
Research on steelmaking-continuous casting production scheduling problem with uncertain processing time based on Lagrangian relaxation framework

Liangliang Sun*, Tianyi Lu, Zhi Li, Ye Li,
Yaqian Yu and Jinyu Liu

School of Information and Control Engineering,
Shenyang Jianzhu University,
Shenyang, China

Email: swinburnsun@163.com

Email: 963387017@qq.com

Email: liyeburn@126.com

Email: yuyaqian_edu@163.com

Email: 1624611774@qq.com

Email: 1094855350@qq.com

*Corresponding author

Abstract: The uncertainty of processing time directly affects the rhythm of steelmaking-continuous casting production, disturbs the dynamic balance between logistics and time, and reduces the flexible matching of product structure and production capacity. How to formulate efficient steelmaking-continuous casting production scheduling optimisation plan with uncertain processing time, which is the key to improve the large steel plant equipment production efficiency, shorten the waiting time of the process, reduce the material consumption and energy consumption. In this paper, aiming at the analysis and description of the processing time uncertainty in the steelmaking-continuous casting production scheduling, an effective Markov chain transfer matrix is established to accurately simulate the uncertain processing time probability, aiming at the inefficiency of traditional Lagrange relaxation algorithm in solving steelmaking-continuous casting production scheduling due to the exact solution of each iteration, an efficiency improved Lagrangian relaxation algorithm is proposed based on gradient direction controllable iterative optimisation strategy without predicting the optimal value. Finally, the method proposed in this paper is verified based on the background of the actual steelmaking and continuous casting schedule in domestic steelmaking plant. The optimisation results guarantee the solution quality and speed of the steelmaking-continuous casting production scheduling optimisation problem with uncertain processing time.

Keywords: steelmaking-continuous casting; scheduling; uncertain processing time; Markov chain; Lagrange relaxation.

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Biographical notes: Liangliang Sun received his PhD in Control Theory and Control Engineering from the Northeastern University, China, in 2015. He is currently an Associate Professor of Information and Control Engineering at the Shenyang Jianzhu University. His current research interests include intelligent manufacturing systems, planning, scheduling, and coordination of design, manufacturing, and service activities intelligence field.

Tianyi Lu received his BS degree in Information and Control Engineering from Shenyang Jianzhu University. He is currently pursuing his MS degree in Control Theory and Control Engineering at Shenyang Jianzhu University.

Zhi Li received his BS in Information and Control Engineering from the Shenyang Jianzhu University. He is currently pursuing his MS in Control Theory and Control Engineering at the Shenyang Jianzhu University.

Ye Li received his BS in Information and Control Engineering from the Shenyang Jianzhu University. He is currently pursuing his MS in Control Theory and Control Engineering at the Shenyang Jianzhu University.

Yaqian Yu received his BS in Information and Control Engineering from the Shenyang Jianzhu University. He is currently pursuing his MS in Control Theory and Control Engineering at the Shenyang Jianzhu University.

Jinyu Liu received his BS in Information and Control Engineering from the Shenyang Jianzhu University. He is currently pursuing his MS in Control Theory and Control Engineering at the Shenyang Jianzhu University.

1 Introduction

In the context of the computerisation strategic deployment of the steel industry in the new era, steel companies must accurately, quickly and sensitively understand customer needs, reduce production costs and shorten production cycles. These problems must be solved by developing and building efficient manufacturing systems. Through the understanding of large steelmaking plants and long-term research on steel production, the steelmaking process can be divided into three parts, namely ironmaking zone, steelmaking zone and rolling zone, of which steelmaking zone is also known as steelmaking-refining-continuous casting area, this area is the bottleneck of the entire steel production system. Formulating scientific scheduling methods can improve production efficiency, reduce process waiting time, increase equipment utilisation and increase yield (Sun et al., 2018). Steel production systems involve a variety of uncertainties, and dispatchers need to efficiently deal with the effects of changes in workpiece specifications, order quantities and delivery dates. At the same time, it is necessary to take account of various uncertainties caused by time fluctuations, product problems, metallurgical processes, order adjustments, equipment failures, and it is difficult to obtain an efficient and feasible solution in a short period of time (Bertsekas, 1999). The uncertainty of processing time in steelmaking-continuous casting process is one of the important uncertainty factors in the disturbed environment (Wolpert and Macready, 1997). With the development of steel market demand in the direction of multi-variety, low-volume and on-time delivery, the traditional steelmaking-continuous casting

production scheduling method cannot guarantee the dynamic balance of logistics and time between large-scale steel production processes due to the uncertainty of processing time. Flexible matching of product structure and production capacity between production and operation control levels, reducing the executability of existing solutions (Held and Karp, 1970). Therefore, it is necessary to formulate an effective steelmaking-continuous casting production scheduling scheme in an uncertain environment. The traditional probability description method is difficult to overcome for the accuracy of the model generated by the correlation between different processes under the uncertain processing time. And uncertain processing time steelmaking-continuous casting production scheduling optimisation problem is in the double dimension of time and space, it is necessary to comprehensively consider the steelmaking process in the steelmaking-continuous casting production scheduling, the production sequence and the machine capability coupling constraints between the refining process and the continuous casting process. Analysis of the associated influence of uncertain processing time on steelmaking-continuous casting production scheduling optimisation problems will cause the difficulty of model solving to increase exponentially with the increase of the number of production line equipment and the increase of orders. The traditional probability description method can significantly reduce the calculation scale of the optimisation problem of processing time uncertainty, due to the uncertainty of processing time of the production scheduling optimisation problem, the processing time correlation effect between the various processes leads to the diversity of the model decomposition and simplification methods, it is difficult to solve the model-based structure and take into account the characteristics of the algorithm. From the perspective of model system analysis, the uncertain mathematical description optimisation method of the existing design is difficult to ensure the accuracy of the model generated by the correlation between processing time and different processes, and it is difficult to reduce the difficulty of solving the optimisation problem of steelmaking-continuous casting production scheduling optimisation. However, due to the high-dimensional, discrete-continuous complex characteristics of large-scale steelmaking-continuous casting production scheduling optimisation problems, the Lagrangian relaxation method has not been overcome due to strict convergence conditions, which appears in the process of finding the optimal Lagrangian multiplier. The phenomenon of boundary oscillating causes the convergence to the optimal Lagrange multiplier iteration time, which leads to slow convergence, and the needs to accurately optimise the relaxation function for each gradient iteration will also lead to low efficiency. It is difficult to obtain an approximate optimised feasible solution that meets the actual production requirements in a short time.

