
Portfolio optimisation of power futures market: evidence from France and Germany

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Abstract: Understanding the nature of power futures is particularly crucial, for non-storable commodities such as wholesale electricity since it has been deregulated. This paper examines different types of optimisation techniques and provides a temporal analysis of energy future prices. In particular, it highlights how one of the well-known elements of modern finance theory could improve the accuracy of evaluating the risk exposure inherent in power futures market via a modified version of the mean-variance portfolio (MVP) theory. The optimisation techniques employed account for the initial capital requirement of the energy futures and estimate the optimal weights needed to mitigate the downside risk inherent in the energy futures market. One major finding of this paper shows that, a portfolio of energy prices with different maturities could provide market players with a less risky investment in the energy market. In addition, the feasibility of the methodologies utilised have been presented.

Keywords: portfolio theory; portfolio optimisation; electricity futures; energy markets; investment risk management.

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1 Introduction

The liberalisation of energy markets has removed the regulatory risk that shields and exposes investors to various risks in different ways and dimensions. For instance, neither is generators assured of their ability to recover all costs from power consumers nor are future prices guaranteed. The privatisation and creation of a competitive spot market for electricity in UK brought in its wake volatile pool of prices and their impacts on different sectors of the economy. Because of this, risk-oriented large investors sought to mitigate these impacts through portfolio of contracts that involve delivery of electricity in future periods. Some example of such contracts include day-ahead contracts, derivatives such as forwards, futures, swaps, variable volumes or swing options, direct or indirect investments strategy in energy production facilities¹. Therefore, proper portfolio management requires a continuous examination of which contracts to invest in (instrument selection) and at which moment to do exactly that (timing) or not. Mostly, the aim of the investor is to invest where there is not too much risk from tremendous fluctuations of prices across space and time. Deregulation in France and Germany likewise resulted in volatile spot market prices. This requires the use of hedging instruments, in particular, forward trades, which are committed to provide electricity upon demand to their customers.

The main focus of the empirical literature on electricity markets has mainly been the analysis and modelling of the behaviour of spots prices². Electricity market in its entirety has intrinsic features that distinguish it from other financial and commodity markets. Because of these features such as non-storability, generation constraints, transmission constraints, seasonality, weather dependence, energy (electricity) futures represent a larger market than spot trading. Energy risk management makes use of futures to hedge against spot price fluctuations during the delivery period. In order to lock in prices in advance for planned generations or consumptions of the next two years, one year, quarters and months, futures contract are traded (bought and sold). As a result, spot trading is only used to optimise procurements and sales of power in the short-run. Energy futures are also the most natural choice for investors willing to take positions in power markets with the absence of underlying physical constraints. However, there are other risk management practices such as the use of portfolio optimisation techniques.

Indeed, the history of mean-variance portfolio (MVP) theory is long and transcends several decades. It has found its application in applied finance and operations research. A detailed application of the portfolio selection model has been carried out by Fabozzi et al. (2002) in the areas of risk management and capital budgeting. MVP theory, based on the seminal work of Markowitz (1952) provides a benchmark that allows portfolio selection to take into account risk and return of an asset. Major results from Lintner (1965); Sharpe (1964) and others have made the role of risk in the portfolio selection for financial assets even more articulate and its implication for portfolio selection on market equilibria in the financial market has been established. However, standard portfolio application in the deregulated energy market is limited. The selection of power plant and fuel portfolios in the electricity industry have been pioneered by Bar-Lev and Katz (1976)³. They applied MVP theory to fossil fuel procurement in the US electricity industry, where they showed that there is efficient diversification of electric utilities. However, their portfolios were generally characterised by a relatively high rate of return and risk. Nazari et al. (2015) provide a decision support framework utilising a combination of conditional value-at-risk (CVaR) portfolio optimisation and real option analysis to assist investors in evaluating

their investment decisions for additional capacity, considering their existing power generation assets and uncertainties regarding the future of climate policies. The CVaR method employed helps to determine the range of efficient portfolios subject to capacity/demand limitations.

From another perspective, Boroumand et al. (2015) define optimal portfolios of hedging retail prices of electricity using value-at-risk (VaR) and CVaR, where they clearly show that the losses of an optimal daily portfolio are at least nine times higher than the losses of optimal intra-day portfolios. Youssef et al. (2015) evaluate VaR and expected shortfall (ES), sometimes referred to as CVaR, for crude oil and gasoline market. Their findings highlight the fact that taking into account long-range memory, asymmetry and fat tails via CVaR in the behaviour of energy commodity price returns coupled with filtering process such as extreme value theory (EVT), are important in improving risk management assessments and hedging strategies in the high volatile energy market. Denton et al. (2003) provide a description of how market risks in operations can be measured and managed using real option models and stochastic optimisation techniques. They then link these results to VaR and related risk metrics such as cash flow earnings and credit risk, which can be used to measure trading risks from weeks to months. Besides, they also detailed how to optimise portfolios from the viewpoint of risk-return relationships. Further analysis has also been carried out for long run phenomena.

On the other hand, Hernandez (2014) compares oil and gas stocks in addition to coal and uranium stocks, to identify their riskiness via optimisation methods and risk measures that produce the best risk-return trade-off. This analysis therefore helps these authors to recognise the stocks in which the optimal weight allocations converge on average. Their conclusion is that vine copula model best accounts for the overall dependences in the energy portfolios. Behboodi et al. (2016) examine the question of what is the optimal installed capacity allocation of renewable resources in conjunction with demand response. Their finding shows that inter-hourly demand response magnitude is much less useful in promoting additional renewables than intra-hourly demand elasticity. That is to say, adding renewable resources can reduce the costs of uncertainty more than production costs. In other words, Behboodi et al. (2016) highlight the essence at which renewable resource level is sensitive in comparison to carbon tax. Following the Heath-Jarrow-Morton (HJM) approach, Benth and Lempa (2014) model the entire futures price curve as a solution to stochastic partial differential equation. Besides, they provide a general formalism to handle portfolio of futures contracts. Benth and Lempa (2014) employ a finite-dimensional realisation to derive a finite-dimensional form of the portfolio optimisation problem and study its solution. Gao et al. (2014) attempt to use portfolio theory to optimise China's overall energy system via the learning curve effect of renewable energy cost and the characteristic of fossil energy cost increasing over time. They also take into account additional factors such as environmental costs of coal consumption and various growth rates of the cumulative research and development (R&D) capacity for solar power. Their major finding is that the development of renewable energy in China has tremendous potential but it will not replace fossil energy in the next decades.

