Operation optimisation of urban rail transit train base on energy saving

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Abstract: To overcome the problem in currently studying train energy saving operation optimisation, the speed is selected as the design variable to establish the objective and constraint functions and train energy saving operation optimisation model. Taking a route optimisation as an example, the program of train energy saving operation optimisation based on MATLAB language is developed, and optimisation solution is accomplished by using the sequential quadratic programming (SQP) algorithm. Then, the distribution of each interval endpoint speed is obtained and the optimum speed-distance curve is designed. Optimisation results show that this handling research method is feasible and effective.

Keywords: urban rail transit; energy saving; operation optimisation; SQP algorithm; MATLAB.


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1 Introduction

Urban rail transit system is the backbone of the urban public transportation. With the characteristics of the energy-saving, land-saving, large capacity, high efficiency, punctuality and so on, urban rail transit system is very popular and is especially suitable for large and medium-sized cities (Lin et al., 2016). The energy consumption of railway transport system refers to the equipment consumption of train traction, ventilation, air conditioning, elevators, lighting, water supply and drainage, weak current equipment and so on. According to statistics, train traction energy consumption accounted for over 40% of the total energy consumption in the rail transit system (Wang, 2011). Under increasing concern in low-carbon environment, energy conservation and emission reduction, reducing the traction energy by energy saving operation optimisation is one of the efficient methods to cut down the energy consumption of subway system (Gu et al., 2014; Tang et al., 2016). Therefore, the train energy saving operation optimisation has become an important research direction in the field of rail transit. In fact, the train energy saving operation is a optimal problem for designing speed-distance curve of the minimum energy consumption under the constraints (Xun et al., 2014). So, the researchers have used many optimisation algorithms to optimise the train energy saving operation model. For example, Jin et al. (1998) proposed a computation model combined local optimisation with global to generate the velocity schema curve of optimisation operation for train, Miyatake et al. (2010) used the three methods [dynamic programming, gradient method and sequential quadratic programming (SQP)] to optimise the train speed profile for minimum energy consumption, Lu et al. (2012) used genetic algorithm for energy saving optimal control of following trains, Chen et al. (2012) proposed the optimisation algorithm of train operation energy consumption based on genetic algorithm, Dominguez et al. (2014) proposed the design method of efficient ATO speed profiles in metro lines based on multi objective particle swarm optimisation, Liu et al. (2016) proposed a novel time-approaching search algorithm for energy-saving optimisation of urban rail train, Lin et al. (2016) proposed multi-train energy saving for maximum usage of regenerative energy by dwell time optimisation in urban rail transit using genetic algorithm, and Huang et al. (2016) proposed energy optimisation for train operation based on an improved ant colony optimisation methodology.

The fmincon, MATLAB optimisation function, integrates the SQP algorithm. SQP method is considered to be one of the most efficient methods to solve the non-linearly constrained optimisation problems (Jian, 2005; Xue et al., 2009). Schittkowski (1986), for example, has implemented and tested a version which outperforms every other tested
method in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems. Accordingly, the SQP algorithm is selected to optimise energy saving control of the train in the paper.

The vast majority of studies (Jin et al., 1998; Liu and Golovitcher, 2003; Chen et al., 2012; Su et al., 2014; Li et al., 2016; Liu et al., 2016; Song et al., 2016; Yang et al., 2016; Tian et al., 2017) on train energy saving operation optimisation methods consider displacement or time as design variables for optimisation solution, but the method has the problem that mathematical relationship is not clear. Because of the increases of ramps, curves and speed limit intervals in route, however, the expressions will become very complicated and difficult to solve. In fact, the relationship – the traction and braking force with the speed – is clear, and there are various speed limits in route. Therefore, the paper selects the speed as design variable to build the relationships of displacement, time and acceleration with the speed achieving the unity of the dependent variable expression. Considering running time, boundary conditions, speed limits, line data, according to the relationships, the constraint functions are set up, establishing the train operation optimisation based on the energy saving. To reduce the difficulty of solution and improve the optimisation efficiency, we reduce the design variables by using equality constraint. The optimisation results, which are reliable, are used to obtain the optimal speed-distance curve and the speed-time and acceleration-distance curve.

