Determination of distance advanced by the waterfront in a border strip land using Laplace transformation

Mahbub Hasan* and Aschalew Kassu

Department of Mechanical, Civil Engineering and
Construction Management,
College of Engineering, Technology and Physical Sciences,
Alabama A&M University,
Normal, AL 35762, USA
Email: Mahbub.hasan@aamu.edu
Email: aschalew.kassu@aamu.edu
*Corresponding author

Abstract: Among various methods of surface irrigation, border method is widely applicable to cultivate most cereal crops like wheat, maize etcetera and rice (Amiri et al., 2016). The waterfront advance data has to be synchronised with the infiltration depth, especially at the head end along with the consideration of sufficiency of infiltrated depth of water at the tail end. With that point of view, this study was undertaken to calculate the waterfront advance distance from the head end using Laplace transformation. Values for the constants for infiltration equation were collected and used in the Laplace transformation equations and found to match the calculated and actual waterfront advance data with a very reasonable discrepancy. The actual waterfront distance was only 0.51 m shorter than that was calculated by Laplace transformation. This might have caused due to soil cover, irregular tillage operation, huge potholes, and discontinuation of the uniformity of slope of the border strip. Application of this method will help to increase the water application efficiency with respect to flow size, border strip slope, depth of water requirement infiltrated in certain time to meet the crop water requirement.

Keywords: border irrigation; Laplace transformation; infiltration; border strip; waterfront advance; optimum irrigation water.

Reference to this paper should be made as follows: Hasan, M. and Kassu, A. (2021) 'Determination of distance advanced by the waterfront in a border strip land using Laplace transformation', *Int. J. Hydrology Science and Technology*, Vol. 11, No. 1, pp.26–37.

Biographical notes: Mahbub Hasan has been teaching in the Department of Mechanical, Civil Engineering, and Construction Management as an Associate Professor at the Alabama A&M University. He has very strong and diversified experiences in teaching, research, extension and subject matter expertise in hydrology, hydraulics, ground water, irrigation/drainage engineering, open channel hydraulics, pipeline water distribution system and water resources and management that are environmentally affirmative. Also, he possesses consulting experiences in Japan on water management, environmentally sound soil and water conservation structures and hydraulic structures design.

Aschalew Kassu is an Associate Professor in the Department of Mechanical, Civil Engineering, and Construction Management at the Alabama A&M University, USA. He was the recipient of the 2016/2017 'Faculty of the Year' award for 'Overall Excellence in Teaching, Research and Services' in the College of Engineering, Technology and Physical Sciences at the Alabama A&M University; and the 2015/2016 'Faculty of the Year' award for 'Excellence in Scholarship, Research, Creativity and Other Professional Contributions', both at the college level and university wide. He is an active member of the American Society of Civil Engineers (ASCE), American Society for Engineering Education (ASEE), and a senior member of the International Society for Optics and Photonics (SPIE).

1 Introduction

Efficient irrigation in a border stripped field needs proper matching and optimisation of different related factors where the knowledge of hydraulic characteristics plays the major role (Zahraei et al., 2017; Zerihun et al., 2005; Alazba, 1997; Parhi, 2016, 2017; Khanjani and Barani, 1999). Basic theory of application of irrigation water at the head end diverted initially into three basic physical components which are

a evaporation or evapotranspiration

b surface runoff along the length of the border strip

c infiltration.

Considering the interest of this research, we may exclude the consideration of evaporation or evapotranspiration. The remaining two other factors, surface runoff and infiltration need to be best optimised to compromise between the depth of root zone water requirement and length of water front advanced distance from the head end. Both these factors are again dependent on

a soil type

b slope along the length of the border strip

c vegetative coverage

d surface characteristics of the field

e crop grown

f crop growing stage

g existing moisture content.

The best management practice (BMP) ensures an application of water into the crop field with highest possible efficiency. The target of applying water into the field should satisfy the infiltration up to the root zone and that should ensure the maximum length of water front advancement simultaneously. In reality, the longer distance of waterfront in the border strip will cause an infiltration much deeper than the root zone depth at the head end of the border strip. That causes a great loss of water causing lower yield if the water holding capacity of the soil is poor. Hence, a careful consideration and decision on the flow size should be based on optimising these two factors of infiltration and length of waterfront advance distance. Recently, Hasan et al. (2015) developed an infiltration model by modified Kostiakov method and reported that the accumulated infiltration can be defined by equation (1). The values of the parameters of the infiltration model, a, α, and b are 9.12, 0.682, and 0.145, respectively.

$$y = at^{\alpha} + b \tag{1}$$

where

y	accumulated infiltration, cm
t	elapsed time, min
a, α, and b	characteristic constants.

