An optimal policy for an integrated vendor-buyer model with two warehouses under vendor's capacity constraint

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Abstract: The paper considers an integrated single-vendor single-buyer supply chain model in which the vendor is assumed to be capacity constrained. The vendor can keep the excess units beyond the capacity of its own warehouse (OW) in a rented warehouse (RW) whose holding cost is higher than that of the own warehouse. The vendor delivers the buyer's order quantity in a number of equal shipments. The proposed integrated model is formulated and some of its characteristics are studied analytically. Considering the vendor's capacity as a control variable, the optimal decisions of the model are obtained for a numerical example. Sensitivity analysis is also carried out to measure the impact of key model-parameters on the outcome of the model.

Keywords: supply chain; vendor-buyer integrated model; rented warehouse; capacity constraint.

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1 Introduction

The traditional vendor-buyer model assumes that the item is stored at the vendor's place or buyer's place in a single own warehouse (OW) having unlimited capacity. This assumption is unrealistic in the sense that not all organisations can afford OWs of large capacities or they may not have the opportunity to avail such warehouses due to several reasons. To resolve the warehouse space crisis, the general practice in the market is to hire a rented warehouse (RW). Inventory model with two warehouses was first proposed by Hartley (1976). Sarma (1987) generalised Hartley's (1976) model to incorporate transportation cost from RW to OW. More works in this direction could be found in Goswami and Chaudhuri (1992), Bhunia and Maiti (1998) and their references. Hariga (2011) developed an inventory model with multi-warehouse and flexible space contract. Liao et al. (2012) discussed the lot sizing decisions with two warehouses and trade credit. A two-warehouse partial backlogging inventory model with permissible delay in payment was discussed by Yang and Chang (2013). Das et al. (2014) developed a two-warehouse inventory model for deteriorating item with finite replenishment over a finite planning horizon. They designed a genetic algorithm to determine the optimal decisions of the model. Bhunia et al. (2016) applied the particle swarm optimisation (PSO) technique to solve a two-warehouse inventory model for deteriorating item under permissible delay in payment.

As far as the supply chain is concerned, a single-vendor and a single-customer integrated model was introduced by Goyal (1976). Assuming that the vendor's production rate is finite, Banerjee (1986) developed a lot-for-lot model where the vendor produces each shipment-sized quantity as a separate batch. Goyal (1988) argued that producing a batch which is made up of equal shipments generally costs lower but the whole batch must be completed before the first shipment is made. Lu (1995) examined the existence of optimal solution of the model with single vendor, single buyer and equal shipments. Later, Goyal (1995) showed that a different shipment size policy could give a better solution. The proposed policy involves successive shipments within a production batch increased by a constant factor which is equal to the ratio of the production rate over the demand rate. Hill (1999) provided another unequal shipment policy for a single-vendor single-buyer integrated production inventory problem. Several researchers (Chakraborty and Martin, 1988; Yang, 2004) showed that the vendor's cost as well as the total cost of the integrated system decrease, while the buyer's cost increases. Huang (2004) developed a model to determine an optimal integrated vendor-buyer policy in just-in-time (JIT) environment with unreliability condition. Ben-Daya and Hariga (2004) showed that co-ordination is effective from vendor's as well as buyer's perspectives for stochastic demand and variable lead time. Huang et al. (2010) considered order processing cost reduction in a single-vendor single-buyer integrated inventory system under permissible delay in payments. Teng et al. (2012) studied a vendor-buyer model with trade credit financing linked to order quantity under both non-cooperative and integrated environments. A vendor-buyer model with stochastic lead time and service level constraint was analysed by Soni and Patel (2014). Jauhari and Winingsih (2016) developed four integrated vendor-buyer models with stochastic demand where the vendor and the shipper offer discounts to the buyer. Recently, Khan et al. (2017) investigated the joint effect of imperfect production, inspection errors and stochastic lead time demand on the optimal cost of a vendor-buyer supply chain. All these works did ignore the capacity restrictions of the vendor and the buyer.