2 Literature review

Steelmaking and continuous casting scheduling problem with uncertain processing time is non-deterministic polynomial (NP) problem. Uncertain parameter optimisation method (Imen et al., 2020; Roshni et al., 2018) and optimisation solution method (Abderrezek et al., 2019) are the core research contents of this problem.

Uncertain parameter optimisation method in steelmaking and continuous casting scheduling optimisation mainly include fuzzy mathematical methods, probability distribution methods and interval number methods. The fuzzy mathematics method abstracts the uncertain parameters into a fuzzy set and establishes a membership function

to deal with the uncertain relationship. Xie et al. (2005) used triangular fuzzy numbers to research the uncertain processing time and used different algorithms to solve and compare, and verify that the triangular fuzzy numbers are more efficient for solving uncertain processing time (Marjan and Milad, 2020) researched the flow shop scheduling problem and developed it combined with hybrid genetic algorithm to verify the effectiveness of the algorithm. Hanene et al. (2020) and Yang et al. (2019) proposed a fuzzy selection mechanism for workshop scheduling methods and applied genetic algorithm to predict uncertain processing time. Some uncertain variables have a distribution law. At this time, the probability distribution method can be used for description. This method has the advantage of simple and small amount of measurement (Ma et al., 2019). In order to describe the uncertain variables more accurately, a large amount of data is often needed for statistics and the size of the data samples directly affects the relationship between the uncertain variables and the accuracy. The probability distribution description method mentioned above requires a large amount of statistical analysis of actual data, consumes a lot of energy, and requires a large-scale of actual data. So as compared the interval number method does not need to count the distribution of all data, just get the data range, which is the upper and lower bounds can simulate the uncertainty of the parameters

The optimisation of steelmaking-continuous casting process is a large-scale mixed flow shop scheduling problem. This kind of problem gives the mathematical model of the scheduling performance index of the production process by establishing the scheduling model of the production process. Under the relevant constraints of the process and resources that satisfy the process scheduling problem, the appropriate optimisation scheduling method is adopted to make the scheduling process multiple performance metrics are optimised or approximated (Pang et al., 2017). The optimal solution methods for production scheduling optimisation under uncertain environment are operational research method, heuristic method, neighbourhood search method and artificial intelligence method (Chang et al., 2019; Salma et al., 2019). However, the existing methods are directly applied to steelmaking-continuous casting production scheduling in large-scale uncertain environments. There are problems in process description and long solution time. It is necessary to further propose a closer to the actual solution method. As compared with operations research methods, heuristic methods, neighbourhood search methods and artificial intelligence methods, the Lagrangian relaxation method has great advantages in solving production scheduling problems. The Lagrangian relaxation method can relax the resource constraints into the objective function and decompose into independent sub-problems, and obtain feasible solutions to the scheduling problem by solving all simple sub-problems. Chen et al. (2015) studied the Lagrangian relaxation algorithm for solving the mixed-flow shop problem with limited waiting time. The relaxation strategy is machine capacity constraint and the Lagrangian relaxation algorithm based on the workpiece decomposition strategy is developed. Sun (2015) abstracted the steelmaking-continuous casting scheduling problem into a flow shop scheduling problem with complex constraints and established a 0–1 mixed integer nonlinear model, using the dual relaxation function descent level control strategy to ensure that the algorithm converges to the dual relaxation problem optimal solution. However, some scholars have studied the method of processing order constrained relaxation strategy based on integer programming model and mixed integer programming model (Mao et al., 2014; Ding et al., 2014). It can be seen that the characteristics of the separable structure based on different scheduling problems, the different relaxation

models under different strategies will lead to the acquisition of the relaxation sub-problem and the difficulty of the subsequent optimisation process, which directly affects the solution quality and the solution speed of the final scheduling problem. Secondly, the Lagrangian relaxation algorithm uses a coordination strategy to find the optimal Lagrangian multiplier, which is to decompose the relaxation problem into some sub-problems (Tolouei et al., 2020; Nishi et al., 2010). The corresponding dual problem can be solved using the sub-gradient optimisation method. When the sub-gradient optimisation method searches for the optimal Lagrangian multiplier termination, the best feasible solution to the dual problem is chosen as the solution to the original problem. The dual theory guarantees that the solution to the Lagrangian relaxation problem can provide the lower limit of the original problem, while the best feasible solution provides the upper limit of the original problem. However, the research on the Lagrangian relaxation iterative optimisation algorithm mainly focuses on the estimation of the lower bound of the original problem and the design of the efficient iterative optimisation algorithm. The existing iterative optimisation method is difficult to guarantee the actual large-scale steelmaking-continuous casting production scheduling requirements due to low search efficiency. Nishi et al. proposed a Lagrangian relaxation method with cut generation to improve the Lagrangian boundary problem by relaxing machine capacity constraints (Ma et al., 2019).

Based on the previous research work, we can get the conclusions, the traditional probability description method describes the uncertainty of processing time in steelmaking-continuous casting production scheduling based on the statistical distribution obeyed by known uncertain factors. However, for the relationship existing between unknown two adjacent processes, the traditional probability description method is difficult to describe accurately. Based on the deterministic solution space with fixed processing time, the uncertainty of the processing time parameter increases the different solution spaces corresponding to different processing times, which will make the scheduling solution space become larger. The existing methods are difficult to guarantee the quality of the solution for the steelmaking-continuous casting production scheduling problem with uncertain processing time and it is difficult to obtain an approximate optimisation feasible solution that meets the actual production requirements in a short time.

3 Model description

3.1 Problem description and basic assumptions

The main part of the steel production studied in this paper is the steelmaking-continuous casting process. As the productivity of the process is usually smaller than the subsequent stage (hot rolling mill process), it is the bottleneck for steel production system (Jiang and Zheng, 2019). Therefore, the coordination between the production stages from steelmaking to continuous casting plays an important role in the efficiency of the workshop. The entire steelmaking-continuous casting production system has a general character of the mixed flow shop problem. Among them, the refining stage includes multiple processes.

Different charge of the same cast needs to meet the following technical restrictions (Liu et al., 1997):

- 1 The adjacent charge of the same continuous casting machine has the same or similar steel grade.
- 2 The slabs of different charge are the same size.
- 3 The width difference of the slabs on the same continuous casting machine is within a certain limit and the width of adjacent charge cannot exceed a certain maximum value.
- 4 The number of charges can be arranged for one cast, depending on the ladle wear degree.
- 5 The delivery time of different charge of the same cast shall be as close as possible.
- 6 For two consecutive charges that are processed on the same machine, the next charge can be processed only after the current charge is processed.

