Multi-objective decision making in macro- and micro-economics with application of the MULTIMOORA method

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Abstract: MULTIMOORA, the most effective method to respond to normalisation and importance of different objectives, substitutes by dimensionless measurements the use of weights; avoids the Condorcet-Arrow paradox by starting from a decision matrix instead of comparison two by two and is composed of three methods controlling each other. The *ratio system of MOORA* goes for a ratio system with each response of an alternative on an objective compared to a positive denominator, representative for all alternatives concerning that objective, ending with a total of objectives to be maximised, the objectives to be minimised subtracted. The reference point approach is the second method in MOORA by applying the ratios found earlier and whereby a maximal objective reference point is chosen. Finally, the MULTIMOORA method adds a third method, the full multiplicative form: all elements per row in the decision matrix are multiplied. Applications in macro and micro economics illustrate the issue.

Keywords: Keynes; weights; normalisation; multi-objective optimisation by ratio analysis; MOORA; multiplicative form; ratio analysis; reference point method; MULTIMOORA; dominance.

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1 Introduction

In the light of multi-objective decision making, macro- and micro-economics are considered as problems of optimisation with many objectives or criteria, criterion being a weaker formulation of an objective. Nevertheless, the term ‘criteria’ is avoided as much as possible. Indeed, this term creates much confusion as being used for objectives as well as for attributes [Hwang and Masud, (1979), pp.6–7]. An attribute is a common characteristic of each alternative such as its economic, social, cultural or ecological significance, whereas an objective consists in the optimisation (maximisation or minimisation) of an attribute. Keeney and Raiffa (1993, pp.32–38) present the example of the objective ‘reduce sulphur dioxide emissions’ to be measured by the attribute ‘tons of sulphur dioxide emitted per year’. An attribute should always be measurable. It is true that that this optimisation is not absolute but is rather called ‘a satisfying solution’ (Wierzbicki, 1982).

The problems posited in macro-economics have to do with the business cycle. At that moment, the name of ‘Keynes’ is heard. The research on Keynes is much facilitated through the publication of his complete works.[1] In addition one could refer to the comments of different authors. However, as so much is published on Keynes reference is

only made to some of the most well known publications, namely by Hansen, Harrod and Klein.

Hansen (1953) only discusses Keynes most famous publication from 1936: '*The General Theory of Employment, Interest and Money*' (reedited in 1973). In a later publication, Hansen (1966) refers to so-called applications of the General Theory in the following 30 years like in the US Employment Act of 1946. This Act represented a modest step toward counter-cyclical fiscal policy. Harrod (1951) as a previous student and later friend of Keynes describes mostly the biography of Keynes. Klein (1950, p.14) mentioned that the opponents of Keynes argued that employment on public works would simply divert employment from private industry. On p.16, he argues, that after Keynes fluctuations in the rate of investment cause the business cycle, which in our opinion does not signify that a decision on new investments have a counter-cyclical effect immediately.

1.1 Keynes and public investment

Already in 1928 Keynes wrote: "Nevertheless, even in such cases where the cheap money stimulus to investment cannot be applied, it is still open to the public authorities to stimulate investment by stepping in with extensive programs of public works" [Keynes, (1973), p.112].

In 1929, Keynes advised Lloyd George, a candidate of the liberal party for next general election concerning "the terrible figures of the workless" [Keynes, (1981), p.804]. The government would invest: 1/4 till 1/3 coming from the disappearance of unemployment allowances, allowances already existing in Great Britain in 1929 and coming from compensation: 1/8 from the gain of the Chancellor of the Exchequer corresponding to the increase of National Income and the remainder from increase of consumption by the employed before being unemployed [Keynes, (1981), p.821].

Keynes thought in 1929 of public investment in: "roads, electricity, telephones, ports, drainage and also on railways and houses" [Keynes, (1981), p.810].

The construction sector usually reacts to economic changes with quite some delay, as current activity is based on orders made months/years earlier. Building permits take mostly a long time. Anyway against Keynes' theory plays the fact that public investments takes a very long time from intention to project, from project to decision, from decision to public subscription with sometimes appeal to a higher court or to a referendum. In that way loosing time will make public investment pro-cyclical instead of anti-cyclical as in the mean time the business cycle went upward.

More specific about the categories Keynes is mentioning:

- *Roads:* In 1929, with rather limited and not heavy traffic, cobble-stones with not much prepared underground were sufficient. Nowadays, beforehand a lot of project management is necessary (Brauers et al., 2008a). Alternatives with bridges or tunnels have to be foreseen and nuisance and pollution for neighbours have to be taken into account.
- *Electricity:* Capacity has to be foreseen long time in advance, being in particular much discussed for atomic plants.
- *Telephones:* Nowadays, many competing products are on the market and consequently telephones are no more considered as a public utility.

- *Ports:* The steamers from 1929 are now replaced by the large transportation military Liberty Ships from after the Second World War and by the large containerships since the closing down of the Suez Canal in 1956. These large ships ask for huge docks or for huge installations at a river or at the seaside to be planned long time beforehand.

- *Drainage:* Is also planned much in advance.

- *Railways*: Tracks are mostly still in hands of the state; but being part of a ten-year plan or even longer. Train companies are more a problem of competition between private companies.