2 Train operation model for energy saving

2.1 Train travelling process

When the train runs in a route, the speed limit that the train is not allowed to exceed is calculated according to the line condition and train characteristics. Under the constraints of speed limit the train travelling processes usually contain four stages: traction, cruise, coasting and braking, as shown in Figure 1. Traction state: the train accelerates and the engine consumes energy; cruise state: the train keeps uniform motion by providing the traction or braking according to the stress state, and the sum of the forces on train is zero; coasting stage: neither drawing nor braking, the engine not consumes energy, and the train running state depends on the total resistance; braking state: the train slows down and the engine consumes no energy. If the distance between the stations is shorter, the train usually uses ‘traction – coasting – braking’ strategy to run. If the distance between stations is longer, after the train reaches the speed limit by traction, the train usually takes the strategy of coasting, cruise and traction until stop when it is close to the next station.

2.2 Train dynamical model

The actual stress state is very complicated when the train is running. The single mass point model is a common simplified method. The train is considered as the single mass point in the method, and its movement satisfies Newton’s kinematics laws, as shown in Figure 2. Its force can be divided into four types: traction $F$, braking force $B$, total resistance $W$ and gravity $G$. The motion differential equations are described as follows.
According to the transformation, \( \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} \), the equation (1) can be transformed into:

\[
\frac{dx}{dv} = \frac{Mv}{u_t F_{max} - u_s B_{max} - W}
\]

(2)

According to the equation (1), the equation can be described as follows:

\[
\frac{dt}{dv} = \frac{1}{a} = \frac{M}{u_t F_{max} - u_s B_{max} - W}
\]

(3)

Figure 1  Speed-distance curve between stations (see online version for colours)

Figure 2  Force analysis diagram on the single mass point

In the equations (1)–(3):

- \( M \)  train mass
- \( v \)  train speed
- \( a \)  train acceleration
- \( t \)  train running time
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\( u_f \) traction coefficient (\( u_f \in [0, 1] \))

\( u_b \) braking force coefficient (\( u_b \in [0, 1] \))

\( F_{\text{max}} \) maximum traction

\( B_{\text{max}} \) maximum braking force

\( W \) total resistance of the train running.

Traction \( F \), which keeps a same direction with train running, is produced by power transmission device to drive train. It is maximum \( F_{\text{max}} \) is determined by speed. The relationship between speed and \( F_{\text{max}} \) depends on the traction characteristic of the train.

Braking force \( B \), which keeps an opposite direction with train running, is produced by the braking device. Braking force \( B \) can be controlled by the driver. The maximum \( B_{\text{max}} \) is determined by speed. The relationship between speed and \( B_{\text{max}} \) depends on the traction characteristic of the train.

Total train resistance \( W \) refers to the outside force on account of the interaction between the train and the outside world, producing the opposite force with the train running direction. The force usually blocks the train running, and can not be control by driver. According to its formation, it can be divided into basic resistance and additional resistance. The basic resistance of the train expresses the resistance without the gradient and curve in the line. The resistance is mainly due to the mechanical friction and air friction. Specifically, it can be divided into five parts: the frictional resistance between axle and bearing, rolling frictional resistance between wheel and rail, impact resistance and aerodynamic resistance. So, the basic resistance relates to many factors, such as the train composition, the state, the running speed, line data, weather situation and so on. The factors interaction are quite complicated. In practice, it is difficult to accurately express the basic resistance. Therefore, the basic resistance usually adopts the empirical formula to calculate as follows:

\[
\frac{w_0}{a+bv+cv^2}
\]  

(4)

where \( w_0 \) is the unit basic resistance, \( a \), \( b \) and \( c \) are the resistance multinomial coefficient defined by the experience, and \( v \) is the train speed.

Under the additional conditions, such as gradient, curve and tunnel, running resistance of the train will increase, and the increased resistance called additional resistance. Additional resistances mainly are made up of the gradient additional resistance and curve additional resistance. The gradient additional resistance is a component force toward the moving direction due to the gravity when the train climbs or descends. In general, the resistance is calculated as follows.

\[
w_i = i
\]  

(5)

where \( w_i \) is the unit gradient resistance coefficient, \( i \) is the track gradient. If \( i > 0 \) means that the train climbs, and if \( i < 0 \) means that the train descends.

