European J. of Industrial Engineering

ISSN online: 1751-5262 - ISSN print: 1751-5254
https://www.inderscience.com/ejie

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DOI: 10.1504/EJIE.2023.10046732

Article History:

Received:	27 October 2020
Accepted:	22 January 2022
Published online:	15 December 2022

Searching for the best profit-sharing allocation in multi-echelon supply chain

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Abstract: In this paper, we propose coordination procedures for a multi-echelon supply chain in which the appropriate profit-sharing rate is allocated for each supply chain member. The search process is first developed to maximise the total compromised profit of a two-member supply chain. From the achieved profit-sharing rate, the best ordering quantity is also determined. Then, a cascading procedure is also proposed for searching the best profit-sharing ratios for each member in the multi-echelon supply chain. Our proposed procedures are validated by comparing it with the fixed profit-sharing scheme. We have also investigated different scenarios to test the effect of demand variations on total cost at different profit-sharing rates. The obtained results are promising. [Submitted: 27 October 2020; Accepted: 22 January 2022]

Keywords: supply chain coordination; profit sharing; multi-echelon supply chain.

Reference to this paper should be made as follows: Van Hop, N. (2023) 'Searching for the best profit-sharing allocation in multi-echelon supply chain', *European J. Industrial Engineering*, Vol. 17, No. 1, pp.148–167.

Biographical notes: Nguyen Van Hop is currently the Dean of the School of Industrial Engineering and Management, International University VNU-HCMC, Vietnam. His research interests are in the areas of fuzzy stochastic optimisation, optimisation for supply chain and/or manufacturing systems.

1 Introduction

Recent studies have shown that information and fixed profit-sharing mechanisms among supply chain members are not enough to coordinate efficiently the supply chain. To deal with the uncertainty of the demand at the retailer and production risks at the manufacturer or the supplier, the appropriate profit-sharing scheme to all supply chain members could give overall benefit for the whole supply chain. Besides, many current literatures only consider a two-layer supply chain configuration instead of multi-echelon settings. In such a case, it is still not easy to coordinate efficiently the three-echelon chain even with a simple price-only contract (Giri and Bardhan, 2017). Due to different interests among firms in the supply chain, managers always put the top priority to maximise their own company's profit, the coordination decisions are still challenging to achieve. Therefore, the firms should adopt an appropriate policy for all supply chain members to work together. From the above observations, there are some gaps needed to fulfill to coordinate the multi-echelon supply chain that concerns appropriate benefit of all supply chain members. In that sense, we propose the coordination frameworks for multi-echelon supply chains that the profit-sharing rates are allocated appropriately to all supply chain members. In these frameworks, we search for the best profit-sharing ratio that could contribute to the maximum compromised profit of the whole supply chain.

The rest of the paper will be organised as follows: the literature review is conducted in the next section. Then, Section 3 summarises some important results of the previous works that will be used for our development. In Sections 4 and 5, we propose our searching procedure to determine the best profit-sharing ratio for supply chain members. Section 6 will investigate the performance of our proposed procedures by numerical experiments. In this section, we also analyse the effects of demand variations on total cost at different profit-sharing rates. Finally, the conclusion and future research recommendations are discussed in Section 7.

2 Literature review

Previous coordination studies for multi-echelon supply chain systems often focus on analysing the centralised and decentralised supply chain systems to overcome the difficulty of uncertain and/or fuzzy demand. Some approaches have used buyback contracts to handle the impact of demand fluctuation (Hu et al., 2010; Zhang et al., 2014; He et al., 2016; Peng and Pang, 2020; Difrancesco et al., 2021). In the study of Hu et al. (2010), the whole supply chain profit can be achieved by a manufacturer's repurchase strategy to deal with fuzzy random demand and imperfect quality products. Zhang et al. (2014) also represented uncertain and fuzzy demand by a two-level buyback contract for a newsvendor model with a single cycle. The expected profit is defuzzified by using a crisp possibilistic mean value. The optimal order quantities in decentralised and centralised systems are analysed and the conditions for supply chain coordination are obtained. He et al. (2016) has investigated reliability and service for the coordination of a two-stage automobile logistics service supply chain (LSSC). The impacts of demand fluctuation are in terms of its magnitude and reliability on the whole supply chain performance is investigated to determine an adjustment strategy. Considering the feature of non-storage and reliability, the buyback contract model was built under the stochastic demand. Compared with no contract, the buyback contract can coordinate the automobile LSSC better. Besides, Peng and Pang (2020) also proposed a buyback and risk sharing (BBRS) contract to reduce the risks of the supplier's yield uncertainty and the distributor's demand uncertainty. Both parties collaborate to improve the total supply chain profit by buying more overproduced products, waiving the shortage penalty, or buying back the unsold products. Recently, Difrancesco et al. (2021) focused on coordinating the supply chain under demand uncertainty, supply disruptions, and random yield. A popular situation of a single buyer using a cheaper but unreliable key supplier and another expensive but reliable backup supplier is investigated. Risks sharing contract and buyback contract are two mechanisms to deal with the risks of demand uncertainty, supply disruption, and random yield.

Profit-sharing could also be an efficient way to motivate the supply chain members to collaborate (Saha et al., 2015; Feng et al., 2018; Fu et al., 2018; Van Hop, 2018; Venegas and Ventura, 2018). Saha et al. (2015) stated that a buyback contract is not effective when demand is realised before the retailer places its orders and after the manufacturer creates its capacity. They concentrated on the long-term contract with a profit-sharing scheme for a multi-item multi-objective manufacturer-retailer supply chain coordination model in a fuzzy stochastic environment. The proposed profit-sharing scheme is fixed in advance. These fixed profit allocation rates are easy to be biased by the decision maker and demotivate the supply chain members due to unfair allocation. In the extension works, Van Hop (2018) has developed new fuzzy stochastic supply chain coordination models for long-term contracts with a profit-sharing scheme. These models include demand variation in the objective to minimise the highest risk aversion factors and avoid decision-maker bias. The models are converted to the corresponding deterministic multi-objective linear programming models by using newly developed fuzzy stochastic measures to de-randomise and de-fuzzify demand, manufacturing cost, and budget values. The proposed model gives better results than Saha's model in both terms of supply chain profit and budget utilisation. Besides, Feng et al. (2018) combined revenue-sharing and buy-back contracts to coordinate a two-stage supply chain in which members experience budget constraints. They have compared the revenue-sharing-and-buy-back (RSBB), revenue-sharing (RS), and buy-back (BB) contracts under budget constraints. They concluded that the combined mechanism of revenue-sharing-and-buy-back could give better economic efficiency to coordinate the supply chain. It could also allocate arbitrarily the supply chain profit between the retailer and manufacturer. Under RSBB, the additional administrative costs are required less than the RS and BB contracts. In other form of profit sharing, Venegas and Ventura (2018) explore the coordination between a supplier and a buyer within a decentralised supply chain, through the use of quantity discounts in a game-theoretic model. In their model, they allow the buyer to charge the final customer a different discount than the one offered by the supplier. Both cooperative and non-cooperative approaches considering that the product traded experiences a price-sensitive demand are considered. The solution obtained from the non-cooperative approach could be improved in a later stage only by allowing the buyer to set a different price to the final customer. In the cooperative model, where decisions are taken simultaneously, emulating a centralised firm, showing the benefits of the cooperation between the players. For the first case, no improvements can be made by fixing the supplier's discount and only changing the buyer's pricing. However, in the second case, when the pricing game is considered as a different scenario, both a better discount and ordering strategies for the players could be found. In another development, a distributional robust Stackelberg game model is developed by Fu et al. (2018) to study the profit-sharing contract design problem with limited information about demand and price distributions. They investigated an ambiguity averse supply chain with price and demand uncertainty. In terms of choosing profit share parameter, one key finding of this paper is that compared with the case without profit sharing, the Stackelberg equilibrium chooses a profit share parameter γ^* in (0, 1), such that:

1 The retailer has a higher worst-case profit.

2 The supplier has a higher worst-case profit, and hence the supply chain is more efficient.

If the profit share of the supplier is smaller than this threshold γ^* then both the retailer and the supplier will be worse off in terms of the profit accrued to each of them. On the other hand, a share higher than γ^* accrued to the supplier will only benefit the supplier but hurt the retailer. Thus, a careful calibration of the contractual elements in the profit-sharing agreements is crucial for the retailer. Furthermore, the worst-case profit for both the retailer and the supplier dominates the pure wholesale price model (with no profit sharing). These results indicate that in an ambiguity averse supply chain, the profit-sharing agreement approach is generally more beneficial to both parties involved, compared with the traditional wholesale price contract. Some other interesting researches in profit/revenue sharing contract for the supply chain are also recently considered in other aspects and applications such as the works of Gamchi and Torabi (2018) with value-added services; Giri et al. (2018) for a closed-loop supply chain with demand dependent on the greening level in addition to the selling price and warranty period; Heydari and Ghasemi (2018) for reverse supply chain coordination under the stochastic quality of returned products and uncertain remanufacturing capacity; Chen et al. (2021) using the profit sharing and option contract to mitigate the risks of yield uncertainty from the supplier and stochastic demand from the retailer.

Most of the previous studies analysed the two-level supply chain, Giri and Bardhan (2017) recently have investigated both a three-layer supply chain and a semi-integration within two members with stochastic demand in which both the productions of the raw-material supplier as well as the manufacturer are subject to random yield. They considered the optimal order and production quantities are obtained for both centralised and decentralised system under price only contract. They observed that the price-only contract in the three-echelon chain is not easy to establish. It is easier to choose some suitable wholesale price at each stage to easily determine optimal order and production batch sizes. It is also observed that the coordination at any stage enhances the total profit of the chain, and such enhancement is higher if the coordination takes place at the upstream level. However, it is not the case that an entity would be benefited locally if he comes under contract with its adjacent channel member. That is why each entity seeks to act alone provided other entities to come under some coordinating contracts.

In short, although efficient coordination can be obtained in some settings, many factors still need to be considered carefully to improve the supply chain performance. To the best of our knowledge, no work addresses a three-stage supply chain with the appropriate profit-sharing rate to motivate all the supply chain members that could give overall benefit for the whole supply chain. In the following sections, we will develop in details such coordination mechanisms and show that the obtained results are very promising.

3 Preliminaries

For ease of reading, we summarise some important results from the work of Saha et al. (2015) and some basic calculations which will be used for our development.

3.1 Multi-objective decision making approaches

Saha et al. (2015) has shown that fuzzy compromise programming method (FCPM) could give the best performance for solving the multi-objective decision-making problems. Consider the following multi-objective decision-making model:

$$Max\ Z_k = \sum_{j=1}^{n} c_{kj} x_j; \qquad k = 1, 2, 3, \ldots, s \tag{1}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad i = 1, 2, \ldots, m \tag{2}$$

$$x_j \ge 0; \qquad j = 1, 2, \ldots, n \tag{3}$$

Let $Z_k^{\min}$, $Z_k^{\max}$ be the aspired level and the highest acceptable level of achievement for the single objective solution of the multi-objective decision-making (MODM) model (while disregarding the other objectives). The FCPM model is based on maximising the least satisfaction level among all objectives as follows:

$$Max\ \pi \tag{4}$$

Subject to

$$\frac{Z_k - Z_k^{\min}}{Z_k^{\max} - Z_k^{\min}} \ge \pi; \qquad k = 1, 2, \ldots, s \tag{5}$$

$$1 \ge \pi \ge 0 \tag{6}$$

$$x \in X \tag{7}$$

Constraint (7) is a set of original constraints (2) and (3).

3.2 The inventory quantities for different demand distributions

In a supply chain system, for order quantity q and demand x, the expected value and the variance of the overstock quantities are defined by

$$E(q-x) = (q-x)^+ = \max(0, q-x) = \int_0^x (q-x) f(x) dx \tag{8}$$

$$\operatorname{var}(q-x) = \int_0^x (q-x) f(x) dx \tag{9}$$

Depending on type of demand distribution, Saha et al. (2015) proved the following results:

- If demand follows a uniform distribution with PDF $f(x)$ and CDF $F(x)$:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

$$E(q-x)=(q-x)^{+}=\frac{(q-a)^2}{2(b-a)} \tag{11}$$

$$\operatorname{var}(q-x)=\frac{a(q-a)^2}{2(b-a)}-\frac{1}{b-a}\left[\frac{2q^3}{3}-aq^2+\frac{a^3}{3}\right]-\frac{1}{4}\left[\frac{(q-a)^2}{2(b-a)}\right]^2 \tag{12}$$

- If demand follows an exponential distribution with PDF $f(x)$ and CDF $F(x)$:

$$f(x)=\lambda e^{-\lambda x},\qquad 0\le x\le\infty \tag{13}$$

$$E(q-x)=(q-x)^{+}=q+\frac{1}{\lambda}\left[e^{-\lambda q}-1\right] \tag{14}$$

$$\operatorname{var}(q-x)=\frac{1}{\lambda^2}\left(1-e^{-2\lambda q}\right)-\frac{2qe^{-\lambda q}}{\lambda} \tag{15}$$

