A modified bat algorithm with torus walk for solving global optimisation problems

Waqas Haider Bangyal*

Department of Computing and Technology,
Iqra University,
Islamabad, Pakistan
Email: waqas_bangyal@hotmail.com
*Corresponding author

Jamil Ahmed

Department of Computing and Technology,
Kohat University of Science and Technology (KUST),
Kohat, Pakistan
Email: vc@kust.edu.pk

Hafiz Tayyab Rauf

Department of Computer Science,
University of Gujrat,
Gujrat, Pakistan
Email: hafiztayyabrauf093@gmail.com

Abstract: Bat algorithm (BA) has been widely used to solve the diverse kinds of optimisation problems. In accordance with the optimisation problems, balance between the two major components: exploitation and exploration, plays a significant role in meta-heuristic algorithms. Several researchers have worked on the performance for the improvement of these algorithms. BA faces one of the major issues in high dimensions. In our work, we proposed a new variant of BA by introducing the torus walk (TW-BA) to solve this issue. To improve the local search capability instead of using the standard uniform walk, torus walk is incorporated in this paper. The simulation results performed on 19 standard benchmark functions depicts the efficiency and effectiveness of TW-BA compared with the traditional BA, directional bat algorithm, particle swarm optimisation, cuckoo search, harmony search algorithm, differential evolution and genetic algorithm. The promising experimental result suggests the superiority of proposed technique.

Keywords: bat algorithm; torus walk; chaotic inertia weight; exploitation; exploration.

Reference to this paper should be made as follows: Bangyal, W.H., Ahmed, J. and Rauf, H.T. (2020) 'A modified bat algorithm with torus walk for solving global optimisation problems', *Int. J. Bio-Inspired Computation*, Vol. 15, No. 1, pp.1–13.

Biographical notes: Waqas Haider Bangyal is a PhD candidate at Iqra University Islamabad, Pakistan. He received his MS degree in Computer Science from Iqra University Islamabad Pakistan. His research interests are in evolutionary computation, machine learning, neural networks and implementation of computational intelligence (CI) algorithms in real world applications. He has been serving as the reviewer of many prestigious journals of Elsevier, Springer and IEEE. He is a senior member of IEEE SMC and member of ACM.

Jamil Ahmad obtained his PhD from the Department of Electrical and Electronics Engineering, Kings' College London. He has over 22 years of post PhD experience of administration, research, teaching and project management at various international academic institutions. He published over 110 papers/book chapters in the areas of science, engineering and technology in international journals and refereed conferences. He is a fellow of the British Computer Society, a senior member of IEEE, CEng, and a member of IET and ACM.

Hafiz Tayyab Rauf received his Undergraduate degree in Computer Science with distinction in 2017 and he is currently enrolled on a Master degree at the University of Gujrat. His research interest is artificial intelligence, evolutionary computing and artificial neural networks.

1 Introduction

The processes of optimisation involve vector space modelling and searching of the optimal solution for an addressed problem. All feasible advantages are recognised as an acceptable solution, while unusual output considered as an optimal solution. Optimisation is a method to achieve an optimum explication by performing specific objective functions (Yılmaz et al., 2014). Usually, optimisation processes are employed to sort out the local and global search optimisation problems (Gandomi et al., 2013).

Nature inspired swarm intelligence (SI)-based optimisation methods have been employed for several years for handling real-world complex problems. Beni and Wang (1993) introduced the term SI motivated by the behaviour of fish, insects, and birds as well as their independent skills to manage the multifaceted variety of problems. Every single individual cooperates with each other to perform complex tasks (Cui et al., 2018a).

In the optimisation process, discrete solutions are produced by using the stochastic algorithms, which do not use gradient and take an initial configuration identical. Although, the algorithm's variables does not affect the minor modifications but provide the best optimal solutions (Runarsson and Yao, 2000). A sub type of meta-heuristic family is population-based algorithms called swarm algorithms that are employed to analyse the common behaviour of swarms (Gandomi et al., 2013).

BA also, belongs to swarm meta-heuristic family, proposed by Yang (2010). This algorithm is simulated with the characteristics of bats called echolocation. Although, bats also have an ability to distinguish each kind of insects that cross their way during hunting process in the darkness. BA has carried out in several artificial intelligence (AI) applications like feature selection, engineering design, image processing, natural language processing, text classification, clustering, malware variants detection (Cui et al., 2018b) and many more.

A searching ability of an algorithm is affected by the two most critical ingredients: exploration and exploitation (Cui et al., 2018a). Exploration has an ability to bring out the solutions from area where they stuck in an algorithm. The best solution is founded by examining the different unknown areas with the help of an algorithm having exploration ability. In literature, many researchers suggest that exploration should be carried out first in order to scan the whole area of search space. However, exploitation has an ability to expand the algorithm's convergence speed and exploitation comes for enhancing the solutions that are taken with the help of exploration (Bangyal et al., 2018a). The balance between exploitation and exploration ability of population-based algorithms may tend to highly increase the performance (Bangyal et al., 2018a).

The major problem with BA and other swarm-based evolutionary algorithms is a convergence of swarm (Cai et al., 2016). Before obtaining a global best solution, BA may converge prematurely and due to this premature behaviour, BA becomes stabbed into the local minima. Researchers have tackled this problem by proposing the improved techniques to resolve local optima problem (Kora and Kalva, 2015). In case of BA, the targeted parameters to handle local minima problem is: swarm size, velocity, pulse rate, loudness and frequency (Meng et al., 2015).

Different optimisation methods including gene expression programming (GEP), differential evolution (DE), evolutionary algorithms (EAs), genetic programming (GP), particle swarm optimisation (PSO) and genetic algorithm (GA) also faced premature convergence problem (Eberhart and Shi, 1998). PSO performs better than GA in terms of exploration, reported by Kennedy and Eberhart (1995).

In this study, we have carried out two major modifications in standard BA and proposed new version of BA called TW-BA. The first modification is the use of chaotic random inertia weight for updating the velocity and in second modification, we employed torus walk (TW-BA) to improve exploitation capability of BA. The proposed TW-BA is compared with Standard BA, dBA (Chakri et al., 2017), PSO (Kennedy and Eberhart, 1995), CS (Yang and Deb, 2009), HS (Geem et al., 2001), DE (Das and Suganthan, 2011) and GA (Davis, 1991). The experimental results proved the superiority of TW-BA over other variants.

The article has been divided into different sections as: Section 2 contains the related work. In Section 3 the working of standard BA is elaborated. Methodology is discussed in Section 4. In Section 5 experimental setup and their characteristics are presented. Results and discussion is shown in Section 6. Section 7 describes the conclusion.

2 Related work

The local search ability of BA is better than the global search ability, however, the basic scale to measure the performance of evolutionary algorithms is highly dependable on the balance between local search and global search ability. In order to improve the exploitation and exploration ability, several researchers have proposed novel variants of BA (Bangyal et al., 2018a).

Multiobjective bat algorithm (MOBA) has been introduced (Yang, 2011) to sort out multi-objective optimisation. A novel mutation operator is carried out in Wang and Guo (2013) named as the robust hybrid optimisation (HS/BA) in order to improve the convergence rate.

Fister et al. (2013) hybridised BA with differential evolution (DE) to generate the new population-based BA variant called HBA that gives the more optimal global solution. In Mirjalili et al. (2014), binary bat algorithm (BBA) is used to handle binary optimisation problems.

BA based on opposition learning (OBA) is suggested by Saha et al. (2013) for solving infinite impulse response (IIR) classification problem. Yilmaz and Kucuksille (2013) carried out three basic modifications in BA named as IBA to cover the drawbacks of standard BA. IBA includes the modification of local search by improving the exploitation and exploration capabilities of bats.

A new variant of BA called modified bat algorithm (MBA) proposed by Yılmaz et al. (2014). In MBA the exploration process is modified by mutating the emission rate and loudness of bats in the search space.

The updated version of BA is proposed in Fister et al. (2014), described as hybrid self-adaptive bat algorithm (HSABA). HSABA is composed of adaptive bat algorithm (SABA) and DE. This unique method is executed as the heuristic for exploitation ability. An improved version of chaotic behaviour-based BA introduced in Abdel-Raouf et al. (2014), termed as IBACH. Acoustic monopoly incorporated along with chaotic distribution in order to generate the bat solution. IBACH is useful for solving integer programming problems.

BA with the Gaussian walk (BAGW) (Cai et al., 2014) is one of the major contributions in the field of optimisation. In BAGW the authors targeted the high dimensionality of the multi-objective and nonlinear problem. Chaotic BA is carried out by Gandomi and Yang (2014), for implementing stable, obvious and balance optimisation.

For increasing the population diversity a novel complex-valued BA is presented by Li and Zhou (2014). Kabir et al. (2014) proposed adaptive BA (NABA) to increase the searching capability of BA in exploratory features. NABA contains two mutation operators to enhance the search strategy of BA. Furthermore, detailed comparison of different BA mutation has been carried out in Bangyal et al. (2018b).

Cloud model BA (CBA) is carried out (Zhou et al., 2014) to improve the echolocation process of BA. CBA follows the concept 'Bats overtake their victim'. Moreover, Levy fight mode and swarm learning information also induced in CBA for better diversity and robust search abilities. The authors added a new parameter in BA by proposing the inertia weight factor (Cui et al., 2015) to develop the strong exploitation abilities. Inertia weight is employed to regulate the bat's prior velocity.