The train curve resistance mainly depends on the track curvature radius. When the train runs in the route with curve, the force of sliding friction between longitudinal and transverse in wheel-rail and the force of friction between the parts of the bogie will
increase, and the increased resistance is called the curve resistance. In general, the formula of the curve resistance is given by

\[ w_c = \frac{C}{R} \quad (6) \]

where \( w_c \) is the unit curve resistance coefficient, \( R \) is the curvature radius, and \( C \) which comprehensively reflects from the curve resistance factors is the empirical constant. In the rail transit of China, the value \( C \) generally is equal to 600.

In order to calculate conveniently, sometimes, when gradient additional resistance and additional curve resistance appear simultaneously, according to the principle of the equal force, the curve additional resistance is converted into gradient combined with the virtual gradient as the total gradient.

In conclusion, the train running total resistance can be calculated according to the following formula.

\[ W = (w_0 + w_1) \cdot g \cdot M / 1,000 \quad (7) \]

Here, \( W \) is the line resistance, \( w_0 \) is the unit resistance coefficient, the \( w_1 \) is the unit resistance coefficient, \( M \) is the train mass, and \( g \) is the gravitational acceleration.

In the equations (4)–(7), the unit of physical quantity is explained as shown Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The explanation on the symbol unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>( w_0, w_1, w_c, w_i )</td>
</tr>
<tr>
<td>Unit</td>
<td>N/kN, km/h, %, m, N, kg</td>
</tr>
</tbody>
</table>

2.3 The train operation strategy based on energy saving

When the train is running between stations, there are speed-distance curves which can be chosen by us to control the train. Different speed-distance curves between stations are corresponding to different running time and energy consumption, as shown in Figure 3. In Figure 3, the train can finish the same distance using the four operation schemes, but running time and energy consumption is different. In addition, even if the station running time is same, there are multiple speed-distance curves for the train to use.

Figure 3  Different speed-distance curves (see online version for colours)
Generally, there is an approximation relation between the running time and the energy consumption, as shown in Figure 4. It is worth noting that to increase the running time does not reduce the same amount of energy consumption. Accordingly, to find a optimal speed-distance curve, which makes the energy consumption least under maintaining the same running time, is the objective of the train operation optimisation.

**Figure 4** Energy consumption and running time curve

By using the maximum principle, Asnis et al. (1985) proved the optimal speed-distance curve that consists of four working conditions: maximum traction, cruise, coasting and maximum braking. Hence, under the constraints of the speed limit, based on energy saving the train operation strategy is designed as follows.

The traction acceleration stage: the train runs by using full traction to accelerate where $u_f = 1$ and $u_b = 0$.

The cruise stage: the train keeps uniform motion by using partial traction or braking where $0 < u_f < 1$ and $0 < u_b < 1$.

The coasting stage: the train neither draws nor brakes where $u_f = 0$ and $u_b = 0$.

The braking stage: the train uses the maximum braking force to brake where $u_f = 0$ and $u_b = 1$.

3 Optimisation model of the train operation based on energy saving

3.1 Design variable

The relationship between speed with time, displacement and outside force, such as traction, braking force and basic resistance, is clear. So, the interval endpoint speed is selected as the design variable in this paper, and SQP algorithm is applied to optimise the train operation model based on energy saving. According to equations (1)–(3), the distribution of each interval endpoint time, distance and acceleration is calculated, which obtains the speed and distance curve of energy consumption minimisation. Moreover, considering that the speed is constant in the cruise stage, we can not obtain the time and distance by using speed. So the design variables need to add time variable $t$ (cruise duration). To sum up, the design variable is expressed as follows.
where $n$ is the number of demarcation interval, and there are same definitions below.

### 3.2 Objective function

Ignoring other current consuming apparatus of the train, we adopt the minimum energy consumption in train traction as objective function as below.

$$\min E = \int_0^T F(t)v(t)dt$$

(9)

According to equation (3), $v$ is selected as the independent variable to express total energy consumption of the train as below.

$$\min E = \sum_{i=1}^{n} \int_{v_i}^{v_{i+1}} \frac{F(v)}{a_i(v)} dv$$

$$= \sum_{i=1}^{n} \int_{v_i}^{v_{i+1}} \left( M\lambda_i(v) + W_i(v) \right) \frac{v}{a_i(v)} dv$$

$$= \sum_{i=1}^{n} \int_{v_i}^{v_{i+1}} \left( M + \frac{W_i(v)}{a_i(v)} \right) \frac{dv}{v} + \sum_{i=n_1+1}^{n_2} W_i t_i$$

(10)

where $1 \sim n_1$ expresses the acceleration interval, $n_1 + 1 \sim n_2$ expresses the cruise interval and $\sum_{i=n_1+1}^{n_2} t_i = t$. The equation (10) defines the traction by using the acceleration and resistance in order to express the traction included the acceleration constraint clearly.