However, capacity or space constraint has a major effect on the optimal decisions of an integrated system. Most of the studies on integrated vendor-buyer inventory system, as mentioned above, ignores the capacity constraint. The reason is perhaps to avoid the complexity of the problem. Hoque and Goyal (2000) developed a single-vendor single-buyer production inventory system with unequal and equal-sized shipments from the vendor to the buyer under capacity constraint of the transport equipment. Lee and Wang (2008) studied the impact of buyer's capacity constraint on the optimal decisions in case of a consignment stock policy. Xu and Leung (2009) focused a stocking policy in a two-party vendor managed supply chain with space restriction. Hariga et al. (2013) considered a supply chain model with vendor-managed inventory (VMI) contract for multiple retailers with storage constraint and unequal shipment policy. Hariga et al. (2013) developed a supply chain model under a VMI contract considering buyer's warehouse space limitation. Hariga et al. (2014) studied a single vendor multiple retailers supply chain system where a maximum stock level is allowed by each retailer. Ouyang et al. (2015) considered an integrated inventory model with retailer's capacity constraint and order size dependent trade credit. Giri and Bardhan (2015) investigated the effect of buyer's space constraint on the optimal decision in a vendor-buyer model with stock-dependent demand and consigned inventory.

Though some works have been done on integrated vendor-buyer system with a limited capacity, much attention has not been paid to the capacity constraint of the vendor as evident from above. Further, no attempt has been made to consider a RW in JELP (joint economic lot size problem). In this paper, we propose a generalised JELP assuming that the vendor is capacity constrained, i.e., the capacity of its OW is limited. The vendor can keep the excess units beyond the capacity of its OW in a RW. If the vendor has to meet the buyer's demand (order quantity) then the question arises, whether the vendor will go for accommodating all the produced quantities in its OW whatever may be the size of the production lot or it will decide the optimal capacity of OW and accommodate the excess units in the RW, even though the holding cost of the RW is more than the OW. The novelty of the paper is to find the answer of this question. The rest of the paper is organised as follows: notations and assumptions for developing the model are given in the following section. The description of the model is given in Section 3. The proposed model is formulated and the solution methodology is presented in this section. In Section 4, a numerical example is taken to obtain numerically the optimal solution of the developed model. A sensitivity analysis is also carried out to examine the effect of changes of key model-parameters on the optimal solution. Finally, in Section 5, the paper is concluded with some remarks and future scope of research.

2 Notations and assumptions

The following notations are used to develop the proposed integrated vendor-buyer model.

D	demand rate at the buyer
$P(> D)$	production rate at the vendor
Q	vendor's production lot size
n	number of shipments per cycle from the vendor to the buyer, a positive integer
q	shipment size
S_v	vendor's setup cost per setup
S_b	buyer's ordering cost per order
h_v	vendor's unit holding cost in OW
h'_v	additional cost of holding per unit per unit time in the RW compared to OW
h_b	buyer's unit holding cost
W	vendor's (own) warehouse capacity
$I_{\max}$	maximum on-hand inventory at the vendor
T	cycle length
F	transportation cost per shipment.

The following assumptions are made to develop the proposed model.

1 The supply chain consists of a single-vendor and a single-buyer for trading a single product. The vendor follows multiple equal-shipments policy for delivering the order to the buyer.

2 The buyer's demand rate is constant. The vendor's production rate is also constant and greater than the buyer's demand rate.

3 The vendor is capacity constrained. The maximum stocking capacity of the vendor's OW is limited to W which we take as a decision variable.

4 The vendor can keep the excess units produced beyond the capacity of its OW in a RW whose holding cost is higher than that of the OW.

5 At the buyer, replenishments are instantaneous and lead time is zero. Shortages are not allowed in the buyer's inventory.

6 It is assumed that $h_b > h'_v + h_v$ (Hill, 1999) as the value of the product usually increases when the product moves down the supply chain, and accordingly the holding cost is more at the buyer's place than that at the vendor's RW. Though this assumption is not to be true in all cases, it is made here to establish a relationship of the holding costs at the buyer and the vendor (own and RWs).