In another paradigm, De Oliveira et al. (2011) adjust the CVaR metric to the mix of contracts on the energy markets. Their approach uses mixture design of experiments

(MDE). With a shift from the traditional linear programming, the concept of desirability function is used to combine the multi-response, nonlinear objective functions for mean with the variance of a specific portfolio obtained through MDE. The maximisation of the desirability function is implied in the portfolio optimisation results in the generation of an efficient recruitment frontier. Overall, their contribution includes the following: it incorporates risk aversion in the optimisation routine, assesses interaction between contracts and curbs down the computational effort required to solve the constrained nonlinear optimisation problem. In a paper by Delarue et al. (2011), the author presents a portfolio theory model that explicitly differentiates between installed capacity electricity and actual instantaneous power delivery. They indicated that, the variability of wind power and ramp limits of conventional power plants are correctly included in the investment optimisation. The question of optimal portfolio weight allocation with electricity futures has, however, hardly been addressed. The management of portfolios of electricity futures have provided the possibility to fix delivery prices leading to the reduction of the exposure to price fluctuations in the day-ahead market.

Simply put, buying power with electricity futures implies hedging the risk inherent in day-ahead market. The expected hedging cost is usually equal to the risk premium embedded in the forward price. With a set of electricity futures contract that can be traded every day, the task of management is to quantify the optimal selection of electricity futures to accommodate various delivery periods. The target of the portfolio manager is to keep the portfolio that yields lowest expected costs for electricity consumption while respecting his/her risks orientation or appetite. Finally, standard methods for optimal allocation of shares in a financial portfolio are determined by second-order conditions, which are very sensitive to outliers according to Grossi and Laurini (2011). The problem in Markowitz's mean-variance model is to minimise risk, which is defined as the variance of a portfolio for a given return. This paper, thus explicitly applies an improved version of the well-known MVP, which is based on the input of a mean vector and a covariance matrix. The standard approach seems to provide questionable results in financial management, since small changes of inputs might lead to irrelevant portfolio allocations. The replacement of the variance, as a measure of risk with the CVaR, which is an alternate risk measure lies in the fact that, it is coherent in nature and overcomes the shortcomings of the use of variance in MVP, thereby accurately minimising the risk encountered by investors in the energy futures market.

The contribution of this paper to the literature is empirical in nature. First, the optimisation techniques have been employed to ascertain which technique accurately accounts for the risk inherent in the futures power markets. Nonetheless, generation plant owners (operators) are mostly interested in and thus focus on generation portfolios. However, in this paper, I focus on electricity future portfolios that are of interest to financial operators, who want to invest capital on energy exchanges. This analysis consider two important energy exchanges in central Europe; the European Energy exchange (EEX) and Powernext and therefore provide a benchmark for analysing any other future exchanges such as other European energy markets, which have similar features. So far, portfolio optimisation in the energy futures market has been carried out using variance as a measure of risk, but variance have been criticised widely. I have therefore replaced the variance with the CVaR, which makes the optimisation procedure more complex. To this end, the results obtained are more reliable, (see Rockafellar and Uryasev, 2002 for an overview).

In particular, three optimisation techniques have been employed namely:

- 1 the Markowitz's optimisation method
- 2 optimisation of the CVaR
- 3 optimisation of the CVaR under Gaussian distribution.

These techniques help to solve problems of asset allocations in the energy futures markets. The main finding of this paper reveals that even though utilising the second approach; that is, optimisation of CVaR only, provides a better estimate than the Markowitz's approach, the last optimisation technique, that is, the optimisation of the CVaR under Gaussian Assumption, provide best accurate estimates among the three techniques examined. This is because of its coherency property and its ability to capture downside risk coupled with a known underlying distribution. These characterisations therefore provide accurate risk quantification and hence a good risk management strategy.

In consequence, analyses of these kinds constitute a temporal diversification. In addition, portfolio diversification in the power market proved to be a good source of risk management strategy in the sense that it leads to an improvement in the efficient frontier of German energy futures after the inclusion of other assets such as stock and bonds. Moreover, these estimation methods revealed that utilising energy futures with various maturities provide a less risky environment for investors or market players in the energy futures exchange. The rest of this paper is organised as follows: in Section 2, the markets under consideration have been described. Section 3 presents the model set up. Section 4 provides the computational analysis of data and the results obtained using the models under consideration while Section 5 concludes.

2 The electricity market

Electricity (both power and energy) in economics term is a commodity capable of being bought and sold. Most electricity markets provide two types of markets in which energy is traded namely: the spot market and the (physical) forward/futures market. An electricity market is a system for making purchases, through bids to buy; sales, through offers to sell; and short-term trades, generally in the form of financial or obligation swaps. Bids and offers use supply and demand principles to set the price. Long-term trades are contracts similar to power purchase agreements and generally considered private bilateral transactions between counter parties. As a result, many countries have established over-the-counter (OTC) and centralised markets to facilitate power trading.

The two most commonly used markets are the day-ahead and forward/futures market. Concerning the day-ahead market, traders can submit bids and offers for amounts of electricity to be delivered in different hours in the next day. This market is the closest equivalent to the spot market. Vendors of electricity use day-ahead markets for buying part of their electricity consumption, however, the amount of price variation in these markets remains substantial. Electricity in its own right as a commodity is not directly and efficiently storable. Under most circumstances, this leads to increased volatility in prices, seasonal variations and frequent price spikes. The fact that the issue of risk is prevalent in the day-head market is not questionable and could be envisaged in the sense that vendors of electricity prefer to access insurance depending on their risk orientation

rather than tackling the risk of price variations in the day-ahead market. In fact, these investors manage portfolio of derivative contracts that entail future deliveries. Taking position in portfolios of these contracts, the purchasers can already lock in the acquisition of (some of) the expected future consumptions ahead of time before the actual maturity. Henceforth, the investors are able to manage the risk encountered due to price variations in the day-ahead market.

Wholesale transactions (bids and offers) in electricity are typically cleared and settled by market operators or a special-purposed independent entity charged exclusively with this function. Market operators do not clear trades. Nevertheless, they often require knowledge of the trade in order to maintain generation and load balance. The commodities within an electric market generally consist of two types: power and energy. Power is the metered net electrical transfer rate at any given moment and is measured in megawatts (MW). Energy is electricity that flows through a metered point for a given period and is measured in megawatt hours (MWh). Markets for energy related commodities are net generation output for a number of intervals usually in increments of 5, 15 and 60 minutes. Markets for power related commodities required and managed by market operators to ensure reliability, are considered ancillary services and include such names as spinning reserve, non-spinning reserve, operating reserves, responsive reserve, regulation up, regulation down, and installed capacity. Moreover, majority of operators have the possibility to buy and sell on markets for transmission congestion and electricity derivatives, such as electricity futures and options, which are actively traded. These markets developed because of the restructuring of electric power systems around the world. This process has often gone on in parallel with the restructuring of natural gas markets⁴.