This equation is valid for $0 \le \alpha \le 1$ and $t \ne 0$.

2 Hydraulics of border irrigation

BMPs in a border irrigation requires and depends on the technical know – how of the hydraulic characteristics of the irrigation stream. Flow in a border strip acts like an unsteady open channel flow with decreasing flow size. Flow size at the downstream decreases due to infiltration, and downward movement of water. The advance of waterfront is a function of

a elapsed time

b soil type (Dagadu and Nimbalkar, 2012)

c slope of the strip surface

d vegetative cover characteristic offing resistance to the flow.

A valid and viable analysis of border irrigation would require correct information on

a land slope

b infiltration

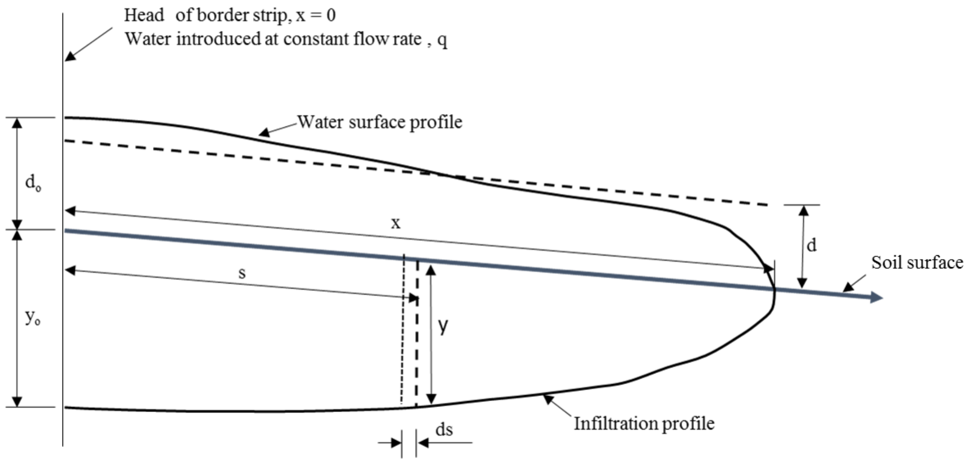
c roughness of soil surface

d vegetation

e depth of water on the velocity flow down the slope.

Prediction of advance rate of the border stream consists of the understanding and logical approach that the advance of the waterfront is a function of the elapsed time, the infiltration-time function, $y(t)$, the size of entrance stream, q, and the depth of flow, d, are known. As irrigation water moves forward into a field and approaches down its length, a continuously decreasing portion of the entire volume of water applied flows above the ground, while the rest infiltrates into the soil and constitutes the subsurface storage. The basic approach to equate the total volume of water discharged at the head end to be equal to the sum of subsurface storage that is infiltrated in to the soil profile and the surface storage which is represented by the runoff water on top of the soil surface and moves forward towards the tail end of the strip. Figure 1 shows a schematic diagram of water movement on top of the land and subsurface storage.

Figure 1 Schematic diagram of infiltration-advance of waterfront in a border strip



Source: Michael (1997)

In Figure 1,

q	rate of flow per unit width available from the head end of the strip, cm^3/min/cm, t = time since irrigation started, min
x	irrigation waterfront advance distance, cm
d	over the ground surface water average depth, cm
t_s	value of t at which $x(t) = s$, min
$y(t - t_s)$	accumulated infiltration depth at $x = s$ at time t_s, cm
s	value of x at $t = t_s$, cm
$x'(t_s)$	first derivative of waterfront advance distance with respect to time, that is $\frac{dx}{dt}$ at $t = t_s$.

Referring to Figure 1

$$\textit{Total volume of water applied on the ground surface per unit width of the border strip} = q \times t \quad (2)$$

$$\textit{Total volume of water stored on the ground surface per unit width of the strip} = d \times x \quad (3)$$

$$\textit{Volume infiltrated into the soil} = \int_0^x y\,ds \quad (4)$$

Hence, a relationship can be established as follows:

$$qt = d.x + \int_0^x yds. \tag{5}$$

Here, y is a function of $(t - t_s)$, and x is function of t_s.