3 Model formulation

We assume that the buyer's order quantity is $Q(= nq)$ which is delivered by the vendor in n regular shipments of size q. Obviously, for each replenishment, the buyer's inventory will be depleted after q/D time. Since $P > D$, more than q quantity is produced by the time q/D but only q quantity is delivered to the buyer. This process continues up to the time instant when the vendor's inventory level reaches the capacity W. To find the vendor's inventory holding areas in its OW and RW, we first assume that the vendor's OW capacity W is greater than the maximum on-hand inventory level $I_{\max}$. Then the total holding area will be equal to the holding area of its OW. On the other hand, if $W < I_{\max}$, then we have to consider the holding areas in the OW as well as the RW. If, however, $h'_v = 0$ then our proposed model coincides with that of Ha and Kim (1997).

3.1 Vendor's cost

To find the vendor's holding area, we divide the total inventory area into some sub-areas. The inventory holding area before the first shipment at the vendor's warehouse is $A_1 = \dfrac{q^2}{2P}$. The time period after the first shipment and before the completion of production is $\dfrac{(n-1)q}{P}$. So, the number of shipments in this time period is

$$m_1 = \left\lfloor \frac{(n-1)q}{P}\left(\frac{D}{q}\right) \right\rfloor = \left\lfloor \frac{(n-1)D}{P} \right\rfloor,$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

If IL_i^+ and IL_i^- denote respectively the inventory levels of the vendor just before and after the time of i^{th} shipment then we have

$$IL_i^+ = q(i-1)\frac{P}{D} - (i-2)q, \quad \text{and} \quad IL_i^- = (i-1)q(P/D-1), \quad i = 2,3\ldots m_1$$

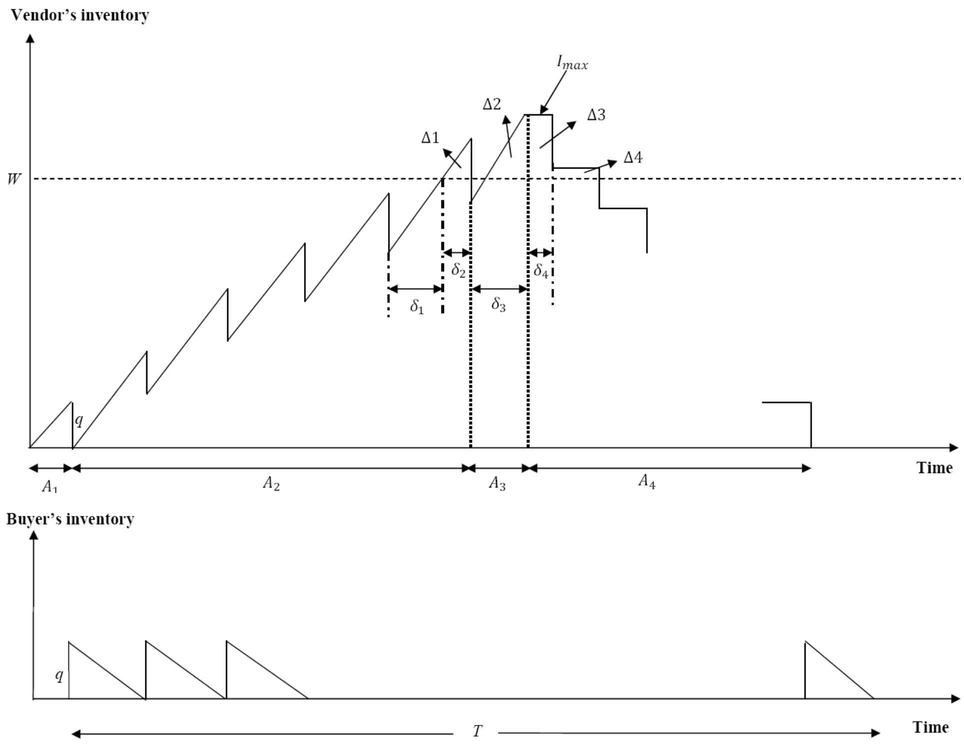
Clearly, we have $IL_i^+ - IL_i^- = q > 0$. Let Θ_i denote the inventory holding area at the vendor between $(i-1)^{\text{th}}$ and i^{th} shipments. Then we have,