Problem hypothesis:

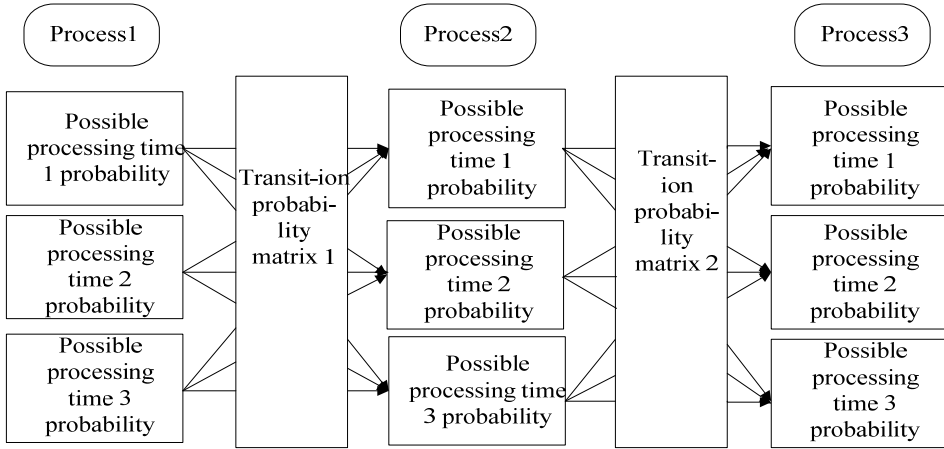
- 1 All charges follow the same route: at each stage, for each charge, each piece of machine can be machined, with the same properties for each machine.
- 2 A machine can only process one workpiece at a time.
- 3 A workpiece can only be processed by one machine at any one time.
- 4 The processing sequence of all charge on the continuous casting machine is completed in advance.
- 5 For two consecutive operations of the same charge, only when the current operation is completed can the next operation be started.
- 6 For two consecutive charge processed on the same machine, only when the current charge is finished can the next charge be processed.
- 7 Processing time does not depend on the charge properties.

3.2 Markov chain description processing time uncertainty

A Markov chain is a random model that describes a series of possible events, where the probability of each event depends on the state obtained in the previous event (Nishi et al., 2009). In this paper, the processing time uncertainty is formulated as a discrete Markov process. By using the historical data of the processing time of the adjacent processes, the transition probability matrix is built-up to describe the correlation of processing time for the same charge in different processes. The processing time of the current process could be got by the transition probability matrix with the processing time of the former process. Assume that the steelmaking-continuous casting process has three processes, each of which is numbered 1, 2 and 3. Specifically, as shown in Figure 1, is a parameter model of the processing time uncertainty for a charge. The first process has three processing times, which are 2, 3 and 4. The subsequent processes are described in turn, and the relationship between the two adjacent processes is described by a state transition matrix. For example, both process 1 and process 2 have three implementation modes, so the state transition

matrix 1 has a size of 3×3 , representing process 1 under the premise that a certain processing time is completed, the probability of turning to a certain processing time in the process 2 is performed. As it can be seen from Figure 1, each state transition matrix is of order 3×3 .

Figure 1 Uncertain processing time parameter model



The matrix consisting of all transition probabilities π_{yz} is a state transition matrix, and the processing time is discretised into three states. Both y and z represent possible processing times, as shown in the following formula:

$$\pi_{yz(3 \times 3)} = \begin{bmatrix} \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix} \quad (1)$$

The state transition matrix π_{yz} is related not only to the processing times y and z but also to the two adjacent processes. If the processing time of the initial process of a certain process and the state transition matrix between two adjacent processes can be obtained, the processing time of any process can be obtained, and the processing time of this process in all processes can be finally obtained.

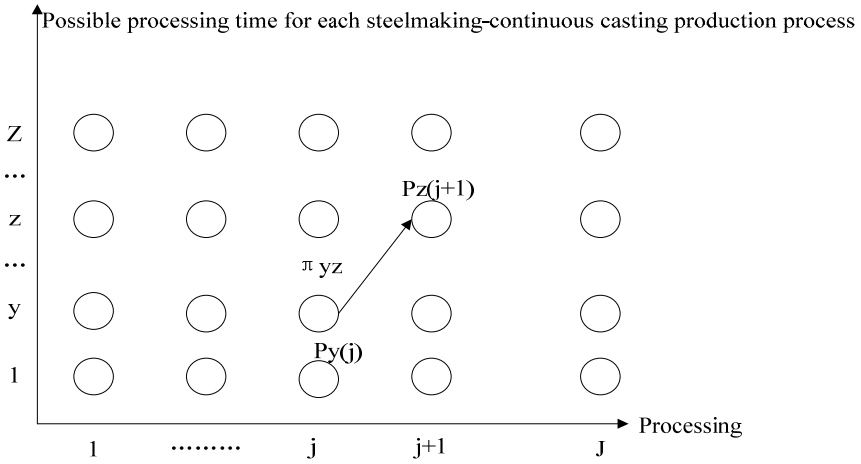
The core issue is how to determine the state transition matrix with uncertain processing times. The probability that the processing time of adjacent processes changes from y to z is established based on historical data.

$$\pi_{yz} = \frac{\text{Number of times the processing time changes from } y \text{ to } z}{\text{Number of occurrences of processing time } z}. \quad (2)$$

The possible processing time of each process is established according to the following formula, as shown in Figure 2.

$$P_z(j+1) = \sum_{z=2}^4 \pi_{yz} P_y(j). \quad (3)$$

Figure 2 Processing time transfer probability



3.3 Parameters and decision variables of steelmaking-continuous casting production scheduling problem with uncertain processing time

3.3.1 Parameter description of steelmaking-continuous casting scheduling problem with uncertain processing time

To facilitate the description, variables and parameters are defined as follows:

- i serial number, to contain all the charge, $i \in \Omega$, $|\Omega|$ for the total number of charges
- K total number of sprays
- k cast number, $k = 1, 2, 3, \dots, K$
- j process number, $1 < j < J$
- M_j number of equipment for the j^{th} process (integer), $M_j \geq 1$; Ω_k : an ordered collection of all the charge in the k^{th} cast, $|\Omega_k|$ means the number of charge in the k^{th} cast, and has $\Omega_{k_1} \cap \Omega_{k_2} = \emptyset, \forall k_1, k_2 = 1, 2, \dots, K, k_1 \neq k_2, \Omega_1 \cup \Omega_2 \cup \dots \cup \Omega_k = \Omega$
- $s(k)$ the last charge number of the k^{th} cast: $s(k) = s(k - 1) + |\Omega_k|, s(0) = 0, k = 1, 2, \dots, K, s(K) = |\Omega|$
- B_m order set of assigned casts on the m^{th} continuous casting machine, $|B_m|$ represents the number of the cast on the M^{th} continuous casting machine, $1 \leq m \leq M_s$
- $b(m)$ the last cast number on the m^{th} continuous casting machine, $b(m) = b(m - 1) + |B_m|$
- $b(0)$ $0, 1 \leq m \leq M_s, b(M_s) = K$
- P_{ij} operating time of charge i in the j^{th} process

$T_{j,j+1}$	the standard transport time of the charge between the j^{th} process and the $j^{\text{th}} + 1$ process
S_u	minimum interval time required for replacing crystalliser between adjacent cast
W_1, W_2	waiting time penalty
W_3, W_4	time to start casting penalty
T	constant large enough.

3.3.2 Decision variable description of steelmaking-continuous casting scheduling problem with uncertain processing time

Manual scheduling based on empirical methods can obtain good scheduling results, but steelmaking-continuous casting production scheduling optimisation can avoid human error and reduce unnecessary consumption. The purpose of our research on steelmaking-continuous casting scheduling is to derive decision variables and plan a schedule for optimising performance indicators.

$$\delta_{ijt} = \{0, 1\}, i \in \Omega, j = 1, 2, \dots, J, t = 1, 2, \dots, T \quad (4)$$

$$t_{i,j} = \{0, 1, 2, \dots, T\}, i \in \Omega, j = 1, 2, \dots, J. \quad (5)$$

δ_{ijt} if for 1 represents the T^{th} time node, charge i in the first j process by processing. If 0, represents charge i in the first j process has not been processed. $t_{i,j}$ on behalf of the charge i at the beginning of the first j process time.

3.4 Mathematical model for steelmaking-continuous casting scheduling problem with uncertain processing time

3.4.1 Constraints

- 1 The charge order constraint (Kaskavelis and Caramanis, 1998), which is the processing starting time of the charge i in the $j + 1^{\text{th}}$ process is later or equal to the processing starting time of the charge i in the j^{th} process plus the processing time of the charge i in the j^{th} process and the transportation time of the charge i from the j^{th} process to the $j + 1^{\text{th}}$ process

$$t_{i,j} + T_{j,j+1} + p_{i,j} \leq t_{i,j+1}, i \in \Omega, j = 1, 2, \dots, J - 1. \quad (6)$$

- 2 Sequence constraint of cast processing, which is the starting time difference between adjacent charge i and charge $i + 1$ of the same cast in the J^{th} process is greater than the processing time of $i + 1$ in the J^{th} process

$$t_{i,J} \leq t_{i+1,J} - p_{i+1,J}, i, i + 1 \in \Omega. \quad (7)$$

- 3 Machine capacity constraint (Luh et al., 1998)

$$\sum_{i \in \Omega} \delta_{ijt} \leq M_{jt}, i \in \Omega, j = 1, 2, \dots, J - 1, t = 1, 2, \dots, T. \quad (8)$$

Due to processing time uncertainty, what this article needs to look for is not a ‘static’ schedule. Instead, look for a ‘scheduling strategy’ that considers various achievable

situations. The ‘executable schedule’ is a scheduling strategy that satisfies any random parameter implementation of equations (6)–(8). Due to the uncertainty of processing time, it is difficult to deal with machine capability constraint (9) mathematically for all possible implementations of random events. Therefore, machine capacity constraints are required to be met in the expected sense.

$$E\left[\sum_{i \in \Omega} \delta_{ijt} \leq M_{jt}\right], i \in \Omega, j = 1, 2, \dots, J-1, t = 1, 2, \dots, T. \quad (9)$$

Constraint (9) is an approximate description under uncertain conditions.

3.4.2 Objective function

In steel production, it is strictly required that molten iron cannot produce temperature drop, so it is additionally required to be heated, resulting in an increase in production costs. The sum of all the charge waiting for the processing time penalty in the refining stage is set to Z_1 , and the sum of the processing time penalty between all the charge in the steelmaking and refining stages is set to Z_2 . The sum of the actual opening time of all the charge in the continuous casting stage and the penalty value of the pre-arranged time deviation is set to Z_3 . Therefore, in this paper, the performance index ‘the sum of the waiting times of all the charge in different processes’ and ‘the sum of the actual opening time and the pre-arranged time deviation of all the charge in the cast’ are taken as the optimisation targets, and the following mathematical functions are established. ($\min Z$) represents the sum of the minimum penalty of Z_1 , Z_2 and Z_3 .

$$\min Z = Z_1 + Z_2 + Z_3 \quad (10)$$

$$Z_1 = \sum_{m=1}^{M_j} \sum_{k=b(m-1)+1}^{|B_m|} \sum_{i=s(b(m-1)+k-1)+1}^{s(b(m-1)+k-1)+|B_{b(m-1)+k}|} W_1 (t_{i+1,J} - t_{i,J} - p_{i,J}) \quad (11)$$

$$Z_2 = E\left[\sum_{i=1}^{|\Omega|} \sum_{j=1}^{J-1} W_2 (t_{i,j+1} - p_{i,j} - T_{j,j+1} - t_{i,j})\right] \quad (12)$$

$$Z_3 = E\left[W_3 \sum_{i=1}^{|\Omega|} \sum_{j=1}^{J-1} \max(0, t_{i,J} - d_i) + W_4 \sum_{i=1}^{|\Omega|} \max(t_{i,J} - d_i, 0)\right]. \quad (13)$$

4 Research on improved Lagrangian relaxation optimisation algorithm

4.1 Coupling constrained relaxation strategy

In the previous section’s model, the cast processing order constraint (6) and the machine capacity constraint (8) couple different types of variables or different workpieces on the same machine. Therefore, we consider the relaxation constraints (6) and (8) to decompose the Lagrangian relaxation problem into two sub-problems of the furnace or two tractable sub-problems of each type of variable.