- If demand follows a normal distribution with PDF $f(x)$ and CDF $F(x)$:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}};\qquad -\infty<x<\infty \tag{16}$$

$$\begin{aligned}E(q-x)&=(q-x)^{+}\\&=\frac{q}{2}+\frac{1}{\sqrt{2\pi}}\left[\frac{\sigma}{2}\left\{e^{\frac{-\mu^2}{2\sigma^2}}-e^{\frac{-(q-\mu)^2}{2\sigma^2}}\right\}+\frac{1}{2\sigma}\left\{\frac{(q-\mu)^2}{2}-\frac{\mu^2}{2}\right\}\right]\end{aligned} \tag{17}$$

$$\begin{aligned}\operatorname{var}(q-x)&=2qE(q-x)-[E(q-x)]^2\\&-\left(\left(\frac{q}{2}\right)^2+\frac{1}{\sqrt{2\pi}}\left[q^2\left(\frac{q-\mu}{2\sigma}\right)e^{\frac{-(q-\mu)^2}{2\sigma^2}}+\frac{1}{\sigma}\left\{\frac{q^3}{3}-\frac{uq^2}{2}\right\}\right]\right)\end{aligned} \tag{18}$$

In addition, we also need to determine the $E(yA-B)$ and $\operatorname{var}(yA-B)$ for uniform distribution of production yield y. We have:

$$E(yA-B)=\int_{B/A}^{\delta}(yA-B)f(y)dy=\int_{B/A}^{\delta}(yA-B)\frac{1}{\delta-\gamma}dy=\frac{\frac{A\delta^2}{2}+\frac{B^2}{2A}-B\delta}{\delta-\gamma} \tag{19}$$

where

$$f(y)=\begin{cases}\dfrac{1}{\delta-y}, & \gamma\le y\le\delta\\ 0, & \text{otherwise}\end{cases}$$

$$\begin{aligned}\operatorname{var}(yA-B)&=\int_{\frac{B}{A}}^{\delta}[yA-B-E(yA-B)]^2f(y)dy\\&=\frac{\delta A-B}{3A(\delta-\gamma)}\left[(\delta A-B)^2-3E(yA-B)(\delta A-B-E(yA-B))\right]\end{aligned} \tag{20}$$

These results will be used in our experimental experiments.

4 Profit-sharing allocation in two-layer supply chain

To develop an appropriate profit distribution framework, we first investigate the case of a supply chain with two players only. Most of the current works also focus on studying the two-member supply chain. In this case, we also consider a supply chain system that includes one manufacturer and one retailer. In T periods, the retailer sells n products to the market with stochastic demand $x_{i,t}$ (i = 1, 2, …, n) with probability density function (PDF) $f(x_{i,t})$ and cumulative density function (CDF) $F(x_{i,t})$. The retail price for a unit of product i is p_i. The product i is bought at a unit wholesale price of w_i from the manufacturer. The unit production cost is c_i. At period t (t =1, 2, …, T), the retailer places an order quantity $q_{i,t}$ that will be received at previous period ($t + L$) with L is the supplying lead time. In case of no order, the value of $q_{i,t} = 0$. If the receiving quantity $q_{i,t} + L$ is less than the demand, then the retailer is incurred lost sales at the end of each period. Otherwise, the remaining quantity of the item is sold at salvage value v_i. It is noticed that $p_i > w_i \geq c_i > v_i$. We assume that these prices do not change in the planning horizon. Our system modifies the settings of Saha et al. (2015) by relaxing some assumptions of lead time, production cost c_i, and dynamic profit-sharing allocation. The order quantities are placed by shifting the corresponding lead time L. Without loss of generality, we specify the production cost c_i to be deterministic values instead of fuzzy numbers. The study of Saha et al. (2015) has assumed that the retailer kept a profit fraction α_i and share $(1 - \alpha_i)$ portion to the manufacturer. This profit-sharing ratio α_i is fixed and given in advance. This could make the performance of the supply chain to be suffered and not at the best results. Moreover, profit-sharing decision is often applied in a long-term contract. It could significantly affect the total performance of the supply chain. At this point, we try to allocate dynamically the profit-sharing ratios in a fair manner. It is used to answer the question that if a profit-sharing scheme is applied what could be the best sharing ratio that the retailer could give to the manufacturer? To simplify our investigation, the budget is assumed to be a deterministic value. Besides, we still assume that the expected value of demand never changes when the transaction is conducted. With these changes, the profit models of retailer and manufacturer will be:

$$Max\ Z_R = \sum_{i=1}^{n} \sum_{t=1}^{T} \left[\alpha_i \left[\left(p_i - w_i \right) q_{i,t+L} - \left(p_i - v_i \right) E \left(q_{i,t+L} - x_{i,t+L} \right) \right] \right] \tag{21}$$

$$Max\ Z_M = \sum_{i=1}^{n} \sum_{t=1}^{T} \left(w_i - C_i \right) q_{i,t+L} + \left(1 - \alpha_i \right) Z_R \tag{22}$$

Subject to

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left[(1-\alpha_i) \left(p_i - v_i \right) \sqrt{\operatorname{var} \left(q_{i,t+L} - x_{i,t+L} \right)} \right] \leq A_M \tag{23}$$

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left[\alpha_i \left(p_i - v_i \right) \sqrt{\operatorname{var} \left(q_{i,t+L} - x_{i,t+L} \right)} \right] \leq A_R \tag{24}$$

$$\sum_{i=1}^{n} w_i q_{i,t+L} \leq B_R; \qquad t = 1, 2, \ldots, T \tag{25}$$

$$q_{i,t} \geq; 1 \geq \alpha_i \geq 0; \qquad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T \tag{26}$$

where

T number of planning periods

p_i the retail price per unit item, $i = 1, 2, \ldots, n$

w_i the wholesale price per unit item of the manufacturer

v_i the salvage value per unit item.

c_i the manufacturing cost per unit item.

α_i the profit-sharing fraction ($0 < \alpha_i < 1$).

$q_{i,t}$ the ordering quantity of the item i at period t

$x_{i,t}$ the random market demand

Z_M the total expected profit of the manufacturer in n periods

Z_R the total expected profit of the retailer in n periods

B_R the budget for the retailer.

In this model, objective functions (21) and (22) are total expected profit in n periods of the manufacturer and the retailer, respectively. Constraints (23) and (24) limit the standard deviation of the profit for the manufacturer and the retailer, respectively, in n periods. In our model, we relax the assumptions of fuzzy production cost and stochastic budget for the retailer which leaves to be considered in future research. So, constraint (25) is the limitation on the available budget. Constraint (26) is the condition of variables. Our main focus is to investigate a practical approach that could help to coordinate the supply chain better by allocating the appropriate profit for each member in the supply chain.