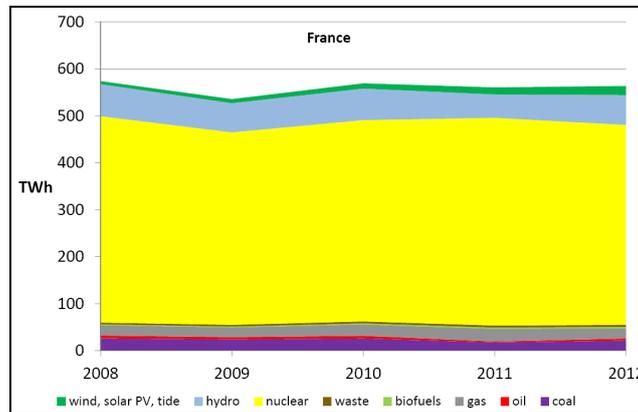
2.1 Electricity futures pricing of Powernext and the European energy exchange

Futures contracts are standardised forward contracts that are traded on exchanges and no physical delivery is necessary. Forward contracts are agreements to buy/sell an agreed amount of a commodity at a specified price at a designated time. However, a forward contract is not an investment in a strict sense that funds are paid for an asset. It is only a commitment today to make transactions in the future. According to Fama and French (1987), expectation theory teaches that forward prices for non-storable commodities reflect the expectation of markets participant on the (spot) price in the delivery period and risk premium that compensates producers for enduring the uncertainty of committing to sell against fixed prices.

However, the French power exchange (Powernext electricity trading market) was launched in France in November 2001 as initiative of the European stock exchange Euronext. This initiative was supported by Nord Pool, as was the case of APX and LPX. Powernext, a regulated investment firm, began with a day-ahead trading in 2002 and opened a future market two years later. Powernext's main competition comes from the Nord Pool, which offers spot electricity trading and futures. Figure 1 reports the electricity generation mix of France in the period 2008–2012. The main difference with respect to the German market is the large prevalence of Nuclear among all the other sources of energy. Coal and gas, on the other hand, play a very limited role, while the contribution of renewable sources has increased, but the share of generation from these sources is clearly lower than in Germany. The first German power exchange, the Leipzig Power exchange (LPX) was launched and backed by Nord Pool, regional banks and

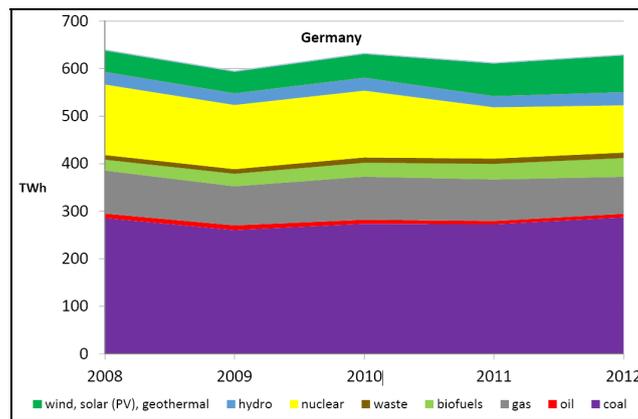
regional governments in mid-2000. The LPX was the first market in which prices were quoted hourly. Until then, trading was only done OTC.

Figure 1 Generation mix of electricity in France, 2008–2012 (see online version for colours)



Source: see Fianu and Grossi (2015) for a brief overview

Figure 2 Generation mix of electricity in Germany, 2008–2012 (see online version for colours)



Source: Refer to Fianu and Grossi (2015) for an overview

In August 2000, a second power exchange started operation; the European Energy Exchange (EEX) in Frankfurt am Main, as an initiative of the German futures exchange Eurex. Both exchanges, the LPX and EEX merged and created a single European Energy Exchange based in Leipzig. EEX operates a day-ahead market with hourly products (anonymous, bilateral auction) and block products (continuous trading). Here, market participants start to submit their offers (bids) before 12:30 p.m. of the day before delivery. The market results are published by the EEX until 12:30 p.m. and become binding half an hour later. The exchange operates futures market, where contracts can be traded for delivery up to six years in advance. It also offers OTC clearing services and introduced trading with options in power futures in 2004. EEX currently offers spot and derivative trading for natural gas and CO₂ emission rights as well as trading in financial coal futures.

In addition, the price of EEX serves as a benchmark for the entire market including OTC wholesale and retail business. The German electricity market is the largest in Europe. A quick glimpse at Figure 2 reveals the sources mainly used to generate electricity in Germany in the period 2008–2012.

As can be noticed, the main source of energy is coal. Note that the decrease of nuclear generation followed by the nuclear phase-out decided by the German Government after the Fukushima accident has mostly been replaced by increases in renewable energy sources, specifically wind and many others. The European Energy Exchange and its French counterparts, Powernext created through a merger the EPEX spots market for France, Germany/Austria and Switzerland in 2008. Together, these countries account for more than one third of the European power consumption. EPEX SPOT is the exchange for the power spot markets at the heart of Europe. In 2010, the European Electricity Index (ELIX) was launched in the quest for an integrated European market.

3 Methodology

The general problem of portfolio selection can be put in the following settings. Consider, k risky assets, whose observed prices for T periods are p_{it} , $t = 1, \dots, T$, $i = 1, \dots, k$ and let $\omega = (\omega_1, \omega_2, \dots, \omega_k)$ be the vector of portfolio weights. Given assets, $Y = (y_1, \dots, y_k)$ where $y_i = (y_{i1}, \dots, y_{iT})'$ the asset return is $y_{it} = \ln(p_{it}/p_{i,t-1}) \approx (p_{it}/p_{i,t-1} - 1)$, which is a $k \times 1$ vector depicted as μ and has $k \times k$ covariance matrix denoted with the symbol, Ω . The portfolio return and variance can be written as $\mu_p = \omega'\mu$ and $\sigma_p^2 = \omega'\Omega\omega$, respectively. Mathematically, we have:

$$\Omega_{k,k} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,k} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{k,1} & \sigma_{k,2} & \cdots & \sigma_{k,k} \end{pmatrix},$$

$$\omega_p = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_k \end{pmatrix}$$

$$\mu_p = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_k \end{pmatrix}.$$

The classical mean-variance optimisation problem can be formulated as:

$$\min_{\omega} (\omega'\Omega\omega - \gamma\omega'\mu). \quad (1)$$

subject to the constraints $\omega \geq 0$ (implying that all weights are strictly non-negative) and $\omega'l_k = 1$ where l_k is a $k \times 1$ vector of ones and γ is a risk tolerance parameter. The principle of no-short selling is allowed and would be followed throughout the paper. This is mostly imposed because many funds and institutional investors are not allowed to sell stocks short. Without loss generality, using the Lagrange method, for any $\gamma \geq 0$, the

maximisation problem has an analytical solution. Gradually increasing γ from zero and for each episode solving the optimisation problem, produce portfolios along the efficient frontier. The set of all possible portfolios that can be constructed is called the feasible set. Minimum variance portfolios are called the mean variance efficient portfolios. The set of all possible mean-variance efficient portfolios for different desired levels of expected return is called the efficient frontier.