In this paper, the Lagrangian multiplier is used u_i and v_{jt} relax cast processing order constraint and machine capacity constraint. The objective of the relaxed optimisation problem is to minimise the Lagrangian function J_{LR} :

$$\begin{aligned} \min Z_{LR} \equiv & Z_1 + Z_2 + Z_3 + \sum_{k=1}^{M_s} \sum_{n=1}^{|B_k|} \sum_{i=s(b(k-1)+n-1)+\Omega_{b(k-1)+n-1}}^{s(b(k-1)+n-1)+\Omega_{b(k-1)+n-1}} u_i (t_{i+1,J} - t_{i,J} - p_{i+1,J}) \\ & + \sum_{t=1}^T \sum_{j=1}^J v_{jt} \left(\sum_{i \in \Omega} \delta_{ijt} - M_{jt} \right). \end{aligned} \quad (14)$$

The objective of the relaxed optimisation problem is to minimise the Lagrangian function J_{LR} , the corresponding Lagrange duality problem is:

$$Z_{LD} = \max_{u_i, v_{jt}} \left\{ \min_{t_{ij}, \delta_{ijt}} [z_1 + z_2 + z_3] \right\}. \quad (15)$$

4.2 Sub-problem optimisation

In the Lagrangian Z_{LR} , to collect all the items related to charge i , sub-problems that make up charge i :

$$\begin{aligned} \min Z_i = & E \left[\sum_{j=1}^{J-1} W_2 (t_{i,j+1} - p_{i,j} - T_{j,j+1} - t_{i,j}) \right] + E \left[W_3 \max(0, t_{i,J} - d_i) \right] \\ & + E \left[\sum_{t=1}^T \sum_{j=1}^J v_{jt} \delta_{ijt} \right] + E \left[W_4 \max(0, d_i - t_{i,J}) \right]. \end{aligned} \quad (16)$$

Among them, $\varphi(i)$ as shown below

$$\varphi(i) = (u_i - W_1) t_{i,J}, i = 1 \quad (17)$$

$$\varphi(i) = (u_i - W_1) t_{i,J} + (W_1 - u_{i-1})(t_{i,J} - p_{i,J}), i = 2, \dots, |\Omega| - 1 \quad (18)$$

$$\varphi(i) = (W_1 - u_{i-1})(t_{i,J} - p_{i,J}), i = |\Omega|. \quad (19)$$

The sub-problem is a multi-process random optimisation problem. In this paper, the maximum value of the sum of the process is set to J , and the sub-problem is solved by using the backward dynamic programming method. Taking the j^{th} stage of the charge i as an example, the decision variables t_{ij} and δ_{ijt} are used. Find the optimal decision t_{ij}^*, δ_{ijt}^* among the feasible values, and minimise the value function $J_{ij}(t_{ij}, \delta_{ijt})$ of the j^{th} process. In the j^{th} process, the optimal decision t_{ij}^*, δ_{ijt}^* and its optimal value function can be obtained by:

$$Z_{ij}^*(t_{ij}, \delta_{ijt}) = \min \left\{ S_{ij}(t_{ij}, \delta_{ijt}) + Z_{i,j+1}^*(t_{i,j+1}, \delta_{i,j+1,t}) \right\}. \quad (20)$$

Among them, $Z_{i,j+1}^*(t_{i,j+1}, \delta_{i,j+1,t})$ is the $j + 1^{\text{th}}$ process of the optimal value function, $S_{ij}(t_{ij}, \delta_{ijt})$ is in t_{ij} and δ_{ijt} , j^{th} process of current penalty value, can be calculated by the next formula:

$$\begin{aligned} S_{ij}(t_{ij}, \delta_{ijt}) = & W_2 (t_{i,j+1} - p_{i,j}) + \sum_{t=1}^T v_{jt} \delta_{ijt} + \left[W_3 \max(0, t_{i,J} - d_i) \right. \\ & \left. + W_4 \max(0, d_i - t_{i,J}) + \sum_{t=1}^T \sum_{j=1}^J v_{jt} \delta_{ijt} + \varphi \right] * \alpha_j. \end{aligned} \quad (21)$$

When $j = J$, $\alpha_j = 1$, otherwise $\alpha_j = 0$. The backward dynamic programming method starts from the J step (the continuous casting stage), and iteratively calculates the optimal decision and optimal penalty value function in each stage, up to the first step (the

steelmaking stage). The optimal penalty value function $J_{i1^*}(t_{i1^*}, \delta_{i1^*t})$, in the first step is the optimal penalty value for the sub-problem $J_{i1^*}(t_{i1^*}, \delta_{i1^*t})$ corresponds to the optimal decision $t_{ij^*}, \delta_{ij^*t}, j = 1, \dots, J$ is the optimal solution for the sub-problem.

The following example illustrates the process of stochastic dynamic programming to solve the sub-problem of uncertain processing time. For the convenience of description, the furnace i has three processing steps. Assume that parameters W_1, W_2, W_3, W_4 are all 1, and there is no transportation time between processes. The process times $p_{i,1}, p_{i,2}, p_{i,3}$ of the three processing steps are 0.5 with a probability of 0.5 and 0.5 with a probability of 2. Operation 1 may be performed on machine type 1 or it may be performed on machine type 2. Operation 2 needs to be performed on machine type 2 and operation 3 is performed on machine type 1.

The solution time of the sub-problem is determined by the time complexity of the dynamic programming algorithm. At each stage of dynamic programming, it is necessary to traverse all possible combinations of decision variables and calculate the current stage cost for each combination. The time complexity of a dynamic programming algorithm is the product of the number of stages, the size of the state space (that is, the number of discrete values that the state variable can take), and the size of the decision space (the number of discrete values that the decision variable can take).