### 3.3 Constraint function

According to the equation (3), the time constraint is given by

$$\sum_{i=1}^{n_1} \int_{v_i}^{v_{i+1}} \frac{dv}{a_i(v)} + t + \sum_{i=n_1+1}^{n_2} \int_{v_i}^{v_{i+1}} \frac{dv}{a_i(v)} + \sum_{i=n_1+1}^{n_2} \int_{v_i}^{v_{i+1}} \frac{dv}{a_i(v)} = T$$

(11)

According to the equation (2), the distance constraint is defined as

$$\sum_{i=1}^{n_1} \int_{v_i}^{v_{i+1}} \frac{v}{a_i(v)} dv + v_{n_2} t + \sum_{i=n_1+1}^{n_2} \int_{v_i}^{v_{i+1}} \frac{v}{a_i(v)} dv + \sum_{i=n_1+1}^{n_2} \int_{v_i}^{v_{i+1}} \frac{v}{a_i(v)} dv = L$$

(12)
In the equations (10) and (11), \( n_2 + 1 \sim n_3 \) expresses the coasting interval and \( n_3 + 1 \sim n \) expresses the braking interval.

Acceleration constraint is expressed as below:

\[
|a_i(v)| \leq 1 \quad (i = 1, 2, \ldots, n+1) \tag{13}
\]

Speed limit is written by

\[
v_i \leq \bar{v}_i \quad (i = 1, 2, \ldots, n+1) \tag{14}
\]

The speed constraint of each station is described as

\[
v_i = 0, \quad v_{i+1} = 0 \tag{15}
\]

## 4 Train analysis of operation optimisation based on energy saving

### 4.1 Train characteristic parameter and the railway data

The study on operation optimisation based on energy saving is to solve the problem that a train passes though a route (A7–A6 station). The distance of the route is 1,344 m, and the run time is 110 s. The train characteristic parameters are shown as Table 2. The line data is shown as Table 3. The relationships between maximum traction and braking force with speed are shown as equations (16) and (17).

\[
F_{\text{max}} = \begin{cases} 
203 & 0 \leq v \leq 51.5 \text{km/h} \\
-0.002032v^3 + 0.4928v^2 - 42.13v + 1343 & 51.5 \leq v \leq 80 \text{km/h}
\end{cases} \tag{16}
\]

\[
B_{\text{max}} = \begin{cases} 
203 & 0 \leq v \leq 77 \text{km/h} \\
0.1343v^2 - 25.07v + 1,300 & 77 \leq v \leq 80 \text{km/h}
\end{cases} \tag{17}
\]

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Characteristic parameters of train</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Train mass</td>
</tr>
<tr>
<td>Symbol</td>
<td>Value</td>
</tr>
<tr>
<td>( a )</td>
<td>194.295 t</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Line data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval range (m)</td>
<td>Gradient (%)</td>
</tr>
<tr>
<td>0–50</td>
<td>0</td>
</tr>
<tr>
<td>50–670</td>
<td>3.5</td>
</tr>
<tr>
<td>670–1,050</td>
<td>−1.8</td>
</tr>
<tr>
<td>1,050–1,234</td>
<td>0</td>
</tr>
<tr>
<td>1,234–1,354</td>
<td>0</td>
</tr>
</tbody>
</table>
4.2 Qualitative analysis and assumption of the train energy saving operation optimisation

To simplify the calculation, according to the line data and train parameters the qualitative analysis and assumptions are given as follows.

1. Assume that the train uses ‘traction – coasting – braking’ strategy to run, considering that the distance of two stations is nearest.

2. Assume that the train uses the maximum acceleration to accelerate or brake. At the same time, ensure the ride comfort: \( |a| \leq 1 \).