The uniqueness of this paper lies in the fact that it replaces, as a risk measure, covariance matrix with ES (or CVaR). This is possible because of the properties of CVaR against its variant the VaR, which is the most used risk measure. As a reminder, the ES or CVaR of a random variable, X of order $\alpha \in (0, 1)$ is given as:

$$CVaRE_\alpha(X) = \frac{1}{\alpha} \left[\int_0^\alpha VaR_u(x) du \right], \quad (2)$$

where $VaR_u(X) = \inf\{x: P(X \geq x)\}$ is the VaR of X of order u . The definition of a risk measure can be put in the following framework.

Definition 3.1 (risk measure). Let X be a random variable. Then ψ is a risk measure if it satisfies the following properties.

- (monotonicity): if $X \geq 0$, then $\psi(X) \leq 0$
- (positive homogeneity): $\psi(\alpha X) = \alpha\psi(X) \forall \alpha \geq 0$
- (translational invariance): $\psi(X + b) = \psi(X) - b \forall b \in \mathcal{R}$.

The rationale behind this definition is that a positive number implies that one is at risk of losing capital and should have that positive number of cash balance at his disposal to offset this potential loss. However, a negative number would imply that the company has enough capital to take on more risk or to return some of its cash to other operations or to its shareholders. Fixing a confidence level p such as 99% and a time horizon (e.g., two weeks) the α -VaR of a given portfolio is the loss in the market value that is exceeded with probability $1 - p$. A more formal definition can be stated as follows. The α -VaR of the loss associated with a position X is the value,

$$VaR_\alpha(X) = \inf \{x: P(X > x) \leq 1 - \alpha\}. \quad (3)$$

Since Markowitz's seminal paper in 1952, many have esteemed the usefulness of diversification. VaR, however, does not account for this preference. That is to say, VaR can discourage diversification, which is one of its setbacks. In effect, Dowd (2003) and the references therein provide an example. For instance, suppose there are 100 possible future states of the world, each with the same probability. There are 100 different assets, each earning reasonable money in 99 states, but suffering a big loss in one state. Each of these assets loses in a different state, so we are certain that one of them will suffer a large loss. If we invest in one of these assets only, then our VaR will be negative at, say, the 99% confidence level, because the probability of incurring a loss is 1%. However, if we diversify our investments and invest in all assets, then we are certain to incur a big loss. The VaR of the diversified portfolio is therefore much larger than the VaR of the non-diversified one.

In consequence, the VaR measure can discourage diversification of risks because it fails to take into account the magnitude of losses in excess of VaR. These deficiencies led Artzner et al. (1997) to refer to the VaR as incoherent. According to these authors a risk measure that is coherent favours diversification and can be defined as follows: if X_1, X_2 are random variables, then $\psi(X_1 + X_2) \leq \psi(X_1) + \psi(X_2)$. This is the property of sub-additivity. Any risk measurement that is Translational-equivalent, positively homogeneous, sub-additive and monotonic can be called coherent risk measure. Thus, *monotonicity* means that the risk is greater for more negative random outcomes. *Positive homogeneity* implies that if we increase the size of the portfolio by α with the same weights, we increase the risk measure by the same factor α . *Translational invariance* implies that, if we add a sure amount b to the position, it will decrease the risk measure by b . *Sub-additivity* means that the risk of a portfolio made up of two sub-portfolios is smaller than the sum of the risk of the two sub-portfolios.

An implication of coherency is convexity, which is economically meaningful: it corresponds to the fact that diversification reduces risk. An example of a coherent risk measure is the ES or CVaR. Since, VaR is not sub-additive for a general loss distribution; this deters it to be a convex function. In Rockafellar and Uryasev (2000), the authors point out the difficulty in optimising VaR even when this is done under different scenarios. In this case, VaR is non-convex, non-smooth as a function of positions and it has local extrema. As a result, many researchers thought it wise to define new risk measures which overcome these drawbacks – in particular the lack of sub-additivity and the low sensitivity to the extreme events (i.e., leptokurtic nature of the returns distributions). This led Artzner et al. (1999) to define two new risk measures called the tail conditional expectation (TCE) and worst conditional expectation.

In order to overcome this problem, Rockafellar and Uryasev (2002) introduced a further risk measure known as CVaR. These authors as well as Gaivoronski and Pflug (2005) outlined the proof that, the CVaR is a coherent measure in general. Another advantage of the CVaR deduced by Rockafellar and Uryasev highlights the fact that, by solving a simple convex optimisation problem in one dimension, it is possible to obtain simultaneously $CVaR_\alpha(X)$ and $VaR_\alpha(X)$ of the portfolio, say X . The importance of this result lies in the fact that one can calculate the VaR of a position without necessarily having knowledge of the VaR. Rockafellar and Uryasev expressed the CVaR as a functional denoted as:

$$F_\alpha(x, b) = b + \frac{1}{1-\alpha} \mathcal{E} \{ [f(x, y) - b]^+ \} \quad \text{and} \quad (4)$$

$$[f(x, y) - b]^+ = \max(f(x, y) - b, 0),$$

where $b \in (0, 1)$. Thus, the α -CVaR, implies the mean of α -tail distribution of the function, $f(x, y)$ adequately quantifies the losses of the return distribution, which is the amount one would expect to lose on average that exceeds the VaR.

3.1 Portfolio optimisation of the CVaR

The well-proven result of the formulation in equation (4) is useful for the computation of the CVaR. The function, $F_\alpha(x, b)$ has the following properties:

- convexity
- continuously differentiable
- minimum value $F_\alpha(x, b)$ is $CVaR_\alpha(x)$.

