Random fatigue analysis of drill-pipe threaded connection

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Abstract: Threaded connections are widely used in connecting drill pipes into a drill string. During drilling operations, the threaded connections are subjected to various external loads, including the load from the unevenness of rock formation at the bottom, impact from the well bore and axial hook load etc. These loads are primarily random, and will cause fatigue damage to the connections during drilling operation. In this paper, a standard 4.5” API line pipe threaded connection is analysed to investigate the fatigue effect of the random loads. A static stress analysis is first conducted considering ‘make-up’ and ‘tensile load’ steps. Then modal analysis and random vibration analysis are carried out, with the excitation considered as random. The fatigue damage is predicted using the so called three-band technique based on the stress results obtained. Effects of parameters, such as the random excitation, location of the connection, and friction coefficient, on the fatigue are discussed.

Keywords: drill string; fatigue; random loading.


Biographical notes: Jiahao Zheng is currently working as a Mechanical Engineer at ConMico Inc. Canada. He obtained his Master of Engineering degree from Memorial University. His research area includes random vibration and fatigue analysis of the drill-string.

Jianming Yang is currently an Assistant Professor at Memorial University. Before his current appointment, he holds several positions in several Chinese universities and industries. His main research interests lie in machinery dynamics, random vibration and fatigue prediction of mechanical systems.

1 Introduction

In the oil and gas industry, a drill-string, which is made up of numerous drill-collars and drill-pipes, is used to drill a well for extraction of the resources. Threaded connections serve as connections between adjacent drill-pipes or drill-collars to form a whole drill string. There are several different types of connections used in the industry; the most
commonly used type shown in Figure 1 is called threaded and coupled connection. This type of connections mainly contains three parts: two pins and one box. The two pins serve as master parts and the box serves as a slave part in the coupling. The analysis in this paper is based on this specific type of connections; however, the method and procedures used in this paper can be easily extended to fatigue prediction of other types.

Fatigue of drill string parts is a prominent problem encountered by the oil and gas industry, which causes significant economic losses and down time. Two-thirds of drill-string failures reported are related to threaded connections (Schwind, 1998). This fact has motivated many researchers to investigate the fatigue issue of drill string both experimentally and theoretically. Experimental tests are relatively costly and time consuming, and also limited by assumptions of boundary conditions and equipment. However, they are considered indispensable given that numerical simulation needs validation through test data. In addition, the S-N curves, which are the basis of theoretical fatigue prediction, can only be obtained through physical tests. In recent decades, finite element method (FEM) has been widely accepted in modelling threaded connections. With this method, various connections can be investigated relatively faster and cheaper compared with experimental tests. Two-dimensional axisymmetric model is widely accepted owing to its trade-off between time-saving and accuracy (Zhong, 2007). In this model, the following two simplifications are generally made (Van Wittenbergh, 2011). First, the thread helix angle is neglected. Secondly, thread run-in and run-out regions are not modelled. Research works using this model include Tafreshi and Dover (1993), Dvorkin and Toscano (2003), Yuan et al. (2006) and Macdonald and Deans (1995). Vibration is unavoidable in drilling and it induces fatigue damage to the drill string. However, very few researchers have considered vibration’s effect on fatigue of the drill string. In this paper, the 2D axisymmetric model is used, but with inclusion of the vibration effect. A full threaded connection pair, including the upper pin, box and lower pin, is considered. The boundary conditions are assumed as two equivalent springs connecting to the upper and lower ends. Besides, random excitation from the bottom is considered. Based on random vibration theory and Gaussian distribution assumption, equivalent stresses are obtained in probability sense. Then cumulative damage is calculated according to the Miner’s damage law.
The paper is organised as follows: Section 2 describes the model in detail, which is analysed with the ANSYS workbench 15.0. Section 3 gives static stress analysis considering ‘make-up’ and ‘tensile load’ steps. Then Section 4 explains modal analysis and random vibration theory in connection with ANSYS. In Section 5, the fatigue is computed and simulation results are given. Finally in Section 6 the results are analysed and conclusions are drawn.