In literature, the common way to improve supply chain performance is to centralise decision making by sharing information. However, sharing information is not always feasible. Besides, as we have also known from Giri and Bardhan (2017) that it is almost impossible to control the entire supply chain centrally, particularly in a highly competitive market. Therefore, a centralised model is often used as the benchmark case for supply chain coordination. In practice, profit-sharing among supply chain members could help the overall supply chain profit to be better. Saha et al. (2015) used a fixed profit-sharing ratio in advance. However, this could lead to an unfair allocation that will demotivate the coordination efforts of the supply chain members. To improve coordination performance, a fair profit-sharing framework is needed to be developed. There are different methods to fairly distribute profit among players. The simplest approach could be allocating equally profit gain between all parties. However, it is not fair because of the different contributions of each player to the total profit gain. Another promising approach could be a searching technique to allocate the appropriate profit ratio to the supply chain members. In this case, the supply chain member, especially, the retailer, will share a certain profit fraction with the manufacturer at which the total profit of the whole supply chain to be maximised. To search efficiently the best profit-sharing fraction α_i in the range of (0, 1), the iterative process of the dichotomous section could be applied as in the following pseudo-code:

PROCEDURE Profit-sharing allocation (PSA)

Begin

Step 0 Initialisation. Set: $k = 0$, the current interval $I_0 = (L, R) = (0, 1)$.

Repeat

Step 1 The current interval is $I_{k-1} = (L_L, R_R)$. Define a, b such that $L< a < b < R$:

$$a = \frac{1}{2}(R + L - \Delta) \tag{27}$$

$$b = \frac{1}{2}(R + L + \Delta) \tag{28}$$

Step 2 Solving the model (20)–(25) with $\alpha = a$ or $\alpha = b$ by using the FCPM method, we have: $\Pi^*(\alpha) = Z_R^*(\alpha) + Z_M^*(\alpha)$

Step 3 The next interval, $k = k + 1$, I_k is determined as follows.

if $\Pi^*(a) > \Pi^*(b)$ then $L < \alpha_M < b$, set $I_k = (L, R = b)$

if $\Pi^*(a) < \Pi^*(b)$ then $a < \alpha_M < R$, set $I_k = (a= L, R)$

if $\Pi^*(a) = \Pi^*(b)$ then $a < \alpha_M < b$, set $I_k = (L= a, R = b)$

Until ($I_k \leq \varepsilon$)

Step 4 The best profit-sharing fraction will be:

$$\alpha^* = \frac{|I_k|}{2} = \frac{(R - L)}{2} \tag{29}$$

End

Note: ε = user-defined level of accuracy, Δ = predefined value, say, 0.1.

Here, we propose to coordination frameworks to identify the appropriate profit-sharing ratio for each member of the supply chain while it is still sharing relevant information to make sure that the total supply chain profit is maximised. The proposed procedures could help the decision-makers to allocate fairly and efficiently the profit-sharing ratio for each supply chain member and calculate the appropriate order quantity at each level that maximising the total supply chain profit. The first approach is a constructive method that utilises Shapley value to distribute profit gain among supply chain members. The second procedure is an iterative process to search for the appropriate profit-sharing ratios for each member. In this case, the whole supply chain profit is compared between the case when all entities act separately to maximise their profits with the case of a centralised model where all members' profit takes place together. Then, the gained profit of the whole supply chain is used to determine the appropriated profit-sharing level for each member in the supply chain. Finally, the collaborative profit-sharing model is developed to give the best solution for the whole system.

5 Cascading profit-sharing allocation in a multi-echelon supply chain

In this section, we extend our consideration to the case of a three-layer supply chain. The general case of a multi-layer supply chain could be applied by a similar approach. The considered supply chain includes one supplier, one manufacturer, and one retailer. The system is similar to the setting in the previous section with multiple products, T planning periods, and stochastic market demand for finished goods. All parameters such as the retail price p_i, manufacturing wholesale price of w_i, the production cost c_i, salvage value of unsold items v_i, and zero lost sales ($p_i > w_i \geq c_i > v_i$; $i = 1, 2, \ldots, n$) are the same as before. At period t ($t = 1, 2, \ldots, T$), the retailer places an order quantity $q_{i,t}$ that will be received at period ($t + L_M$) with L_M is manufacturing lead time. In case of no order, the value of $q_{i,t} = 0$. If the receiving quantity $q_{i,t+L_M}$ is less than the demand, then the retailer is incurred lost sales at the end of each period. Otherwise, the remaining quantity of an item is sold at salvage value v_i. It is also noticed that $p_i > w_i \geq c_i > v_i$ and do not change in the planning horizon. At the next level, the manufacturer then decides the production quantity $d_{i,t}$ after receiving order quantity order $q_{i,t}$ at time t from the retailer. The manufacturer also orders $e_{i,t}$ amount of raw materials at time t with the wholesale price u_i of raw material buying from the supplier. Similar to Giri and Bardhan (2017), the manufacturer's output at time ($t + L_M$) will be $y_{i,t+L_M} d_{i,t+L_M}$ with stochastic production rate $y_{i,t+L_M}$ due to machine breakdown, defects, processing variation, operator skills, etc. If the production output is less than the order quantity $q_{i,t+L_M}$ then the manufacturer has incurred zero lost sales. Otherwise, the excess quantity is salvaged at the same value of v_i. With these basic settings, the expected profit model of the retailer does not change.

$$Max\ Z_R = \sum_{i=1}^{n} \sum_{t=1}^{T} \left[\left[(p_i - w_i) q_{i,t+L_M} - (p_i - v_i) E\left(q_{i,t+L_M} - x_{i,t+L_M}\right) \right] \right] \tag{30}$$

Subject to

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left[(p_i - v_i) \sqrt{\operatorname{var}\left(q_{i,t+L_M} - x_{i,t+L_M}\right)} \right] \leq A_R \tag{31}$$

$$\sum_{i=1}^{n} w_i q_{i,t+L_M} \leq B_R; \qquad t = 1, 2, \ldots, T \tag{32}$$

$$q_{i,t} \geq 0; \qquad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T \tag{33}$$

The expected profit model of the manufacturer will be:

$$\begin{aligned} Max\ Z_M = \sum_{i=1}^{n} \sum_{t=1}^{T} \big[& w_i q_{i,t+L_M} - c_i d_{i,t+L_M} - u_i e_{i,t+L_S} \\ & + v_i E\left(y_{i,t+L_M} d_{i,t+L_M} - q_{i,t+L_M}\right) \big] \end{aligned} \tag{34}$$

Subject to

$$\sum_{i=1}^{n} \sum_{t=1}^{T} v_i \sqrt{Var\left(y_{i,t} d_{i,t+L_M} - q_{i,t+L_M}\right)} \leq A_M; \tag{35}$$

$$\sum_{i=1}^{n} \left(c_i d_{i,t} + u_i e_{i,t}\right) \leq B_M; \qquad t = 1, 2, \ldots, T \tag{36}$$

$$q_{i,t} \geq 0; d_{i,t} \geq 0; \qquad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T \tag{37}$$

For the supplier, after receiving order quantity $e_{i,t}$ of raw materials to deliver at time ($t + L_S$) with L_S is supplying lead time. Similar to Giri and Bardhan (2017), the supplier runs production to produce an amount of $z_{i,t+L_S} o_{i,t+L_S}$ with stochastic rate $z_{i,t+L_S}$. The unit production cost at the supplier is r_i. If the supplier output is less than the order quantity $e_{i,t+L_S}$, then the supplier has incurred a zero lost sales. Otherwise, the extra items are sold at the salvage price of s_i. The expected profit model of the supplier is:

$$\text{Maximise } Z_S = \sum_{i=1}^{n} \sum_{t=1}^{T} \left[u_i e_{i,t+L_S} - r_i o_{i,t+L_S} + s_i E\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S} \right) \right] \tag{38}$$