As mentioned earlier on, the goal of this paper is to implement CVaR, a risk measure as an instrument incorporated in portfolio optimisation. This is carried out under various sensitivity analysis as the distribution of the futures contracts, i.e., the market factor X is normally unknown. Consider that there exist in the portfolio k varieties of futures contracts, with weights given as $\omega = (\omega_1, \dots, \omega_k)^T$ such that $\sum_{i=1}^k \omega_i = 1$ and $\omega_i \geq 0$. The optimisation problem therefore becomes:

$$\begin{cases} \min_{\omega} CVaR_\alpha(x) - \gamma \omega' \mu \\ \sum_{i=1}^n \omega = 1 \\ \omega_i \geq 0 \quad \forall \quad i \end{cases} \quad (5)$$

3.2 Portfolio optimisation of the CVaR under multivariate Gaussianity

In fact, if asset returns assume multivariate normal distribution, then the returns on portfolio are normally distributed with variance $\omega' \Sigma \omega$; where ω is the vector of weights and Σ is the covariance matrix between stock returns; see Rachev et al. (2008) for a brief overview. The mean of the normal distribution is then given as:

$$\mathcal{E}r_p = \sum_{i=1}^n \omega_i \mathcal{E}X_i \quad (6)$$

where in all cases \mathcal{E} stands for the mathematical expectation. Henceforth, under this assumption the CVaR of portfolio return at the tail probability, say α , can be expressed in closed form through:

$$\begin{aligned} CVaR_\alpha(r_p) &= \frac{\sqrt{\omega' \Sigma \omega}}{\alpha \sqrt{2\pi}} \exp\left(\frac{-(VaR_\alpha(X))^2}{2}\right) - \mathcal{E}r_p \\ &= C_\alpha \sqrt{\omega' \Sigma \omega} - \mathcal{E}r_p, \end{aligned} \quad (7)$$

where the random variable, X has the standard normal distribution, $X \in N(0, 1)$ and C_α is a constant, independent of the portfolio composition and can therefore be calculated in advance. The decision problem encountered by the investor is deduced by putting $CVaR_\alpha(r_p)$ in place of $CVaR_\alpha(x)$ in equation (5). In effect, due to the limitations of the multivariate normal assumption, the portfolio CVaR appears symmetric and is representable as the difference between the properly scaled standard deviation of the random portfolio returns and the expected return of the portfolio. Thus, one can compute the optimal portfolio that solves (with no short selling) equation (5); with the assumption that the portfolio returns are Gaussian with respect to the mean and covariance of the assets.

4 Data analysis and empirical results

The identification of investment risks in the electrical energy industry may be straightforward; for instance, investors in power generation attempt to understand the relative importance of different risks by quantifying them where possible. For the most important risks, it is usually prudent to adopt risk management strategies that can cost-effectively reduce exposure to such risks. Therefore, estimating the profitability of an investment must rely on modelling, requiring knowledge of the future costs and revenues from generating project and their variation. However, in electricity markets, what matters to the investor is the profitability of the investment against the risk to the capital used. Financial hedging instruments such as electricity futures and forward markets are important tools in the development of efficient electricity markets. As a highly capital-intensive industry, a key sign of an efficiently functioning electrical energy industry is an efficient allocation of capital. The key drivers of investment decisions in a market are prices as they signal potential rewards to investors. High prices relative to the cost of building new plants signal the need for new investment. Nonetheless, low prices usually discourage investment.

In this section, I therefore detail how empirically, the optimisation procedures can be implemented to minimise the CVaR of portfolio of energy futures price. In order to employ the model, I consider a time series data of electricity futures from German and France Power Exchanges from 6 January 2009 to 28 November 2011. The load profile depicts at which delivery rate (volume of power per hour) the power delivery based on the futures contract takes place. Load profiles which can be traded on EEX are the base load, peak load and the off-peak load. The German and French energy futures are both operated by the EEX. The base load consists of a constant delivery rate on all delivery days from Monday to Sunday and during all 24-delivery hours of a delivery day during the delivery period. Peak load consists of a constant delivery rate on all delivery days from Monday to Friday during all 12-delivery hours from 08:00 am (CET) to 08:00 pm (CET) of a delivery day during the delivery period. The choice to use approximately three years sample of data provides answer to the jumps inherent in the series, which are deemed to be severely affected by the optimal portfolio choice.

However, this has been removed by applying an error correction factor. Accounting for correlation between the spot market and the future market, I separately considered the correlation co-efficient between the spot prices and the different future returns with varying maturities. A linear relationship between the spot and futures returns are not clearly observed as correlations ranges between -0.00276 and 0.00838 for the German electricity futures returns only. The clear-cut absence of a linear relationship between the spot returns and the future returns motivates us to model the futures returns separately from the spot market return. For the France futures market, I observe similar pattern since correlation co-efficient ranges between -0.06589 and 0.028183 . In order to provide an in-depth understanding and analysis of the different optimisation procedures, I compare the outcome of the various risk measurement methodology via optimisation techniques. The results of these comparisons are depicted in Tables 1, 2, 3, and 4). I assume that investors have a lump sum of money at hand, which they deem to invest in the electricity market according to their risk preferences.

Table 1 Optimisation of the condition VaR only in French power futures market

| $\theta_{Sym,price}$ | $\theta_{1-month-ahead}$ | $\theta_{2-month-ahead}$ | $\theta_{1-quarter-ahead}$ | $\theta_{2-quarter-ahead}$ | $\theta_{1-year-ahead}$ | $\theta_{2-year-ahead}$ | θ_{PDF} | $\theta_{THECOLA}$ | $\theta_{SCHIELENE}$ | $\theta_{GDF-SIEZ}$ | θ_{FEDLA} ENVIRONNEMENT | θ_{TOTAL} | θ_{MAUREL} ET PROM | $\theta_{BENFRANCE.2007}$ -DEFAULT PRICE | $\theta_{GATFRANCE}$ PRCLSTRIP | γ | \mathcal{E}_p | $CVaR_p(r_p)$ |
|----------------------|--------------------------|--------------------------|----------------------------|----------------------------|-------------------------|-------------------------|----------------|--------------------|----------------------|---------------------|-----------------------------------|------------------|------------------------------|---|-----------------------------------|-----------|-----------------|---------------|
| 0.00037 | 0.00034 | 0.00000 | 0.00025 | 0.00025 | 0.00467 | 0.03291 | 0.00427 | 0.00069 | 0.00081 | 0.00535 | 0.00000 | 0.00782 | 0.00406 | 0.93821 | 0.00000 | 0.00010 | -0.00009 | 0.00201 |
| 0.00012 | 0.00000 | 0.00000 | 0.00023 | 0.00084 | 0.00000 | 0.01579 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00017 | 0.01785 | 0.96364 | 0.00137 | 11.44588 | -0.00006 | 0.00211 |
| 0.00000 | 0.00000 | 0.00000 | 0.00032 | 0.00315 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.02462 | 0.97060 | 0.00131 | 23.49406 | -0.00005 | 0.00226 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00349 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.03254 | 0.96397 | 0.00000 | 35.54224 | -0.00005 | 0.00244 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00610 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.04564 | 0.94826 | 0.00000 | 47.39043 | -0.00004 | 0.00285 |
| 0.00008 | 0.00000 | 0.00000 | 0.00000 | 0.01046 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.05934 | 0.93013 | 0.00000 | 59.63861 | -0.00003 | 0.00343 |
| 0.00083 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.08262 | 0.89638 | 0.00634 | 71.68680 | -0.00001 | 0.00456 |
| 0.00505 | 0.00000 | 0.00000 | 0.00000 | 0.15300 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.77991 | 0.00000 | 0.06205 | 83.73498 | 0.00051 | 0.04499 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.04837 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.95163 | 0.00000 | 0.00000 | 95.78317 | 0.00059 | 0.05254 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.04837 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.95163 | 0.00000 | 0.00000 | 107.83135 | 0.00059 | 0.05254 |

