A fuzzy economic order quantity model with credibility induced demand and shortages

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Abstract: The purpose of this study is to explore the research for a decision-maker to minimise the total cost of the newly launched products of good quality and to control the inventory introduced in the market. This research provides the modification way of the stock to regulate the inventory management of the retailer. Due to the uncertainty in the marketplace, various parameters like a shortage, ordering, and deteriorating cost not fixed in nature. So, it is difficult to obtain the proper estimate of such costs. In this paper, we develop a traditional economic order quantity model with a fuzzy approach and provide a suitable structure to handle such uncertain parameters, improving the exactness and computational efficiency of the inventory system. The optimal solution of the problem is done with the method of graded mean integration. The model illustration is demonstrated through the appropriate numerical and sensitivity analysis.

Keywords: graded mean integration technique; credibility induced demand; procurement time; order quantity; deterioration; uncertainty; optimisation.

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1 Introduction

In general, inventory management indecisions may depend on several relevant costs, the demand rate and lead-time. Although, in predictable inventory-models, uncertainties considered as randomness, which is handled by stochastic-techniques. Many decision-making problems become too complicated due to uncertainty in the form of vagueness. The fuzzy set approach (FSA) is a useful tool to control uncertainty. FSA helps to
incorporate unpredictability in decision-making process. Nowadays, FSA used in many fields of science and technology. In particular, researchers take different parameters of an inventory model for the expansion of the economic order quantity models either as constant or probabilistic in nature. However, these parameters can have minor deviations from the accurate value in real-life situations. So the probability distribution is not suitable in such types of conditions. But in this situation, the parameters can be treated with fuzzy, and it will become more realistic, and these types of problems handled by using an appropriate fuzzy technique, and the solution procedure may be achieved as usual. Due to uncertainty in various mathematical formulations in the inventory model, researchers introduced the fuzzy inventory model by using different fuzzy numbers. A fuzzy number is an amount whose values are vague, rather than exact. Zadeh’s Extension Principle introduced the normal arithmetic operations with real numbers, which extended to fuzzy numbers. Due to fault in measuring method, machine fault.

Several records in our observation cannot be correctly found. This leads to a new type of Fuzzy number called the pentagonal fuzzy numbers (PFN’s) being introduced. A fuzzy pentagonal number (PFN) is a 5-tuple subset with five points of a real number R.

This research comprises a fuzzy inventory model under shortages with a constant deterioration rate of the product, whereas; demand is directly proportional to the time and Credibility factor. In order to optimise the standard total cost (STC), shortage-cost, deterioration-cost, ordering-cost, and holding costs are fuzzified by using a PFN, and graded mean technique is used. Further, the sensitivity analysis helps to analysis the effect of unalike parameters on STC in both fuzzy and crisp sense.

The structure of the paper as follows: the literature review presented in Section 2. Some crucial definitions and preliminaries related to this research work described in Section 3. Section 4 introduced the notations and assumption related to the paper. The mathematical fuzzy inventory model developed in Section 5. A computation algorithm described in Section 6. The computational analysis through a numerical example with a comparison graph between the crisp and fuzzy inventory model presented in Section 7, and sensitivity analysis showing the effect of various parameters in Section 8, and the conclusion is in the last section.