Subject to

$$\sum_{i=1}^{n} \sum_{t=1}^{T} s_i \sqrt{Var\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S} \right)} \le A_S \tag{39}$$

$$\sum_{i=1}^{n} r_i o_{i,t+L_S} \le B_S; \qquad t = 1, 2, \ldots, T \tag{40}$$

$$d_{i,t} \ge 0; \, o_{i,t} \ge 0; \qquad i = 1, 2, \ldots, n; \, t = 1, 2, \ldots, T \tag{41}$$

It is noticed that our individual profit models (31)–(42) adapted the models of Saha et al. (2015) and Giri and Bardhan (2017) for the general three-echelon supply chain structure. We focus on handling the decision of profit-sharing rate instead of pricing which is normally the result of the negotiation process in practice. In literature, the common way to improve supply chain performance is to centralise decision making by sharing information. Theoretically, we can allocate multiple profit-sharing ratios for all members together based on a centralised framework. However, centralised coordination is very difficult to achieve in practice as mentioned by Giri and Bardhan (2017). Therefore, we could develop some coordination framework in decentralised mode by sharing profit between partners in the supply chain in some ways to improve the performance of the whole supply chain. For the case of a multi-echelon supply chain, the coordination is eventually more difficult. However, to motivate each supply chain member, logically we have to consider the case that supply chain members will contribute partially or totally to the supply chain profit. Intuitively, the cascading allocation approach could be an appropriate way to distribute profit ratios among supply chain members. The pairwise coordination framework for two members proposed in the previous section is the basic framework. At first, we try to fix the first profit-sharing ratio between two first members, say, retailer and manufacturer. Then, we search for allocating the next profit ratio by considering the next supply chain member with the allocated member by using the basic PSA procedure above. In this way, we can obtain the best performance for the whole supply chain. The process repeats until all members are linked together in the appropriate profit-sharing allocation framework. Actually, we may apply another way that considers the independent pairs of supply chain members, say, retailer-and-manufacturer and manufacturer-and-supplier. Or we may use the hybrid approach like the case of Giri and Bardhan (2017), say, r-ms or s-mr in which we apply PSA for the centralised components and the remaining component of the supply chain. However, the pairwise approach does not have the total link or coordination between the pairs. Here, we only present the cascading allocation process of profit-sharing ratios for a three-layer supply chain system. In this procedure, we first apply PSA procedure for the pair of a manufacturer and a retailer in which the manufacturer could share profit to the retailer or vice versa. In practice, the manufacturer could share profit to retailer for the case they offer some promotion campaign. In the reverse case, retailer could share profit to manufacturer for the case of retailer wants the large production lot-size for high demand season, say, year-end season. After that, we update the profit model of the manufacturer with a profit allocation faction with the retailer. Finally, we consider the pair of manufacturer and supplier in which the supplier could share profit with the manufacturer or vice versa. Similarly, the supplier shares profit to the manufacturer in the form of a quantity discount. In contrast, the manufacturer could share the profit to the supplier in the form that they secured a big order quantity. For ease of understanding, we will present our cascading profit-sharing allocation procedure in the form that the supplier shares profit to the manufacturer with a fraction of β and the manufacturer shares a profit fraction of α to the retailer.

PROCEDURE Cascading profit-sharing allocation (CPSA)

Begin

Step 1 For the manufacturer-retailer pair, consider the case that the manufacturer shares a profit fraction of α_i to the retailer. The profit-sharing model of the pair of manufacturer and retailer will be:

$$\begin{aligned} Max\ Z_R = & \sum_{i=1}^{n}\sum_{t=1}^{T}\left[\left(p_{iw}-w_i\right)q_{i,t+L_M}-\left(p_i-v_i\right)E\left(q_{i,t+L_M}-x_{i,t+L_M}\right)\right] \\ & +\alpha_i Z_M \end{aligned} \tag{42}$$

$$\begin{aligned} Max\ Z_M = & \sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i)\left[w_i q_{i,t+L_M}-c_i d_{i,t+L_M}-u_i e_{i,t+L_S}\right. \\ & \left.+v_i E\left(y_{i,t+L_M}d_{i,t+L_M}-q_{i,t+L_M}\right)\right] \end{aligned} \tag{43}$$

Subject to

$$\begin{aligned} & \sum_{i=1}^{n}\sum_{t=1}^{T}\left[\left(p_i-v_i\right)\sqrt{\operatorname{var}\left(q_{i,t+L_M}-x_{i,t+L_M}\right)}\right] \\ & +\sum_{i=1}^{n}\sum_{t=1}^{T}\alpha_i v_i\sqrt{\operatorname{var}\left(y_{i,t}d_{i,t+L_M}-q_{i,t+L_M}\right)}\le A_R \end{aligned} \tag{44}$$

$$\sum_{i=1}^{n} w_i q_{i,t+L_M}\le B_R;\qquad t=1,2,\ldots,T \tag{45}$$

$$\sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i)v_i\sqrt{\operatorname{var}\left(y_{i,t}d_{i,t+L_M}-q_{i,t+L_M}\right)}\le A_M \tag{46}$$

$$\sum_{i=1}^{n}\left(c_i d_{i,t}+u_i e_{i,t}\right)\le B_M;\qquad t=1,2,\ldots,T \tag{47}$$

$$q_{i,t}\ge 0;\ d_{i,t}\ge 0;\ e_{i,t}\ge 0;\qquad i=1,2,\ldots,n;\ t=1,2,\ldots,T \tag{48}$$

Apply the PSA procedure for the above model, we can obtain the best profit-sharing ratios α_i^*, the best order quantity $q_{i,t}^*$, the best production quantity $q_{i,t}^*$, and the associated profit values $\Pi^*(\alpha)=Z_R^*(\alpha)+Z_M^*(\alpha)$

Step 2 Updating the manufacturing model with α_i^* as follows:

$$\begin{aligned} Max\ Z_M = & \sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i)\left[w_i q_{i,t+L_M}^*-c_i d_{i,t+L_M}-u_i e_{i,t+L_S}\right. \\ & \left.v_i E\left(y_{i,t+L_M}d_{i,t+L_M}-q_{i,t+L_M}^*\right)\right)\right] \end{aligned} \tag{49}$$

Subject to

$$\sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i^*)v_i\sqrt{\operatorname{var}\left(y_{i,t}d_{i,t+L_M}-q_{i,t+L_M}^*\right)}\le A_M \tag{50}$$

$$\sum_{i=1}^{n}(c_i d_{i,t} + u_i e_{i,t}) \le B_M; \qquad t = 1, 2, \ldots, T \tag{51}$$

$$q_{i,t} \ge 0; d_{i,t} \ge 0; e_{i,t} \ge 0; \qquad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T \tag{52}$$