$$\begin{aligned}\Theta_i &= \frac{q}{2D}\left[IL_{i-1}^{-} + IL_i^{+}\right] \\ &= \frac{q}{2D}\left[(2i-3)\frac{qP}{D} - (2i-4)q\right] \\ &= \frac{q^2}{2D}\left[(2i-3)\frac{P}{D} - 2i + 4\right]\end{aligned}$$

So, the total holding area up to the m_1^{th} shipment is

$$A_2 = \sum_{i=2}^{m_1} \frac{q^2}{2D}\left[(2i-3)\frac{P}{D} - 2i + 4\right]$$

Figure 1 The inventory diagram when vendor is capacity constrained



The remaining time between m_1^{th} shipment and completion of production is (see Figure 1).

$$\delta_3 = \frac{(n-1)q}{P} - \frac{m_1 q}{D} = q\left[\frac{n-1}{P} - \frac{m_1}{D}\right].$$

The corresponding holding area for the time period δ_3 is $A_3 = \frac{\delta_3}{2}[IL_{m_1}^{-} + I_{\max}]$. During the time of production, the highest inventory level is given by

$$I_{\max} = q\left(n - m_1 - P/D\right)$$

The idle time i.e., the time between completion of production and $(m_1 + 1)^{\text{th}}$ shipment is given by $\delta_4 = \left(\frac{q}{D} - \delta_3\right)$. So, the corresponding inventory holding area will be $\delta_4 I_{\max}$. After time δ_4, the inventory level drops down to $I_{\max} - q$. So, the remaining number of shipments is $m_2 = \frac{I_{\max - q}}{q}$. In this time duration, the inventory area is given by $\frac{q}{D}\sum_{k=1}^{m_2}(I_{\max} - kq)$. So, the total holding area after the completion of production is

$$A_4 = \delta_4 I_{\max} + \frac{q}{D}\sum_{k=1}^{m_2}\left(I_{\max} - kq\right)$$

We now find the holding area in the RW. Note that if $W \geq I_{\max}$ then there is no need of a RW. Let us assume that at the time of k^{th} shipment, the vendor's inventory level curve crosses (or touches) the capacity level W. Then we have

$$IL_k^+ = (k-1)\frac{Pq}{D} - (k-2)q \geq W$$

$$\Rightarrow k \geq \frac{W + \left(\frac{Pq}{D} - 2q\right)}{\frac{Pq}{D} - q}$$

$$\Rightarrow k = \left\lfloor \frac{W - q}{\frac{Pq}{D} - q} + 1 \right\rfloor, \quad \text{if} \quad \frac{W - q}{\frac{Pq}{D} - q} + 1 \text{ is an integer}$$

$$\text{or,} \quad k = \left\lfloor \frac{W - q}{\frac{Pq}{D} - q} + 1 \right\rfloor + 1$$

The intersection of the vendor's inventory level curve with its capacity level line will occur if $W < I_{\max}$. However, the number of intersections plays a vital role in calculating the inventory holding area in the RW. Therefore, it is important to consider the conditions on which the number of intersections depends.

If the intersection occurs after $(k - 1)^{\text{th}}$ shipment then $IL_k^+ > W$. Again, if the vendor's inventory level drops down to W after k^{th} shipment then $IL_k^- < W$. These two conditions simultaneously hold when $(P/D - 1) > 0$ i.e., $P > D$. Along with these conditions, if the condition $IL_{k+1}^+ > W + q$ is true then one and only one intersection is guaranteed.

Proposition 1. Two consecutive intersections of the vendor's inventory level curve with the capacity level line do not arise provided that $P > 2D$.