Therefore,

$$\int_0^x yds = \int_0^t y(t - t_s)x'(t_s)dt_s \tag{6}$$

Now equation (5) can be rewritten as follows:

$$qt = dx + \int_0^t y(t - t_s)x'(t_s)dt_s \tag{7}$$

Equation (7) was primarily proposed by Lewis and Milne (1938). Later Philip and Farrel studied the validity of this equation in 1964. They suggested that equation (7) is valid if, x, has a monotonic characteristic and an increasing function of, t. This conditional function $x(t)$ places a restriction of the form of $y(t)$ for which the analysis is valid and conditions are as follows:

$$y \geq 0,$$

$$\frac{dy}{dt} \geq 0, \text{ and}$$

$$\frac{\delta^2 y}{\delta t^2} \geq 0$$

Philip and Farrel (1964) used the Faltung or convolution theorem of Laplace transformation and yielded the general solution of equation (7) as follows and the Laplace transformation was used as

$$L\{y(t)\} = \int_0^\infty e^{-st} y(t)dt = f(s) \tag{8}$$

and

$$L^{-1} = \{f(s)\} = y(t) \tag{9}$$

Faltung theorem states,

$$\begin{aligned} &\text{If } L\{F(t)\} = f(s) \text{ and } L\{G(t)\} = g(s) \text{ then} \\ &L^{-1}\{f(s)g(s)\} = \int_0^t F(\lambda)G(t-\lambda)d\lambda \end{aligned} \tag{10}$$

Here, $G(t - \lambda)$ and $F(\lambda)$ represent the corresponding values of $y(t - t_s)$ and $x'(t - t_s)$ of equation (7).

Considering the Laplace transformation of equation (7) and using the relationship in equation (10):

$$\frac{q}{s^2} = dL\{x\} + L\{x'\}L\{y\} \tag{11}$$

Since, $x(0) = 0$, equation (7) can be written as

$$\frac{q}{s^2} = dL\{x\} + sL\{x\}L\{y\} = L\{x\}\left[d + sL\{y\}\right] \tag{12}$$

Therefore,

$$L\{x\} = \frac{q}{s^2\left[d + sL\{y\}\right]} = \frac{q}{\left[ds^2 + s^3L\{y\}\right]} \tag{13}$$

Or,

$$\frac{x}{q} = L^{-1}\left[\frac{1}{s^3L\{y\} + ds^2}\right] \tag{14}$$

Here equation (14) represents the general solution of equation (7) in terms of Laplace transformation which can be applied to the specific interest of the form of $y(t)$.

Michael (1997) suggested that the solution of equation (14) to determine the water front advance when the accumulated infiltration and the elapsed time are expressed as shown in equation (1). Considering Laplace transformation of equation (1):

$$L\{y\} = \frac{a\Gamma(\alpha+1)}{s^{(\alpha+1)}} + \frac{b}{s} \tag{15}$$

Here Γ denotes gamma function.

Equation (14) can be written as follows:

$$\frac{x}{q} = L^{-1}\left[\frac{1}{s^3\left\{\frac{a\Gamma(\alpha+1)}{s^{(\alpha+1)}} + \frac{b}{s}\right\} + ds^2}\right] \tag{16}$$

Assuming the value of $a\Gamma(\alpha + 1) = k$ then equation (16) becomes:

$$\frac{x}{q} = L^{-1}\left[\frac{1}{ks^{(2-\alpha)} + (b+d)s^2}\right] \tag{17}$$

There may be two different scenario of elapsed time, t. One is when, t is a smaller value, and the other is when t is a larger value.

Case 1: When t is smaller.

Considering $\frac{k}{b+d} = \beta,$ then we can write the portion of equation (17) as follows:

$$\frac{1}{ks^{(2-\alpha)} + (b+d)s^2} = \left[\frac{1}{1 + \frac{k}{b+d}s^{-\alpha}}\right]\frac{1}{(b+d)s^2} = \frac{1}{(b+d)s^2}\left(1 + \beta s^{-\alpha}\right)^{-1}$$

$$= \frac{1}{(b+d)s^2}\sum_{n=0}^{\infty}\left(-\beta s^{-\alpha}\right)^n \text{ for } s > \beta^{1/\alpha}$$