The authors combined the Cauchy mutation operator along with elite opposition-based learning behaviour to the proposed new version of BA (Paiva et al., 2017). This enhanced BA develops a controlled diversity of an algorithm as well as high convergence speed.

3 Bat algorithm

BA is a population-based metaheuristic approach, inspired by the echolocation rule of bats. Every single individual called a bat denotes the candidate solution in the swarm.

The processes of bat echolocation involve the following characteristic rules (Kabir et al., 2014):

- *Distance:* every individual used echolocation to estimate the distance between their current position and the estimated position of prey. Bats also send omens to their surroundings to acknowledge their food.
- *Frequency:* for each iteration t in dimension, d bats move randomly with the velocity v_{ij}^t and the position x_{ij}^t, where the frequency of bats $f_{ij}^{(t)}$ follows the linear decreasing behaviour of wavelength λ. Bat loudness is determined by $A_{ij}^{(t)}$.
- *Loudness:* loudness $A_{ij}^{(t)}$ diversifies from A_0 to A_m. Where A_0 is the maximum value and $A_{\min}$ is minimum constant.

Algorithm 1 Standard bat algorithm

Require: $x_{ij}^t = (x_{i1} \ldots x_{ij})^t$ population of bats
Ensure: $\bar{x}^{best}$& $\bar{\bar{f}}_{x^{\min}}$ global best solution and minimal fitness score
1: x_{ij}^t =Bat_Init($\bar{x}_i$); initialise particles
2: New_population_Evluation(x_{ij}^t)
3: $Calculate(\bar{x}^{best})$; current best solution
4: **for** each Itration ($t < t_{\max}$) **do**
5: **for** $i = 1 \ldots N_p$ **do**
6: Compute $f_{ij}^{(t)}$ by equation (1)
7: Compute $v_{ij}^{(t+1)}$ by equation (2)
8: Compute $x_{ij}^{(t+1)}$ by equation (3)
9: **if** $\bar{R} > r_{ij}^t$ **then**
10: $xij_{new}^t =$ replace current solution by equation (4)
11: **end if**
12: Calculate xij_{new}^t; produce new solution
13: Eva = Eva +1;
14: **if** $\bar{f}_{\min}^{new} < \bar{f}_{old}$ **&** $\bar{R} < A_{ij}^t$ **then**
15: $\bar{x}_i = xij_{new}^t; \bar{f}_{old} = \bar{f}_{\min}^{new}$;
16: **end if**
17: $\bar{f}_{\min} = \bar{f}_{x^{\min}}$;
18: **end for**
19: **end for**

For each bat, x_{ij}^t population is initialised with random distribution to adjust their initial positions in the dimension d. In Algorithm 1, line 1 corresponds to the bat initialisation where the primitive loop starts from line 1 to line 19. The initial best solution in line 2, the current best solution is determined from line 6 to 9 and the solution evaluation in line 10. The process from generating the new solution to retain global optimum is expressed by lines 11 to 19. Following equation are applied to retain the new solution:

$$f_{ij}^{(t)} = f_{\min} + (f_{\max} - f_{\min}).\bar{R} \tag{1}$$

$$v_{ij}^{(t+1)} = v_{ij}^t + (x_{ij}^t - B\bar{e}st)f_{ij}^t \tag{2}$$

$$x_{ij}^{(t+1)} = x_{ij}^t + v_{ij}^t \tag{3}$$

where a Gaussian distribution is used to produce the random number$\bar{R}$, The standard deviation and mean of number$\bar{R}$ are fixed to one and zero respectively. The frequency for determining the change of the rate of velocity is as: $f_{ij}^{(t)} = [fij_{\min}^{(t)}, fij_{\max}^{(t)}]$. The current best solution is improved using the following equation.

$$xij_{new}^t = B\bar{e}st + \varepsilon A_{ij}^{(t)}.\bar{N} \tag{4}$$

In the above equation $A_{ij}^{(t)}$ recognised as the bat loudness factor where ε referred as scaling factor helps in local searching,$\bar{N}$ representing random number following the uniform distribution. Generally, the loudness of bats $A_{ij}^{(t)}$ leads to decline and the pulse rate r_{ij}^{t} leads to raise when the bats approximately near to their current best solution (Wang et al., 2019). The equation for both factors loudness and pulse rate are as:

$$A_{ij}^{t+1} = \bar{\alpha} A_{ij}^{t} \tag{5}$$

$$r_{ij}^{t} = r_{ij}^{0}[1 - exp(-\gamma^{t})] \tag{6}$$

where $\bar{\alpha}$ and $\gamma^{t}$are constant number generated using the uniform distribution randomly. Algorithm 1 contains the pseudo code of standard BA.

4 Methodology

Many researches implemented different strategies of inertia weights to improve the performance of BA. Yang and Le (2015) proposed time varying inertia weight in original BA to balance the searching process of BA. Thus, this study contains two major modifications in standard BA. Firstly, we have carried out chaotic inertia weight (Feng et al., 2007) in order to maintain the balance between local search and global search ability of BA. Secondly, we modified the random walk with torus walk in order to improve the exploitation process. These two improvements chaotic inertia and the torus walk together develop a new variant of BA termed as modified bat algorithm with torus walk (TW-BA).

4.1 The chaotic inertia weight

Chaotic inertia weight includes chaotic mapping to set weights of inertia. In this study, we carried out a logistic mapping for controlling the parameter of inertia weights. The equation for logistic mapping is given below:

$$m = \delta m(1 - m) \tag{7}$$

where $3.57 < \delta < 4$, leads to the chaotic phenomena and the interval $[0, 1]$ is be-sprinkled by the chaotic result. The combination $[0, 0.1]$ and $[0.9, 1]$ shows the times happening very high. The maximum and mean time for the time happening interval of $[0.1, 0.9]$ is more than 1,500 and 200 respectively.

TW-BA contains chaotic random inertia weight; the strategy for chaotic random inertia weight is given below:

Step 1 Chose a number m randomly over the interval of $(0, 1)$. Select a number R randomly in the interval $[0, 1]$.

Step 2 Calculate logistic mapping as: $m = 3.58m(1 - m)$.

Step 3 Compute chaotic random inertia weight as (Feng et al., 2007): $w_{ij}^{t} = 0.5R + 0.5m$.

The standard version of BA has no inertia weight, so we altered the velocity equation $v_{ij}^{(t+1)}$of standard BA. Now the following equation is used to update the velocity of bats.

$$v_{ij}^{(t+1)} = v_{ij}^{t} w_{ij}^{t} + (x_{ij}^{t} - \bar{Best}) f_{ij}^{t} \tag{8}$$

Algorithm 2 Proposed bat algorithm (TW-BA)

Require: $x_{ij}^{t} = (x_{i1} \dots x_{ij})^{t}$ population of bats
Ensure: $\bar{x}^{best}$& $\bar{\bar{f}}_{x^{\min}}$ global best solution and minimal fitness score
x_{ij}^{t} =Bat_Init($\bar{x}_i$); initialise particles
New_population_Evluation(x_{ij}^{t})
$Calculate(\bar{x}^{best})$; current best solution
for each iteration ($t < t_{\max}$) **do**
  for $i = 1 \dots N_p$ **do**
    Compute $f_{ij}^{(t)}$ by equation (1)
    Compute $v_{ij}^{(t+1)}$ by equation (2)
    Compute $x_{ij}^{(t+1)}$ by equation (8)
    if $\bar{R} > r_{ij}^{t}$ **then**
      xij_{new}^{t} = replace current solution by equation (9)
    end if
    Calculate xij_{new}^{t}; produce new solution
    Eva = Eva +1;
    if $\bar{f}_{\min}^{new} < \bar{f}_{old}$ **&** $\bar{R} < A_{ij}^{t}$ **then**
      $\bar{x}_i = xij_{new}^{t}; \bar{f}_{old} = \bar{f}_{\min}^{new}$;
    end if
    $\bar{f}_{\min} = \bar{f}_{x^{\min}}$;
  end for
end for

Table 1 Parameter setting

Algorithm	*Parameters*
BA	$N_p = 40$, r_{ij}^{t}, $[0,1]$, A_{ij}^{t}, $[0,2]$
PSO	c1 = c2 = 1.49, w = linearly decreasing
TW-BA	r_{ij}^{t}, $[0,1]$, A_{ij}^{t}, $[0,2]$, $\bar{L}$, $[0,1]$, T, $(0,1)$

4.2 The torus walk

In the traditional BA, population adopted the random walk during the search process of global minima. But this random walk may lead to premature convergence as particles searching food randomly without any specific pattern. We replaced this random walk into a torus walk. In the original BA, by (4) the local search only concentrates the knowledge of their neighbours, so if neighbour's bats get stuck in local minima then the capability of local search may useless. This inadequate exploitation is not supposed to give the best optimal solution. We modified this random walk by torus, which increases the local search capability to handle the local minima problem. The local search equation for TW-BA is given below:

$$xij_{new}^{t} = \bar{Best} + 1A_{ij}^{(t)}(\bar{L}T(0,1)(x_{ij}^{t} - iter\bar{Best})) \tag{9}$$

In the above equation, $\bar{L}$ is referred as controlling parameter, which controls the speed of each bat by following linearly decreasing phenomena helps to improve their searching capability. Where $T(0,1)$ denotes to a random number generated using torus distribution over the interval of $[0,1]$. x_{ij}^t is current local best position of i^{th} bats at j^{th} dimension where $iter$ representing the current epoch number. Our improved exploitation strategy is introduced for conquering this local search problem, i.e., enhances local search technique throughout the global solution, maintains population heterogeneity, and reaches to a uniform result. The proposed technique for TW-BA is presented in Algorithm 2.