Proof: We have

$$IL_k^+ > W \Rightarrow qk(P/D-1) - q(P/D-2) > W$$
$$\text{and} \quad IL_{k+1}^+ > W + q \Rightarrow qk(P/D-1) > W$$

Thus the conditions $IL_k^+ < W$ and $IL_{k+1}^+ > W+q$ jointly hold only when $\frac{P}{D} - 2 > 0$ giving $P > 2D$.

Proposition 2. Three consecutive intersections of the vendor's inventory level curve with the capacity level line do not arise provided that $P > \frac{3D}{2}$.

Proof: If $IL_k^+ > W$, then $IL_{k+1}^+ > W$. So, three consecutive intersections of the vendor's inventory level curve and capacity level line is possible only when $IL_{k+1}^- < W$ and $IL_{k+2}^- < W$. The condition for satisfying this is given by

$$IL_{k+2}^- = (k+1)q\left(\frac{P}{D} - 1\right) \geq q\left(\frac{2P}{D} - 3\right) \geq W$$

which holds if $\frac{2P}{D} - 3 > 0$.

Thus we can formulate a sequence of the ratio of P and D depending on which the number of intersections can be calculated. Here, we have the sequence $\left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots\right\}$ for the ratio P/D and the corresponding sequence of number of intersections is {1, 2, 3, 4, 5, …}. This is represented graphically in Figure 2.

From above, we have the following observations:

1 as the ratio P/D tends to 1, the number of intersections increases

2 when $P = D$, the total number of intersections either becomes zero or equal to the total number of shipments

3 the ratio P/D decreases implying that the usage of the RW also decreases.

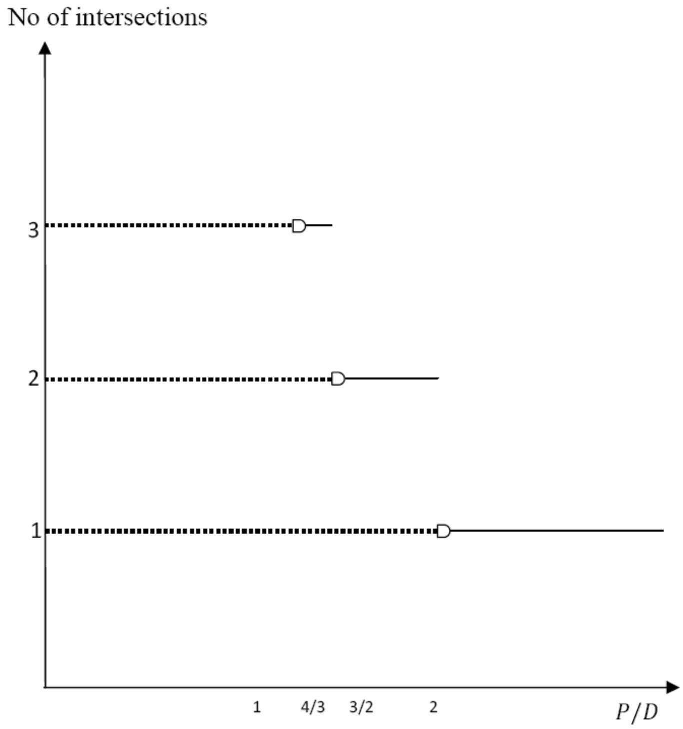
To further analyse the model, we assume that $P > 2D$, i.e., the number of intersections between the vendor's inventory level curve and the capacity level line is only one. For the other cases, calculations can be done in a similar manner.

In the following, we first find the holding area at the RW. If the vendor's inventory level curve first touches the capacity level line after the $(m_1 - 1)^{\text{th}}$ shipment then the holding area at the RW in $(q/D - \delta_1)$ time is

$$\Delta_1 = \frac{1}{2}\left(\frac{q}{D} - \delta_1\right)\left[(m_1 - 1)\frac{Pq}{D} - (m_1 - 2)q - W\right],$$

$$\text{where} \quad \delta_1 = \frac{W - (m_1 - 2)q\left(\frac{P}{D} - 1\right)}{P}.$$