- *Houses*: As official investment for social planning are also foreseen for the long-term.

Moreover, Keynes himself sees it more as a structural problem by remarking that at least one tenth of the working population was out of work for eight years (1921–1929). In April 1929 1,140,000 were unemployed [Keynes, (1972), p.92] and on p.97: "the liberal policy would provide in 10 years a million houses to meet the needs for the slums alone" and a bit further on p.98: "at the same time it would provide continuous employment for an additional 150,000 men".

Knowing that public investments, in fact public works, taking a long time between decision and execution, one may conclude that they are not an anti-cyclical instrument in recession years.

1.2 Keynes and private investment

In his book: '*The General Theory of Employment, Interest and Money*' of Keynes (1936) wrote: "I conclude that the duty of ordering the current volume of investment cannot safely be left in private hands" [Keynes, (1973), p.320] which would include private investment.

After the death of Keynes the United Nations Industrial Development Organization started its studies on feasibility of industrial investments showing the different time-consuming stages to set up these investments (UNIDO, 1968, 1978a, 1978b). Also, the theories on net present value (NPV) and internal rate of return (IRR) were developed inside industrial project management [a case study on NPV and IRR is presented in Brauers (1990)]. In the macro-economic field maximising economic net present value (ENPV) in money terms means ENPV, i.e., discounted revenues before national taxes, minus discounted investments, exclusive of subsidies. In the micro-economic field maximisation of NPV, expressed in money means discounted revenues incl. local taxes but after direct and indirect government taxes plus rent on industrial land minus investments. There is also minimisation of the Payback Period of NPV for the users, expressed in years and months.

Anyway, a private as well as a public investment policy takes too long to be considered as a counter-cyclical instrument during an economic crisis. All these investments have rather a structural behaviour than a cyclical one.

1.3 Could Keynes have done it otherwise?

Aside caeteris paribus for the Money Economy Keynes stresses also the importance of an increase in the propensity to consume in years of economic crisis. On p.325 of the General Theory (1936, but reprinted in 1973) he declares: "I should support at the same time all sorts of policies for increasing the propensity to consume. For it is unlikely that full employment can be maintained, whatever we may do about investment with the existing propensity to consume. There is room for both policies to operate together" and he continues: "If to take round figures for the purpose of illustration-the average level of output of today is 15 percent below what it would with continuous full employment, and if 10% of this output represents net investment and 90% of it consumption if, furthermore net investment would have to rise 50% in order to secure full employment with the existing propensity to consume, so that with full employment output would rise from 100 to 115, consumption from 90 to 100 and net investment from 10 to 15; then we might aim, perhaps, at so modifying the propensity to consume that with full employment consumption would rise from 90 to 103 and net investment from 10 to 12".

One has to notice that in the time of Keynes VAT had not the general influence it has in Europe today. Compared to a doubtful increase of investment in time of recession consumption can increase immediately by diminishing VAT rates on private consumption. It is even possible that by decreasing VAT rates the economy will be influenced in a positive way, even without lowering government revenue due to a Laffer curve influence (Laffer, 1981). In this sense one could conclude that the reasoning above of Keynes is out of date.

Brems (1959, p.51) is right by mentioning that instead of looking at aggregates like C, I and Y disaggregation would make economic analysis much more connected with reality. He refers therefore for example to Leontief Input-Output notations. For instance, Keynes sees a multiplier effect on Total income as a function from Total investment and a general Propensity to Consume, whereas Leontief considers production multipliers per branch of activity [Brauers, (1995), p.37].

In order to conclude Keynes considered mainly huge aggregates such as general public investment and private consumption. It was Leontief who entered more in details by introducing sector analysis.

An important remark has to be made about the aggregates discussed above. If a strong correlation exists between two or more objectives, this correlation will be the cause that the dependent objectives will be excluded from multi-objective optimisation. A side model can take these correlation issues into account. Let us elaborate on this point further on.

1.4 Correlations do not need multi-objective optimisation

Concerning the large aggregates of Keynes one may say that for Belgium the historical proportions between consumption, exports and Investment as motors of the economy are still valuable. Indeed after taking into consideration all upstream effects as translated in the production multipliers one may conclude that Belgian economic growth is due to 1/3 Private Consumption, 1/3 Exports and 1/3 Public Consumption and Investment [Brauers, (1995), p.63]. It does not mean that Investment is unimportant. On the contrary investment is a necessity, for the size of consumption, exports and stocks.

The strength of the relationship between two variables can be measured by a correlation coefficient, being a purely mathematical interpretation and is completely devoid of any cause or effect implications. Both sides of a correlation have not to be expressed in the same unit. Hoel (1971, p.188), for instance, cites the following examples: between smoking and heart ailments, between radio reception and sun-spot activity, between beauty and brains.

If a strong correlation exists between two or more objectives, this correlation will be the cause that the dependent objectives will be excluded from multi-objective optimisation. A side model can take these correlation issues into account. Indeed, in multi-objective optimisation there is no correlation at all or the correlation cannot be expressed in quantity terms. Similar to correlation the objectives in multi-objective optimisation are expressed in different units.