5 Experiments

The core objective of our simulations was to demonstrate that TW-BA has superior results over standard BA and similarly, the results were compared with other famous approaches like Standard BA, PSO DE, GA, CS, HS and dBA. All these described techniques were tested for standard function optimisation problem that relate to the part of combinatorial problems.

5.1 Benchmark suite

Nine famous standard benchmark functions were taken from the literature. Detail of these well- known functions for the researchers can be studied in Chakri et al. (2017). Standard functions detail is provided in the Table 2, which contains attributes like function name, its definition and function label f.

In Table 4, each function is labelled with serial number from 1 to 9. Generally, as the dimension size of the problem increases, the problem becomes tough to solve. Consequently, in this simulation, benchmark functions with higher dimensions are optimised.

5.2 Experimental setup

During the experiments, features of TW-BA have been seen and tested. The results of standard BA are compared with TW-BA and PSO. The findings of the others famous algorithms were also shown in the comparative study. Where parameter setting is concerned, the best parameters were selected to keep the unbiased comparison. Parameter settings suites were discovered after hit and trial method. The experimental setup of simulation is set as with swarm size 40 and the dimensions of the problem for all standard benchmark function are 10, 20, 30 and 50. Against each dimension size, the total number of epochs is 1,000, 2,000, 3,000 and 5,000 sequentially. For moderately fair comparison, TW-BA is compared with standard BA and six other algorithms on comparable parameters. All methods were observed for 30 numbers of runs to analyse the performances.

6 Results and discussion

The proposed variant of TW-BA is executed on HP Compaq with configuration Intel Core i7-3200, with speed 3.8 GHZ with RAM 4 GB. In order to validate the integrity and effectiveness of the suggested variant, a set of 19 uni-modal and multi-modal standard benchmark functions have been carried out to compare the TW-BA with standard BA, dBA (Chakri et al., 2017), PSO (Kennedy and Eberhart, 1995), CS (Yang and Deb, 2009), HS (Geem et al., 2001), DE (Das and Suganthan, 2011) and GA (Davis, 1991). In terms of convergence rate, local search, and global search, the performance of elocutionary algorithms is tested on standard nonlinear benchmark functions. The detailed description and the characteristics of these benchmark functions are listed in Table 2 and Table 3, where Table 4 contains experimental results. In Table 3 the possible lowest value is represented by x^* and f^* indicates the global least value of fitness function f.

The goal of this study is to develop the strong aptitude of standard BA by introducing two strategies chaotic random inertia weight and torus walk. The chaotic random inertia weight controls the scale of global search and local search of BA while torus walk enhances the exploitation ability of BA. The purpose of this study continues to observe the unique characteristics of experimental results that rely on dimensions of these standard benchmark functions.

In the experiments, three simulation experiments were performed and following features of TW-BA were observed

- impact of TW-BA
- impact of dimension's nature for problems
- a comparative analysis.

During the first experiment, inertia weight and torus walk was integrated in TW-BA. The objective of second simulation is to find the nature of dimension regarding standard function optimisation. Lastly, the simulation results of TW-BA were compared with standard BA and dBA along with famous approaches like PSO, DE, GA, CS and HS. In the rest of the paper, simulation results were discussed in detail.

6.1 Impact of TW-BA

In this model, torus walk is incorporated with the standard BA rather than uniform walk, as well as, inertia weight is embedded. The introduced algorithm, which is referred as TW-BA, can be deliberated as outstanding algorithm from all others seven comparable algorithms as displayed in Table 4. The introduced TW-BA is capable to work effectively in both problems of low and high dimensions. Due to the incorporation of torus walk with respect to standard BA, the introduced algorithm is appropriate for the solution of low and high dimensional problems as compared to other algorithms. The functionality of proposed TW-BA in low and high dimensional problems explains that the TW-BA is remarkable.

The proposed algorithm TW-BA gives superior results on all 19 test functions in higher and lower dimensions. Similarly, it depicts that the proposed TWBA are better than PSO and standard BA algorithms, is shown in Table 5.

Table 2 Nineteen standard benchmark functions

f	*Function name*	*Definition*
f_1	Sphere	$Minf(x) = \sum_{i=1}^{n} x_i^2$
f_2	Axis parallel hyper-ellipsoid	$Minf(x) = \sum_{i=1}^{n} i.x_i^2$
f_3	Schumer Steiglitz	$Minf(x) = \sum_{i=1}^{n} x_i^4$
f_4	Schwefel 2.23	$Minf(x) = \sum_{i=1}^{n} x_i^{10}$
f_5	Powell singular 2	$Minf(x) = \sum_{i=1}^{D-2} (x_{i-1} + 10x_i)^2 + 5(x_{i+1} - x_{i+2})^2 + (x_i - 2x_{i+1})^4 + 10(x_{i-1} - x_{i+2})^4$
f_6	Schwefel 2.21	$Minf(x) = \binom{\max \lvert x_i \rvert}{1<i<D}$
f_7	Rastrigin	$Minf(x) = \sum_{i=1}^{n} [x_i^2 - 10cos\ .\ (2\pi x) + 10]_i$
f_8	Ackley	$Minf(x) = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} [x_i^2} - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$
f_9	Schwefel 1.2	$Minf(x) = \sum_{i}^{D} (\sum_{j=1}^{i} x_j)^2$
f_{10}	Rotated hyper-ellipsoid	$Minf(x) = \sum_{i}^{n} (\sum_{j=1}^{i} x_j)^2$
f_{11}	Moved axis parallel hyper-ellipsoid	$Minf(x) = \sum_{i=1}^{n} 5i.x_i^2$
f_{12}	Sum of different power	$Minf(x) = \sum_{i=1}^{n} \lvert x_i \rvert^{i+1}$
f_{13}	Noisy function	$Minf(x) = \sum_{i=1}^{n} (i+1).x_i^2 + rand[0,1]$
f_{14}	Salomon function	$1 - cos(2\pi\sqrt{\sum_{i=1}^{n} x_i^2}) + 0.1(\sqrt{\sum_{i=1}^{n} x_i^2})$
f_{15}	Schwefel 2.22	$Minf(x) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$
f_{16}	Sum squares function	$Minf(x) = \sum_{i=1}^{n} i.x_i^2$
f_{17}	Zakharov function	$Minf(x) = \sum_{i=1}^{n} x_i^2 + (1/2\sum_{i=1}^{n} i.x_i)^2 + (1/2\sum_{i=1}^{n} i.x_i)^4$
f_{18}	Dixon and price function	$Minf(x) = (x_i - 1)^2 + \sum_{i=2}^{n} i.(2x_i^2 - x_{i-1})^2$
f_{19}	Cigar	$Minf(x) = x_i^2 + 10^6 \sum_{i=2}^{n} x_i^2$

Table 3 Characteristics of standard benchmark functions

f	x	*f*	*Domain*
f_1	(0, 0, 0 …, 0)	0.00	$-5.12 \leq x_i \leq 5.12$
f_2	(0, 0, 0 …, 0)	0.00	$-5.12 \leq x_i \leq 5.12$
f_3	(0, 0, 0 …, 0)	0.00	$-5.12 \leq x_i \leq 5.12$
f_4	(0, 0, 0 …, 0)	0.00	$-10 \leq x_i \leq 10$
f_5	(0, 0, 0 …, 0)	0.00	$-4 \leq x_i \leq 5$
f_6	(0, 0, 0 …, 0)	0.00	$-100 \leq x_i \leq 100$
f_7	(0, 0, 0 …, 0)	0.00	$-5.12 \leq x_i \leq 5.12$
f_8	(0, 0, 0 …, 0)	0.00	$-35 \leq x_i \leq 35$
f_9	(0, 0, 0 …, 0)	0.00	$-100 \leq x_i \leq 100$
f_{10}	(0, 0, 0 …, 0)	0.00	$-65.536 \leq x_i \leq 65.536$
f_{11}	$(5 * i)$	0.00	$-5.12 \leq x_i \leq 5.12$
f_{12}	(0, 0, 0 …, 0)	0.00	$-1 \leq x_i \leq 1$
f_{13}	(0, 0, 0 …, 0)	0.00	$-1.28 \leq x_i \leq 1.28$
f_{14}	(0, 0, 0 …, 0)	0.00	$-100 \leq x_i \leq 100$
f_{15}	(0, 0, 0 …, 0)	0.00	$-100 \leq x_i \leq 100$
f_{16}	(0, 0, 0 …, 0)	0.00	$-10 \leq x_i \leq 10$
f_{17}	(0, 0, 0 …, 0)	0.00	$-5 \leq x_i \leq 10$
f_{18}	$(f(2(\frac{2^i-2}{2^i})))$	0.00	$-10 \leq x_i \leq 10$
f_{19}	(0, 0, 0 …, 0)	0.00	$-100 \leq x_i \leq 100$