$$= \frac{1}{(b+d)}\sum_{n=0}^{\infty}(-\beta)^n s^{-(n\alpha+2)}$$

Hence,

$$\frac{x}{q} = L^{-1}\left[\frac{1}{ks^{(2-\alpha)} + (b+d)s^2}\right] = \frac{1}{b+d} L^{-1} \sum_{n=0}^{\infty} (-\beta)^n s^{-(n\alpha+2)}$$

$$= \frac{1}{b+d} \sum_{n=0}^{\infty} (-\beta)^n \frac{t^{n\alpha+1}}{\Gamma(n\alpha+2)} = \frac{1}{b+d} \sum_{n=0}^{\infty} \frac{\left(-\beta t^{\alpha}\right)^n}{\Gamma(n\alpha+2)}$$

Therefore,

$$x = \frac{qt}{b+d}\left[\begin{array}{l} \frac{1}{\Gamma(2)} - \frac{\beta^2 t^{2\alpha}}{\Gamma(2+\alpha)} + \frac{\beta^2 t^{2\alpha}}{\Gamma(2+2\alpha)} \\ -\frac{\beta^3 t^{3\alpha}}{\Gamma(2+3\alpha)} + \frac{\beta^4 t^{4\alpha}}{\Gamma(2)+4\alpha} \end{array} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \right] \tag{18}$$

Case 2: When t is larger.

$$\frac{1}{ks^{(2-\alpha)} + (b+d)s^2} = \frac{1}{ks^{(2-\alpha)}}\left[\frac{1}{1+\left\{\frac{(b+d)}{k} s^{\alpha}\right\}}\right] = \frac{1}{ks^{(2-\alpha)}}\left(1+\beta^{-1}s^{\alpha}\right)^{-1}$$

$$= \frac{1}{ks^{(2-\alpha)}} \sum_{n=1}^{\infty} \left(-\beta^{-1}s^{\alpha}\right)^{-1} \text{ for } s < \beta^{1/\alpha}$$

$$= \frac{1}{k} \sum_{n=0}^{\infty} \left(-\beta^{-1}s^{\alpha}\right)^n = \frac{1}{k} \sum_{n=0}^{\infty} \left(-\beta^{-1}\right)^n \frac{1}{s^{[2-(n+1)\alpha]}}$$

Hence,

$$\frac{x}{q} = L^{-1}\left[\frac{1}{ks^{2-\alpha} + (b+d)s^2}\right] = \frac{1}{k} L^{-1} \sum_{n=0}^{\infty} \left(-\beta^{-1}\right)^n \frac{1}{s^{[2-(n+1)\alpha]}}$$

$$= \frac{1}{k} L^{-1} \sum_{n=0}^{\infty} \frac{t^{[1-(n+1)\alpha]}\left(-\beta^{-1}\right)^n}{\Gamma[2-(n+1)\alpha]} = \frac{1}{\beta(b+d)} \sum_{n=0}^{\infty} \frac{t^{[1-(n+1)\alpha]}\left(-\beta^{-1}\right)^n}{\Gamma[2-(n+1)\alpha]}$$

$$= \frac{-t}{(b+d)} \sum_{n=0}^{\infty} \frac{t^{-(n+1)\alpha}(-1)^{n+1}(\beta)^{-(n+1)}}{\Gamma[2-(n+1)\alpha]} = \frac{-t}{(b+d)} \sum_{n=1}^{\infty} \frac{\left(-\beta t^{\alpha}\right)^{-n}}{\Gamma(2-n\alpha)}$$

Or,

$$x = \frac{qt}{(b+t)}\left[\frac{1}{\left(\beta t^{\alpha}\right)\Gamma(2-\alpha)} - \frac{1}{\left(\beta t^{\alpha}\right)^2 \Gamma(2-2\alpha)} + \frac{1}{\left(\beta t^{\alpha}\right)^3 \Gamma(2-3\alpha)} \right. \\ \left. - \frac{1}{\left(\beta t^{\alpha}\right)^4 \Gamma(2-4\alpha)} + \ldots\ldots \right] \tag{19}$$