2 Literature review

Chang et al. (1998) presented the economic order and backorder quantity model, Chauhan and Singh (2014) investigate optimal replenishments and ordering policy for the time-dependent demand rate. Chauhan and Singh (2015) developed a replenishment policy of the integrated demand pattern. Goyal et al. (2015) discussed the EPQ model for selling price-dependent demand. Several inventory models for decaying goods with different characteristics of the inventory organisations have been described; it is not feasible to list them individually. For an extensive study of inventory models with fading products, concerned researchers may pass on to review articles by Raafat (1991), and Janssen et al. (2016). Zadeh (1965) introduced the concept of fuzzy numbers in 1965 and gives an extension principle for arithmetic operations of fuzzy numbers. Further study of operations of fuzzy numbers is done by Dubois and Prade (1978). Due to uncertainty, they used fuzziness in their inventory model. Then Park (1987) introduced the EOQ technique with a fuzzy environment. Chang et al. (1998) presented the economic order and backorder quantity with the fuzzy approach, the model in the fuzzy environment; and
observed that total cost is more significant than with the crisp model; on the other hand, it permits better use of the economic fuzzy quantities arising with changes in sales, deliveries, and orders. An optimal total cost for healthcare industries for three types of variable demand with fuzzy parameters considered by Rajput et al. (2019). Kazemi et al. (2017) established a fuzzy model in which, fuzzified the input parameters, and used a grated mean integration technique. Lee and Yun (2014) define the fuzzy pentagonal sets and simplify the addition, subtraction, multiplication, and division for two fuzzy pentagonal sets. Panda and Pal (2015) introduced a PFN with its arithmetic operations. They also described a particular type of pentagonal fuzzy matrices. Raj and Karthik (2016) use a PFN to choose the best machine for a job by feed-forward neural network. A model with a shortage that completely backlogged is introduced by Rajalakshmi and Rosario (2017) with a fuzzy approach and compared the total cost using triangular, trapezoidal, PFN’s, and Graded mean method used for defuzzification. Kamble (2017) described different operations of a particular type of PNF’s described by Mondal and Mandal’s (2017) research work, also carried out pentagonal fuzzy solutions for the fuzzy equation. Shekarian et al. (2017) studied the Comprehensive review of the fuzzy inventory models.

3 Definition and preliminaries

**Definition 3.1 (Fuzzy set (Rajput et al., 2019)):** A set $\tilde{N}$ is a fuzzy set, if his membership function, from the domain $U$ mapped into $[0, 1]$, i.e., $\tilde{N} = \{u, \mu_\tilde{N}(u): u \in U\}$. Here the membership function is $\mu_\tilde{N}: \tilde{N} \to [0,1]$ and the grade value of $u \in U$ is $\mu_\tilde{N}(u)$ in a fuzzy set $\tilde{N}$.

**Definition 3.2 (Fuzzy number (Rajput et al., 2019)):** Let $\tilde{N}$ is a subset of real set R, with the membership function $\mu_\tilde{N}$ is said to be a fuzzy number if

- $\mu_\tilde{N}(u)$, piecewise continuous in its domain.
- $\tilde{N}$ (normal), i.e., $\mu_\tilde{N}(u_o) = 1$ for $u_o \in \tilde{N}$.
- $\tilde{N}$ (convex), i.e., $\mu_\tilde{N}(\lambda u_1 + (1-\lambda)u_2) \geq \min(\mu_\tilde{N}(u_1), \mu_\tilde{N}(u_2)) \quad \forall u_1, u_2 \in U$.

**Definition 3.3 (Pentagonal fuzzy number (PNF’s) (Mondal and Mandal, 2017)):**

The following conditions should be satisfied for a fuzzy pentagonal number $\tilde{N} = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$

- $\mu_{\tilde{N}}(x)$, a continuous function in the interval $[0,1]$.
- $\mu_{\tilde{N}}(x)$, strictly increasing in $[\eta_1, \eta_2]$ and $[\eta_2, \eta_3]$.
- $\mu_{\tilde{N}}(x)$, strictly decreasing in $[\eta_3, \eta_4]$ and $[\eta_4, \eta_5]$.

The membership function of a PFN $\tilde{N} = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$ is of the form
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In the case of PFN, Figure 1, the membership value is obtained for a particular range.