Table 4 Comprehensive results for TW-BA with other variants

F		*BA*	*dBA (Chakri et al., 2017)*	*PSO*	*CS (Chakri et al., 2017)*	*HS (Chakri et al., 2017)*	*DE (Chakri et al., 2017)*	*GA (Chakri et al., 2017)*	*TW-BA*
F1	Best	8.27E-08	1.927E-03	0.00E+00	2.340E+02	5.919E+03	2.481E+01	5.517E+00	0.00E+00
	Median	1.51E-07	1.408E-02	0.00E+00	4.357E+02	9.621E+03	4.120E+01	6.560E+02	2.04E-317
	Worst	2.05E-07	2.233E+00	1.77E-25	6.119E+02	1.568E+04	8.028E+01	7.964E+03	5.66E-304
	Mean	1.49E-07	2.256E-01	5.91E-27	4.153E+02	9.618E+03	4.411E+01	1.678E+03	**2.42E-305**
	SD	2.90E-08	4.869E-01	3.18E-26	9.518E+01	2.226E+03	1.259E+01	2.032E+03	**0.00E+00**
F12	Best	5.51E-04	1.011E+06	1.53E-228	3.229E+17	2.573E+33	9.080E+08	7.488E+04	3.32E-237
	Median	1.02E-03	8.171E+09	1.00E-223	7.654E+19	7.580E+37	1.177E+11	9.245E+29	3.17E-207
	Worst	2.00E-03	1.713E+13	9.82E-123	2.433E+22	8.664E+42	1.553E+12	2.390E+41	**1.77E-182**
	Mean	1.11E-03	1.363E+12	3.27E-124	2.263E+21	3.533E+41	3.051E+11	1.049E+40	**5.89E-184**
	SD	3.50E-04	4.261E+12	1.76E-123	5.976E+21	1.697E+42	4.102E+11	4.671E+40	0.00E+00
F10	Best	1.43E+07	1.634E-02	0.00E+00	1.062E+03	4.124E+04	9.877E+01	8.280E+01	0.00E+00
	Median	1.73E+08	3.115E-01	0.00E+00	1.996E+03	5.220E+04	1.618E+02	5.373E+03	0.00E+00
	Worst	6.74E+08	1.256E+02	6.16E-132	3.409E+03	7.472E+04	3.850E+02	3.294E+04	0.00E+00
	Mean	2.16E+08	1.461E+01	2.05E-133	2.138E+03	5.336E+04	1.742E+02	8.130E+03	**0.00E+00**
	SD	1.55E+08	3.456E+01	1.11E-132	5.493E+02	8.132E+03	6.173E+01	8.472E+03	**0.00E+00**
F7	Best	1.06E+02	6.812E+01	–8.70E+04	1.129E+02	1.330E+02	2.998E+01	2.994E+01	0.00E+00
	Median	1.74E+02	1.057E+02	–8.70E+04	1.378E+02	1.625E+02	1.575E+02	5.895E+01	0.00E+00
	Worst	2.54E+02	2.471E+02	0.00E+00	1.644E+02	1.845E+02	2.047E+02	9.913E+01	0.00E+00
	Mean	1.85E+02	1.193E+02	–8.60E+04	1.366E+02	1.580E+02	1.551E+02	5.746E+01	**0.00E+00**
	SD	3.96E+01	4.023E+01	5.09E+03	1.349E+01	1.558E+01	3.368E+01	1.825E+01	**0.00E+00**
F8	Best	2.30E+00	3.214E+00	2.72E+00	8.691E+00	1.366E+01	2.302E+00	2.595E+00	4.44E-16
	Median	3.13E+00	5.681E+00	2.72E+00	1.200E+01	2.384E+01	3.191E+00	5.744E+00	4.44E-16
	Worst	3.88E+00	8.801E+00	2.72E+00	1.750E+01	3.540E+01	3.648E+00	1.145E+01	4.44E-16
	Mean	3.22E+00	5.839E+00	2.72E+00	1.209E+01	2.417E+01	3.191E+00	5.920E+00	**4.44E-16**
	SD	3.75E-01	1.730E+00	8.88E-16	1.753E+00	5.004E+00	2.904E-01	2.453E+00	**4.93E-32**
F17	Best	1.02E+00	7.536E+01	1.07E-209	1.337E+02	2.960E+02	1.414E+02	1.122E+01	4.73E-275
	Median	2.21E+01	1.561E+02	2.32E-166	2.190E+02	4.023E+02	1.879E+02	7.707E+06	1.39E-247
	Worst	6.23E+03	2.506E+02	2.73E-14	3.009E+02	5.713E+02	2.352E+02	9.316E+08	4.02E-234
	Mean	3.31E+02	1.515E+02	9.11E-16	2.214E+02	4.052E+02	1.937E+02	9.884E+07	**2.26E-235**
	SD	1.21E+03	4.105E+01	4.90E-15	4.094E+01	6.898E+01	2.433E+01	2.076E+08	**0.00E+00**
F18	Best	2.96E+05	7.448E-01	6.67E-01	1.059E+02	4.057E+04	2.650E+01	1.207E+01	7.56E-01
	Median	7.46E+05	5.528E+00	6.67E-01	2.200E+02	7.688E+04	6.164E+01	1.836E+03	9.32E-01
	Worst	1.87E+06	1.044E+02	6.67E-01	6.159E+02	1.282E+05	1.438E+02	3.631E+04	9.95E-01
	Mean	8.43E+05	1.911E+01	**6.79E-01**	2.611E+02	7.853E+04	6.790E+01	6.494E+03	9.16E-01
	SD	3.43E+05	2.917E+01	6.72E-02	1.384E+02	2.813E+04	2.559E+01	9.520E+03	6.44E-02
F19	Best	1.96E+05	4.499E+01	1.03E-259	1.542E+08	6.110E+07	1.283E+05	1.474E+06	-3.86E-12
	Median	2.86E+05	3.283E+02	7.65E-138	3.709E+08	8.407E+07	2.388E+05	1.061E+07	-1.26E-29
	Worst	5.92E+05	2.518E+03	7.01E-06	6.179E+08	1.111E+08	3.528E+05	8.335E+07	0.00E+00
	Mean	3.19E+05	4.926E+02	2.34E-07	3.760E+08	8.696E+07	2.392E+05	1.803E+07	**-1.52E-12**
	SD	9.74E+04	5.304E+02	1.26E-06	1.192E+08	1.489E+07	6.522E+04	2.213E+07	**7.50E-12**
F14	Best	9.80E+00	3.554E-01	1.56E-135	2.426E+01	6.724E+02	3.377E+00	9.307E+00	9.99E-02
	Median	1.59E+01	1.328E+00	2.52E-91	4.450E+01	9.243E+02	5.082E+00	1.757E+02	9.99E-02
	Worst	2.08E+01	2.357E+00	8.00E-01	8.646E+01	1.277E+03	8.308E+00	7.373E+02	9.99E-02
	Mean	1.61E+01	1.417E+00	**3.67E-02**	4.591E+01	9.074E+02	5.193E+00	2.319E+02	9.99E-02
	SD	2.55E+00	4.826E-01	1.47E-01	1.265E+01	1.325E+02	1.241E+00	1.940E+02	**5.55E-17**