4.3 Improved Lagrange multiplier update strategy

This paper proposes an improved surrogate Lagrangian relaxation iterative optimisation algorithm based on surrogate sub-gradient algorithm (Ruiz and Vazquez-Rodriguez, 2010). First, this paper will be based on the non-smooth properties of the Lagrangian relaxation dual function. Design improved surrogate Lagrangian relaxation sub-gradient iterative optimisation process, so that

$$L(\lambda^n, x^n) < \hat{L}(\lambda^n, x^{n-1}). \tag{22}$$

Secondly, combined with the non-smooth convex optimisation theory (Tang et al., 2006), the selection method of the algorithm step size and the convergence condition of the algorithm are studied. In this paper, a step selection method based on the concept of contraction mapping is proposed. In the process of two consecutive iterations, the distance between the multipliers is guaranteed to satisfy the decreasing relationship, and the rational selection of the decreasing rate parameter α^k is made.

$$\|\lambda^{n+1} - \lambda^n\| = \alpha^n \|\lambda^n - \lambda^{n-1}\|. \tag{23}$$

Establish a step:

$$c^n = \alpha^n c^{n-1} \|\hat{g}(x^{n-1})\| / \|\hat{g}(x^n)\| = \alpha^n \alpha^{n-1} c^{n-2} \|\hat{g}(x^{n-1})\| \|\hat{g}(x^n)\| \tag{24}$$

In order to ensure that the proposed algorithm will not lead to diminishing because step too fast algorithm premature convergence problem, make the α^n and c^n :

$$\lim_{n \rightarrow \infty} (1 - \alpha^n) / c^n = 0 \tag{25}$$

$$\hat{d}^n = \hat{g}^n + \gamma^n \hat{d}^{n-1} \tag{26}$$

$$\gamma^n = \max \left[0, -\beta \frac{(\hat{d}^{n-1})^T \hat{g}^n}{(\hat{d}^{n-1})^T \hat{g}^{n-1}} \right], 0 \leq \beta \leq 2 \tag{27}$$

One of the reasons for the low efficiency of the surrogate sub-gradient algorithm is the sawtooth oscillation when the algorithm searches in the feasible domain (Tang and Xuan, 2006).

For this reason, a Lagrangian relaxation iterative strategy based on gradient direction control is designed to judge the gradient of two adjacent iterations. The angle of the direction, if it is an obtuse angle which is $\gamma^n > 0$, introduces an offset sub-gradient, so that the two adjacent iteration directions are acute, as shown in Figure 3. If the gradient direction of two adjacent iterations is an acute angle, which is $\gamma^n = 0$, the gradient direction does not change, as shown in Figure 4, so that the convergence can be achieved more quickly.

Figure 3 The gradient direction of two adjacent iterations is obtuse

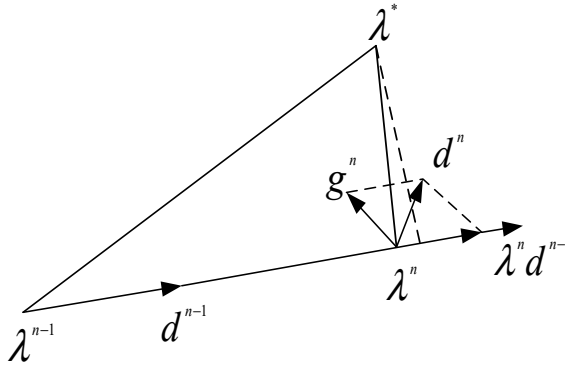
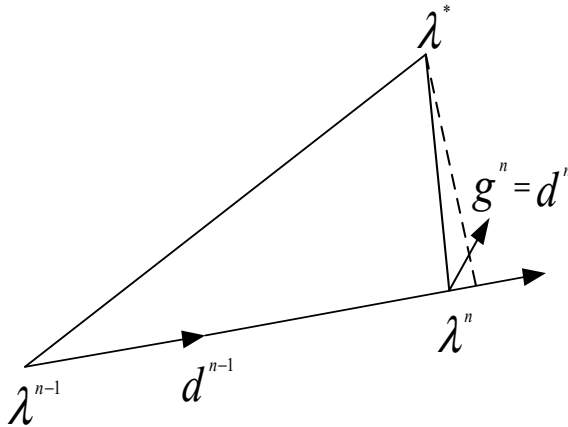


Figure 4 The gradient direction of two adjacent iterations is acute



The improved surrogate sub-gradient algorithm updates the Lagrangian multiplier steps as follows:

1 Set the initial value of Lagrange multiplier.

2 Solve the sub-problem get x^0 ; among them,

$$x^0 = \arg \min \left\{ \sum_{i=1}^I J_i(x_i) + (\lambda^0)^T \left(\sum_{i=1}^I a_i x_i - b \right) \right\}. \quad (28)$$

3 The initial value setting step c^0 , among them,

$$c^0 = \frac{L^* - \hat{L}(\lambda^0, x^0)}{\|\hat{d}(x^0)\|^2} \quad (29)$$

$$d^0 = g^0 = Ax^0 - b. \quad (30)$$

4 Calculated the agent gradient \tilde{g}^n .

5 The modified agent sub-gradient \tilde{d}^n ; given by the following formula:

$$\hat{d}^n = g^n + \gamma^k \hat{d}^{n-1}, n = 1, 2, \dots \quad (31)$$

$$\gamma^n = \max \left[0, -\beta \frac{(\hat{d}^{n-1})^T \hat{g}^n}{(\hat{d}^{n-1})^T \hat{g}^{n-1}} \right], 1 \leq \beta \leq 2. \quad (32)$$

6 Update step c^n , the step of c^n content,

$$c^n = \alpha_n \frac{c^{n-1} \|\hat{d}(x^{n-1})\|}{\|\hat{d}(x^n)\|}, 0 < \alpha_n < 1, n = 1, 2, \dots \quad (33)$$

$$\alpha_n = 1 - \frac{1}{M * (n^p)}, M \geq 1, 0 < p < 1. \quad (34)$$

7 The Lagrangian multiplier is updated according to the following formula;

$$\lambda^{n+1} = \lambda^n + c^n \hat{d}^n. \quad (35)$$

8 Perform approximate optimisation; given λ^{n+1} , execute the approximate optimisation with x^{n+1} , make x^{n+1} content,

$$L(\lambda^{n+1}, x^{n+1}) < \hat{L}(\lambda^{n+1}, x^n) = \sum_{i=1}^I J_i(x_i) + (\lambda^{n+1})^T (Ax^n - b). \quad (36)$$

If not x^{n+1} , has made the $x^{n+1} = x^n$.

9 Check whether the stop criterion is met. If the stop criterion is met, stop the multiplier update; otherwise, go to Step 2 to update the next multiplier iteration.
Stop criteria:

$$\|\lambda^{n+1} - \lambda^n\| < \varepsilon_1 \quad (37)$$

$$\|\lambda^{n+1} - x^n\| < \varepsilon_2. \quad (38)$$

4.4 Dual problem solving

The original problem is transformed into a dual problem, and the lower bound of the original problem is transformed into the upper bound of the dual problem:

$$Z_D = \max_{u_i, v_{jt}} \left\{ \sum_i Z_{LR}(i)^* - \sum_{t=1}^T \sum_{j=1}^J v_{jt} \left(\sum_{i \in \Omega} \delta_{ijt} - M_{jt} \right) \right\}. \quad (39)$$

Among them, $Z_{LR}(i)^*$ on behalf of $\min Z_{LR}(i)$.