Multi-objective optimisation counts more than one objective, whereas at least two solutions are present, leading to a Decision Matrix. For instance different solutions are represented whereas governments but also enterprises have to take into account general well being of the population, the workers in particular, with problems of education and health and finally respect for the environment. As conclusion one may say that multiple objectives optimisation is a necessity in macro- and micro-economics, but also for general well being.

1.5 The government budget in the business cycle

A government budget always yearly in equilibrium is acceptable if the Sovereign Debt is not too high, meaning in EU terms 60% of GDP or less. Otherwise the government budget has to be cyclical with a budget surplus in the upturn of the business cycle and a budget deficit in the downturn but if necessary in such a way that on the long term the Sovereign Debt decreases.

Nevertheless public investment can stay out of this cyclical movement being rather structural than cyclical and certainly not anti-cyclical. Indeed nowadays public investment asks for a long time of preparation given necessary consulting of different stakeholders and many administrative formalities. If these arguments are not accepted an additional public investment could become rather pro-cyclical than anti-cyclical in a recession period (Brauers et al., 2014, 2013).

If public investment remains outside the cyclical movement of the government budget as said above its financing can be done by borrowing. Then, it is acceptable that public investments increase the Sovereign Debt on condition that these investments remain useful for the long run.

However, is there still room for public investment? Nowadays, highways, government buildings, prisons, public utilities and all items mentioned by Keynes in 1929 [Keynes, (1981), p.810]: roads, electricity, telephones, ports, drainage and also railways and houses, are also core business for private investment. In addition there is even public-private investment as a new form to finance public infrastructure.

One could put the question if public investment is not in competition with private investment. Agénor (2013) maintains that if public investment uses more resources to invest there will less capital available for the private sector and even it will become more costly for the private sector to borrow.

To conclude one may say that public investment comes in competition with Private Investment and public-private investment. Therefore, these three forms of investment

have to fulfil the different objectives set by all the interested stakeholders, as summarised in Table 1.

Table 1 Decision matrix of multiple objective optimisations concerning the construction of a 100 km highway

	A	*B*	*C*	*D*	*E*	*F*
Public investment	-	-	-	-	-	-
Private investment	-	0	-	-	-	-
Public-private investment	-	-	-	-	-	-

Notes: *A*: Minimisation of cost price in €.
B: Minimisation of negative influence on interest rate for private investment due to the public investment in %.
C: Minimisation time of construction in years and months.
D: Minimisation time of preparation in years and months.
E: Minimisation of average solid pollution emissions per km^2 in kg; eventually a significance coefficient for each form of solid pollution can be given by all stakeholders interested in the issue.
F: Maximisation employment in person-hours.

Some remarks about Table 1:

- Typical for any construction investment decision matrix: the number of minima is always larger than the number of maxima.

- For the choice of the objectives the delegates of all stakeholders interested in the issue are brought together for an ameliorated nominal group technique meeting. For the ameliorated nominal group technique see Appendix.

- In case of public investment the government owns the highway.
  In case of private investment the sector concerned owns the highway.
  In case of public-private investment the government owns partly the highway.

2 Choice of a method for multi-objective optimisation

The objectives are expressed in different units. Synthesis is necessary by a multi-objective method, for which MULTIMOORA is chosen for its robustness and superiority from many points of view (Brauers and Zavadskas, 2011; Balezentis et al., 2011; Chakraborty, 2011; Brauers and Ginevicius, 2010; Brauers, 2004).

MULTIMOORA is composed of three methods controlling each other: multi-objective optimisation by ratio analysis (MOORA) composed of the ratio system and the reference point method and finally the multiplicative method. Once agreement reached about alternative solutions and objectives, a decision has to be taken how to read a decision matrix.

A *Decision Matrix* assembles raw data with vertically numerous objectives, criteria (a weaker form of objectives) or indicators and horizontally alternative solutions, like projects. In order to define an objective better we have to focus on the notion of attribute.

2.1 Horizontal reading of the decision matrix

Simple additive weighting (SAW) followed by many other methods, reads the response matrix in a horizontal way. The additive weighting procedure [MacCrimmon (1968, pp.29–33), which was called SAW method by Hwang and Yoon (1981, p.99)] starts from:

$$Max.U_j = w_1 x_{1j} + w_2 x_{2j} + \ldots + w_i x_{ij} + \ldots + w_n x_{nj} \quad (1)$$

U_j overall utility of alternative j with $j = 1, 2, \ldots, m$, m the number of alternatives

$i = 1, 2, \ldots, n$ n the number of attributes or objectives

x_{ij} response of alternative j on attribute i

w_i weight of attribute I indicates as well as normalisation as the level of importance of an objective, with:

$$\sum_{i=1}^{i=n} w_i = 1$$

As the weights add to one a new super-objective is created and consequently it becomes difficult to speak of multiple objectives.

With weights importance of objectives is mixed with normalisation. Indeed, weights are mixtures of normalisation of different units and of importance coefficients.

2.2 Vertical reading of the decision matrix

Vertical reading of the decision matrix means that normalisation is not needed as each column is expressed in the same unit. In addition if each column is translated in ratios dimensionless measures are created and the columns become comparable to each other. Indeed they are no more expressed in a unit. Different kind of ratios are possible but Brauers and Zavadskas (2006) proved that the best one is based on the square root in the denominator.