Table 5 Comprehensive results from F1 to F10 for TW-BA, BA and PSO

F#	*IT*	*DIM*	*BA*				*PSO*				*TW-BA*			
			Best	*Worst*	*Mean*	*Std. dev.*	*Best*	*Worst*	*Mean*	*Std. dev.*	*Best*	*Worst*	*Mean*	*Std. dev.*
F1	1,000	10	3.57E-08	3.37E-07	1.70E-07	7.48E-08	3.20E-129	2.33E-73	7.76E-75	4.18E-74	1.14E-205	1.26E-172	**4.37E-174**	**0.00E+00**
	2,000	20	8.45E-08	2.78E-07	1.53E-07	3.87E-08	1.97E-241	1.02E-83	3.40E-85	1.83E-84	1.00E-272	2.31E-250	**8.71E-252**	**0.00E+00**
	3,000	30	8.27E-08	2.05E-07	1.49E-07	2.90E-08	0.00E+00	1.77E-25	5.91E-27	3.18E-26	0.00E+00	5.66E-304	**2.42E-305**	**0.00E+00**
	5,000	50	8.63E-08	1.85E-07	1.28E-07	2.19E-08	3.48E-243	2.62E+01	8.74E-01	4.71E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F2	1,000	10	1.98E-07	1.07E-06	6.45E-07	2.28E-07	4.49E-130	1.30E-80	4.34E-82	2.33E-81	7.47E-193	4.79E-174	**2.02E-175**	**0.00E+00**
	2,000	20	6.42E-07	3.02E-06	1.51E-06	5.35E-07	7.50E-242	1.16E-89	3.85E-91	2.07E-90	9.08E-269	7.41E-248	**2.48E-249**	**0.00E+00**
	3,000	30	2.02E-06	5.89E-06	3.42E-06	1.18E-06	0.00E+00	2.50E-65	8.34E-67	4.49E-66	0.00E+00	7.04E-303	**2.47E-304**	**0.00E+00**
	5,000	50	1.24E-05	2.55E-05	1.77E-05	3.50E-06	6.72E-244	1.10E+03	3.67E+01	1.98E+02	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F3	1,000	10	7.18E-17	1.43E-14	5.22E-15	3.50E-15	8.25E-250	2.23E-137	7.45E-139	4.01E-138	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	2,000	20	8.13E-16	8.76E-15	3.20E-15	1.76E-15	0.00E+00	3.79E-147	1.26E-148	6.80E-148	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	3,000	30	9.24E-16	4.14E-15	2.22E-15	7.39E-16	0.00E+00	4.43E-125	1.48E-126	7.96E-126	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	5,000	50	6.88E-16	2.82E-15	1.51E-15	5.55E-16	0.00E+00	1.40E-26	4.68E-28	2.52E-27	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F4	1,000	10	3.60E-41	1.98E-35	2.01E-36	3.77E-36	0.00E+00	3.11E-316	1.03E-317	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	2,000	20	5.52E-39	1.98E-36	1.79E-37	3.57E-37	0.00E+00	9.05E-273	3.02E-274	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	3,000	30	4.26E-40	4.51E-37	9.54E-38	1.29E-37	0.00E+00	3.52E-213	1.17E-214	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	5,000	50	5.19E-39	8.58E+03	3.89E+02	1614.56	0.00E+00	2.95E-141	9.83E-143	5.29E-142	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F5	1,000	10	5.69E+00	4.55E+03	7.03E+02	1.02E+03	1.00E+05	1.00E+05	1.00E+05	2.91E-11	8.36E-168	2.91E-120	**9.69E-122**	**5.22E-121**
	2,000	20	1.05E+02	1.12E+04	2.64E+03	2.65E+03	1.00E+05	1.00E+05	1.00E+05	2.91E-11	1.77E-235	1.52E-208	**5.06E-210**	**0.00E+00**
	3,000	30	3.29E+02	3.36E+04	5.55E+03	6.70E+03	1.00E+05	1.00E+05	1.00E+05	2.91E-11	1.15E-286	1.99E-255	**7.86E-257**	**0.00E+00**
	5,000	50	2.66E+03	3.82E+04	1.20E+04	8.30E+03	1.00E+05	1.00E+05	1.00E+05	2.91E-11	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F6	1,000	10	1.40E+01	4.79E+01	3.15E+01	9.01E+00	2.50E-65	1.57E-38	5.23E-40	2.82E-39	2.07E-93	3.84E-85	**1.29E-86**	**6.89E-86**
	2,000	20	3.31E+01	5.38E+01	4.58E+01	4.44E+00	2.69E-124	1.22E-16	4.06E-18	2.19E-17	1.31E-134	1.64E-122	**7.71E-124**	**2.98E-123**
	3,000	30	3.12E+01	6.47E+01	5.22E+01	7.59E+00	6.85E-129	8.94E-04	2.98E-05	1.61E-04	1.25E-165	2.83E-149	**1.92E-150**	**6.34E-150**
	5,000	50	5.00E+01	7.06E+01	6.08E+01	5.39E+00	6.79E-67	1.26E+00	4.23E-02	2.26E-01	4.24E-201	8.02E-180	**2.68E-181**	**0.00E+00**
F7	1,000	10	9.95E+00	8.26E+01	4.39E+01	1.84E+01	–9.00E+03	0.00E+00	–8.97E+03	1.61E+02	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	2,000	20	7.56E+01	1.83E+02	1.15E+02	2.33E+01	–3.80E+04	0.00E+00	–3.77E+04	1.50E+03	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	3,000	30	1.06E+02	2.54E+02	1.85E+02	3.96E+01	–8.70E+04	0.00E+00	–8.60E+04	5.09E+03	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	5,000	50	2.28E+02	4.81E+02	3.37E+02	4.63E+01	–2.45E+05	0.00E+00	–2.41E+05	2.09E+04	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F8	1,000	10	1.75E+00	3.19E+00	2.36E+00	3.75E-01	2.72E+00	2.72E+00	2.72E+00	8.88E-16	4.44E-16	4.44E-16	**4.44E-16**	**4.93E-32**
	2,000	20	2.33E+00	4.13E+00	3.13E+00	3.86E-01	2.72E+00	2.72E+00	2.72E+00	8.88E-16	4.44E-16	4.44E-16	**4.44E-16**	**4.93E-32**
	3,000	30	2.30E+00	3.88E+00	3.22E+00	3.75E-01	2.72E+00	2.72E+00	2.72E+00	8.88E-16	4.44E-16	4.44E-16	**4.44E-16**	**4.93E-32**
	5,000	50	2.85E+00	4.21E+00	3.56E+00	3.74E-01	2.72E+00	2.72E+00	2.72E+00	8.88E-16	4.44E-16	4.44E-16	**4.44E-16**	**4.93E-32**
F9	1,000	10	5.09E+02	9.97E+03	3.50E+03	2.26E+03	6.04E-91	4.77E-64	3.78E-02	2.03E-01	8.31E-144	9.80E-100	**3.27E-101**	**1.76E-100**
	2,000	20	1.08E+04	1.32E+05	4.00E+04	2.82E+04	1.43E-105	1.38E+02	4.62E+00	2.47E+01	2.76E-152	1.17E-91	**3.90E-93**	**2.10E-92**
	3,000	30	2.66E+04	8.84E+05	2.87E+05	2.36E+05	1.15E-101	7.44E+01	2.48E+00	1.34E+01	1.28E-159	1.68E-87	**5.74E-89**	**3.01E-88**
	5,000	50	3.05E+05	8.47E+06	1.54E+06	1.67E+06	3.78E-109	1.12E+02	3.74E+00	2.01E+01	4.21E-176	1.93E-09	**6.42E-11**	**3.46E-10**
F10	1,000	10	1.84E-15	9.64E+05	1.74E+05	2.59E+05	2.71E-249	1.14E-147	3.81E-149	2.05E-148	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	2,000	20	3.66E+05	9.41E+07	1.98E+07	1.68E+07	0.00E+00	7.91E-153	2.64E-154	1.42E-153	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	3,000	30	1.43E+07	6.74E+08	2.16E+08	1.55E+08	0.00E+00	6.16E-132	2.05E-133	1.11E-132	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
	5,000	50	2.93E+08	3.57E+09	1.47E+09	6.73E+08	0.00E+00	6.76E-39	2.25E-40	1.21E-39	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**