Notes: *This table shows the estimation outcomes of optimising only the conditional value at risk in Powernext with stocks and bonds inclusive using daily data. With a given tolerance parameter, it provides estimates for the optimal weights: (ω_i) , the expected return, \mathcal{E}_p , and the risk, $CVaR_p(r_p)$ of the portfolio.

Table 2 Optimisation of the CVaR under the Gaussian assumption in French power futures market

| $\theta_{\text{Spot price}}$ | $\theta_{1\text{-month-ahead}}$ | $\theta_{2\text{-months-ahead}}$ | $\theta_{1\text{-quarter-ahead}}$ | $\theta_{1\text{-year-ahead}}$ | $\theta_{2\text{-year-ahead}}$ | θ_{EJP} | θ_{THEGOLLA} | $\theta_{\text{SICHELLENSE}}$ | $\theta_{\text{GDF-SEZ}}$ | $\theta_{\text{THEGOLLA ENVIRONNEMENT}}$ | θ_{TOTAL} | $\theta_{\text{MAUREL ET PROM}}$ | $\theta_{\text{BYNFRANCE 300P}}$ | $\theta_{\text{OATFRANCE PKLS STRIP}}$ | γ | \mathcal{E}_p | $\text{CVaR}_p^N(r_p)$ |
|------------------------------|---------------------------------|----------------------------------|-----------------------------------|--------------------------------|--------------------------------|-----------------------|----------------------------|-------------------------------|---------------------------|--|-------------------------|----------------------------------|----------------------------------|--|----------|-----------------|------------------------|
| 0.00012 | 0.00110 | 0.00045 | 0.00000 | 0.00265 | 0.01727 | 0.00386 | 0.00029 | 0.00190 | 0.00165 | 0.00000 | 0.00898 | 0.00255 | 0.95918 | 0.00000 | 0.00010 | -0.00008 | 0.00165 |
| 0.00010 | 0.00000 | 0.00005 | 0.00233 | 0.00072 | 0.00832 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00502 | 0.01245 | 0.97101 | 0.00000 | 11.44588 | -0.00006 | 0.00172 |
| 0.00010 | 0.00000 | 0.00000 | 0.00451 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.02106 | 0.97433 | 0.00000 | 23.49406 | -0.00006 | 0.00187 |
| 0.00008 | 0.00000 | 0.00000 | 0.00593 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.02850 | 0.96549 | 0.00000 | 35.54224 | -0.00005 | 0.00203 |
| 0.00004 | 0.00000 | 0.00000 | 0.00824 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.04001 | 0.94991 | 0.00180 | 47.59043 | -0.00004 | 0.00240 |
| 0.00000 | 0.00000 | 0.00000 | 0.01274 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.05910 | 0.92415 | 0.00401 | 59.63861 | -0.00003 | 0.00318 |
| 0.00000 | 0.00000 | 0.00000 | 0.10654 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.54042 | 0.28963 | 0.06341 | 71.68680 | 0.00033 | 0.02821 |
| 0.00000 | 0.00000 | 0.00000 | 0.13412 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.86588 | 0.00000 | 0.00000 | 83.73498 | 0.00056 | 0.04499 |
| 0.00000 | 0.00000 | 0.00000 | 0.09259 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.90741 | 0.00000 | 0.00000 | 95.78317 | 0.00058 | 0.04642 |

Notes: *Table 2 depicts the estimation outcomes of optimising the CVaR under the Gaussian assumption in Powernext with stocks and bonds inclusive. With a given tolerance parameter, γ it provides estimates for the optimal weights; (θ_1) , the expected return, \mathcal{E}_p and the risk, $(\text{CVaR}_p(r_p))$ of the portfolio.

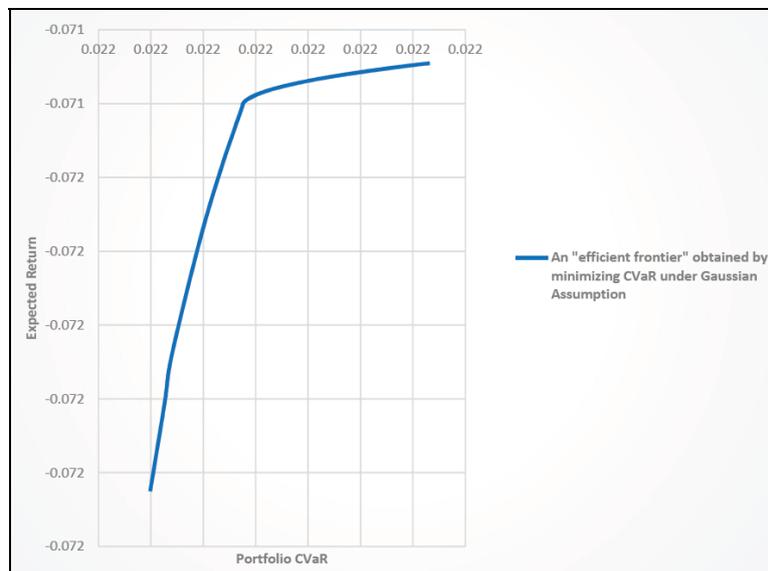
Table 3 Optimisation of the CVaR only in German power futures market