**Figure 1** Graphical representation of PFN’s

**Definition 3.4** (α-Cut of PFN’s (Mondal and Mandal, 2017)): Let PFN \( \hat{N} = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5) \) then the α-Cut operation on PFN’s is defined as

\[
L_\alpha(\hat{N}) = [2\alpha(\eta_2 - \eta_1) + \eta_1, -2\alpha(\eta_4 - \eta_3) + \eta_3],
\]

\[
R_\alpha(\hat{N}) = [2\alpha(\eta_3 - \eta_2) + 2\eta_2, 2\alpha(\eta_4 - \eta_3) - \eta_3 + 2\eta_4].
\]

**Definition 3.5** (Graded mean method (GMI) (Rajput et al., 2019)): Let \( \hat{N} = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5) \) be a PFN, then GMI representation of \( \hat{N} \) as

\[
G(\hat{N}) = \frac{1}{2} \int_0^1 \alpha[L_\alpha(\hat{N}) + R_\alpha(\hat{N})] d\alpha
\]

**Definition 3.6** (Fuzzy arithmetical operations for PFN’s (Rajput et al., 2019)): Let \( \tilde{\tau} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) \) and \( \tilde{\omega} = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) \) are two PFN’s where \( \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5 \) are positive real number. Note that every PFN is associated with two weights: \( v_1 \) and \( v_2 \).

- \( \tilde{\tau} \oplus \tilde{\omega} = (\tau_1 + \omega_1, \tau_2 + \omega_2, \tau_3 + \omega_3, \tau_4 + \omega_4, \tau_5 + \omega_5) \),
  with \( v_i(\tau_i + \omega_i) \geq \max(v_{i\text{old}}, v_{i\text{new}}) \) for \( i = 1, 2 \)
• $\ddot{\tau} \otimes \ddot{\omega} = (\tau, \omega, \tau, \omega, \tau, \omega, \tau, \omega, \tau, \omega)$, with $V_{i(t, \omega)} \geq \max(V_{i(t)}, V_{i(\omega)})$ for $i = 1, 2$

• $\ddot{\tau} \Theta \ddot{\omega} = (\tau - m_1, \tau - m_2, \tau - m_3, \tau - m_4, \tau - \omega_5)$,
with $V_{i(t, \omega)} \geq \max(V_{i(t)}, V_{i(\omega)})$ for $i = 1, 2$

• $\ddot{\tau} + \ddot{\omega} = \left(\frac{\tau}{\omega_1}, \frac{\tau}{\omega_2}, \frac{\tau}{\omega_5}, \frac{\tau}{\omega_3}, \frac{\tau}{\omega_4}\right)$

• $\alpha \ddot{\tau} = (\alpha \tau_1, \alpha \tau_2, \alpha \tau_3, \alpha \tau_4, \alpha \tau_5)$, where $\alpha$ is any positive real number.

4 Notation and assumption

Assumptions

• The system has a finite-time horizon.
• The model deals with a single type of product.
• The replenishment time of items is instantaneous and finite.
• Demand influenced by time as well as credibility, i.e., $D(t) = \rho t \kappa$ here $\rho$ and $\kappa$ are the demand parameter.
• Deterioration rate is assumed to be constant, and no replacement provision for such item.
• Shortages completely backlogged.
• Ordering, holding, shortage, and deterioration cost are considered as fuzzy parameters.
• The lead time (delivery time) is negligible.

Notations

• $I(t)$: Inventory level at any time $t$ in the interval $[0, T]$.
• $I_{\text{max}}$: The greatest level of inventory.
• $T$: Fixed duration for the ordering-cycle.
• $I_0$: Initial ordering-quantity.
• $\ddot{\delta}$: Fuzzy shortage cost per unit time.
• $\ddot{\beta}$: Fuzzy deterioration cost per unit time.
• $t_0$: Procurement time.
• $\ddot{\gamma}$: Fuzzy holding cost per unit time.
• $\ddot{\alpha}$: Fuzzy ordering-cost per order.
• $\rho$: Shape parameter of demand factor.
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- $\kappa$: Credibility factor of item for the inventory system, i.e., $\kappa$ of perfect item present in the inventory.
- $\phi$: Deterioration rate; ($0 < \phi < 1$)
- STC ($t_n$): Standard total cost per unit time.
- $t_n^*$: Optimal procurement time.
- STC* ($t_n^*$): Optimal standard total cost per unit time.