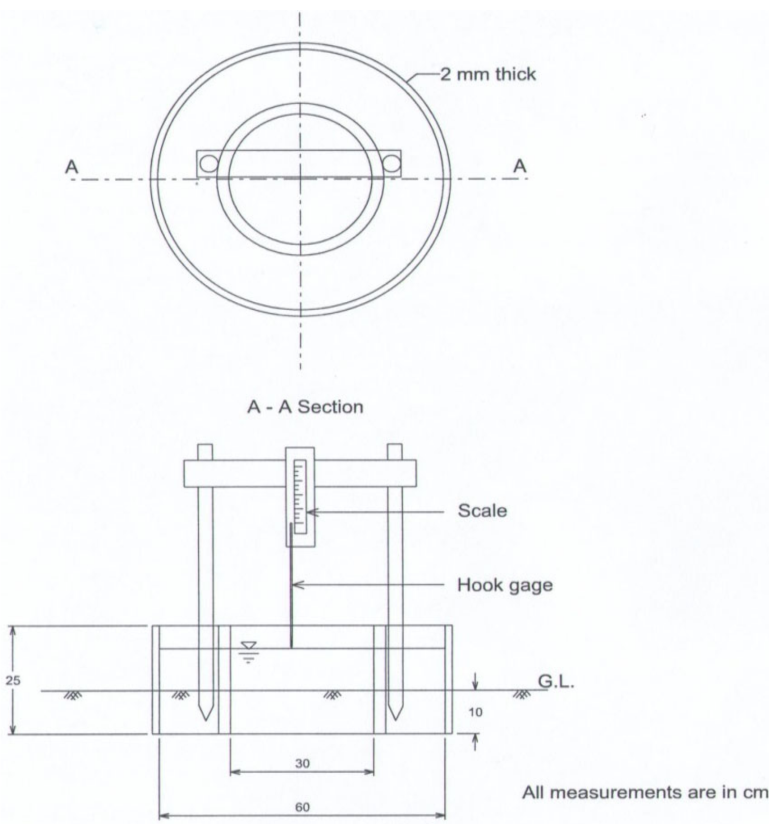
Michael (1997) reported that the equation (18) is suitable when $\frac{\beta t^{\alpha}}{\Gamma(2+\alpha)} < 1$ and equation (19) is suitable when $\frac{\beta t^{\alpha}}{\Gamma(2+\alpha)} > 1$.

3 Study area and methodology

A field study was carried out to check the validity of equations (18) and (19) in the irrigation and water management demonstration field of Bangladesh Agricultural Research Institute (BARI) in summer 2012. The atmospheric temperature during the infiltration and waterfront advancement were measured at a temperature of 85°F, the dynamic viscosity of water was 0.8076 mPa·s, and density was 0.9958 gm/cm^3. This field is used as the demonstration purposes and to exhibit different irrigation methods for training the farmers and agricultural extension officials. Sometimes, these plots are used for both exhibition and crop water requirement related research purposes too.

The field was kept fallow at the time when the study was conducted. It was sandy-loam soil, without any tillage practices done and had common soil vegetation. Four infiltrometers were installed lengthwise in the border strip. Figure 2 shows the dimensions of a common infiltrometer. There are two cylinders of diameters 30 cm and the 60 cm driven concentrically into the soil surface about 10 cm and the total height of these two ring cylinders is 25 cm.

Figure 2 Plan and cross-sectional views of a cylindrical infiltrometer



Source: Hasan et al. (2015)

Cylindrical infiltrometer method has been identified as the most advantageous while compared to other methods for determining infiltration rate (Michael, 1997). The other methods are merely considered and avoided because of the cumbersome procedure in collecting correct data from the field while estimating from waterfront advance data. There is necessity of considering the evaporation loss due to atmospheric influences on the large basin while measurement of water depletion is considered. The cylindrical infiltrometer is comparatively reliable for measuring the infiltration rate and accumulated infiltration. Infiltration characteristics can be measured by using a metal cylindrical round shaped hollow drum driving to a certain length into the soil surface and then pounding this cylinder with water and simultaneously record the time required to deplete water and enter into the soil surface. In early days, only one cylinder was used to measure the height of water lowered in the cylinder. That procedure yielded several drawbacks and a higher degree of variability due to the uncontrollable movement of lateral seepage and movement of water to and from the cylinder. This lateral movement of water has been well controlled by another concentric cylinder similarly with pounded water as the inner cylinder.

4 Results and discussion

The study conducted by Hasan et al. (2015) was to estimate cumulative infiltration using modified Kostiakov method and deriving the constant values of modified Kostiakov method for the soil under consideration. The derived equation for cumulative infiltration is:

$$y = 9.12\, t^{0.682} + 0.145 \tag{20}$$

where

y cumulative infiltration, cm

t elapsed time, minutes.