2 Finite element model

A FE model of a standard 4:500 API line pipe connection is built through ANSYS workbench 15.0. In this model the nonlinearities from large deformation and the interaction between the pin and box are considered (Van Wittenbergh, 2011). The material of the connection is taken as a bi-linear elastic-plastic structural steel with 208 GPa of Young’s modulus and 0.3 of Poisson ratio (API 5L Specifications, 2007). The yield strength is taken 241 MPa.

Figure 2 Pin geometry (see online version for colours)

Figure 3 Box geometry (see online version for colours)
The 2-D axisymmetric model contains three parts: pin 1, the box and pin 2 as shown in Figure 1. The basic geometries of the pin and half of the box are shown in Figures 2 and 3 respectively. Detail of the threads is shown in Figure 4. According to API specification standard 5B and 5L (API 5L Specifications, 2007, 2008), the dimensions of the model are listed in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Dimensions of pin, box and thread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_1$</td>
</tr>
<tr>
<td>Pin (in)</td>
<td>0.844</td>
</tr>
<tr>
<td>Box (in)</td>
<td>2.250</td>
</tr>
<tr>
<td>Thread (in)</td>
<td>1.250</td>
</tr>
</tbody>
</table>

A meshing strategy similar to the one used in Van Wittenberghe (2011) is adopted as shown in Figure 5. The elements at the root are meshed with the finest size, while the elements at the thread crests are with coarser size. The thread flanks are divided by 35 elements with a biased sizing ratio equal to 5. Besides, given the fact that the pin serves as a master part and the box as a slave in the contact pair, the box is meshed coarser than the pins. Also to avoid possible convergent problems in ANSYS simulation, the contact regions are meshed with nearly identical element size. The meshing parameters are listed in Table 2 and the mesh diagram is shown in Figure 6. As a result, the whole model contains 35,920 elements.
Figure 5    Thread mesh (see online version for colours)

![Thread mesh](image)

Table 2    Mesh sizes of the model

<table>
<thead>
<tr>
<th></th>
<th>Global</th>
<th>Root</th>
<th>Crest</th>
<th>Flank elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin (in)</td>
<td>WT / 6 = 0.0358</td>
<td>0.00358</td>
<td>0.00179</td>
<td>35</td>
</tr>
<tr>
<td>Box (in)</td>
<td>0.0358</td>
<td>0.0007</td>
<td>0.0035</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 6    Mesh of the threaded connection (see online version for colours)

![Mesh of the threaded connection](image)
3 Static stress analysis

3.1 Tensile load

A drill-string shown in Figure 7 is subjected to its self-weight, hook load, hydrostatic force and weight on bit. Initially, it is under static condition and these forces act only in the longitudinal direction. An initial longitudinal deformation will be expected under the static load. The static stress inside the drill string will vary with the position and is calculated by the following two equations:

\[
\sigma_{\text{pipe}}(x) = \frac{F_h - \rho A_p g (L - x)}{A_p} \quad (1)
\]

\[
\sigma_{\text{collar}}(x) = \frac{\rho A_c g x - W_s - \rho_{\text{mud}} g L A_c}{A_c} \quad (2)
\]

where \( x \) represents the location in the drill-string, \( F_h \) is the hook load and assigned the value of \( 7 \times 10^5 \) N, the static weight on bit is represented as \( W_s \) with a assigned value of \( 1 \times 10^5 \) N. \( A_c \) and \( A_p \) are the cross-section areas of drill-collar and drillpipe respectively. They have the values of 0.0188 m\(^2\), and 0.0028 m\(^2\). \( \rho = 7.85 \times 10^3 \) kg/m\(^3\) and \( \rho_{\text{mud}} = 1.5 \times 10^3 \) kg/m\(^3\) represents the densities of steel and drilling mud, respectively.