5 Formulation fuzzy inventory model

Initially, there is no inventory to the retailer, there are only advance booking of item. Due to the continuous effect of demand, the shortages occur in $[0, t_n]$. At time $t_n$, the inventory is replenished. Then, the deterioration of the product will start in $[t_n, T]$. Finally, the inventory point reaches zero at $T$, shown in Figure 2.

**Figure 2** Inventory level with respect to time $t_n$.

Governing differential equations are

$$\frac{dI(t)}{dt} = -\rho t \kappa', \quad 0 \leq t \leq t_n$$

(1)

$$\frac{dI(t)}{dt} + \phi I(t) = -\rho t \kappa', \quad t_n < t \leq T$$

(2)

Subject to boundary conditions are $I(0) = 0$ and $I(T) = 0$. Solution of the equations (1) and (2) we obtain:

$$I(t) = -\rho [\kappa'(t \log[k] - 1) + 1]$$

$$\log[k]^2$$

(3)
\[ I(t) = \frac{\rho((\kappa^+ + \kappa^-) - (\log[\kappa] + \phi)(t\kappa^+ + T\kappa^-))}{(\log[\kappa] + \phi)^2} \]  

(4)

at \( t = t_n \) we get the maximum inventory level \( I_{\text{max}} \) for each ordering rotation, i.e.,

\[ I_{\text{max}} = \frac{\rho((\kappa^+ + \kappa^-) - (t_n\kappa^+ + T\kappa^-)(\phi + \log[\kappa]))}{(\log[\kappa] + \phi)^2} \]  

(5)

Backlogged demand \( (b) = \int_{t_n}^{t} \rho t \kappa' dt = \frac{\rho}{\log[\kappa]} [\kappa^+ (-1 + t_n \log[\kappa]) + 1] \]  

(6)

Initial inventory \( I_0 = I_{\text{max}} + b = \frac{\rho((\kappa^+ + \kappa^-) - (\phi + \log[\kappa])(t_n\kappa^+ + T\kappa^-))}{(\phi + \log[\kappa])^2} + \frac{\rho [\kappa^+ (t_n \log[\kappa] - 1) + 1]}{(\log[\kappa])^2} \]  

(7)

The relevant costs associated with model are such as:

- The ordering cost (OC):
  \[ \text{OC} = \tilde{d} \]  
  (8)

- The deteriorating cost (DC) for the interval \([t_n, T]\)
  \[ = \tilde{\beta} \otimes \left\{ \frac{\rho((\kappa^+ + \kappa^-) - (\phi + \log[\kappa])(T\kappa^+ + t_n\kappa^-))}{(\phi + \log[\kappa])^2} \right\} \int_{t_n}^{T} \rho t \kappa' dt \]
  \[ = \tilde{\beta} \otimes \left\{ \frac{\rho((\kappa^+ + \kappa^-) - (\phi + \log[\kappa])(t_n\kappa^+ + T\kappa^-))}{(\phi + \log[\kappa])^2} + \frac{\rho (\phi + \log[\kappa])(t_n\kappa^+ + T\kappa^-)}{\log[\kappa]^2} \right\} \]  
  (9)

- The inventory holding cost (HC) for the interval \([t_n, T]\)
  \[ \tilde{\gamma} \otimes \int_{t_n}^{T} I(t) dt = \tilde{\gamma} \otimes \int_{t_n}^{T} \frac{\rho((\kappa^+ + \kappa^-) - (\phi + \log[\kappa])(t\kappa^+ + T\kappa^-))}{(\phi + \log[\kappa])^2} dt \]
  \[ = \tilde{\gamma} \otimes \rho \left( T\kappa^+ + \kappa^- - \kappa^+ \right. \]
  \[ \left. \frac{\kappa^+ - \kappa^+}{\log[\kappa]} + \frac{(\phi + \log[\kappa])(\kappa^+ (\log[\kappa]t_n - 1) + \kappa^+ (1 - T \log[\kappa](1 + T \log[\kappa]) + T \log[\kappa] t_n))}{(\log[\kappa])^2} \right) \]  
  (10)

- The shortage cost (SC) in \([0, t_n]\)
  \[ = \tilde{\delta} \otimes \rho \left( 2 - 2\kappa^+ + (1 + \kappa^-) \log[\kappa] t_n \right) \]  
  (11)
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By using equations (8), (9), (10) and (11) we get the standard total cost (STC).