Hence, comparing equation (1) with equation (20), values of a, α and b equal to 9.12, 0.682, and 0.145, respectively in the same field. The authors further derived the average percentage of error between observed and the infiltration calculated using equation (21) was found to be 0.134, which is promising.

$$Error = \sum_{i=0}^{n} \frac{AI_a - AI_c}{AI_a} \times 100 \tag{21}$$

where AI_a is the actual accumulated infiltration, AI_c is the calculated value of accumulated infiltration, and, i, is the index number of data that varies from 1 to n (n = total number of data). For selecting the appropriate equation to determine the distance of the advance of waterfront [either equation (18) or equation (19)], using the conditions stated by Michael (1997), equation (18) is suitable when $\frac{\beta t^{\alpha}}{\Gamma(2+\alpha)} < 1$ and equation (19) is suitable when $\frac{\beta t^{\alpha}}{\Gamma(2+\alpha)} > 1$.

Here,

$$\beta = \frac{\alpha\Gamma(\alpha+1)}{(b+d)} = \frac{9.12\times\Gamma(0.682+1)}{(0.145+2.35)} = 3.309339$$

Test of validity whether the derived equations (18) and (19) will be useful or not were done by using the pump maximum discharge rate of water, q of 2,880 cm^3/min/sm delivered at the headed of the border strip, and the average the average depth of water depth of water on the strip surface, d was 2.35 cm (refer Figure 1). The inflow rate is one of the important factor which governs the performance of border irrigation (Michael et al., 2015; Salahou et al., 2018). The distance advanced by the waterfront was measured 26 minutes after the delivery of inflow of water was started at the head end.

Then,

$$\frac{\beta t^{\alpha}}{\Gamma(2+\alpha)} = \frac{3.309339\times 26^{0.682}}{\Gamma(2+0.682)} = 20.0498 > 1$$

Here,

$$\left(\beta t^{\alpha}\right) = 30.53185, \left(\beta t^{\alpha}\right)^2 = 932.19369, \left(\beta t^{\alpha}\right) = 810,062,418.8, \text{ and}$$

$$\left(\beta t^{\alpha}\right)^4 = 4.305999131\times 10^{35}$$

$$\Gamma(2-\alpha) = \Gamma(2-0.682) = 0.894905$$

$$\Gamma(2-2\alpha) = \Gamma(2-2\times 0.682) = 1.412066$$

$$\Gamma(2-3\alpha) = \Gamma(2-3\times 0.682) = -22.3639$$

$$\Gamma(2-4\alpha) = \Gamma(2-4\times 0.682) = -4.55588$$

Now, plugging these values in equation (19)

$$\frac{2,880\times 26}{(0.145+2.35)}\left[\frac{1}{30.53185\times 0.894905} - \frac{1}{932.19369\times 1.412066}\right.$$
$$\left. + \frac{1}{810,062,418.8\times(-)22.3639} + \frac{1}{4.305999131\times 10^{35}\times(-)4.55588}\right]$$
$$= 30,012.02405\times[0.036599061 - 0.000759694 - 5.51993513\times 10^{-11}$$
$$-5.09745952\times 10^{-37}]$$

Neglecting, $5.51993513 \times 10^{11}$, and $5.09745952 \times 10^{-37}$, the value of x becomes

$$30,012.02405\times[0.036599061 - 0.000759694] = 1,075.611944 \text{ cm} \approx 10.76 \text{ cm}$$

The total width of the border strip was equally divided into five sub-sections and the data were recorded on waterfront advances in those sub-sections. Considering the average of that waterfront advanced data, the actual data was recorded to be 10.25 m. As the actual measurement of the distance of waterfront advanced was 10.25 m, and the calculated waterfront was 10.76 m. The difference between the actual and the calculated values of waterfront advances is 0.51 m shorter than that of the calculated value, which is very promising. This difference, even not a very big one, was due to the unevenness of soil surface and a huge number of soil depressions, and some places it was observed that there discontinuation of slope uniformity, and existence of soil cover.