Figure 2 Relation between number of intersections and P/D



According to our assumption, the minimum inventory level at the time of m_1^{th} shipment is less than $I_{\max}$ and it again reaches the capacity level after $\frac{W - IL_{m_1}^{-}}{P}$ time, and the vendor's inventory level curve crosses the capacity level line and reaches $I_{\max}$ until completion of production after δ_3 time. The point to be noted here is that $\delta_1 + \delta_2 = q/D$ and $\delta_3 + \delta_4 = q/D$ (see Figure 1). The area for the time $\left(\frac{q}{D} - \delta_3\right)$ is given by

$$\Delta_2 = \frac{1}{2}\left(\delta_3 - \frac{W - IL_{m_1}^{-}}{P}\right)\left[I_{\max} - W\right].$$

The area in the RW for the time interval between the completion of production and the first shipment after it is

$$\Delta_3 = \delta_4\left(I_{\max} - W\right), \quad \text{where} \quad \delta_4 = q\left[\frac{m_1 + 1}{D} - \frac{n-1}{P}\right].$$

Here $\frac{nq}{P}$ is the time when production is completed. It is obvious that the maximum on-hand inventory level at the vendor is the same as the inventory level at the completion of production. The remaining holding area in the RW is given by

$$\Delta_4 = \frac{q}{D}\sum_{i=1}^{s}\left[I_{\max} - iq - W\right], \quad \text{where} \quad s - \left\lfloor \frac{I_{\max} - q - W}{q} \right\rfloor + 1.$$

So, the average total cost of the vendor is given by

$$TC_v(n, q, W) = \frac{h_v'\left(HC_r\right) + h_v\left(HC_w\right) + S_v}{T} \tag{1}$$

where HC_r is the total inventory area at the RW and HC_w is the total inventory area at the OW. So, $HC_r = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$ and $HC_w = A1 + A2 + A3 + A4$.

3.2 Buyer's cost

The buyer's ordering cost is S_b and total transportation cost is nF. Therefore, the annual total cost of the buyer is

$$TC_b(n, q) = \frac{S_b + nF}{T} + \frac{h_b q}{2} \quad \text{where} \quad T = \frac{nq}{D}. \tag{2}$$

3.3 Integrated system cost

From (1) and (2), the annual total cost of the vendor-buyer integrated system is given by

$$TC(n, q, W) = TC_v(n, q, W) + TC_b(n, q) \tag{3}$$

Our objective is to minimise $TC(n, q, W)$ subject to the condition $I_{\max} > W > 0$.

Proposition 3. For fixed capacity level W and number of shipments n, the cost function $TC(n, q, W)$ is piecewise continuous in q.

Proof: We have from (3), $TC(n, q, W) = TC_v(n, q, W) + TC_b(n, q)$. For fixed n, $TC_b(n, q)$ is a continuous function in q. In order to examine the nature of the cost function $TC_v(n, q, W)$, we see that $HC_r = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$ where Δ_4 depends only on s where

$$\begin{aligned} s &= \left\lfloor \frac{I_{\max} - q - W}{q} + 1 \right\rfloor \\ &= \left\lfloor \frac{q\left(n - m_1 - P/D\right) - q - W}{q} \right\rfloor + 1 \\ &= \left\lfloor \left(n - m_1 - P/D\right) - 1 - W/q \right\rfloor + 1 \\ &= \left\lfloor n - m_1 - P/D - W/q \right\rfloor \\ &= n - m_1 + \left\lfloor -P/D - W/q \right\rfloor \end{aligned}$$

As $\lfloor -P/D - W/q \rfloor$ always gives an integer value, and P, D and W are fixed, therefore, $-1 < -P/D - W/q < 0$ gives $\lfloor -P/D - W/q \rfloor = -1$.

This shows that Δ_4 is piecewise continuous and hence the cost function $TC_v(n, q, W)$ is piece-wise continuous in q for given n and W.

In the following, the convexity of the cost function $TC(n, q, W)$ is established when the vendor's capacity constraint is an integer multiple of the shipment size.

Proposition 4. The average cost function $TC(n, q, W)$ is convex in q when the vendor's capacity W is an integer multiple of shipment size q.