**Standard total cost** $STC(t_n)$

$$STC(t_n) = \frac{1}{12} \left( \frac{\alpha_1 + 3\alpha_2 + 4\alpha_3 + 3\alpha_4 + \alpha_5}{T} \right) + \frac{\beta_1}{T} + \beta_2 + \beta_3 + \beta_4 + \beta_5 \right) \left( \frac{\kappa^2 + \kappa^4}{(\phi + \log[\phi])^3} \right) \left( \phi^2 + \phi^4 - \rho \left( T \phi^2 + t_n \phi^4 \right) \log[\phi] \right)$$

$$+ \left( \frac{\gamma_1 + 3\gamma_2 + 4\gamma_3 + 3\gamma_4 + \gamma_5}{(\theta + \log[\phi])^2} \right) \left( T \phi^2 + \phi^4 - \phi^6 \right) \log[\phi]$$

$$- \phi^6 t_n \left( \left( \log[\phi] + \theta \right) \left( \phi^4 \log[\phi] t_n - 1 \right) + \phi^2 \left( 1 - T \log[\phi] + (T \log[\phi])^2 + T \log[\phi]^2 t_n \right) \right)$$

$$+ \left( \delta_1 + 3\delta_2 + 4\delta_3 + 3\delta_4 + \delta_5 \right) \frac{\rho (2 - 2\phi^4 + (1 + \phi^4) \log[\phi] t_n)}{(\log[\phi])^3}$$

(12)

6 **Computational algorithm for optimal criterion**

To get the optimal value of the standard total cost $STC(t_n)$ with respect to $t_n$ the following steps are used to minimise $STC(t_n)$.

**Step 1**: Start with the first derivative of $STC(t_n)$ of equation (12) and equate it to zero and get the critical point $t_n$.

**Step 2**: Take second derivative of $STC(t_n)$, i.e., $\frac{\partial^2 STC(t_n)}{\partial t_n^2}$.

**Step 3**: Use a critical point in step 2 we get the second derivative of total average cost will be positive at critical point, i.e., $\frac{\partial^2 STC(t_n)}{\partial t_n^2} > 0$, and hence $STC(t_n)$ is minimum at a critical point $t_n$. Shown in Appendix.

**Step 4**: Find the minimum value of $STC(t_n)$ by putting the value $t_n$ in equation (12).

7 **Computational analysis**

The advance booking of mobiles is started before it launched. After a fixed duration of time, the mobile phone comes to the market to replenish. The buyer wants the durability of the product for a long time. If mobile store for a long time, both system parameters like holding and ordering cost have highly impacted the order to customer service and the retailer’s profit. So we study the credibility with uncertain parameters such as holding-cost, deterioration-cost, ordering-cost, and shortage-cost. The numeric data result from a mobile retailer shop obtained from different resources in both cases as follows: the demand rate does not only depend on time but also on the credibility of the mobile quality. Therefore, the demand rate is taken as $D(t) = \rho t k^2$, where let $\rho = 0.01$. 
Thus, let a new product be presented in the marketplace, of which 80% are of acceptable quality. The ordering cost is $20 per unit per year. To hold the item, the retailer spends $2 per unit per year. The item undergoes deterioration with time at the rate of 1%, which costs $10 per unit per year. During shortage, part of the customers wait for item to come in the market with cost $5 per unit. The total system cycle is considering for four years.