Vertical reading of the decision matrix and the Brauers-Zavadskas ratios are practiced in the MOORA method.

Let us illustrate the decision matrix with a concrete example. Suppose a national government being very active with enterprise creation under the form of an investor's guide, launching project ideas, giving legal assistance and of other forms of coaching (Industry Promotion Agency). Suppose the government would have the choice to promote one of three projects. The following objectives are proposed:

1 Maximisation of NPV expressed in money terms (in million €):

   NPV = discounted revenues exclusive local and direct and indirect government taxes, inclusive rent on industrial land and depreciation, but minus investments.

2 Maximisation of the IRR expressed as a % interest rate, considering NPV equal to zero at the end of the project period.

3 Minimisation of the payback period of NPV, expressed in years and months.

4 Maximisation of government income: Local and direct and indirect government taxes in €100,000.

5 Maximising direct and indirect local and national employment; indirect employment found by local and national input-output tables in person-years

6 Maximising the increase in gross domestic product in €1 million.

7 Minimisation of the risk on 5 and 6 in %.

8 Maximisation of increase in €100,000 in the balance of payments.

9 Maximisation of hard currency to be provided by foreign sources for investment, expressed in money terms (in €1 million).

Next, Table 2 presents the decision matrix for the three projects.

Table 2 Decision matrix of multiple objective optimisation

Objectives	*1* *MAX.*	*2* *MAX*	*3* *MIN.*	*4* *MAX*	*5* *MAX.*	*6* *MAX.*	*7* *MIN.*	*8* *MAX.*	*9* *MAX.*
A	1	14	9	200	600	20	20	3.5	2.5
B	1.6	16	7	150	800	13.5	25	4	1.5
C	2	17	5	80	1200	10	30	3.8	1.25

2.3 The MOORA method

2.3.1 Ratio system of MOORA

We go for a ratio system in which each response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{j=i}^{m} x_{ij}^{2}}} \tag{2}$$

x_{ij} response of alternative j on objective i

$j = 1, 2, \ldots, m$ m the number of alternatives

$I = 1, 2, \ldots, n$ n the number of objectives

x_{ij}^{*} this time a dimensionless number representing the response of alternative j on objective i.[2]

$$y_{j}^{*} = \sum_{i=1}^{i=g} x_{ij}^{*} - \sum_{i=g+1}^{i=n} x_{ij}^{*} \tag{3}$$

with

$i = 1, 2, \ldots, g$	as the objectives to be maximised
$i = g + 1, g + 2, \ldots, n$	as the objectives to be minimised
y_j^*	the total assessment of alternative j with respect to all objectives
y_j^*	can be positive or negative depending of the totals of its maxima and minima.

For the concrete example we obtain in Tables 3 and 4:

Table 3 Sum of squares and their square roots

A	1	196	81	40,000	360,000	400	400	12.25	6.25
B	2.56	256	49	22,500	640,000	182.25	625	16	2.25
C	4	289	25	6,400	1,440,000	100	900	14.44	1.5625
Σ	8	741	155	68,900	2,440,000	682	1,925	43	10
Root	2.7495	27.221	12.45	262.488	1,562.05	26.12	43.875	6.5338	3.1721

Table 4 Objectives divided by their square roots and MOORA

										Sum	*Rank*
A	0.3637	0.5143	0.7229	0.76194	0.384	0.766	0.4558	0.536	0.788	2.9348	1
B	0.5819	0.5878	0.5623	0.57145	0.51215	0.5168	0.5698	0.61221	0.4729	2.7230	2
C	0.72739	0.6245	0.4016	0.30478	0.76822	0.3828	0.6838	0.5816	0.3941	2.6980	3

Cardinal numbers can be compared in an ordinal ranking after Arrow (1974): "Obviously, a cardinal utility implies an ordinal preference but not *vice versa*".

An ordinal ranking of the y_i^* shows the final preference in a descending order.

A second part of MOORA consists of the reference point approach which uses the ratios found in the ratio system of MOORA.

2.3.2 Reference point approach of MOORA

A second method in MOORA is the reference point approach which will use the ratios found earlier and whereby also a maximal objective reference point is used. The maximal objective reference point approach is called realistic and non-subjective as the coordinates (r_i), which are selected for the reference point, are realised in one of the candidate alternatives. In the example with A (10; 100), B (100; 20) and C (50; 50), the maximal objective reference point R_m results in: (100; 100). Per objective the coordinates of the corresponding ratio are subtracted from the coordinates of the reference point.

Then this matrix is subject to the *Metric of Tchebycheff* [Karlin and Studden, (1966), p.280]:

$$\underset{(j)}{Min}\left\{\max_{(i)}\left|r_i - x_{ij}^*\right|\right\} \tag{4}$$

with $|r_i - x_{ij}^*|$ the absolute value necessary if x_{ij}^* is larger than r_i

r_i the i^{th} coordinate of the reference point

x_{ij}^* the dimensionless measurement of objective i for alternative j

$i = 1, 2 \ldots n$ n the number of objectives

$j = 1, 2 \ldots m$ m the number of alternatives.