Proof: Suppose that $W = Lq$, where L is a positive integer. Then from equation (1), $TC_v(n, q, W)$ takes the form $aq + b/q$, where a and b are constants. Similarly, from equation (2), $TC_b(n, q)$ can be transformed into the same form. Further, we check that

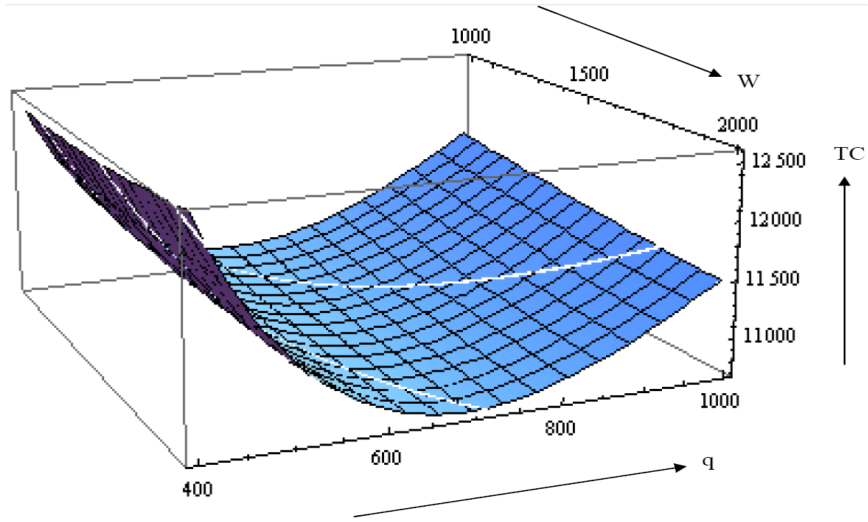
$$\frac{d^2}{dq^2}\left(TC_v(n, q, W)\right) = \frac{d^2}{dq^2}\left(TC_v(n, q, Lq)\right) > 0$$

implying that TC_v is convex in q. Hence $TC(n, q, W)$ is convex in q.

4 Numerical example

To illustrate the proposed model numerically, we consider the following data, a part of which is taken from Salameh and Jaber (2000): P = 160,000 units/year, D = 50,000 units/year, S_v = Rs. 300/cycle, S_b = Rs. 100/cycle, h_v = Rs. 2/unit/year, h'_v = Rs. 1/unit/year, h_b = Rs. 5/unit/year and F = Rs. 25/delivery.

Figure 3 The convexity of $TC(n, q, W)$ for $n = 8$ (see online version for colours)



We find that for any given n, the average cost function $TC(n, q, W)$ represents a convex surface. One instance is shown in Figure 3. We, therefore, perform a line search on n and find the optimal values of q and W and the corresponding average cost TC for each value of n. The results are shown in Table 1.

Table 1 shows that the capacity level W increases and the lot size q decreases with the increase in the number of shipments n. The cost function attains the minimum value for n = 8, and the corresponding optimal values are q^* = 701.13, W^* = 1,963.18 and TC^* = 10,696.8.

Table 1 Optimal results

n	q^*	W^*	TC^*
5	971.26	777.01	10,809.2
6	855.21	1,539.33	10,718.6
7	766.65	1,609.96	10,714.6
8	*701.13*	*1,963.18*	*10,696.8*
9	645.93	2,454.54	10,748.9
10	599.85	2,579.37	10,836.0

Table 2 Optimal results when W is fixed

W	n^*	q^*	TC^*
500	6	832.94	10,843.7
1,000	6	845.02	10,752.4
1,500	7	765.26	10,715.3
2,000	8	702.64	10,697.2
2,500	8	710.48	10,716.1
3,000	8	742.61	10,788.5

We now obtain the outcome of the proposed model when the vendor's capacity is fixed. Table 2 shows that, as W increases, TC^* first decreases and then increases and the minimum value of TC^* is attained for $W \in (1{,}500, 2{,}500)$. We have verified that the minimum value TC^* is 10,696.8 which is obtained for $W = 1{,}963.18$. This result exactly matches with the result given in Table 1.