Table 6 Comprehensive results from F11 to F19 for TW-BA, BA and PSO

F#	*IT*	*DIM*	*BA*				*PSO*				*TW-BA*			
			Best	*Worst*	*Mean*	*Std. dev.*	*Best*	*Worst*	*Mean*	*Std. dev.*	*Best*	*Worst*	*Mean*	*Std. dev.*
F11	1,000	10	9.89E-07	5.35E-06	3.23E-06	1.14E-06	2.25E-129	6.50E-80	2.17E-81	1.17E-80	3.73E-192	2.39E-173	**1.01E-174**	**0.00E+00**
	2,000	20	3.21E-06	1.51E-05	7.55E-06	2.68E-06	3.75E-241	5.78E-89	1.93E-90	1.04E-89	4.54E-268	3.70E-247	**1.24E-248**	**0.00E+00**
	3,000	30	1.01E-05	2.94E-05	1.71E-05	5.89E-06	0.00E+00	1.25E-64	4.17E-66	2.25E-65	0.00E+00	3.52E-302	**1.24E-303**	**0.00E+00**
	5,000	50	6.19E-05	1.27E-04	8.85E-05	1.75E-05	3.36E-243	5.51E+03	1.84E+02	9.88E+02	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F12	1,000	10	1.09E-04	1.10E-03	5.04E-04	2.39E-04	2.44E-83	1.05E-64	3.49E-66	1.88E-65	2.29E-136	2.49E-104	**8.30E-106**	**4.47E-105**
	2,000	20	2.86E-04	1.86E-03	8.59E-04	3.20E-04	1.03E-153	3.03E-107	1.01E-108	5.44E-108	6.56E-192	.37E-147	**2.46E-148**	**1.32E-147**
	3,000	30	5.51E-04	2.00E-03	1.11E-03	3.50E-04	1.53E-228	9.82E-123	3.27E-124	1.76E-123	3.32E-237	1.77E-182	**5.89E-184**	**0.00E+00**
	5,000	50	6.28E-04	1.89E-03	1.16E-03	3.35E-04	0.00E+00	1.13E-142	3.77E-144	2.03E-143	1.62E-268	1.27E-213	**4.24E-215**	**0.00E+00**
F13	1,000	10	3.24E-03	7.46E-02	2.84E-02	1.62E-02	4.01E-11	1.63E-03	6.75E-05	2.94E-04	7.30E-07	7.16E-04	**1.34E-04**	**1.61E-04**
	2,000	20	2.86E-02	1.69E-01	7.70E-02	3.50E-02	1.01E-10	3.41E-03	1.28E-04	6.12E-04	8.42E-07	3.08E-04	**8.15E-05**	**6.59E-05**
	3,000	30	5.14E-02	2.62E-01	1.70E-01	4.55E-02	4.24E-14	1.46E-02	5.00E-04	2.62E-03	2.08E-07	1.94E-04	**6.40E-05**	**5.52E-05**
	5,000	50	2.02E-01	7.53E-01	4.33E-01	1.24E-01	2.47E-14	2.15E+01	7.21E-01	3.86E+00	5.40E-08	6.34E-04	**8.51E-05**	**1.45E-04**
F14	1,000	10	4.30E+00	1.11E+01	7.21E+00	1.53E+00	2.90E-59	2.00E-01	**9.99E-03**	3.96E-02	1.56E-82	9.99E-02	9.32E-02	**2.49E-02**
	2,000	20	9.50E+00	1.63E+01	1.28E+01	1.84E+00	2.35E-107	3.00E-01	**2.00E-02**	6.53E-02	9.99E-02	9.99E-02	9.99E-02	**5.55E-17**
	3,000	30	9.80E+00	2.08E+01	1.61E+01	2.55E+00	1.56E-135	8.00E-01	**3.67E-02**	1.47E-01	9.99E-02	9.99E-02	9.99E-02	**5.55E-17**
	5,000	50	1.63E+01	2.71E+01	2.20E+01	2.50E+00	2.22E-126	1.20E+00	9.99E-02	2.55E-01	9.99E-02	9.99E-02	**9.99E-02**	**5.55E-17**
F15	1,000	10	2.11E+02	2.27E+06	2.54E+05	4.94E+05	2.50E-65	1.57E-38	5.23E-40	2.82E-39	3.71E-94	1.47E-85	**7.14E-87**	**2.71E-86**
	2,000	20	5.43E+06	1.10E+22	6.30E+20	2.20E+21	2.69E-124	1.22E-16	4.06E-18	2.19E-17	1.00E-133	4.29E-126	**4.60E-127**	**1.12E-126**
	3,000	30	1.44E+03	3.46E+36	1.51E+35	6.42E+35	6.85E-129	8.94E-04	2.98E-05	1.61E-04	2.39E-163	6.85E-150	**2.66E-151**	**1.23E-150**
	5,000	50	2.44E+03	3.98E+64	1.44E+63	7.15E+63	6.79E-67	1.26E+00	4.23E-02	2.26E-01	6.49E-202	4.78E-184	**1.60E-185**	**0.00E+00**
F16	1,000	10	2.00E-07	9.62E-07	6.65E-07	1.83E-07	5.55E-129	4.14E-78	1.38E-79	7.44E-79	1.42E-191	8.95E-172	**3.15E-173**	**0.00E+00**
	2,000	20	8.07E-07	2.16E-06	1.45E-06	4.14E-07	2.49E-243	8.45E-89	2.82E-90	1.52E-89	3.00E-276	4.47E-249	**1.76E-250**	**0.00E+00**
	3,000	30	1.93E-06	7.65E-06	3.46E-06	1.35E-06	0.00E+00	2.18E-29	7.25E-31	3.91E-30	0.00E+00	3.05E-296	**1.02E-297**	**0.00E+00**
	5,000	50	9.19E-06	1.10E+02	9.84E+00	2.27E+01	1.11E-234	4.85E-03	1.62E-04	8.71E-04	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**
F17	1,000	10	1.46E-07	5.68E-07	3.11E-07	1.02E-07	2.18E-117	2.91E-50	9.69E-52	5.22E-51	7.99E-181	3.05E-161	**1.02E-162**	**5.44E-162**
	2,000	20	1.79E-07	1.84E+00	6.15E-02	3.31E-01	4.06E-182	1.21E-26	4.05E-28	2.18E-27	8.42E-237	1.34E-211	**4.47E-213**	**0.00E+00**
	3,000	30	1.02E+00	6.23E+03	3.31E+02	1.21E+03	1.07E-209	2.73E-14	9.11E-16	4.90E-15	4.73E-275	4.02E-234	**2.26E-235**	**0.00E+00**
	5,000	50	1.79E+02	1.10E+06	7.82E+04	2.70E+05	9.25E-192	1.12E-06	3.74E-08	2.02E-07	1.08E-276	2.71E-241	**9.04E-243**	**0.00E+00**
F18	1,000	10	4.60E+03	7.34E+04	3.04E+04	1.84E+04	3.20E-31	3.20E-31	**3.20E-31**	**1.31E-46**	6.77E-01	9.63E-01	7.65E-01	7.52E-02
	2,000	20	4.15E+04	4.83E+05	2.67E+05	1.18E+05	6.67E-01	6.67E-01	6.67E-01	0.00E+00	7.17E-01	9.59E-01	**3.11E-01**	**6.12E-02**
	3,000	30	2.96E+05	1.87E+06	8.43E+05	3.43E+05	6.67E-01	6.67E-01	6.79E-01	6.72E-02	7.56E-01	9.95E-01	**2.16E-01**	**6.44E-02**
	5,000	50	1.29E+06	4.95E+06	3.25E+06	8.40E+05	6.67E-01	1.22E+01	1.05E+00	2.07E+00	8.68E-01	1.00E+00	**9.64E-01**	**3.02E-02**
F19	1,000	10	4.22E+03	4.43E+04	2.02E+04	1.07E+04	4.10E-126	3.23E-78	**1.08E-79**	**5.80E-79**	-2.13E-16	0.00E+00	-4.31E-12	2.32E-11
	2,000	20	4.77E+04	1.54E+05	9.23E+04	3.06E+04	3.28E-238	3.54E-88	**1.18E-89**	**6.36E-89**	-7.68E-12	0.00E+00	-2.79E-12	1.32E-11
	3,000	30	6.01E+04	2.68E+05	1.64E+05	4.75E+04	0.00E+00	3.55E-37	**1.18E-38**	**6.38E-38**	-2.78E-19	1.40E-307	-5.16E-14	2.78E-13
	5,000	50	1.96E+05	5.92E+05	3.19E+05	9.74E+04	1.03E-259	7.01E-06	2.34E-07	1.26E-06	-3.86E-12	0.00E+00	**-1.52E-12**	**7.50E-12**

6.2 Impact of nature of dimension of problems

The core objective is to find the superiority of results depending upon the functions' dimension that are to be optimised. In experiments, four dimensions for benchmark functions D = 10, D = 20, D = 30 and D = 50 were used. Simulation results were presented in Table 5. With the increase of dimension (up to 50 dimensions), the performance of TW-BA in all 19 test functions is better than PSO and standard BA algorithms that can be seen in Table 6. With the increase of dimensions, the advantages of TW-BA become more and more obvious. The TW-BA is very suitable to solve high-dimensional numerical optimisation problems. In Figures 1 to 12, the convergence of PSO, traditional BA and proposed algorithm TW-BA have been illustrated. These figures verified that proposed TW-BA can converge most appropriately in the functions of both uni-modal and multi-modal as compared to traditional BA and PSO, meanwhile, disclosed that TW-BA is efficient than PSO and traditional BA within these functions.

From these simulation results, it was identified that functions are having larger dimensions found toughest to optimise, which can be seen from Table 4, where dimension size is D = 30. Table 4 gives the results of comparison with these seven algorithms in 30 dimensions. From Table 4, we can see that TW-BA is superior to standard BA and seven test functions for PSO.

6.3 Comparative study

In order to validate the efficiency of proposed algorithm, the introduced algorithms, traditional BA and PSO are examined through benchmark test functions with various dimensions as displayed in Tables 2 and 3. In Table 5, the estimated values of best, worst, mean and standard deviation are illustrated. As can see in Table 5, the performance of proposed TW-BA technique increases as the dimensions of the function expands. Thus, it can also be seen that the performance of traditional BA and PSO decreases as the dimensions expands. It has been observed that proposed TW-BA outperforms than traditional BA and PSO among all the standard test functions with various dimensions other than f14 and f18 functions with low dimension (d = 10, 20, 30) and f19 function only on d = 10.

The second part of experiment compares the performance of the TW-BA algorithm that used to verify the diversity and convergence with other algorithms. In order to compare these algorithms in a better way, we analysed the performance comparison for dimension size d = 30. From Table 4, we can see that TW-BA is superior to BA, dBA, CS, HS, DE and GA in all nine test functions and seven test functions are superior to PSO.