Figure 7 Static forces of a drill-string (see online version for colours)
A diagram demonstrating this varying static force against the drill-string’s length is depicted in Figure 8. In this paper, six representative locations are analysed. Considering that the length of the threaded connection is relatively small compared to the drill-pipe, the forces at the two ends of the threaded connection are considered to have the same value. As can be seen on Figure 8, the drill-pipe section is in tension while the drill-collar is in compression.

**Figure 8** Static stress inside a drill-string (see online version for colours)

### 3.2 Contact

The pin and the box are pre-tightened up; this is called the make-up step. The make-up turns can be simulated by an initial interference between the contact regions of the pin and the box (Zhong, 2007). The initial interference δ is related to the make-up turns t, through the equation below (Van Wittenberghhe, 2011):

\[
\delta = tP \tan(\alpha)
\]

(3)

where \(P\) denotes the thread pitch and \(\alpha\) the taper angle whose value is 0.0625° (API 5B Specifications, 2008). The number of make-up turns can be in the range of one to two turns (API 5B Specifications, 2008). In this paper, it is set as two turns, giving \(\delta = 0.15625°\).
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The pin and the box experience friction between the threads under tensile loads. This is considered in ANSYS by setting the contact type as ‘frictional’. Researchers have used various values for the coefficient of friction (COF), ranging from 0.02–0.20. In this study, it is set as 0.12 which lies in the ISO 10407-1 range of 0.06–0.14. This COF can effectively avoid separation of the threads.

3.3 Boundary conditions

The boundary conditions are modelled by two equivalent springs, representing the drill-pipes linked to the two ends of the threaded connection. The value of the stiffness $K$ of the equivalent spring in drill-pipe section depends on the length of the linked drill-pipes, which can be calculated by:

$$K_{upper} = K_{pipe}(L-x)$$  \hspace{1cm} (4)$$

$$K_{lower} = \frac{1}{\frac{1}{K_{collar}} + \frac{1}{K_{pipe}(x-L_c)}}$$  \hspace{1cm} (5)$$

where $K_{collar}$ and $K_{pipe}$ are the equivalent stiffness of the drill-collar and drill-pipe respectively. They are given by:

$$K_{collar} = \frac{A_cE}{L_c}$$  \hspace{1cm} (6)$$

$$K_{pipe} = \frac{A_pE}{x}$$  \hspace{1cm} (7)$$

where $x$ denotes the location of the connection. For the six different locations considered in this paper, the stiffness linked to the two ends are listed in Table 3.

<table>
<thead>
<tr>
<th>Locations $x$ (m)</th>
<th>300</th>
<th>700</th>
<th>1,100</th>
<th>1,500</th>
<th>1,900</th>
<th>2,300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper spring ($10^6$ N/m)</td>
<td>2.635</td>
<td>3.220</td>
<td>4.140</td>
<td>5.796</td>
<td>9.660</td>
<td>28.980</td>
</tr>
<tr>
<td>Lower spring ($10^6$ N/m)</td>
<td>14.567</td>
<td>7.264</td>
<td>4.839</td>
<td>3.627</td>
<td>2.901</td>
<td>2.417</td>
</tr>
</tbody>
</table>

3.4 Results of static stress analysis

The ANSYS simulation contains two loading steps:

Step 1 Make up, which sets the initial interference between the threads of pin and box.

Step 2 Tensile load, which simulates the internal static loadings added to the two ends of the threaded connection.

The calculation results at the position of $x = 1,500$ m are given in Figures 9 and 10. Figure 9 represents the results of step 1 and Figure 10 gives results of step 2.
Figure 9: Stresses at location $x = 1,500$ m after loading step 1 (see online version for colours)
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Figure 10  Stresses at location $x = 1,500$ m after loading step 2 (see online version for colours)
Several observations can be made on step 1 results:

1. The axial stress at the root of the LET of the two pins are the highest while the centre part of the pin has compressive stress. It is also found that a high axial stress exists at the root of the pin’s run-out region.