| $\omega_{Spot, price}$ | $\omega_{1-month-ahead}$ | $\omega_{1-quarter-ahead}$ | $\omega_{2-quarter-ahead}$ | $\omega_{1-year-ahead}$ | $\omega_{2-year-ahead}$ | ω_{NORDEX} | $\omega_{PNE-WIND}$ | $\omega_{SOLARSTROM}$ | ω_{D-EKT} | ω_{E-ON} | ω_{RWE} | $\theta_{BUNDESREPUB/DTL-2006}$ | γ | \mathcal{E}_p | $CVaR_{\alpha}(r_p)$ |
|------------------------|--------------------------|----------------------------|----------------------------|-------------------------|-------------------------|-------------------|---------------------|-----------------------|------------------|-----------------|----------------|---------------------------------|----------|-----------------|----------------------|
| 0.00000 | 0.00643 | 0.00000 | 0.00000 | 0.00000 | 0.12537 | 0.00407 | 0.00765 | 0.01360 | 0.01169 | 0.03317 | 0.01346 | 0.78457 | 0.00010 | -0.00003 | 0.00442 |
| 0.00047 | 0.00000 | 0.00391 | 0.01161 | 0.00628 | 0.07730 | 0.00000 | 0.00862 | 0.01406 | 0.01782 | 0.02549 | 0.00000 | 0.83444 | 11.44588 | 0.00001 | 0.00460 |
| 0.00019 | 0.00000 | 0.00759 | 0.01712 | 0.00000 | 0.00000 | 0.00000 | 0.00584 | 0.01530 | 0.03725 | 0.00000 | 0.00000 | 0.91671 | 23.49406 | 0.00006 | 0.00547 |
| 0.00043 | 0.00000 | 0.00000 | 0.01656 | 0.00000 | 0.00000 | 0.00000 | 0.00562 | 0.01476 | 0.04047 | 0.00000 | 0.00000 | 0.92216 | 35.54224 | 0.00007 | 0.00555 |
| 0.00109 | 0.00000 | 0.00000 | 0.00060 | 0.00000 | 0.00000 | 0.00000 | 0.00556 | 0.02100 | 0.05366 | 0.00000 | 0.00000 | 0.91809 | 47.59043 | 0.00008 | 0.00586 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00188 | 0.01281 | 0.07772 | 0.00000 | 0.00000 | 0.90759 | 59.63861 | 0.00008 | 0.00637 |
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.01241 | 0.09045 | 0.00000 | 0.00000 | 0.89713 | 71.68680 | 0.00009 | 0.00670 |
| 0.00168 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.01181 | 0.10142 | 0.00000 | 0.00000 | 0.88509 | 83.73498 | 0.00009 | 0.00704 |
| 0.00268 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.01056 | 0.12515 | 0.00000 | 0.00000 | 0.86161 | 95.78317 | 0.00010 | 0.00785 |

Notes: * This table shows the estimation outcomes of optimising the CVaR only in European energy exchange with stocks and bonds inclusive. With a given tolerance parameter, γ it provides estimates for the optimal weights: $(\omega_{i,t})$, the expected return, \mathcal{E}_p , and the risk, $(CVaR_{\alpha}(r_p))$ of the portfolio.

There are different cases under consideration. In the first instance, I consider investing a lump sum of money by the different investors in the electricity market only. However, this appears trivial because diversification is not exhaustive. Moreover, I consider electricity futures with different maturities. The so called ‘efficient frontier’ obtained as a result of plotting expected return against CVaR appears in the negative orthant which could be due to economic slowdown in the countries under consideration. This phenomena is quite evident in Figure 3. This was a period where the European energy exchange first experience negative prices due to the inception of the wind energy technology. To intensify our investigation or analysis, I again build a portfolio of the different assets.

Figure 3 Efficient frontier via portfolio optimisation of CVaR under Gaussian assumption: German power futures (see online version for colours)



Note: This figure shows an efficient frontier obtained with the application of the optimisation of the CVaR under the Gaussian assumption technique utilising the German electricity futures daily data.

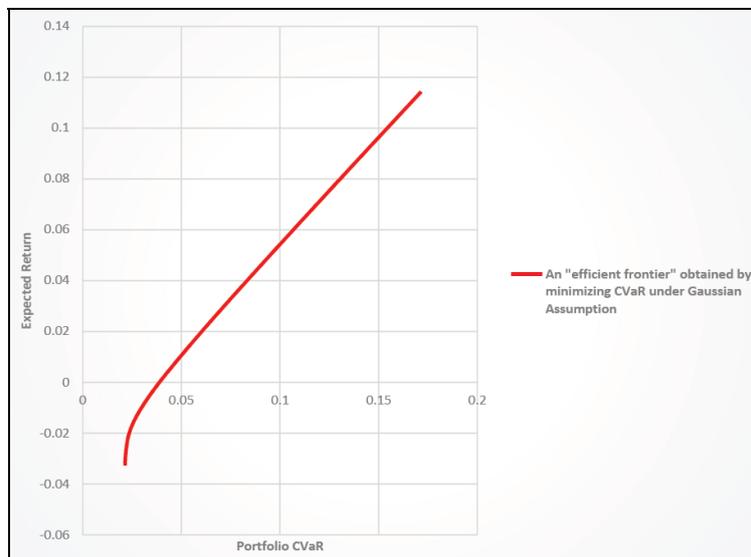
In addition to the electricity futures, assets such as stocks and bonds have also been included as a motivation based on the results of Tobin (1958), Sharpe (1964) and Lintner (1965), where the authors showed that the inclusion of a risk-free asset in the investment universe is superior to the one available to the investors without a risk-free asset. An initial analysis of this data reveals that the data on assets in the year 2008 and some individual ones in particular perform badly. This is no surprise since it is a period where the whole world was experiencing a global economic downturn that has affected various sectors of the economy including the energy sector. The same applies to the expected return on the various assets, which appears negative and is consistent with the results of Bauwens et al. (2013).

In view of the brevity of the findings in this paper, further analyses therefore focus on the weekly and monthly data (obtained using aggregation of daily returns) of electricity

futures, stocks and bonds. This is to provide an overview of the various scenarios occurring at these periods. The optimal investment portfolio is then determined via the three approaches proposed and the portfolio construction is done in two steps. In the first place, the most efficient portfolio is determined (referred to as the market portfolio by Markowitz). Secondly, the investors select between investing in the market portfolio and the risk-free interest rate in such a way that their resulting portfolio reflects their risk orientation under the assumption of a representative agents (see Chabi-Yo et al., 2008 for details). Electricity prices exhibit strong higher moments' characterisations, which is most noteworthy of strong leptokurtosis and skewness. This is consistent with the results of Bunn and Karakatsani (2003), Huisman and Mahieu (2003) and many others.