Tables 5 and 6 follow the concrete example:

Table 5 Reference point theory with ratios: coordinates of the reference point equal to the maximal objective coordinates

r_i	0.727393	0.6245	0.4016	0.761939	0.76822	0.766	0.4558	0.612205	0.788110

Table 6 Reference point theory: deviations from the reference point

										Rank	
										Max.	*Min.*
A	0.364	0.1102	0.3213	0	0.38411	0	0	0.0765	0	0.38411	2
B	0.145479	0.0367	0.1606	0.19049	0.25607	0.2489	0.1140	0	0.3152	0.31524	1
C	0	0	0	0.45716	0	0.3828	0.2279	0.0306	0.3941	0.45716	3

An ordinal ranking of the results shows the final preference in an ascending order [for more details on MOORA, see: Brauers (2004, 2008b)].

2.3.3 The problem of importance

With weights importance of objectives is mixed with normalisation. On the contrary the dimensionless measures of MOORA do not need external normalisation. However, the problem of importance remains. Therefore, in the ratio system of MOORA to give more importance to an objective its response on an alternative under the form of a dimensionless number could be multiplied with a significance coefficient s_i.

$$y_j^* = \sum_{i=1}^{i=g} s_i x_{ij}^* - \sum_{i=g+1}^{i=n} s_i x_{ij}^* \tag{5}$$

with

$i = 1, 2, \ldots, g$ as the objectives to be maximised

$i = g + 1, g + 2, \ldots, n$ as the objectives to be minimised

y_j^* the total assessment of alternative j with respect to all objectives

y_j^* can be positive or negative depending of the totals of its maxima and minima.

If this would be done for the reference point approach the outcome will not change due to *Tchebycheff* formula (4). If the importance coefficients are only used for the ratio system the double control system of MOORA would disappear. Therefore, another approach has to be followed applicable in both methods. Replacement of an objective by some sub-objectives, as valuable as the original objectives, will solve the problem of importance for the original objective. For instance, employment is replaced separately by direct and indirect employment.

2.3.4 MOORA can it be called robust? Characteristics of robustness in multi-objective optimisation (Brauers, 2010; Brauers and Zavadskas, 2012, 2010, 2009; Brauers and Ginevičius, 2010, 2009)

1 All stakeholders are involved (assistance can be given by the ameliorated nominal group technique, see Appendix).

2 Respect for consumer sovereignty (Brauers, 2008b).

3 All non-correlated objectives are involved, as much as possible (see, e.g., Brauers et al., 2008b).

4 All interrelations between objectives and alternatives are considered at the same time and for instance not two by two [otherwise a victim of the Condorcet-Arrow Paradox, see: Condorcet (1785), Arrow (1963),[3] Brauers, (2004), pp.118–124].

5 Non-subjective as much as possible:

- In the choice of the objectives (assistance can be given by the ameliorated nominal group technique, see Appendix).
- To give importance to an objective [assistance can be given by the Delphi method, see: Dalkey and Helmer (1963) and Brauers (2008a, 1976)].
- Omitting normalisation: Dimensionless measurements as used here are preferred to weights, which need normalisation [for normalisation, see: Brauers and Zavadskas (2007) and Brauers (2007a, 2007b)].

6 Based on cardinal numbers is more robust than on ordinal numbers. The rank correlation method of Kendall is based on ordinal numbers. He argues [Kendall, (1948), p.1]: "we shall often operate with these numbers as if they were the cardinals of ordinary arithmetic, adding them, subtracting them and even multiplying them", but he never gave a proof of this statement. In his later work this statement is dropped (Kendall and Gibbons, 1990).

7 The use of the most recent available data.

8 The use of two different methods of MOO is more robust than using a single one.

Already in 1983 at least 96 methods for multi-objective optimisation existed (Despontin et al., 1983). Since then numerous other methods appeared. Therefore, we only cite the probably most used methods for multi-objective optimisation.

First, we mention the Methods of Partial Aggregation, name given by Schärlig (1985, 1996): the Electre Group (Electre I, Electre Iv, Electre Is, Electre TRI, Electre II, Electre III and Electre IV) and Prométhée. As the study under consideration asks for total aggregation we cannot use methods based on partial aggregation.

The analytic hierarchy process (AHP) of Saaty (1988) uses weights. The use of weights in operational research was introduced by Churchman and Ackoff (1954) and Churchman et al. (1957). The additive weighting procedure called SAW was already mentioned. Also, the methods of partial aggregation use weights. In addition, all these methods are expert oriented with qualitative statements as a basis.

Reference point methods like TOPSIS (Hwang and Yoon, 1981) and VIKOR (Opricovic and Tzeng, 2004) do not use weights but rather dimensionless measures but they are overtaken by MOORA which is composed of two different dimensionless based methods, each controlling each other.

An interesting example of MOORA being compared with other methods is what Chakraborty has done for industrial management. Chakraborty (2011) checked six famous methods of multi-objective decision making for decision making in manufacturing. Next, Table 7 shows the results.

