	359	14	270	0	67	55	63	44	451	185	1339
6	0	314	0	24	0	0	0	95	14	170	0	200	152	63	434	0	600	866
Total	0	628	432	145	394	588	170	643	449	661	404	401	427	376	609	475		

Table 6 Order allocation with confidence level $\alpha = 0.9$

	Order quantity QO with α = 0.9																	
	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	0	312	0	0	0	0	0	44	0	92	0	52	0	0	0	500	0
2	0	105	66	24	301	586	4	110	44	110	0	67	22	164	113	95	550	1260
3	0	0	0	0	0	0	96	0	176	0	60	0	22	0	247	0	600	0
4	0	49	54	0	0	0	0	329	172	0	171	0	52	0	0	182	450	560
5	0	266	66	97	75	60	70	110	44	441	81	67	52	149	113	210	500	1400
6	0	207	21	24	0	2	4	110	44	110	81	267	245	63	205	83	600	866
Total	0	628	518	145	376	648	174	658	522	661	485	401	446	376	678	570		

Table 7 Order allocation with confidence level $\alpha = 0.95$

	Order quantity QO with α = 0.95																	
	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	105	152	97	0	118	0	296	44	110	211	67	37	31	56	95	500	918
2	0	0	280	0	18	470	34	110	44	0	81	0	37	282	56	108	550	969
3	0	0	0	0	0	0	0	0	0	0	112	0	37	0	451	0	600	0
4	0	0	0	0	0	0	0	0	348	0	0	0	101	0	1	0	450	0
5	0	105	0	24	329	118	34	252	44	426	0	67	37	31	57	367	500	1390
6	0	418	86	24	0	0	136	0	44	125	81	267	197	31	57	0	600	866
Total	0	628	518	145	346	706	204	658	522	661	485	401	446	376	678	570		

Table 8 Order allocation with confidence level $\alpha = 0.99$

	Order quantity QO with α = 0.99																	
	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	0	238	24	0	118	0	93	0	0	262	0	0	69	0	48	500	352
2	0	0	194	0	198	470	34	16	44	0	81	0	0	185	0	48	550	719
3	0	0	0	0	0	0	0	0	0	0	0	0	207	0	394	0	600	0
4	0	418	0	0	0	0	0	341	392	43	0	67	0	0	59	48	450	917
5	0	105	86	97	149	118	34	207	44	148	0	67	74	121	113	428	500	1289
6	0	105	0	24	0	0	136	0	44	470	142	267	166	0	113	0	600	866
Total	0	628	518	145	346	706	204	658	522	661	485	401	446	376	678	570		

Table 9 Order allocation with confidence level $\alpha = 0.995$

	Order quantity QO with α = 0.995																	
	J1		*J2*		*J3*		*J4*		*J5*		*J6*		*J7*		*J8*		*Total*	
	h1	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*h1*	*h2*	*H1*	*H2*
1	0	52	148	24	0	118	0	43	0	20	323	67	29	160	0	32	500	516
2	0	0	284	0	0	460	34	0	123	0	81	0	29	153	0	32	550	645
3	0	0	0	0	0	0	0	0	142	0	0	0	17	0	441	0	600	0
4	0	471	0	0	0	0	0	127	0	20	0	267	326	0	124	32	450	917
5	0	52	86	3	346	128	67	421	0	90	0	0	0	63	0	443	500	1200
6	0	52	0	118	0	0	103	67	258	531	81	67	46	0	113	32	600	866
Total	0	628	518	145	346	706	204	658	522	661	485	401	446	376	678	570		

5.1 Local and global disruption risks

In this section, we treat the case of selection suppliers under local and global disruption risks. The set of disruption affects all the suppliers simultaneously. An example of a super event may include an economic crisis, a terrorist attack, a widespread labour strike in a transportation sector, etc. In fact, the probability of such disaster events is usually very low, and their consequences may be very high. Equation (2) presents the probability of simultaneous global disruption of all the suppliers due to some super catastrophic events.