The dual function is a concave function and consists of many concave surfaces. Due to the combined nature of discrete optimisation, the realisation of uncertainties further exacerbates this characteristic, so the number of concave surfaces is very large for practical problems. The smoothing property of this dual function can be solved iteratively by improving the surrogate sub-gradient method.

The specific dual problem-solving steps are as follows:

- 1 Setup the Lagrange multiplier u_i^n and v_{jt}^n initial value, n represents the number of iterations, the initial set to 1, $u_i^0 = 0$, $v_{jt}^0 = 0$ including charge $i \in \Omega$, process $j = 1, 2, \dots, J$, time node $t = 0, 1, \dots, T$.
- 2 Using the backward dynamic programming method to solve the scheduling sub-problems established by several charge.
- 3 If the following conditions are satisfied, proceed to Step 4. If not, go back to Step 2 to solve the scheduling sub-problem established for the remaining batch times until the agent optimal conditions are satisfied.

$$Z_D(u^n, v^n, t_{ij}^n, \delta_{ijt}^n) < Z_D(u^n, v^n, t_{ij}^{n-1}, \delta_{ijt}^{n-1}). \quad (40)$$

- 4 Set the gradient direction of updated Lagrange multiplier, the N^{th} iterative Lagrange multiplier of u^n, v^n in the first j working procedure for the gradient direction

$$g^n(u_i^n) = t_{i+1,j}^n - t_{i,j}^n - p_{i+1,j} \quad (41)$$

$$g^n(v_{jt}^n) = \sum_{i \in \Omega} \delta_{ijt}^n - M_{jt}. \quad (42)$$

- 5 Judge the angle of the gradient direction of the two adjacent iterations. If it is an obtuse angle, the offset sub-gradient is introduced to make the direction of the two adjacent iterations acute. The judgment mode and the calculation method of introducing offset sub-gradient are as follows:

$$d^n(u_i^n) = g^n(u_i^n) + \varphi^n d^{n-1}(u_i^{n-1}) \quad (43)$$

$$d^n(v_{jt}^n) = g^n(v_{jt}^n) + \varphi^n d^{n-1}(v_{jt}^{n-1}) \quad (44)$$

Among them,

$$\varphi^n = \max \left[0, -\beta \left(\frac{d^n(u^{n-1})^T d^n(u^{n-1}) + d^n(v^{n-1})^T d^n(v^{n-1})}{d^n(u^n)^T d^n(u^n) + d^n(v^n)^T d^n(v^n)} \right) \right], 1 \leq \beta \leq 2. \quad (45)$$

- 6 Calculate the update step of Lagrange multiplier in the N^{th} iteration.

$$c^n = c^{n-1} \gamma^n \frac{d^n (u^{n-1})^T d^n (u^{n-1}) + d^n (v^{n-1})^T d^n (v^{n-1})}{d^n (u^n)^T d^n (u^n) + d^n (v^n)^T d^n (v^n)} \gamma^n \quad (46)$$

$$= 1 - \frac{1}{M * (n^p)}, M \geq 1, 0 < p < 1$$

- 7 Based on gradient direction and step size, the Lagrangian multiplier is updated.

$$u_i^{n+1} = \max[0, u_i^n + c^n g(u_i^n)] \quad (47)$$

$$v_{jt}^{n+1} = \max[0, v_{jt}^n + c^n g(v_{jt}^n)]. \quad (47)$$

- 8 Determine whether meet the stopping criteria $\|d^n\| < \varepsilon_1$. If meet, the stopping criteria, stop the multiplier updating, get the optimal decision t_{ij}^* and δ_{ijt}^* , enter Step 14, otherwise, $n = n + 1$, turned to Step 7, the next update multiplier iteration.

4.5 Constructing feasible solutions

Since the expected processing order constraint and the expected machine capability constraint are relaxed during the Lagrangian relaxation process, the solution of the original problem composed of the optimal solutions of all sub-problems is usually not feasible. That is, at a certain time node, the ordering constraints or machine capacity constraints are not met. To obtain a feasible scheduling plan, we need to establish a heuristic rule based on the chosen dual solution and random event implementation. Among them, choosing a good dual solution is crucial.

Given the heuristics of feasible plan construction, dual solutions with high penalties are not necessarily a good feasible scheduling schedule. Therefore, we must try to find a feasible scheduling schedule among several candidate dual solutions with high penalty. In a random environment, each dual or feasible solution is actually a strategy. In order to obtain the expected value of the objective function, it is very time-consuming to simulate each dual solution. A short simulation of ordinal optimisation using selected candidate dual solutions to determine their ‘ranking’ of expected costs. Then, choose the winner’s dual solution of the short test to generate an implementable schedule, and perform a strict simulation run to obtain performance statistics (Cui and Luo, 2017).

Through the iterative solution of the sub-problem and the dual problem, the optimal solution of the sub-problem can be converged, but it is judged whether the entire steelmaking-continuous casting process optimised scheduling scheme obtained by the single-slot optimised scheduling scheme violates the ‘pouring order constraints, machine capacity constraints’ situation, if any, according to the start time cost rule to construct an optimal scheduling scheme, the specific steps are as follows (Tang et al., 2006):

- 1 From time node 0, determine whether all the furnaces satisfy the equation of ‘pouring process order constraint, machine capacity constraint’. If it is satisfied, proceed to Step 4, if not, proceed to Step 2.
- 2 For furnaces that do not meet the ‘processing order constraints, machine capacity constraints’ equation, calculate the cost of delaying one unit for the time node and

delaying the processing start time for the unit with the lower cost by one unit. The announcement of the cost of the delayed time is as follows:

$$f(i, j) = \min Z_{ij}(t_{ij} + 1, \delta_{ijt}) - \min Z_{ij}(t_{ij}, \delta_{ijt}) \tag{49}$$

- 3 Repeat Steps 1–2 until all the furnaces meet the requirements.
- 4 Determine the next time node, if it reaches the last time, stop and get a feasible scheduling plan, otherwise go to Step 1.

5 Numerical results

Numerical results are tested based on a large steel company in China This test is simulated by C language programming and executed on the Intel Core i5 5200 CPU 4 GB RAM, Windows 10/64-bit operating system PC.