Table 7 Comparative performance of some MODM methods

MODM	*Computational time*	*Simplicity*	*Mathematical calculations*	*Stability*	*Information type*
MOORA	Very less	Very simple	Minimum	Good	Quantitative
AHP	Very high	Very critical	Maximum	Poor	Mixed
TOPSIS	Moderate	Moderately critical	Moderate	Medium	Quantitative
VIKOR	Less	Simple	Moderate	Medium	Quantitative
ELECTRE	High	Moderately critical	Moderate	Medium	Mixed
PROMETHEE	High	Moderately critical	Moderate	Medium	Mixed

2.4 The MULTIMOORA method

2.4.1 The full multiplicative form and MULTIMOORA

To the two methods of MOORA a third method is added: the full multiplicative form. The use of three different methods of MOO is more robust than using of two, making MULTIMOORA superior to all existing methods of multiple objectives optimisation.

$$U_j = \prod_{i=1}^{n} x_{ij} \tag{6}$$

with

$j = 1, 2, \ldots, m$ m the number of alternatives

$i = 1, 2, \ldots, n$ n being the number of objectives

x_{ij} response of alternative j on objective i

U_j overall utility of alternative j.

The overall utilities (U_j), obtained by multiplication of different units of measurement, become dimensionless.

2.4.2 Objectives moving in a different direction

How is it possible to combine a minimisation problem with the maximisation of the other objectives? Therefore, the objectives to be minimised are denominators in the formula:

$$U'_j = \frac{A_j}{B_j} \tag{7}$$

with

$$A_j = \prod_{i=1}^{g} x_{ij}$$

g The number of objectives to be maximised.

with

$$B_j = \prod_{i=g+1}^{n} x_{ij}$$

$n - g$ The number of objectives to be minimised.

With U'_j : the utility of alternative j with objectives to be maximised and objectives to be minimised.

In the full multiplicative form a problem may arise for zero and negative values making the results senseless. Therefore, the index number 100 replaces the zero number. At that moment for instance 96.6 substitutes the negative value of minus 3.4. Consequently, 103.4 represent the positive value of 3.4. As these operations are made per objective, vertically in the decision matrix, this solution is acceptable.

Table 8 completes the concrete example.

Table 8 simulation by the full multiplicative form

	1	2	3	4	5	6	7	8	9
	MAX.	MAX.		MIN.		MAX.		MAX.	
			3 = 1 × 2		5 = 3:4		7 = 5 × 6		9 = 7 × 8
A	1	14	14	9	1.55555556	200	311.111111	600	186,666.667
B	1.6	16	25.6	7	3.65714286	150	548.571429	800	438,857.143
C	2	17	34	5	6.8	80	544	1,200	652,800
10	11	12	13	14	15	16	17	18	
MAX.		MIN.		MAX.		MAX.			
	11= 9 × 10		13 = 11:12		15 = 13 × 14	,	17 = 15 × 16	Result	Projects
20	3,733,333.33	20	186,666.667	3.5	653,333.333	2.5	1,633,333	1	*A*
13.5	5,924,571.43	25	236,982.857	4	947,931.429	1.5	1,421,897	2	*B*
10	6,528,000	30	217,600	3.8	826,880	1.25	1,033,600	3	*C*

Finally, three methods are combined in MULTIMOORA. In this way, all methods using dimensionless measures are explicitly present and consequently do escape from the subjective choice of weights.

2.5 The theory of dominance

In the most of the not too complicated cases a synthesis of ranking of the three MULTIMOORA methods was made on view. For very large matrices, Brauers and Zavadskas (2011) developed a theory of dominance.

2.5.1 Axioms on ordinal and cardinal scales

1 A deduction of an ordinal scale, a ranking, from cardinal data is always possible (Arrow, 1974).

2 An ordinal scale can never produce a series of cardinal numbers (Arrow, 1974).

3 An ordinal scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.

In application of axiom 3, the ordinal scale of the three methods of MULTIMOORA is translated in another ordinal one based on dominance, being dominated, transitivity and equability.

2.5.2 Dominance, being dominated, transitiveness and equability

2.5.2.1 Dominance

- *Absolute dominance:* Means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).
- *General dominance in two of the three methods with:* a P b P c Pd (P preferred to) is, for instance, of the form:

  (d-a-a) is generally dominating (c-b-b)

  (a-d-a) is generally dominating (b-c-b)

  (a-a-d) is generally dominating (b-b-c)

  and further on transitiveness plays fully.

2.5.2.2 Transitiveness

If a dominates b and b dominates c then also a will dominate c.

2.5.2.3 Overall dominance of one alternative on another

For instance, (a-a-a) is overall dominating (b-b-b) which is overall being dominated by (a-a-a).

2.5.2.4 Equability

- *Absolute equability*: Has the form: for instance, (e-e-e) for two alternatives.
- *Partial equability*: Of 2 on 3 exists, e.g., (5-e-7) and (6-e-3).

2.5.2.5 Circular reasoning

Despite all distinctions in classification some contradictions remain possible in a kind of circular reasoning, like, for instance:

Object A (11-20-14) dominates generally object B. (14-16-15)

Object A (11-20-14) dominates generally object B. (14-16-15)

but Object C (15-19-12) dominates generally Object A (11-20-14).

In such a case the same ranking is given to the three objects.

Table 9 illustrates this outcome for the given concrete example.