Average value of, *d*, which is 2.35 cm could have been shorter distances instead of taking it by averaging the head end and the tail end points only. Unevenness of the soil surface, due to the lack of appropriate tillage operation, had a considerable number of potholes which might have had stored the water and resulting into shortening in the length of the waterfront distance. Some more trials in different soil types might give even closer values between the calculated and actual field observation of the length of waterfront advance. It is really a very useful information for deciding the length of time of pump operation which may further help for increasing the water application efficiency. Once the data for field infiltration is available, and the calculated values of the characteristics values of a, α, and b are available, the infiltration model equation given by equation (1) is established, it is really easy to calculate the length of the waterfront advanced in the border strip. Hence, the same experiment can serve for calculating soil infiltration characteristic, cumulative infiltration of water into the soil, and length of waterfront advanced.

5 Conclusions

The results obtained reasonably agree with the field measurements conducted in 2012 at the BARI, Joydebpur, Gazipur, Bangladesh. This study adopted the concepts of Laplace transformation to determine the theoretical waterfront distance in border strip along with the uses of the values of constants as determined by Hasan et al. (2015). Adoption of Laplace transformation shows a reasonable approach to determine the waterfront advanced distance in a border strip matches agreeably with the actual field data which is the average value of the waterfront advances from equally spaced five sub-sections of the strip widthwise. Difference between the calculated and the measured values of the waterfront was within the acceptable range. The actual distance of waterfront was 0.51m shorter due to soil cover, huge potholes, unevenness of the surface, discontinuation of the slope uniformity might have caused the minor discrepancy of the calculated value. The result was further promising while considering the synchronisation of the waterfront advance distance and the infiltration depth. The infiltrated depth should not be deeper than the root zone and thereby causing lower water application efficiency.

References

Alazba, A.A. (1997) 'Design procedure for border irrigation', *Irrigation Science*, Vol. 18, No. 1, pp.33–43.

Amiri, M.J., Hamidifar, H., Bahrami, M. and Eslamian, S. (2016) 'Optimisation of deficit-irrigation under variable seasonal rainfall and planning scenarios for rice in a semi-arid region of Iran', *International Journal of Hydrology Science and Technology*, Vol. 6, No. 4, pp.331–343.

Dagadu, J.S. and Nimbalkar, P.T. (2012) 'Infiltration studies of different soils under different soil conditions and comparison of infiltration models with field data', *International Journal of Advanced Engineering Technology*, Vol. 3, No. 2, pp.154–157.

Hasan, M., Chowdhury, T., Drabo, M., Kassu, A. and Glenn, C. (2015) 'Modeling of infiltration characteristics by modified Kostiakov method', *Journal of Water Resources & Protection*, Vol. 7, No. 6, pp.1309–1317.

Khanjani, M.J. and Barani, G.A. (1999) 'Optimum design of border irrigation system', *Journal of American Water Resources Association*, Vol. 35, No. 4, pp.787–792.

Lewis, M.R. and Milne, W.E. (1938) 'Analysis of border irrigation', *Agricultural Engineering*, Vo1. 19, No. 6, pp.267–272.

Michael, A.M. (1997) *Irrigation: Theory and Practice*, Vikas Publishing House PVT Ltd., Delhi.

Michael, R.M., Hussain, A., Gillies, M.H. and O'Halloran, N.J. (2015) 'Inflow rate and border irrigation performance', *Agricultural Water Management*, Vol. 155, pp.76–86.

Parhi, P.K. (2016) 'An improvement to the waterfront advance-time relation for irrigation borders', Vol. 65, No. 4, pp.568–573.

Parhi, P.K. (2017) 'Improving water application efficiency of irrigation borders using modified water front advance-time relation', *International Journal of Agricultural Research*, Vol. 12, No. 3, pp.115–122.

Philip, J.R. and Farell, D.A. (1964) 'General solution of the infiltration-advance problem in irrigation hydraulics', *Journal of Geophysical Research*, Vol. 69, No. 4, pp.621–631.

Salahou, M.K., Jiao, X. and Lü, H. (2018) 'Border irrigation performance with distance-based cut-off', *Agricultural Water Management*, Vol. 201, pp.27–37.

Zahraei, A., Saadati, S. and Eslamian, S. (2017) 'Irrigation deficit: farmlands', in Eslamian, S. and Eslamian, F. (Eds.): *Chapter 16 in Handbook of Drought and Water Scarcity*, *Management of Drought and Water Scarcity*, Vol. 3, pp.343–358, Francis and Taylor, CRC Press, USA.

Zerihun, D., Sanchez, C.A., Farrell-Poe, K.L. and Yitayew, M. (2005) 'Analysis and design of border irrigation systems', *Transactions of the ASAE*, Vol. 48, No. 5, pp.1751–1764.