2. Owing to the bending effect, the stress changes from tensile inside to compressive outside.

3. The radial stress is relatively insignificant compared to the axial and the hoop stress, since no restraint is assumed to the right-hand side of the box.

4. The hoop stress in the pins is compressive while in the box it is tensile. This is because the pins serve as master part of the contact pair, which is forced to move toward the box. Thus, the pins are compressed while the box expands in the z direction.

5. The equivalent stress of the box is generally higher mainly due to the large hoop stress.

From Figure 10, the following observations are made:

1. The axial stress at the root of the LET of the two pins is higher than that of the make-up step. However, the axial stress at the run-out region of the pins is relieved smaller than that of the make-up step. This is the consequence of tensile load.

2. The maximum stress is located in the lower pin. This is due to the asymmetric springs attached to the two ends. The lower spring is stiffer than the upper one which results in larger deformation inside the lower pin.

3. The hoop stress affects the equivalent stress intensively.

4. Critical position of the threaded connection is the LET of the lower pin in terms of the maximum equivalent stress.

The results of equivalent stress from the static stress analysis will be used as mean stress for the fatigue calculation in the following sections.

4 Random vibration analysis

4.1 Modal analysis

The free vibration of the drill string is represented by:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

(8)

where $[M]$, $[K]$ are mass matrix and stiffness matrix of the structure. For the linear system, it can be assumed that $\{u\} = \{\phi\}e^{i\omega t}$, where $\{\phi\}$ is the mode shape vector and $\omega$ is the circular natural frequency. Thus, equation (8) becomes:

$$(-\omega^2[M] + [K])\{\phi\} = \{0\}$$

(9)
Solution to the determinant: \(|-\omega^2[M] + [K] = 0|\) gives the natural circular frequency \(\omega\) of the threaded connection. Then the natural frequency \(f\) with unit in Hz is given by:

\[
f = \frac{\omega}{2\pi}
\]

For the threaded connections at the six specific locations, the first three natural frequencies are listed in Table 4.

### Table 4 Stiffness values of equivalent springs linked to the threaded connection

<table>
<thead>
<tr>
<th>Locations (m)</th>
<th>300</th>
<th>700</th>
<th>1,100</th>
<th>1,500</th>
<th>1,900</th>
<th>2,300</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st N.F (Hz)</td>
<td>225.63</td>
<td>176.28</td>
<td>163.16</td>
<td>167.14</td>
<td>192.9</td>
<td>304.26</td>
</tr>
<tr>
<td>2nd N.F (Hz)</td>
<td>5,728.9</td>
<td>8,637.1</td>
<td>8,393.7</td>
<td>8,230.3</td>
<td>8,058.6</td>
<td>7,849.2</td>
</tr>
<tr>
<td>3rd N.F (Hz)</td>
<td>12,812</td>
<td>12,693</td>
<td>12,290</td>
<td>12,002</td>
<td>11,708</td>
<td>11,348</td>
</tr>
</tbody>
</table>

### 4.2 Random vibration method

During drilling, the unevenness of the rock formation at bottom constitutes a source of random excitation to the drill string. We use the “base excitation” in the following analysis. First, the dynamic equations of motion is expressed as the free and the restrained DOF (Kohnke, 2003):

\[
\begin{bmatrix}
M_{ff} & M_{fr} \\
M_{rf} & M_{rr}
\end{bmatrix}
\begin{bmatrix}
\{\ddot{u}_f\} \\
\{\ddot{u}_r\}
\end{bmatrix}
+
\begin{bmatrix}
C_{ff} & \{C_{fr}\} \\
C_{rf} & C_{rr}
\end{bmatrix}
\begin{bmatrix}
\{\ddot{u}_f\} \\
\{\ddot{u}_r\}
\end{bmatrix}
+
\begin{bmatrix}
K_{ff} & \{K_{fr}\} \\
K_{rf} & K_{rr}
\end{bmatrix}
\begin{bmatrix}
\{u_f\} \\
\{u_r\}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
\{F\}
\end{bmatrix}
\tag{11}
\]