3. Assume that the interval is divided into eight sections, as shown in Figure 5. In Figure 5 \( v_1 \sim v_9 \) express the speeds of each interval endpoint. According to the total distance and time, the average speed is 44.0 km/h (12.3 m/s). If the train begins to brake when the distance is 1,234 m from the next station, \( v_9 = 53.5 \) km/h. Comparing the \( v_9 \) with the average speed, we find that it is unreasonable. So, the braking stage must be within the range of 1,234 to 1,354 m. Moreover, comparing the max speed (≥ 51.5 km/h) with the average speed, we can see that the cruise stage will not be too long, otherwise the constraints of the total running time and distance will not be met. From the perspective of energy consumption, the smaller the traction stage is, the lower the energy consumption is. From the discussion above, the interval divisions are assured as shown in Figure 5.

4. Assume that the speed in interval range meets speed limits, if the speed of interval endpoint meets the speed limits.

\[ \text{Figure 5} \quad \text{The process of train running} \]

4.3 The specific optimisation model of operation based on energy saving

1. According to Figure 5, the variable is defined as

\[ X = [v_1, v_2, \ldots, v_9]^T \quad (18) \]
According to Figure 5, the objective function is defined as

\[
\min E = \sum_{i=1}^{3} \int M + \frac{W_i(v)}{a_i(v)} \, dv
\]  

(19)

Constraint function is defined as:

- **Total time constraint:**

\[
\sum_{i=1}^{3} \int \frac{dv}{a_i(v)} + \sum_{i=1}^{7} \int \frac{dv}{a_i(v)} + \int \frac{dv}{a_i(v)} = 110
\]  

(20)

- **Total distance constraint:**

\[
\sum_{i=1}^{3} \int \frac{v}{a_i(v)} dv + \sum_{i=1}^{7} \int \frac{v}{a_i(v)} dv + \int \frac{v}{a_i(v)} dv = 1354
\]  

(21)

- **Intermediate distance constraints:**

\[
\int \frac{v}{a_i(v)} dv = 50
\]  

(22)

\[
\int \frac{v}{a_2(v)} dv + \int \frac{v}{a_3(v)} dv + \int \frac{v}{a_5(v)} dv = 620
\]  

(23)

\[
\int \frac{v}{a_6(v)} dv = 380
\]  

(24)

\[
\int \frac{v}{a_6(v)} dv = 184
\]  

(25)

- **Acceleration limits:**

\[|a_i(v)| \leq 1 \quad (i = 1, 2, \ldots, 8) \]  

(26)

- **Speed limit constraints:**

\[
\begin{cases}
v_i \leq 80 & \quad (i = 1, 2, \ldots, 8) \\
v_{i0} \leq 55 & \quad (i = 1, 2, \ldots, 8) 
\end{cases}
\]  

(27)

- **The speed constraints of A6 and A7 station:**

\[v_1 = 0, \quad v_9 = 0\]  

(28)

- **The speed constraint of segment point on the maximum traction:**

\[v_3 = 51.5\]  

(29)
In the equations (18)–(29), time unit is s, speed unit is km/h, distance unit is m, and acceleration unit is m/s². To ensure that the calculated time, displacement and energy units are s, m and J, we must convert speed unit into m/s in practical computation.

4.4 The simplification of design variable on optimisation model

Considering that intermediate distance constraints are simple equations, we can reduce the number of non-independent design variables by solving those equations before optimisation. On the one hand, reducing design variables can decrease the scale of the optimisation model, thus to reduce the difficulty to optimise. On the other hand, reducing design variables can improve the optimisation efficiency. Because it is hard to use analytical method to solve equations (22)–(25) with integral relation, we use the numerical method to solve those equations. According to the equations (22)–(25), (28) and (29), we select \( v_4 \) and \( v_8 \) as design variable, other variables can be written as follows.

\[
v_2 = f_1(v_1)
\]  
(30)

where \( f_1 \) is the explicit expression equal to equation (22).

\[
v_5 = f_2(v_2, v_3, v_4)
\]  
(31)

where \( f_2 \) is the explicit expression equal to equation (23).

\[
v_6 = f_3(v_5)
\]  
(32)

where \( f_3 \) is the explicit expression equal to equation (24).

\[
v_7 = f_4(v_6)
\]  
(33)

where \( f_4 \) is the explicit expression equal to equation (25).

From equations (28) and (29), we can see that \( v_1, v_3 \) and \( v_9 \) are known. According to equations (30)–(33), the rest of the variables can be calculated by using the \( v_4 \) and \( v_8 \). Accordingly, the optimisation model is simplified to two design variables model.

4.5 Train operation optimisation based on energy saving by using SQP algorithm

The main configurations of the computing environment are mostly relevance with the optimisation speed as follows.