We now perform the sensitivity analysis of the key model-parameters. We change the value of one parameter at a time and keep all other parameters unchanged. Table 3 shows the changes in the optimal results for –20%, –10%, 0%, 10% and 20% changes in the parameter-values.

The following observations are made from Table 3.

1 As the production rate increases, the system cost increases but the capacity level and shipment size decrease.

2 For an increase in demand rate, the capacity level, the shipment size and the average total cost increase.

3 The RW's holding cost exhibits very low sensitivity to the average cost of the system. Similar effects on the capacity level and shipment size are also observed.

4 Effects of the capacity level, shipment size, the OW's holding cost and the buyer's holding cost on the average cost of the supply chain are found similar in nature. The average cost of the supply chain increases in each case. The capacity level and the shipment size decrease in both the cases.

5 The vendor's set-up cost and the buyer's ordering cost have similar effects on the supply chain; they increase the average cost, capacity level and shipment size moderately.

Table 3 Sensitivity analysis

Parameter	*Value*	*Optimal results*		
		q^*	W^*	TC^*
P	129,660	723.10	2,256.06	10,371.1
	144,000	711.01	2,218.34	10,548.4
	160,000	701.13	1,963.18	10,696.8
	176,000	694.04	1,943.32	10,806.5
	193,600	684.31	1,666.98	10,960.0
D	40,500	614.76	1,521.71	9,881.94
	45,000	654.55	1,555.57	10,312.4
	50,000	701.13	1,963.18	10,696.8
	55,000	745.24	2,086.66	11,069.7
	60,500	792.88	2,450.73	11,445.7
h'_v	0.8	689.91	1,931.76	10,870.7
	0.9	700.86	1,962.40	10,696.6
	1.0	701.13	1,963.18	10,696.8
	1.1	701.22	1,963.41	10,697.0
	1.2	701.09	1,963.05	1,0697.3
h_v	1.6	750.69	2,101.82	9,990.8
	1.8	725.97	2,032.71	10,331.2
	2.0	701.13	1,963.18	10,696.8
	2.2	678.71	1,900.38	11,050.3
	2.4	656.34	1,837.87	11,426.5
h_b	4.0	724.03	2,027.29	10,358.4
	4.5	713.14	1,996.79	10,520.1
	5.0	701.13	1,963.18	10,696.8
	5.5	689.91	1,931.75	10,870.7
	6.0	678.24	1,899.06	11,058.8
S_v	243	666.99	1,867.57	10,176.0
	270	683.62	1,914.14	10,426.0
	300	701.14	1,963.20	10,696.8
	330	718.48	2,011.76	10,961.0
	363	736.97	2,063.59	11,244.4
S_b	81	690.02	1,932.05	10,526.1
	90	695.30	1,946.84	10,607.3
	100	701.14	1,963.20	10,696.8
	110	706.96	1,979.48	10,785.6
	121	713.31	1,997.27	10,882.4

5 Conclusions

Buyer's and vendor's capacities play important role in determining the optimal policy of an integrated vendor-buyer inventory system. This paper develops a single-vendor a single-buyer integrated inventory model considering vendor's capacity constraint. The vendor has limited capacity of its OW. It can keep the excess units produced beyond the capacity of OW in a RW whose holding cost is higher than that of the OW. The study suggests that the vendor cannot make an effective production plan for full usage of its OW only. In order to have the minimum average cost of the integrated system, the vendor needs a RW. In the proposed model, the capacity of the vendor's OW is taken as a control variable. Further, it is shown that if the vendor's OW capacity is unlimited then the proposed model coincides with the basic JELP model proposed by Ha and Kim (1997). This indicates that ours is a generalised JELP model. In our model, we have considered the vendor's capacity constraint. One can immediately extend this model by considering the buyer's capacity constraint as well. There are also several other scopes of extending the model such as consideration of imperfectness in the production system, non-constant demand, process improvement by learning, and so on.

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