Figure 1 Convergence curve on f_1 (see online version for colours)

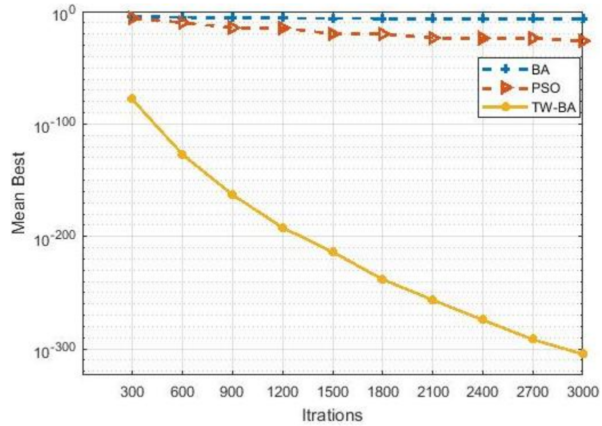


Figure 2 Convergence curve on f_2 (see online version for colours)

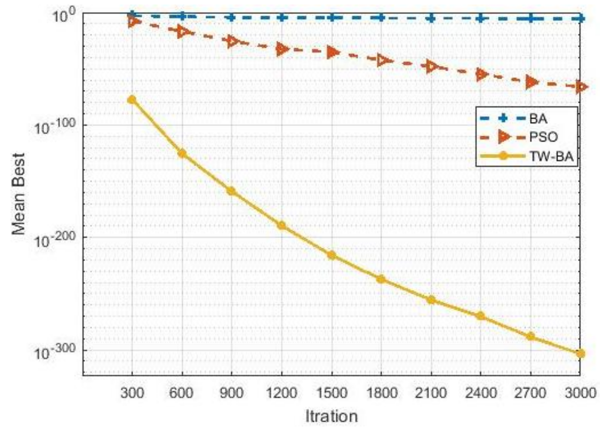


Figure 3 Convergence curve on f_3 (see online version for colours)

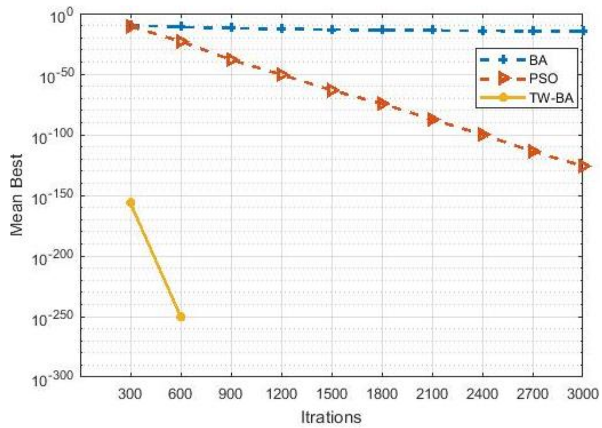


Figure 4 Convergence curve on f_4 (see online version for colours)

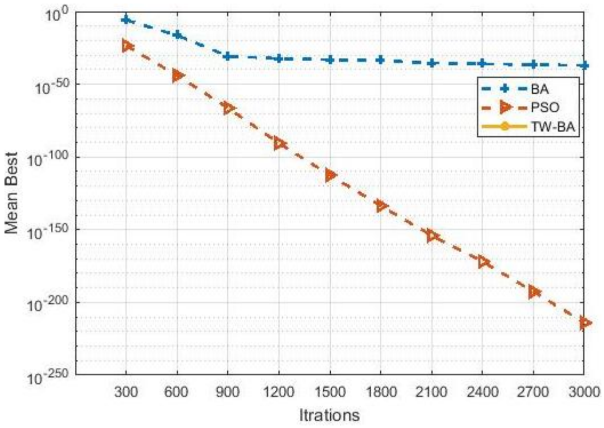


Figure 5 Convergence curve on f_5 (see online version for colours)

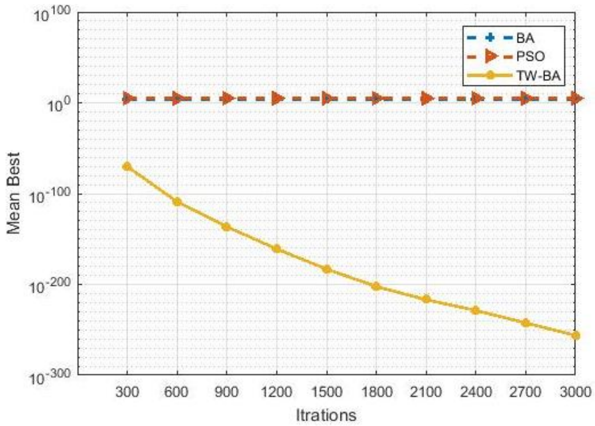


Figure 6 Convergence curve on f_6 (see online version for colours)

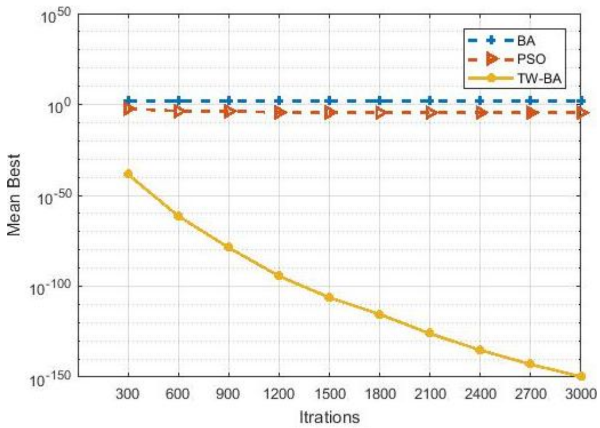


Figure 7 Convergence curve on f_7 (see online version for colours)

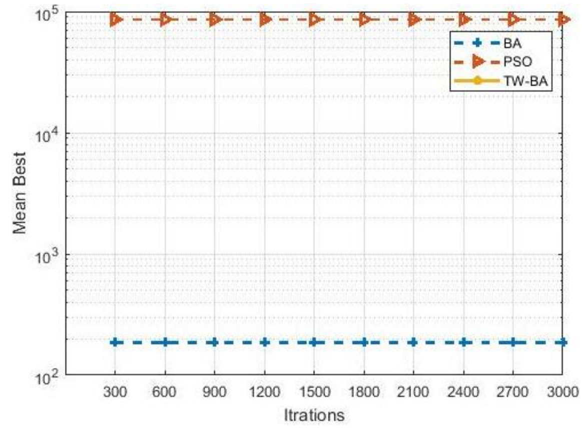


Figure 8 Convergence curve on f_8 (see online version for colours)

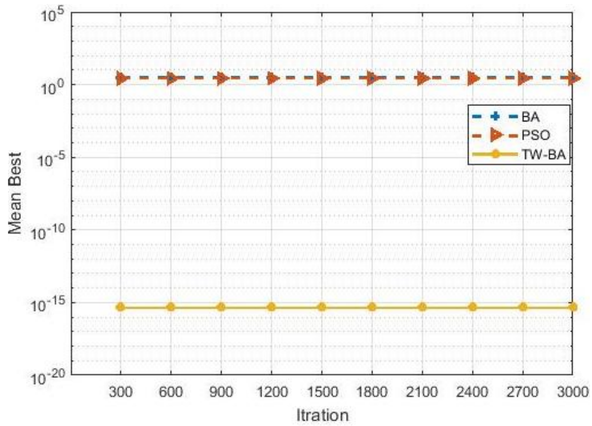


Figure 9 Convergence curve on f_9 (see online version for colours)

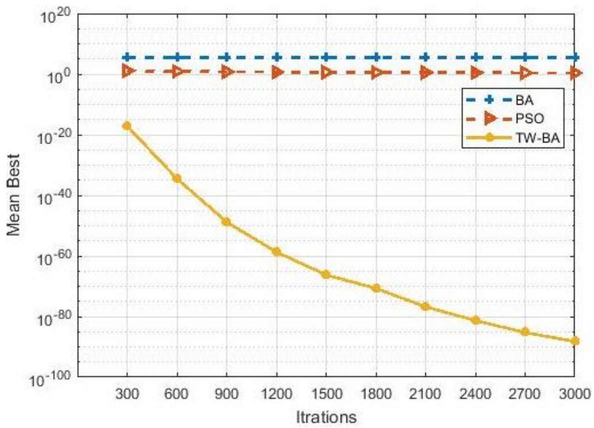


Figure 10 Convergence curve on f_{10} (see online version for colours)

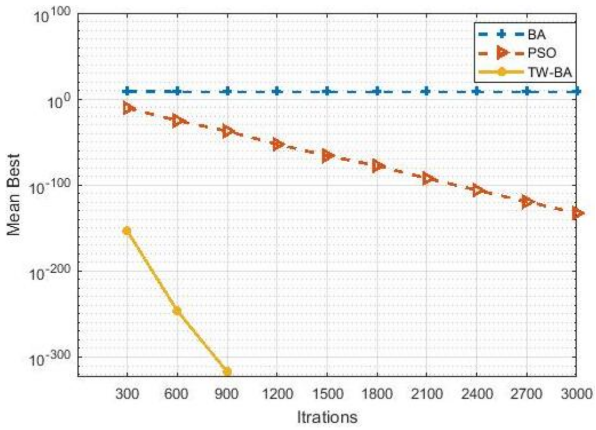


Figure 11 Convergence curve on f_{11} (see online version for colours)

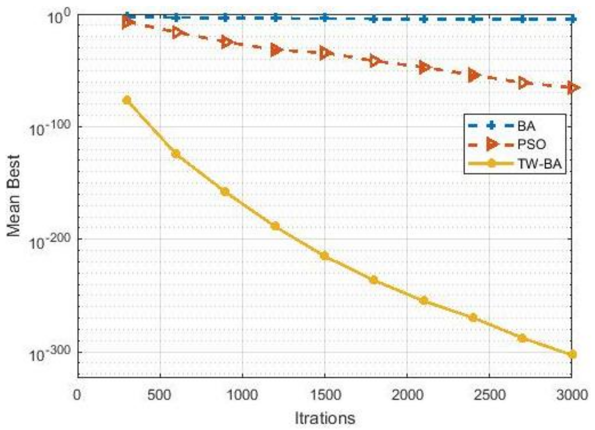
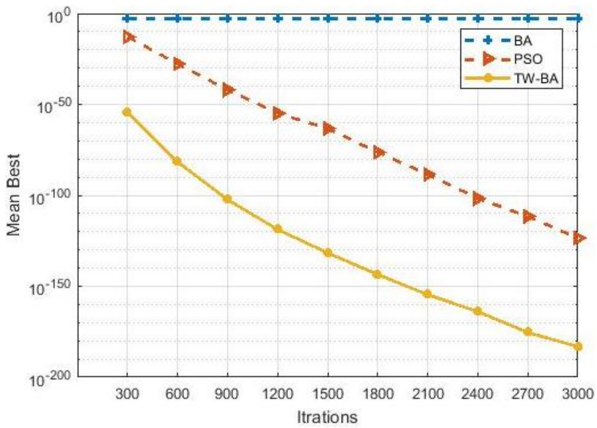