where \(\{u_f\}\) are the free DOF and \(\{u_r\}\) are the restrained DOF (excited by random loading). \(\{F\}\) is the nodal force excitation. The free displacements can be decomposed into pseudo-static and dynamic parts as:

\[
\{u_f\} = \{u_s\} + \{u_d\}
\tag{12}
\]

The pseudo-static displacements can be obtained from equation (11) by excluding the first two terms on the left-hand side of the equation and replacing \(\{u_f\}\) by \(\{u_r\}\):

\[
\{u_r\} = -[K_{ff}]^{-1}[K_{fr}][u_f] = [A][u_r]
\tag{13}
\]

where \([A] = -[K_{ff}]^{-1}[K_{fr}].\) Then substituting equations (12) and (13) into equation (11) and assuming light damping yields:

\[
[M_{ff}][\ddot{u}_s] + [C_{ff}][\ddot{u}_s] + [K_{ff}][u_s] = -([M_{ff}[A] + [M_{fr}])[\ddot{u}_r]
\tag{14}
\]

where the right-hand side represents the equivalent force due to base excitations.

Using the mode superposition method, the dynamic displacement can be written as:

\[
\{u_d(t)\} = [\phi]\{y(t)\}
\tag{15}
\]

where \([\phi]\) is the mode shape vector obtained from modal analysis and \(\{y(t)\}\) is the modal coordinate vector. Using the above equation, then equation (14) is decoupled to be:

\[
\ddot{y}_j + 2\zeta_j\omega_j\dot{y}_j + \omega_j^2y_j = G_j, \quad (j = 1, 2, 3, \ldots, n)
\tag{16}
\]
where \( n \) is the number of mode shapes chosen for evaluation. \( y_j \) is the generalised displacements, \( \omega_j \) is circular natural frequencies and \( \zeta_j \) is modal damping ratios. Also, modal loads \( G_j \) are defined as:

\[
G_j = \{ \Gamma_j \}^T \{ \ddot{u}_j \}
\]

(17)

where \( \{ \Gamma_j \} \) are modal participation factors given by:

\[
\{ \Gamma_j \} = -\left( \begin{bmatrix} M_g \end{bmatrix} \{ A \} + \begin{bmatrix} M_g \end{bmatrix} \{ \phi \} \right) \{ \phi \}
\]

(18)

By assuming PSD of the acceleration \( \{ \ddot{u}_j \} \), response to the random excitation can be evaluated.

### 4.3 Random vibration input

The base excitation is assumed as a combination of two components. One is related to the rotary speed as a deterministic cosine wave; the other represents the random effects.

\[
\ddot{u}_o(t) = A_d \cos(\omega t) + w(t); \quad \omega = 2\pi f_c = 2\pi \frac{3N}{60}
\]

(19)

where \( A_d \) is the amplitude of cosine wave. \( w(t) \) is the random part of the acceleration assumed as a white noise. \( f_c \) represents the rotary table critical frequency. Then its two-sided PSD can be expressed as:

\[
G_w(f) = \frac{A_w^2}{2} \delta(f - f_c) + \frac{A_f^2}{2}; \quad f \in [0, +\infty]
\]

(20)

where \( A_w \) is a constant representing the maximum amplitude of random acceleration. \( f \) denotes the frequency in unit Hz.

Based on existing downhole measurement (Deily et al., 1968), the amplitude of dynamic acceleration ranges from 0–4 G, where G means the gravity acceleration. Then the PSD level of the cosine wave will be \( \frac{A_d^2}{2} \), which ranges from 0–8 G^2/Hz. For the white noise \( A_w \), it can be assumed as 10% of the cosine wave amplitude \( A_d \), as 0–0.4 G, which results in PSD level \( \frac{A_w^2}{2} \) as 0–0.08 G^2/Hz.