Table 9 MULTIMOORA obtained as a summary of the three methods

Projects	*MOORA ratio system*	*MOORA reference point*	*Multiplicative form*	*MULTIMOORA*
A	1	2	1	*1*
B	2	1	2	*2*
C	3	3	3	*3*

There is a small deviation in the reference point part of MOORA but one may conclude for MULTIMOORA that project A is preferred above B, an in between solution. Project C comes in the last position.

Not too many problems arose concerning software used. EXCEL offered sufficient possibilities as software for MULTIMOORA [for Excel see, for instance, Herkenhoff and Fogli (2013) and Quirk (2011)].

In another illustration a comparison is drawn between credit rating agencies like Standard & Poor's, Moody's and Fitch rating countries economic growth with ipso facto countries credit rating on the one side and with the same purpose, MULTIMOORA on the other side. MULTIMOORA presented a ranking of the 27 EU-Countries' economies on basis of 22 objectives but with very comparable results compared to the credit rating agencies ratings. Only MULTIMOORA was the work of maximum four par time researchers against the perhaps 10,000 employees of the credit rating agencies (Brauers and Zavadskas, 2013; Brauers et al., 2012a).

3 MULTIMOORA in the micro economy

Up till now, macro-economics was considered and namely as the economics of the macro-economy. Macro-economy concerns the economy of the world, the nations, the regions, the towns and cities and the industrial and service sectors in their connection with one of the cited higher levels.

Micro-economics on the other hand, study the micro-economy under the form of sectors as professional organisations, of non-profit organisations, of firms, of markets and of the household economy.

An example studies the micro economics of the shareholders of a company, an investment from the point of view of a special kind of investors.

3.1 Investment in shares by MULTIMOORA multiple objectives optimisation

Shareholders participate in the capital of a company. In a company quoted on a stock exchange, this ownership is rather passive as the shareholders are excluded from any form of management, with exception for the reference shareholders.

Investments in stocks from a selection of companies with a different activity or a different location will diminish the risk after the saying: "you must not put all your eggs in one basket". In addition, the advices of experts will help too. These advices may have different forms, such as from: credit rating agencies like Moody, Standard & Poor's and Fitch, a general theory about investment in stocks such as the Buffet philosophy, the advice of one or different experts based on: their personal experience, company balance analyses, interviews with managers or sampling. Why not on basis of multiple objectives optimisation, which would be new and preferably for leading world indices such as: Dow Jones-Industrial New York, FTSE100 London or Nikkei225 Tokyo?

Taking into consideration for instance six objectives would lead to a matrix of 600 elements for the FTSE100 London and for Nikkei225 Tokyo even more. Therefore, a multi-objective optimisation approach which can handle large matrices has to be chosen. MULTIMOORA responds to this condition, is even composed of three different methods which can control each other and is based on dimensionless measures, excluding the difficult problem of normalisation. In order to summarise the three outcomes the theory of dominance is applied.

To make the application much simpler the Belgian Bel20 Index was used but with eight objectives, specific for Belgium. With the three bank shares excluded, due to the instability of banks at the moment, the matrix involved still counted 102 elements. As a measure of importance a 9th objective was added under the form of the opinion of the analysts for each share which stresses a general opinion on each share.

The analysts lead to the following advices:

1 1.49 buy

1 5–2.49 increase your stock

2 5–3.49 hold

3 5–4.49 decrease your stock

4 5–5 sell.

General remarks: the application concerns the past. Pure extrapolation has no sense for such a fluctuating market. Many other factors have to be taken into consideration. Regularly, revisions are needed. Companies with an effective management are assumed. Finally, there are the unknown unknowns or may we say the economics of uncertainty? For instance, a combination of earthquake, tsunami and atomic plants disasters is certainly fatal for insurance companies.

This investigation is just an exercise how to operate similar studies. It would be preferable that a larger group than our researchers would repeat the application for a larger stock exchange index such as those of Tokyo, London or New York.

3.2 The fields of building and engineering

Following publications can be mentioned:

- the optimal construction of a highway in Eastern Germany (Brauers et al., 2008a)
- the optimal construction of dwellings (Brauers et al., 2008b)
- ranking heating losses in a building by applying MULTIMOORA (Kracka et al., 2010)
- exploratory analysis of project selection through MCDM (Bakshi and Sarkar, 2011)
- Lithuanian case study of masonry buildings from the Soviet period (Brauers et al., 2012b)
- sustainability of electricity generation technologies in the EU (Dapkus and Streimikiene, 2013)
- the interval-valued intuitionistic fuzzy MULTIMOORA method for group decision making in engineering (Zavadskas et al., 2015).

4 Conclusions

Conducting research on the macro- and micro-economy on basis of multiple objectives was the topic of this research. This evaluation is made by a method called MULTIMOORA. MOORA composed of two approaches, namely ratio system and reference point theory, the last one starting from the previous found ratios, solves the difficult problem of normalisation, whereas the importance of the objectives is treated separately. If MOORA is joined with the full multiplicative form for multiple objectives, also with the importance of the objectives treated separately, a total of three methods are formed under the name of MULTIMOORA. The MULTIMOORA method can consider all the attributes along with their relative importance, and hence, it can provide a better accurate evaluation of the alternatives. Finally, a dominance theory is applied to summarise the three approaches and to evaluate and rank the different alternative solutions.