Figure 12 Convergence curve on f_{12} (see online version for colours)



7 Conclusions

In this paper, to enhance the local search capability of standard BA, a new variant TW-BA is introduced to solve the high dimension problems. Standard BA has inefficiency of exploitation in solving the high dimension multi-modal function optimisation problems. In this research, BA is enhanced with incorporating the torus walk instead of random walk for local searching and also integrating the chaotic inertia weight to balance the major two components of exploration and exploitation. From the experimental simulation results, it is depicted that TW-BA has better convergence rate and consistent as compared to standard BA, dBA, PSO, CS, HS, DE and GA. In the future, performance comparison of the proposed approach can be examined with other meta-heuristic algorithms. Furthermore, hybridisation of proposed technique with other approaches may also be fruitful.

References

Abdel-Raouf, O., Abdel-Baset, M. and El-Henawy, I. (2014) 'An improved chaotic bat algorithm for solving integer programming problems', *International Journal of Modern Education and Computer Science*, Vol. 6, No. 8, p.18.

Bangyal, W.H., Ahmad, J., Rauf, H.T. and Pervaiz, S. (2018a) 'An improved bat algorithm based on novel initialization technique for global optimization problem', *International Journal of Advanced Computer Science and Applications*, Vol. 9, No. 7, pp.158–166.

Bangyal, W.H., Ahmad, J., Rauf, H.T. and Pervaiz, S. (2018b) 'An overview of mutation strategies in bat algorithm', *International Journal of Advanced Computer Science and Applications*, Vol. 9, No. 8, pp.523–534.

Beni, G. and Wang, J. (1993) 'Swarm intelligence in cellular robotic systems', *Robots and Biological Systems: Towards a New Bionics?*, pp.703–712, Springer, Berlin, Heidelberg.

Cai, X., Gao, X-Z. and Xue, Y. (2016) 'Improved bat algorithm with optimal forage strategy and random disturbance strategy', *International Journal of Bio-Inspired Computation*, Vol. 8, No. 4, pp.205–214.

Cai, X., Wang, L., Kang, Q. and Wu, Q. (2014) 'Bat algorithm with Gaussian walk', *International Journal of Bio-Inspired Computation*, Vol. 6, No. 3, pp.166–174.

Chakri, A., Khelif, R., Benouaret, M. and Yang, X-S. (2017) 'New directional bat algorithm for continuous optimization problems', *Expert Systems with Applications*, Vol. 69, No. 1, pp.159–175.

Cui, Z., Li, F. and Kang, Q. (2015) 'Bat algorithm with inertia weight', *2015 Chinese Automation Congress (CAC)*, pp.792–796, IEEE.

Cui, Z., Li, F. and Zhang, W. (2018a) 'Bat algorithm with principal component analysis', *International Journal of Machine Learning and Cybernetics*, Vol. 10, No. 3, pp.1–20.

Cui, Z., Xue, F., Cai, X., Cao, Y., Wang, G-G. and Chen, J. (2018b) 'Detection of malicious code variants based on deep learning', *IEEE Transactions on Industrial Informatics*, Vol. 14, No. 7, pp.3187–3196.

Das, S. and Suganthan, P.N. (2011) 'Differential evolution: a survey of the state-of-the-art', *IEEE transactions on evolutionary computation*, Vol. 15, No. 1, pp.4–31.

Davis, L. (1991) *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York.

Eberhart, R.C. and Shi, Y. (1998) 'Comparison between genetic algorithms and particle swarm optimization', *International conference on evolutionary programming*, pp.611–616, Springer.

Feng, Y., Teng, G-F., Wang, A-X. and Yao, Y-M. (2007) 'Chaotic inertia weight in particle swarm optimization', *Second International Conference on Innovative Computing, Information and Control, 2007. ICICIC'07*, pp.475–475, IEEE.

Fister, I., Fong, S. and Brest, J. (2014) 'A novel hybrid self-adaptive bat algorithm', *The Scientific World Journal*, Vol. 2014, pp.709–738.

Fister Jr., I., Fister, D. and Yang, X-S. (2013) *A Hybrid Bat Algorithm*, arXiv preprint arXiv:1303.6310.

Gandomi, A.H. and Yang, X-S. (2014) 'Chaotic bat algorithm', *Journal of Computational Science*, Vol. 5, No. 2, pp.224–232.

Gandomi, A.H., Yang, X-S., Talatahari, S. and Alavi, A.H. (2013) 'Metaheuristic algorithms in modeling and optimization', *Metaheuristic Applications in Structures and Infrastructures*, pp.1–24, Elsevier.

Geem, Z.W., Kim, J.H. and Loganathan, G.V. (2001) 'A new heuristic optimization algorithm: harmony search', *Simulation*, Vol. 76, No. 2, pp.60–68.

Kabir, M.W.U., Sakib, N., Chowdhury, S.M.R. and Alam, M.S. (2014) 'A novel adaptive bat algorithm to control explorations and exploitations for continuous optimization problems', *International Journal of Computer Applications*, Vol. 94, No. 13, pp.5–20.

Kennedy, J. and Eberhart, R. (1995) 'Particle swarm optimization', *Proceedings, IEEE International Conference on Neural Networks*, Vol. 4, pp.1942–1948.

Kora, P. and Kalva, S.R. (2015) 'Improved bat algorithm for the detection of myocardial infarction', *SpringerPlus*, Vol. 4, No. 1, p.666.

Li, L. and Zhou, Y. (2014) 'A novel complex-valued bat algorithm', *Neural Computing and Applications*, Vol. 25, No. 6, pp.1369–1381.

Meng, X-B., Gao, X., Liu, Y. and Zhang, H. (2015) 'A novel bat algorithm with habitat selection and Doppler effect in echoes for optimization', *Expert Systems with Applications*, Vol. 42, Nos. 17–18, pp.6350–6364.

Mirjalili, S., Mirjalili, S.M. and Yang, X-S. (2014) 'Binary bat algorithm', *Neural Computing and Applications*, Vol. 25, Nos. 3–4, pp.663–681.

Paiva, F.A., Silva, C.R., Leite, I.V., Marcone, M.H. and Costa, J.A. (2017) 'Modified bat algorithm with cauchy mutation and elite opposition-based learning', *2017 IEEE Latin American Conference on Computational Intelligence (LA-CCI)*, pp.1–6, IEEE.

Runarsson, T.P. and Yao, X. (2000) 'Stochastic ranking for constrained evolutionary optimization', *IEEE Transactions on Evolutionary Computation*, Vol. 4, No. 3, pp.284–294.

Saha, S.K., Kar, R., Mandal, D., Ghoshal, S.P. and Mukherjee, V. (2013) 'A new design method using opposition-based bat algorithm for IIR system identification problem', *International Journal of Bio-Inspired Computation*, Vol. 5, No. 2, pp.99–132.

Wang, G. and Guo, L. (2013) 'A novel hybrid bat algorithm with harmony search for global numerical optimization', *Journal of Applied Mathematics*, Vol. 2013, pp.1–21.

Wang, Y., Wang, P., Zhang, J., Cui, Z., Cai, X., Zhang, W. and Chen, J. (2019) 'A novel bat algorithm with multiple strategies coupling for numerical optimization', *Mathematics*, Vol. 7, No. 2, p.135.

Yang, N-C. and Le, M-D. (2015) 'Multi-objective bat algorithm with time-varying inertia weights for optimal design of passive power filters set', *IET Generation, Transmission & Distribution*, Vol. 9, No. 7, pp.644–654.

Yang, X-S. (2010) 'A new metaheuristic bat-inspired algorithm', *Nature Inspired Cooperative Strategies for Optimization (NICSO 2010)*, pp.65–74, Springer.

Yang, X-S. (2011) 'Bat algorithm for multi-objective optimisation', *International Journal of Bio-Inspired Computation*, Vol. 3, No. 5, pp.267–274.

Yang, X-S. and Deb, S. (2009) 'Cuckoo search via Lévy flights', *World Congress on Nature & Biologically Inspired Computing, 2009. NaBIC 2009*, pp.210–214, IEEE.

Yilmaz, S. and Kucuksille, E.U. (2013) 'Improved bat algorithm (IBA) on continuous optimization problems', *Lecture Notes on Software Engineering*, Vol. 1, No. 3, p.279.

Yılmaz, S., Kucuksille, E.U. and Cengiz, Y. (2014) 'Modified bat algorithm', *Elektronika ir Elektrotechnika*, Vol. 20, No. 2, pp.71–78.

Zhou, Y., Xie, J., Li, L. and Ma, M. (2014) 'Cloud model bat algorithm', *The Scientific World Journal*, Vol. 2014, pp.1–11.