- 1 Parameter $M = 40, r = 0.7$. Through experimental comparison, the combination of $M = 40$ and $r = 0.7$ can achieve faster convergence than other combinations.
- 2 The parameter $\epsilon_1 = 1e^{-5}$ represents that if the distance of the Lagrangian multiplier of two adjacent iterations is less than the set value, the iteration stops.
- 3 $\beta = 1.5$ and the default initial values for u_i and v_{jt} are 0.

To demonstrate the effective and efficiency of the strategy which is proposed in this paper, two integration method based on Lagrange relaxation framework are selected to compare. Table 1, Figure 5 and Figure 6 list the optimisation results based on surrogate sub-gradient algorithm and the improved surrogate sub-gradient algorithm with the different scale problem.

Table 1 Test results of two methods

No.	Problem scale		Duality gap (%)		CPU (s)	
	B	J	SSG	ISSG	SSG	ISSG
1	2	4	3.31	3.33	3.9	2.6
2	2	5	3.69	3.66	6.2	4.1
3	2	6	4.14	4.01	7.7	5.7
Avg.			3.71	3.67	5.9	4.1
4	4	4	2.15	2.17	9.7	6.9
5	4	5	2.39	2.44	14.1	10.4
6	4	6	2.42	2.45	19.2	14.7
Avg.			2.32	2.35	14.3	10.7
7	5	4	1.62	1.60	16.9	11.3
8	5	5	1.95	1.94	27.4	20.7
9	5	6	1.72	1.76	37.3	26.8
Avg.			1.76	1.76	27.2	19.6

Figure 5 Comparison of dual gaps for solving problems of different sizes

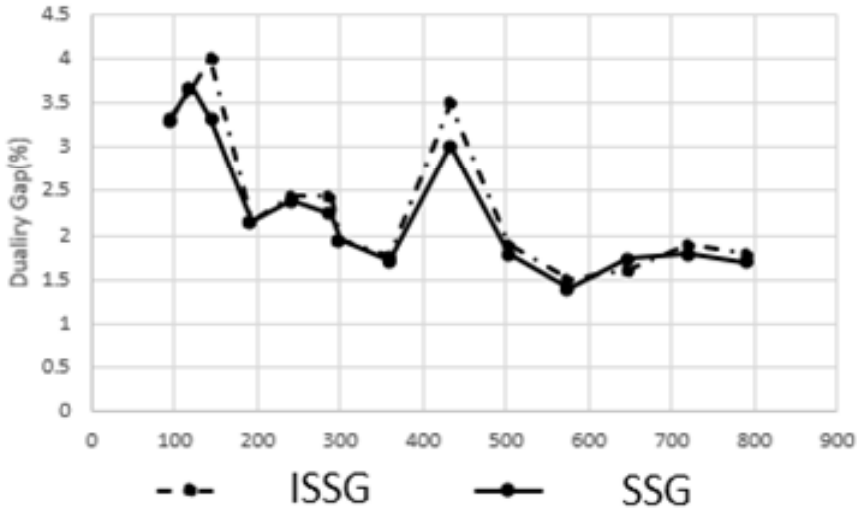
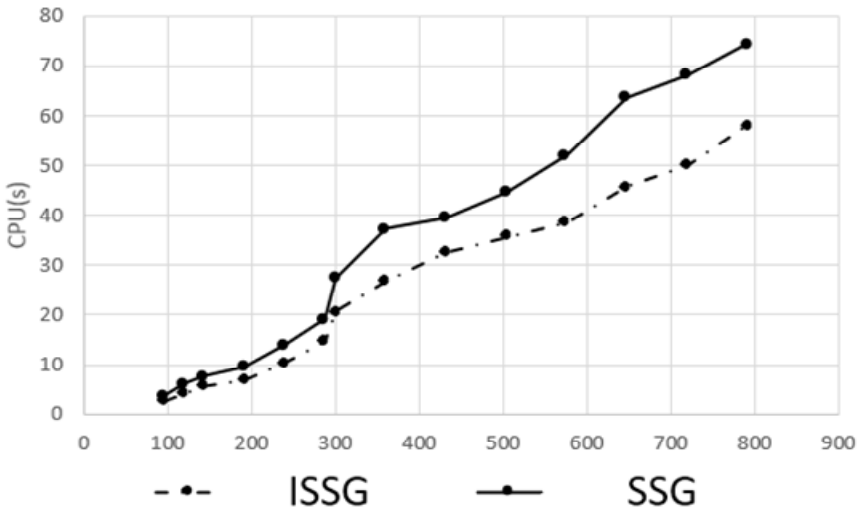


Figure 6 Comparison of runtimes for solving problems of different sizes



From the results, we can see that, the average duality gap of SSG is 2.59% and the average CPU time is 15.8; the average duality gap of ISSG is 2.64% and the average CPU time is 11.4. Although there is not too much difference between the duality gap of two algorithms, the CPU time of ISSG is much better than the CPU time of SSG, which could demonstrate the direction controllable iteration strategy of ISSG has higher efficiency.

6 Conclusions

Steelmaking and continuous casting scheduling is a NP problem according to its complexity. The space involved in the target solution is huge, and the uncertainty of the processing time parameters makes the target solution space larger. Therefore, it turns to consider solving the approximate solution. Because the steelmaking-continuous casting problem has a good mathematical structure, it can be decomposed into sub-problems in units of charge. Therefore, the steelmaking-continuous casting production scheduling based on the mixed integer programming modelling method based on the charge decomposition strategy is proposed. Secondly, the project aims to reduce the scale of scheduling, reduce the difficulty of scheduling, improve the quality of solution and the speed of solve. Based on the efficient Lagrangian relaxation method, this paper proposes an improved Lagrangian relaxation method to study the processing time. Determine the steelmaking-continuous casting production scheduling problem. In finding the optimal Lagrangian multiplier, it is not necessary to obtain the optimal values of all the relaxation problems, and the angle between the adjacent two gradient directions is judged. If the angle is an obtuse angle, a migration sub-gradient is introduced. The gradient direction is changed to ensure that the two adjacent gradient directions are acute. This method can effectively improve the solution quality and response speed of the steelmaking-continuous casting production process scheduling. Finally, this paper conducts verification research on steelmaking-continuous casting production scheduling through the production background of actual steel plants. In order to solve the problem of steelmaking-continuous casting production scheduling optimisation with uncertain processing time; so, this research work provides an effective application in practical complex process industry.

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