Step 3 Next, consider the next pair of supplier-manufacturer in which the supplier share a profit fraction of β_i to the manufacturer. The profit-sharing model of the whole supply chain with three members will be:

$$\begin{aligned} Max\ Z_S = \sum_{i=1}^{n}\sum_{t=1}^{T}(1-\beta_i)\big[&u_i e_{i,t+L_S} - r_i o_{i,t+L_S} \\ &+ s_i E\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S}\right)\big] \end{aligned} \tag{53}$$

$$\begin{aligned} Max\ Z_M = &\sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i^*)\big[w_i q_{i,t+L_M} - c_i d_{i,t+L_M} - u_i e_{i,t+L_S} \\ &+ v_i E\left(y_{i,t+L_M} d_{i,t+L_M} - q_{i,t+L_M}\right)\big] \\ &+ \sum_{i=1}^{n}\sum_{t=1}^{T}\beta_i\big[u_i e_{i,t+L_S} - r_i o_{i,t+L_S} \\ &+ s_i E\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S}\right)\big] \end{aligned} \tag{54}$$

$$\begin{aligned} Max\ Z_R = &\sum_{i=1}^{n}\sum_{t=1}^{T}\big[(p_i - w_i) q_{i,t+L_M} - (p_i - v_i) E\left(q_{i,t+L_M} - x_{i,t+L_M}\right)\big] \\ &+ \sum_{i=1}^{n}\sum_{t=1}^{T}\alpha_i^*\big[w_i q_{i,t+L_M} - c_i d_{i,t+L_M} - u_i e_{i,t+L_S} \\ &+ v_i E\left(y_{i,t+L_S} d_{i,t+L_M} - q_{i,t+L_M}\right)\big] \end{aligned} \tag{55}$$

Subject to

$$\sum_{i=1}^{n}\sum_{t=1}^{T}(1-\beta_i) s_i \sqrt{\operatorname{var}\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S}\right)} \le A_S \tag{56}$$

$$\sum_{i=1}^{n} r_i o_{i,t+L_S} \le B_S; \qquad t = 1, 2, 3, \ldots, T \tag{57}$$

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{t=1}^{T}(1-\alpha_i^*) v_i \sqrt{\operatorname{var}\left(y_{i,t} d_{i,t+L_M} - q_{i,t+L_M}\right)} \\ &+ \sum_{i=1}^{n}\sum_{t=1}^{T}\beta_i s_i \sqrt{\operatorname{var}\left(z_{i,t+L_S} o_{i,t+L_S} - e_{i,t+L_S}\right)} \le A_M \end{aligned} \tag{58}$$

$$\sum_{i=1}^{n}(c_i d_{i,t} + u_i e_{i,t}) \le B_M; \qquad t = 1, 2, \ldots, T \tag{59}$$

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{t=1}^{T}\left[(p_i - v_i)\sqrt{\operatorname{var}\left(q_{i,t+L_M} + x_{i,t+L_M}\right)}\right] \\ &+ \sum_{i=1}^{n}\sum_{t=1}^{T}\alpha_i^* v_i \sqrt{\operatorname{var}\left(y_{i,t+L_M} d_{i,t+L_M} - q_{i,t+L_M}\right)} \le A_R \end{aligned} \tag{60}$$

$$\sum_{i=1}^{n} w_i q_{i,t+L_M} \le B_R; \qquad t = 1, 2, \ldots, T \tag{61}$$

$$d_{i,t} \ge 0; e_{i,t} \ge 0; o_{i,t} \ge 0; q_{i,t} \ge 0; \qquad i = 1, 2, \ldots, n; t = 1, 2, \ldots, T \tag{62}$$

Apply *PSA* again to determine the best profit-sharing ratio β_i^* and other values such as the best order quantities $o_{i,t}^*$, $d_{i,t}^*$, $q_{i,t}^*$.

Step 4 Repeating Step 3 for the next supply chain member, if any.

End

It is noticed that our proposed procedures heuristically determine the best profit-sharing allocation to improve the supply chain member performance. It cannot guarantee to give optimal coordination as per the centralised format that is proved in literature. But it is applicable by simple calculation processes.

6 Numerical experiments

To validate our proposed procedures, we run numerical experiments to compare with the work of Saha et al. (2015) for the two-level supply chain system. We also extend our investigation on the effect of data variation on the profit-sharing ratios. Finally, we illustrate the CPSA procedure for the three-echelon supply chain by testing the effect of different demand distributions on the performance of the best profit-sharing ratios for each echelon and the total performance of the supply chain.

6.1 Two-layer supply chain system

For testing the performance of the two-layer supply chain system, we are revisiting the numerical example from Saha et al. (2015) to investigate numerically our proposed PSA method. Then, a sensitivity analysis is conducted to see the efficiency of the proposed profit-sharing framework. For ease of reading, we summarise the given data as follows: three different types of items (n = 3) are considered. The all parameters are the same as given in Saha's example except the value of c_i is taken the second value of a fuzzy number:

$$\alpha_1 = 0.55;\ p_1 = \$90;\ v_1 = \$6;\ w_1 = \$40;\ c_1 = \$10;$$

$$\alpha_2 = 0.53;\ p_2 = \$100;\ v_2 = \$12;\ w_2 = \$45;\ c_2 = \$15;$$

$$\alpha_3 = 0.51;\ p_3 = \$110;\ v_3 = \$14;\ w_3 = \$50;\ c_3 = \$16;$$

$$T = 50;\ L = 1;\ B = \$15{,}000;\ A_M = 250{,}000;\ A_R = 250{,}000;$$

We also consider three popular types of probability distribution: uniform, exponential, and normal distributions with the same input parameters of Saha et al. (2015) in all periods of time presented in Table 1.