Based on these assumptions, the expected cost is equal to 145756.8 and the selected suppliers to deliver the product h1 are (110101) and for h2 are (110010). With a different scenario, we note that the minimum value is equal to 127286.000 and the maximum value of cost is equal to 314747.000.

Table 10 shows the values of expected cost, VaR, CVaR and the number of selected suppliers.

Table 10 Risk of local and global disruption

Confidence level		0.5	0.75	0.9	0.95	0.99	0.995
Expected cost		145756.8	157511.0	181672.9	181920.2	198814.7	195676.2
VaR		127286.0	149006.2	198380.2	206220.6	225924.2	235812.4
CVaR		162965.9	180071.4	201246.7	209702.6	231885.9	240355.2
Number of selected supplier	h1	4	5	5	5	5	5
	h2	3	4	4	4	5	5
CPU		0.31	1.53	0.33	0.32	1.47	1.06

6 Results and discussion

Based on the different tables, we show that more the confidence level increases the more the values of VaR and CVaR increase and the decision maker becomes more covered against the consequence of supplier disruption risk. Also, each decrease of confidence level makes the decision maker more exposed to the negative effects of disruption risk. The number of suppliers increases with the confidence level, which means that the risk of disruption can mitigate by diversification of the suppliers and the order allocation. Thus, risk of disruption can be mitigated by diversification of the suppliers and the order allocation.

Integrated technique of VaR and CVaR allows to estimate the value of expected cost in worst case. Consequently, the decision maker becomes more aware of the future situation. If the decision maker is neutral risk, they choose lower value of expected with higher probability of risk. If he is aversion risk, he chooses higher value of expected cost but more covered against risk. So, the final decision depends on the preference of decision maker. Hence, the decisions are not univocal because they depend on the risk proneness of the decision maker.

The proposed approach is able to determine and optimise the suppliers and the allocation of ordering problem via calculation of the VaR and minimisation of the conditional value at risk (CVaR).

7 Conclusion

In this paper, we studied the problem of supplier selection and order allocation problems under disruption risk and discount based on a total quantity. We developed two stochastic models based on a scenario analysis for supply chain with three levels composed by a set of suppliers, a purchasing central and a set of customers with a multiproduct. Different criteria have been considered including the purchasing price, the discount on total quantity, the stochastic demand, the transportation cost, the shortage and inventory cost. Our study was based on quantitative techniques of risk such as VaR and CVaR. In fact, taking into consideration the different settings simultaneously make the present problem more realistic.

In a first stage, we developed stochastic mixed integer linear program based on a scenario analysis. In this model, we considered that the decision maker is a neutral risk who seeks to minimise the expected cost without considering the worst case. Based on this model, we determined the VaR with different fixed confidence levels. Then, we minimised the expected cost in the worst case by using CVaR technique. In this model, we considered that the decision maker is an aversion risk who seeks to minimise the expected cost and the expected cost in the worst case simultaneously.

Results show the more the confidence level increases more the values of VaR and CVaR increase and the decision maker becomes more covered against the consequence of supplier disruption risk. The number of suppliers increases with the confidence level, which means that the risk of disruption can mitigate by diversification of the suppliers and the order allocation. Consequently, integrated technique of VaR and CVaR allow to estimate the value of expected cost in worst case. Therefore, the decision maker become more consent for the future situation. If the decision maker is neutral risk, they choose lower value of expected with higher probability of risk. If the decision maker is aversion risk, he chooses higher value of expected cost but more covered against risk. So, the final decision depends on decision maker's preference.

The proposed approach is able to determine and optimise the suppliers and the allocation of ordering problem via calculation of the value at risk (VaR) and minimisation of the conditional value at risk (CVaR).

Future research directions to this work might include the extension of the proposed models for a dynamic model by integrating different periods. Because different disruptions can appear simultaneously, it is better to consider different sets of risk in this stage. For example, we can combine risk delay and disruption. To be more realistic, we can combine different sets of discounts such as an incremental discount, a total volume, and a reduction on total quantity discount simultaneously. In addition, we can extend this model by considering semi-global disruption, which affects a set of suppliers located in the same region. We can also extend this work by using the metaheuristic approach to solve large instances.

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