Table 1  
Computational result and comparison of standard total cost (STC)

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Outcome under crisps</th>
<th>Outcome under fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 20$, $\beta = 10$, $\gamma = 2$, $\delta = 5$, $\rho = 0.01$, $T = 4$, $\kappa = 0.5$, $\phi = 0.01$</td>
<td>Optimal procurement time $t_n^c = 3.3638$ and optimal standard total cost $(STC^c) = $10.1716</td>
<td>Optimal procurement time $t_n^f = 3.0769$ and optimal standard fuzzy total cost $(SFTC^f) = $9.9189</td>
</tr>
<tr>
<td>$\bar{\alpha} = (18,19,19.5,20,21)$</td>
<td>$\bar{\beta} = (8.5,9,9.2,9.4,9.9)$</td>
<td>$\bar{\gamma} = (3,4,6,8,9)$</td>
</tr>
<tr>
<td>$\bar{\delta} = (3,4,6,8,9)$</td>
<td>$\bar{\phi} = (1.9,2,3,2.6,2.7)$</td>
<td>$\bar{\rho} = (0.01,0.01,0.01,0.01,0.01)$</td>
</tr>
</tbody>
</table>

By computational algorithm, the optimum standard total cost $(STC^c)$ for crisp model and fuzzy model obtained shown in Table 1, by using graded mean integration technique from equation (12), we get Optimal procurement time $t_n^c = 3.3638$ and optimal standard total cost $(STC^c) = $10.1716 with crisp model while Optimal procurement time $t_n^f = 3.0769$ and optimal standard fuzzy total cost $(SFTC^f) = $9.9189 for fuzzy. Moreover, the behaviour of the STC for both models shown in Figure 3.

Figure 3  
Behaviour of standard total cost (TC) in crisp and fuzzy (see online version for colours)

8  Sensitivity analysis

From Table 2 and Figure 4, the effect of demand factor $\rho$, deterioration parameter, and procurement time $t_n$ with STC examine and observed that as demand fluctuating, STC and procurement time $t_n$ would be relatively changed for both models. So, the model is
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affected by demand. The variation of the credibility factor $\kappa$, procurement time $t_n$, and STC will also change for both crisp and fuzzy models. Credibility factor has an impact on model. Procurement time $t_n$ and STC moderately affected by the deterioration factor $\phi$. Although, we observed STC is lesser in fuzzy than that of the crisp model.

Table 2  Effect of different parameter on standard total cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% change in parameter</th>
<th>(a) Under crisp model</th>
<th>(b) Under fuzzy model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand factor $\rho$</td>
<td>30%</td>
<td>3.3650 10.2233</td>
<td>3.0778 9.9688</td>
</tr>
<tr>
<td></td>
<td>15%</td>
<td>3.3643 10.1974</td>
<td>3.0773 9.9435</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td><strong>3.3638 10.1716</strong></td>
<td><strong>3.0769 9.9819</strong></td>
</tr>
<tr>
<td></td>
<td>−15%</td>
<td>3.3631 10.1458</td>
<td>3.0763 9.8928</td>
</tr>
<tr>
<td></td>
<td>−30%</td>
<td>3.3625 10.1200</td>
<td>3.0758 9.8676</td>
</tr>
<tr>
<td>Credibility factor $\kappa$</td>
<td>30%</td>
<td>4.7381 10.5265</td>
<td>4.3655 10.2511</td>
</tr>
<tr>
<td></td>
<td>15%</td>
<td>3.9604 10.2968</td>
<td>3.6335 10.0370</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td><strong>3.3638 10.1716</strong></td>
<td><strong>3.0769 9.9819</strong></td>
</tr>
<tr>
<td></td>
<td>−15%</td>
<td>2.8816 10.0998</td>
<td>2.6305 9.8489</td>
</tr>
<tr>
<td></td>
<td>−30%</td>
<td>2.8816 10.0998</td>
<td>2.6305 9.8489</td>
</tr>
<tr>
<td>Deterioration rate $\phi$</td>
<td>30%</td>
<td>3.3705 10.1721</td>
<td>3.0832 9.9186</td>
</tr>
<tr>
<td></td>
<td>15%</td>
<td>3.3671 10.1718</td>
<td>3.0800 9.9184</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td><strong>3.3638 10.1716</strong></td>
<td><strong>3.0769 9.9819</strong></td>
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<td></td>
<td>−15%</td>
<td>3.3603 10.1713</td>
<td>3.0736 9.9179</td>
</tr>
<tr>
<td></td>
<td>−30%</td>
<td>3.3570 10.1711</td>
<td>3.0704 9.9177</td>
</tr>
</tbody>
</table>