Table 1 Distribution and input parameters

Distribution	*E(xi)*	*Var(xi)*	*Input parameters*
Uniform distribution $f(x_i) = \dfrac{1}{b_i - a_i},\ b_i \geq x_i \geq a_i$	$\left[\dfrac{a_i + b_i}{2}\right]$	$\left[\dfrac{(b_i - a_i)^2}{12}\right]$	$a_1 = 0$, $b_1 = 120$, $a_2 = 0$, $b_2 = 160$, $a_3 = 0$, $b_3 = 200$
Exponential distribution $f(x_i) = \lambda_i e^{-\lambda_i x_i},\ x_i \geq 0$	$\dfrac{1}{\lambda_i}$	$\left(\dfrac{1}{\lambda_i^2}\right)$	$\lambda_1 = 0.015$, $\lambda_2 = 0.011$, $\lambda_3 = 0.009$
Normal distribution $f(x_i) = \dfrac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(x_i-\mu_i)^2}{2\sigma_i^2}}$	μ_i	σ_i^2	$\mu_1 = 60$, $\mu_2 = 80$, $\mu_3 = 100$, $\sigma_1 = 24$, $\sigma_2 = 32$, $\sigma_3 = 40$

Table 2 Comparison results

Solution method	*Max/min values and optimal variables*	*Type of distribution for stochastic demand*		
		Uniform distribution	*Exponential distribution*	*Normal distribution*
Saha's solution	$Z_M^{\max}$	712,323.0	611,993.3	642,131.6
	$Z_M^{\min}$	666,321.4	553, 899.2	601,247.5
	$Z_R^{\max}$	217,607.1	159,768.4	174,560
	$Z_R^{\min}$	214,533.2	76,606.41	146,798.5
	α_1	0.55	0.55	0.55
	α_2	0.53	0.53	0.53
	α_3	0.51	0.51	0.51
	π^*	0.6737471	0.9735319	0.5249087
	Z_M^*	697,314.8	610,455.7	622,707.9
	Z_R^*	216,604.2	157,567.3	161,370.8
	Z_{Avg}^* (in $)	*18,278.38*	*15,360.46*	*15,681.57*
	Budget utilised ($)	*14,530.84*	*13,571.68*	*14,022.89*
PSA solution	$Z_M^{\max}$	703,018.4	638,791.7	637,631.3
	$Z_M^{\min}$	662,178.6	563,025.3	597,543.9
	$Z_R^{\max}$	221,750	150,642.2	178,263.6
	$Z_R^{\min}$	206,235	100,810.1	148,690.3
	α_1	0.56	0.526	0.56
	α_2	0.54	0.506	0.54
	α_3	0.52	0.486	0.52
	π^*	0.8973718	0.9240652	0.5417644
	Z_M^*	698827.1	633038.4	619261.8
	Z_R^*	220157.7	146858.2	164712.1
	Z_{Avg}^* (in $)	*18,379.696*	*15,597.93*	*15,681.85*
	Budget utilised ($)	*14,718.18*	*13,972.78*	*14,006.88*

Note: $Z_{Avg}^* = \left(\dfrac{Z_M^* + Z_R^*}{T} \right)$.

The proposed PSA procedure and the fixed profit-sharing scheme of Saha et al. (2015) are tested by using the FCPM approach to obtain the best compromise solution. It is noticed that for Saha case, the profit-sharing rates are given and fixed in advance at: $\alpha_1 = 0.55$, $\alpha_2 = 0.53$, $\alpha_3 = 0.51$. In our proposed PSA, the α_1, α_2 and α_3 are variables. In addition, we assume that demand has the same mean and standard deviation in all periods to compare with the case of Saha et al. (2015). The comparison results are summarised in Table 2.

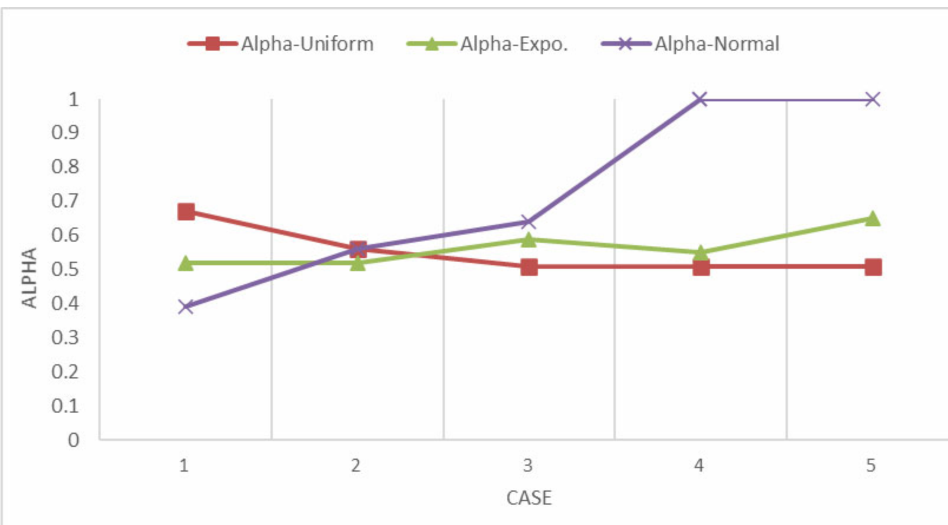
Table 3 Levels of demand uncertainty

Case	*1*	*2*	*3*	*4*	*5*
Uniform	$a_1 = 0, b_1 = 40,$ $a_2 = 0, b_2 = 60,$ $a_3 = 0, b_3 = 80$	$a_1 = 0, b_1 = 120,$ $a_2 = 0, b_2 = 140,$ $a_3 = 0, b_3 = 160$	$a_1 = 0, b_1 = 200,$ $a_2 = 0, b_2 = 220,$ $a_3 = 0, b_3 = 240$	$a_1 = 0, b_1 = 280,$ $a_2 = 0, b_2 = 300,$ $a_3 = 0, b_3 = 320$	$a_1 = 0, b_1 = 360,$ $a_2 = 0, b_2 = 380,$ $a_3 = 0, b_3 = 400$
Exponential	$\lambda_1 = 0.011,$ $\lambda_2 = 0.009,$ $\lambda_3 = 0.007$	$\lambda_1 = 0.015,$ $\lambda_2 = 0.013,$ $\lambda_3 = 0.011$	$\lambda_1 = 0.019,$ $\lambda_2 = 0.017,$ $\lambda_3 = 0.015$	$\lambda_1 = 0.023,$ $\lambda_2 = 0.021,$ $\lambda_3 = 0.019$	$\lambda_1 = 0.027,$ $\lambda_2 = 0.025,$ $\lambda_3 = 0.023$
Normal	$\mu_1 = 20, \sigma_1 = 8,$ $\mu_2 = 40, \sigma_2 = 16,$ $\mu_3 = 60, \sigma_3 = 24$	$\mu_1 = 60, \sigma_1 = 24,$ $\mu_2 = 80, \sigma_2 = 32,$ $\mu_3 = 100, \sigma_3 = 40$	$\mu_1 = 100, \sigma_1 = 40,$ $\mu_2 = 120, \sigma_2 = 48,$ $\mu_3 = 140, \sigma_3 = 56$	$\mu_1 = 140, \sigma_1 = 56,$ $\mu_2 = 160, \sigma_2 = 64,$ $\mu_3 = 180, \sigma_3 = 72$	$\mu_1 = 180, \sigma_1 = 72,$ $\mu_2 = 200, \sigma_2 = 80,$ $\mu_3 = 220, \sigma_3 = 88$

From the obtained results in Table 2, we can see that all our proposed PSA method achieved the best profit-sharing rates for the cases of uniform distribution, exponential distribution, and normal distribution at 0.56, 0.52, and 0.56, respectively. In addition, our proposed approach also gives better the whole supply chain profit (Z^*_{Avg} value) than the Saha's method. Both methods favour the manufacturer while our PSA is better in terms of balancing profits between manufacturer and retailer for uniform and normal demand distribution. Table 2 also shows that our proposed method also utilised the budget better in the case of normal demand distribution. In summary, the proposed PSA could be the preferable method for allocating profit between supply chain members. It means that the total profit of the supply chain is better even though one party may have to sacrifice a little bit of their profit to share for the other party.