### 5 Fatigue damage calculation

#### 5.1 Damage evolution law

Damage evolution law (DEL) is an appropriate method to calculate fatigue damage. It was proposed by Wahab et al. (2001), based on an energy criterion. In Van Wittenberghè’s (2011) work, the material parameters of the DEL were determined using the experimental data from the standard API line pipe connection. The resulting DEL is expressed by:
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\[ N = 2.95 \times 10^6 \cdot \Delta \sigma_{eq}^{-1.26} \cdot R_p^{4.12} \]  \hspace{1cm} (21)

\[ R_v = \frac{2}{3} (1 + \nu) + 3(1 - 2\nu) \left[ \frac{\sigma_H}{\sigma_{eq}} \right]^2 \]  \hspace{1cm} (22)

In the above equation, \( \Delta \sigma_{eq} \) is the nominal equivalent stress satisfying \( \Delta \sigma_{eq} = \Delta / SCF \). Here, the SCF is the stress concentration factor and \( \Delta \sigma \) is the equivalent stress obtained from ANSYS. The SCF is calculated by the ratio between the maximum axial stress value and the applied tensile load. The values calculated for the six positions are given in Table 5. All the values fall in the range between 1.8 to 6.5, which is concluded by Van Wittenberghe’s (2011) work. \( \nu \) denotes the Poisson ratio, \( \sigma_H \) is the hydrostatic stress obtained from ANSYS.

<table>
<thead>
<tr>
<th>Locations (m)</th>
<th>300</th>
<th>700</th>
<th>1,100</th>
<th>1,500</th>
<th>1,900</th>
<th>2,300</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF</td>
<td>5.964</td>
<td>4.246</td>
<td>3.323</td>
<td>3.573</td>
<td>5.268</td>
<td>6.583</td>
</tr>
</tbody>
</table>

5.2 Mean stress correction

The effect of the mean stress is accounted for by the Goodman correction equation:

\[ \left( \frac{S_d}{S_u} \right) + \left( \frac{S_m}{S_u} \right) = 1 \]  \hspace{1cm} (23)

where \( S_d \) denotes the nominal equivalent stress amplitude, the mean stress \( S_m \) is obtained from static stress analysis section, \( S_u \) is the material’s ultimate strength, which is 521 MPa, and \( S_n \) is the corrected fatigue stress amplitude.

5.3 Random fatigue

Three-band technique is used for evaluating the random vibration fatigue (Steinberg, 1988). It assumes that the equivalent stress (E.S) follows the Gaussian distribution; thus, the resulting E.S within \( 1\sigma \) is assumed at the level of 68.3\%, E.S within the \( 2\sigma \) is at the level of 27.1\%, and the E.S within the \( 3\sigma \) is at the level of 4.33\%. The computation results for the connection at different locations under the random intensity of 0.08 G^2/Hz are presented in Figure 11.

Then, the number of cycles causing failure (\( N_i \)) is calculated with the DEL method by substituting the \( 1\sigma, 2\sigma, \) and \( 3\sigma \) corrected equivalent stress ranges into equation (21). According to the damage accumulative law, the fatigue damage ratio is given as:

\[ D = \sum_{i=1}^{3} \frac{n_i}{N_i} \]  \hspace{1cm} (24)

where \( i = 1, 2, 3 \) relates to \( 1\sigma, 2\sigma \) and \( 3\sigma \) stresses. \( n_i \) is the actual number of fatigue cycles generated in specific exciting time \( T \) (in sec):
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\[ n_1 = \nu \cdot T \cdot 0.683 \] (25)
\[ n_2 = \nu \cdot T \cdot 0.271 \] (26)
\[ n_3 = \nu \cdot T \cdot 0.0433 \] (27)

where \( n \) is the stochastic stress cycling number within unit time, namely the equivalent frequency. It is calculated by:

\[ \nu = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \] (28)

where \( m_2 \) and \( m_0 \) are spectral moments of the stress PSD \( G(\omega) \):

\[ m_2 = \int_0^\infty G(\omega) d\omega \] (29)
\[ m_0 = \int_0^\infty \omega^2 G(\omega) d\omega \] (30)