Figure 4  Effect of parameter on STC and FSTC: (a) demand factor; (b) credibility factor and (c) deterioration rate (see online version for colours)
9 Conclusion

This research paper is established for an economic order quantity model with uncertainty in cost factors. The concept of PFN introduced for inventory model due to uncertainty, and we observed that optimal standard total cost (FSTC*) under uncertain environment is a lesser than optimal standard total cost (STC*) in the crisp environment. We determined that uncertainty in decision variables is encouraging the decision maker to minimise the total cost.
We also concluded that the model is affected and worthwhile by the credibility factor in the demand. The study motivates us, how the retailers get additional profit in the inventory system with critical conditions like uncertain demand rate, cost, and the other factors. For, future prospect model will be extended with various factors like inflation, different demand pattern, channel financing with trade credit policy, it gives more advantages for suppliers and manufacturers.

References
Appendix

STC($t_n$) is minimum at critical point

Necessary condition

\[
\frac{\partial \text{STC}(t_n)}{\partial t_n} = \frac{\rho}{12T} \left[ \phi^\prime (\beta_1 + 3\beta_2 + 4\beta_3 + 3\beta_4 + \beta_5)(-1 + \rho)\theta^2 + (-1 + 2\rho)\theta \log(\phi) \right] \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \log(\phi) \right)^2 \left( \rho + \beta_1 \log(\phi) \right) \left( \rho + \log(\phi) + \theta \right) \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \chi + 3\gamma_1 + 4\gamma_3 + 3\gamma_4 + \gamma_5 \right) (T \phi^\prime (\theta + \log(\phi)) + \phi^\prime (-2 + \theta t_n)) \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \delta_1 + 3\delta_2 + 4\delta_3 + 3\delta_4 + \delta_5 \right) (1 + \phi^\prime (-1 + \log(\phi) t_n)) \right] = 0.
\]

As the given consideration $t_n \geq 0$.

Sufficient condition

\[
\frac{\partial^2 \text{STC}(t_n)}{\partial t_n^2} = \frac{\rho}{12T} \left[ \phi^\prime (\beta_1 + 3\beta_2 + 4\beta_3 + 3\beta_4 + \beta_5) \left( \log(\phi) + \theta \right) \right] \\
\log(\phi)(\log(\phi) + \theta)^2 \\
- \phi^\prime (\beta_1 + 3\beta_2 + 4\beta_3 + 3\beta_4 + \beta_5) \left( \rho + 1 + \rho \log(\phi) \right) \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \log(\phi) \right)^2 \left( \rho + \log(\phi) + \theta \right) \left( \rho + \log(\phi) + \theta \right) \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \chi + 3\gamma_1 + 4\gamma_3 + 3\gamma_4 + \gamma_5 \right) (\theta \phi^\prime + 2\phi^\prime \log(\phi) (-2 + \theta t_n)) \\
\log(\phi)(\log(\phi) + \theta)^2 \\
+ \left( \delta_1 + 3\delta_2 + 4\delta_3 + 3\delta_4 + \delta_5 \right) (\phi^\prime \log(\phi) + \phi^\prime \log(\phi) (-1 + \log(\phi) t_n)) \right] > 0
\]

is exits if

\[
0 < t_n < \frac{7 + 2\sqrt{11}}{5\log(\phi)} \quad \text{and} \quad 0 < \theta < \frac{1}{2} (-3\log(\phi) + \sqrt{5}\log(\phi)) \quad \text{where} \quad 0 < \phi < 1.
\]