In addition, we continue to investigate the effects of the demand fluctuation on the supply chain performance and the best profit-sharing rate for each member. For each kind of demand distribution, we consider different levels of demand uncertainty in Table 3. The effect of demand uncertainty on the best profit ratios for different demand distributions are exhibited in Figure 1.

Figure 1 Effect of demand uncertainty on profit-sharing rate (see online version for colours)



In Figure 1, we can see that when the market demand is less variation, with uniform distribution, the demand trend could be predictable. Therefore, the retailer tends to keep its profit-sharing rate is high because they do not need to ask the manufacturer to produce more for them to keep stock high. With the more demand variation in uniform, this rate is reduced a little bit because the duty of keeping more stock for absorbing the demand fluctuation is not so high because the demand pattern is still controllable. In the case of demand change following quite different patterns like exponential and normal distributions, the more demand uncertainty the higher risk of change will be. Therefore, the retailer needs to keep a high stock level to absorb the demand fluctuation. Therefore, when the demand is more fluctuation, the retailer keeps a more profit-sharing rate to compensate for the holding cost.

To show further details the advantage of our proposed method, we use the obtained alpha in above experiments to compare with the Saha's performance at the fixed profit-sharing rates of α_1 = 0.55, α_2 = 0.53, α_3 = 0.51 for three products, respectively. Table 4 illustrates the different performance of the proposed PSA solutions with the Saha's solutions under different demand scenarios of uniform, exponential, and normal distributions, respectively. The obtained results show that the proposed PSA always gives better average supply chain profit compared to the Saha's solutions.

Table 4 Performance of PSA and Saha solutions under different demand distribution

Case		*1*	*2*	*3*	*4*	*5*
Uniform	PSA-Z^*_{Avg} (\$)	7,668.54	18,379.96	21,277.90	23,269.05	24,429.43
	Saha-Z^*_{Avg} (\$)	7,665.37	18,278.38	21,277.85	23,269.00	24,429.38
Exponential	PSA-Z^*_{Avg} (\$)	18,526.34	15,597.93	11,809.60	9,888.10	8,326.84
	Saha-Z^*_{Avg} (\$)	8,163.63	15,360.46	11,617.29	9,888.10	8,311.97
Normal	PSA-Z^*_{Avg} (\$)	10,360.36	15,681.85	17,382.11	16,389.50	16,400.34
	Saha-Z^*_{Avg} (\$)	10,344.24	15,681.57	17,360.62	16,388.09	16,320.32

6.2 Three-echelon supply chain system

For generalisation, we also illustrate the profit-sharing supply chain coordination framework for a three-level supply chain system. We conduct experiments with randomly generated data for three types of demand distribution. Without loss of generality, we consider only one product and assume that the production rate of supplier and manufacturer follow uniform distribution in all periods with the following parameters:

$$T = 50;\ L_M = L_S = 1;\ B_R = B_M = B_S = \$15{,}000;\ A_R = A_M = A_S = 250{,}000;$$
$$p = \$90;\ w = \$40;\ c = \$10;\ u = \$10;\ v = \$6;\ r = \$3;\ s = \$2;$$
$$\gamma_Y = 0.7;\ \delta_Y = 0.9;\ \gamma_Z = 0.75;\ \delta_Z = 0.95;$$

Demand uniform distribution: $a = 0, b = 120$

Demand exponential distribution: $\lambda_1 = 0.015$

Demand normal distribution: $\mu = 180, \sigma = 72$.

We conduct testing our proposed CPSA approach for three types of demand distribution: uniform, exponential, and normal, respectively. The results are expressed in Table 5.

It is observed that the profit-sharing from the manufacturer to the retailer is very limited (α is small). The manufacturer often supports the retailer in case of promotion for increasing demand. So, for the case of demand pattern is stable (uniform) or increasing (exponential), the profit-sharing ratio from manufacturer to the retailer is zero. For the demand pattern is quite a fluctuation as the case of the normal distribution, the manufacturer needs to share a certain limited fraction profit to the retailer. For the relationship between the supplier and the manufacturer, the supplier needs to share a valuable profit ratio (β = 0.3 in this example) with the manufacturer because in this case, the demand of the manufacturer is the production rate in the form of uniform distribution in which it is quite stable. In addition, the supplier needs to support the manufacturer in the form of quantity discount to help the manufacturer for compensating holding cost of keeping more stocks.

Table 5 CPSA solution

Max/min values and optimal variables	*Type of distribution for stochastic demand*		
	Uniform distribution	*Exponential distribution*	*Normal distribution*
$Z_S^{\max}$	1,046,308	1,050,556	732,741.6
$Z_S^{\min}$	58,019.37	35,270.19	425,602
$Z_M^{\max}$	493,461.7	497,836	642,300.4
$Z_M^{\min}$	0.9205131	26,146.77	314,032.1
$Z_R^{\max}$	89,285.71	64,161.62	296,093.3
$Z_R^{\min}$	0	0	0
α^*	0	0	0.1
β^*	0.3	0.3	0.3
π^*	0.827466	0.8075524	0.9765297
Z_S^*	875,794.6	855,166.7	959,972.7
Z_M^*	408,322.9	407,060.5	634,595.8
Z_R^*	73,880.89	51,813.87	28,9143.9
Z_{Avg}^* (in $)	*27,159.97*	*26,280.82*	*37,674.25*
Budget utilised ($)	*14,759.69*	*13,683.09*	*25,222.17*

Note: $Z_{Avg}^* = \left(\dfrac{Z_S^* + Z_M^* + Z_R^*}{T}\right)$.

7 Conclusions

We have proposed two coordination procedures to determine the appropriate profit-sharing ratio for supply chain members. The proposed method could give better supply chain performance compared to the fixed sharing scheme in the work of Saha et al. (2015) for the case of a two-layer supply chain. Our approach could give a better total profit of the supply chain even though one party may have to sacrifice a little bit of their profit to share for the other party. In addition, we also extend our framework for the more general case of a multi-echelon supply chain by a cascading search process. The proposed procedures could also help us to determine the appropriate order quantity at each layer. We also investigate the effects of different demand patterns on profit-sharing rates. This could help each party in the supply chain to have a suitable strategy in cooperation with other members in the system. In this work, we only investigate the profit-sharing factor for supply chain coordination. In the future, the combination of profit-sharing allocation with different coordination contracts could be an interesting research direction.

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