Figure 11  Equivalent stress obtained from ANSYS under random input (see online version for colours)

6  Results analysis and conclusions

All the simulation results indicate that the critical position in a threaded connection is always at the LET of the lower pin; thus, the analyses in this section are all based on this critical position. In this section the effect of the intensity of the random excitation, the location of the connections and the static load to which the connection is subjected are analysed.
Keeping the other factors the same and simulating with three white noise intensity as 0.04, 0.06 and 0.08 G²/Hz, the damage levels for different locations are shown in Figure 12. From the figure, we can make the following observations.

1. The random load intensity has an obvious effect on the fatigue of the connections. The stronger the random load, the severer the damage to the fatigue strength of the connections.

2. Even though the fatigue damage varies with the location; however, in the cases simulated the most damage happens at near \( x = 1,500 \) m. Common sense indicates larger stress level may cause larger fatigue damage. However, the S-N curve used in fatigue evaluation accounts for the tri-axial effect \( R_\nu \) by equation (22), which may have different effects from the individual stress values. The fatigue damage is a combined effect of both stress level and tri-axiality function \( R_\nu \).

In order to check the effect of the static tensile load, we take the connection at \( x = 700 \) m as an example. At this position, the stiffness of the upper spring and the lower spring are given in Table 3. The random input is set as 0.08 G²/Hz, and five different tensile loads (56.3, 84.15, 112.6, 140.75, 168.9 MPa) are simulated. The corresponding results of fatigue damage are illustrated in Figure 13. It can be seen that with the increase of tensile load the fatigue damage increases monotonously. This is in agreement with common sense.

In the last, keeping the static tensile load as 112.6 MPa, we compute the fatigue at the six different locations \( x = 300, 700, 1,100, 1,500, 1,900, 2,300 \) m. Using equations (4) to (7), the stiffness of the upper spring and the lower spring can be calculated and the results are as shown in Table 6. The calculation results of the fatigue are given in Figure 14. Obviously the fatigue damage decreases monotonously with the location changes from the bottom to the top. However, on the upper part, the fatigue damage is much severer than at the bottom. This is logical given that the top part is in general in tension and the bottom in compression and tension stress does more damage to fatigue than compression stresses.
Figure 13  Tensile load effects on threaded connection at the same location $x = 700$ m (see online version for colours)

![Tensile load effects on fatigue damage](image)

Table 6  Examined stiffness pair values

<table>
<thead>
<tr>
<th>Locations (m)</th>
<th>300</th>
<th>700</th>
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<td>3.627</td>
<td>2.901</td>
<td>2.417</td>
</tr>
</tbody>
</table>

Figure 14  Location effects on threaded connection under the same tensile load 112.6 MPa (see online version for colours)

![Location effects on fatigue damage](image)
Based on the above analysis, the following conclusions are drawn:

1. The threaded connection’s critical position lies almost always in LET of the lower pin under random loading.

2. The static tensile load added to the threaded connection affects its fatigue damage monotonously. The more tensile load added, the more fatigue damage it will suffer.

3. The stronger the random excitation, the severer the damage to the fatigue without regard to the location of the connections.

4. The location of the connections also has an obvious effect on the fatigue damage. From the viewpoint of vibration, the stiffness of the equivalent springs determines the natural frequencies of the system, therefore, the transmission rate of the random excitation is affected by the natural frequency. As a result, no general conclusion can be drawn as to the specific position for the most fatigue damage. In the case of the simulation in this paper it is at about $x = 1,500$ m.

References