Applied for macro-economics there is little evidence for Keynes' theory that public or private investment can support immediately a declining economy in a recession period. As further illustration of the macro-economy a simulation makes a ranking for public, public-private and private investment. In addition, reference is made of a study concerning a comparison on the one side credit rating agencies like Standard & Poor's, Moody's and Fitch rating countries economic growth and ipso facto countries credit rating. On the other side with the same purpose MULTIMOORA obtains similar results.

As illustrations for the micro economy, MULTIMOORA multiple objectives optimisation considers the choice problem related to investment in shares by private shareholders and mentions optimisation studies on building and engineering.

Being a tool for multi-objective optimisation in decision theory MULTIMOORA forms a multi-purpose instrument not only for the study of the macro-economy but for the micro economy as well.

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Notes

1 The complete works of Keynes were consulted being possible as his complete works are brought together by different editors in 29 volumes: "the Collected Writings of John Maynard Keynes", London, McMillan and the Cambridge University Press for the Royal Economic Society, 1971–1989.

2 *Dimensionless numbers*, having no specific unit of measurement, are obtained, for instance, by multiplication or division. The Brauers-Zavadskas responses of the alternatives on the objectives belong to the interval *[0, 1]*. However, sometimes the interval could be *[–1, 1]*. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity, i.e., a negative dimensionless number. Instead of a normal increase in productivity growth a decrease remains possible. At that moment the interval becomes *[–1, 1]*. Take the example of productivity, which has to increase (positive). Consequently, a maximisation of productivity is looked for, e.g., in European and US countries. What if the opposite does occur? For instance, take the original transition from the USSR to Russia. Contrary to the other European countries productivity decreased. It means that in formula (2) the numerator for Russia was negative with the whole ratio becoming negative. Consequently, the interval changes to: *[–1, +1]* instead of *[0, 1]*.

3 In the 1963-edition of his book Arrow maintains that in the first edition of 1951 he was not aware of the work of Condorcet: "when I first studied the problem and developed the contradictions in the majority rule system, I was sure that this was no original discovery, although I had no explicit reference, and sought to express this knowledge by referring to the well known 'paradox of voting".

Appendix

The ameliorated nominal group technique

The approach of the nominal group technique, which is explained here, was ameliorated by Brauers (1987, 2004, pp.40–64) but the nominal group technique was first elaborated by Van de Ven and Delbecq (1971).

A.1 The original nominal group technique

The nominal group technique consists of a sequence of steps, each of which has been designed to achieve a specific purpose.

1 The steering group or the panel leader carefully phrases as a question the problem to be researched. Much of the success of the technique hinges around a well-phrased question.

   Otherwise the exercise can easily yield a collection of truisms and obvious statements. A successful question is quite specific and refers to real problems. The question has to have a singular meaning and a quantitative form as much as possible.

2 The steering group or the panel leader explains the technique to the audience. This group of participants is asked to generate and write down ideas about the problem under examination. These ideas too have to have a singular meaning and a quantitative form as much as possible. Participants do not discuss their ideas with each other at this stage. This stage lasts between 5 and 20 minutes.

3 Each person in round-robin fashion produces one idea from his own list and eventually gives further details. Other rounds are organized until all ideas are recorded.

4 The steering group or the panel leader will discuss with the participants the overlapping of the ideas and the final wording of the ideas.

5 The nominal voting consists of the selection of priorities, rating by each participant separately, while the outcome is the totality of the individual votes. A usual procedure consists of the choice by each participant of the n best ideas from his point of view, with the best idea receiving n points and the lowest one point. All the points of the group are added up. A ranking is the democratic result for the whole group.

The original nominal group technique can be characterised as weak robust as the participants expressed too much their personal feeling. For that reason amelioration was proposed.

A.2 The ameliorated nominal group technique

As there was too much wishful thinking even between experts better results were obtained if the group was also questioned about the probability of occurrence of the event, here an objective. In this way the experts became more critical even about their own ideas. The probability of the group is found as the median of the individual probabilities.

Finally, the group rating (R) is multiplied with the group probability (P) in order to obtain the effectiveness rate of the event, (E): $E = R \times P$. The effectiveness rates of the group are ordered by ranking.

In an example of application of the nominal group technique for the facilities sector in Lithuania 15 stakeholders represented all the stakeholders interested in the issue (see Brauers and Lepkova, 2003). A neutral panel leader directed the exercise. Each participant has chosen the most important five objectives from his point of view, with the most important objective receiving five points and the less important one point. The

introduction of probabilities of realisation, introducing a sense of reality and presenting a guaranty against wishful thinking, produces quite some changes in the ranking.

The total 225 is a control figure for the group result. Indeed, each participant could distribute maximum: 5 + 4 + 3 + 2 + 1 = 15 points. With 15 participants, the total has to be not more than 225. It could be less, as each participant is not obliged to allot 15 points. The total of the given points, here namely 225, means that each participant used his rights completely.

Contrary to Delphi, convergence is not aimed at, but final voting is used. In this way, nominal group technique could be considered as exploring any idea about objectives, advisable for a preliminary version of Delphi, where convergence could be reached about the list of objectives after several rounds (Dalkey and Helmer, 1963; Brauers, 2004, 2008a).