- CPU: Intel(R) core(TM) i7-3667U @2.00GHz 2.5GHz
- Memory: 8 G
- Operation system: Windows7 64-bit.

After the objective function and constraint function is established, the optimal solution is solved by calling the fmincon. Then, the SQP algorithm is set as follows: the initial point is \( x_0 = [70, 40]^T \), max iterations are set to 500, nonlinear constraints tolerance is set to \( 1 \times 10^{-3} \), and other parameter settings are maintained the default. After 1.95309 seconds, the optimised solutions are obtained as follows.
\[ x = [67.0047 \quad 98.90]^T, E = 3.8859e+07, \text{exitflag} = 1. \]

The exitflag value indicates that the result converges to the optimal solution. According to the equations (30)-(33) and equations (28) and (29), the speed of each interval end point is shown in Table 4. According to the equations (2) and (3), the time and distance distributions in the range of each interval end point are calculated as shown in Table 5.

### Table 4  The speed of each interval end point

<table>
<thead>
<tr>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( v_3 )</th>
<th>( v_4 )</th>
<th>( v_5 )</th>
<th>( v_6 )</th>
<th>( v_7 )</th>
<th>( v_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (km/h)</td>
<td>0</td>
<td>35.8984</td>
<td>51.5000</td>
<td>67.0047</td>
<td>52.0080</td>
<td>44.5677</td>
<td>40.2035</td>
</tr>
</tbody>
</table>

### Table 5  The time and distance distribution of each interval end point

<table>
<thead>
<tr>
<th>( v_1 )-( v_2 )</th>
<th>( v_2 )-( v_3 )</th>
<th>( v_3 )-( v_4 )</th>
<th>( v_4 )-( v_5 )</th>
<th>( v_5 )-( v_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>10.0034</td>
<td>4.6615</td>
<td>6.8629</td>
<td>27.2961</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>50.0000</td>
<td>56.6459</td>
<td>114.6800</td>
<td>448.6740</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( v_6 )-( v_7 )</th>
<th>( v_7 )-( v_8 )</th>
<th>( v_8 )-( v_9 )</th>
<th>( v_9 )-( v_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>15.6439</td>
<td>5.0393</td>
<td>12.0764</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>184.0000</td>
<td>55.3583</td>
<td>64.6392</td>
</tr>
</tbody>
</table>

### 4.6 Optimised result analysis of the train operation based on energy saving

According to the Table 4 and equation (2), the optimum curve of the speed-distance is plotted as shown in Figure 6. Besides, according to the Table 4 and equations (1) and (3), the speed-time curve and the acceleration-distance can be given as shown in Figure 7 and Figure 8. From Tables 4, 5 and Figure 8, it can be seen that the results meet all constraints, which illustrates the correctness of the optimised results.

**Figure 6**  Optimum curve of speed-distance
5 Conclusions

1. This paper proposes a method, which uses speed as design variable to create the objective function and constraint function. By the method, the train operation optimisation model is established based on energy saving for urban rail transit system, which can make the relation between various physical quantities clear and easily develop the program on MATLAB to solve.

2. An optimisation example is given for the train operation based on energy saving. Using the handling method, illustrates the process of modelling and optimisation solution. By solving the equation constraints, the optimisation model is simplified to the sample model about two design variables; by using the optimisation results, the optimum curve of speed-distance is given. Meanwhile, the speed-time and
acceleration-distance curves are obtained. The optimisation results, which meet all constraints, show that they are reliable. The example shows that the handling method is feasible and effective, which can lay the foundation of multi-train operation optimisation based on energy saving.

3 In this paper, Section 4.2 assumes the eight intervals divided by the line and the location of ‘traction, coasting and braking’ stage by simple qualitative analysis. However, we can not know the specific location of the ‘traction, coasting and braking’ stage in advance. In future studies, for the train energy saving operation model to be better optimised, we will examine how that interval is divided and the location of each stage is determined automatically.

Acknowledgements

This project is supported by Guangxi Scientific and Technological Development Projects of China (Grant No. AC16380078, 1598007-51, 1598007-31), Liuzhou Science, Technology Development Projects of China (Grant No. 2015A010501, 2016B020203) and Guilin Science and Technology Development Project of China (Grant No. 20170104-1). The authors would like to acknowledge the many helpful suggestions from anonymous reviewers on this paper.

References