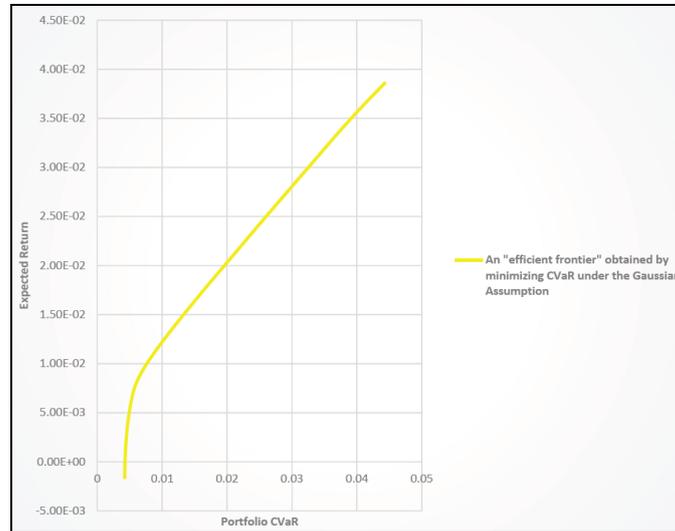
Mostly, an investor's asset allocation decision is typically examined as trade-off between risk and return. The reduction in portfolio risk depends on the degree of co-movement between assets. Naturally, the investor is mostly interested in the short-term portfolio since this addresses the current task of asset allocations. The risk and expected return corresponds to a tolerance parameter, which determines how much risk oriented one is. Thus, the choice of the best portfolio depends on the aversion to risk of the investor. Figures 3, 4, 5, and 6 reveal that investors with different maturities will have different optimal portfolio weights even if they all rely on the same methodology for decision making, i.e., using the same methodology to determine the optimal portfolios. Historical distributions are the best estimates of future distributions. Even with homogeneous beliefs, investors will hold portfolios with different compositions. In effect, this has implications for managers with various clients, as well as for investment theories that will assume that all investors hold the same portfolio of risky assets.

Figure 4 Efficient frontier via portfolio optimisation of CVaR under Gaussian assumption: French power futures (see online version for colours)



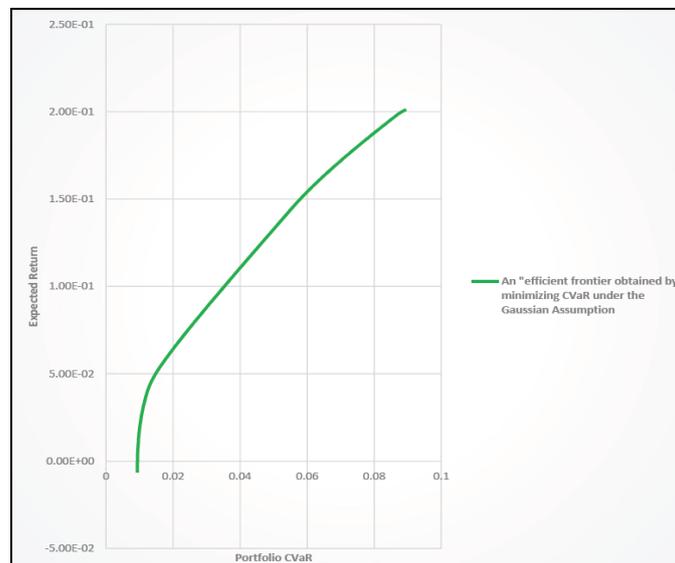
Note: This figure depicts the efficient frontier obtained with the application of the optimisation of the CVaR under the Gaussian assumption technique using the French electricity futures daily data.

Figure 5 Efficient frontier via portfolio optimisation of CVaR under Gaussian assumption: harmonised German daily power futures (see online version for colours)



Note: In this figure, I present the efficient frontier obtained with the application of the optimisation of the CVaR under the Gaussian assumption technique with applications to the new harmonised German electricity futures. In particular, stocks and bonds have been included in the new daily data. Table 4 is an extract of the data resulting in Figure 5.

Figure 6 Efficient frontier via portfolio optimisation of CVaR under Gaussian assumption: harmonised German weekly power futures (see online version for colours)



Note: This figure represents the efficient frontier obtained with the application of the optimisation of the CVaR under the Gaussian assumption technique with applications to the new harmonised German electricity futures. In particular, stocks and bonds have been included but a weekly data.

Consequently, the various optimisation techniques have been carried out as follows: in particular, I consider minimising a coherent risk measure; the CVaR under various criteria such as:

- 1 using Markowitz approach
- 2 minimising CVaR only
- 3 minimising CVaR under Gaussianity.

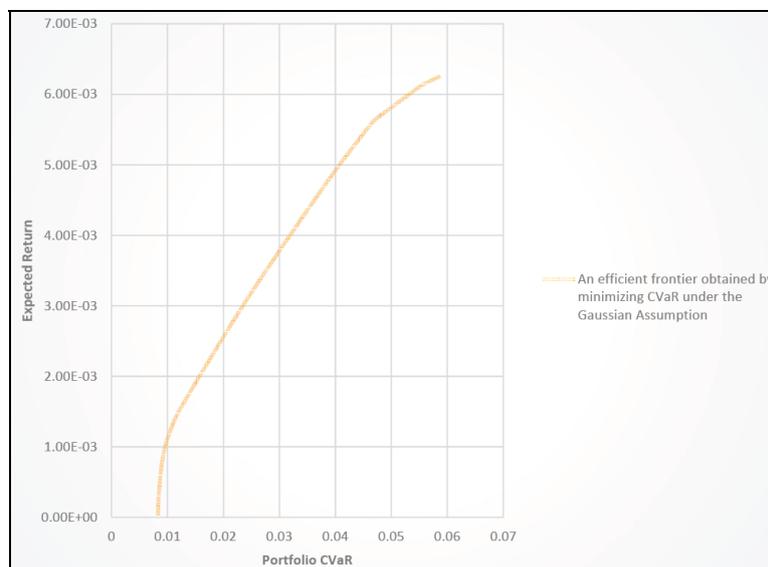
The minimisation of the CVaR model provides a relatively good efficient frontier compared to the Markowitz approach. However, the outputs of the minimisation of the CVaR constitute a robust result of the overall output of the optimisation procedures. This is evident in Figures 4 to 6. In Figures 3 and 4, I dealt with only the electricity futures without stocks and bonds for both the German and the French electricity market. The so called 'efficient' frontiers are not representations of the best of frontiers as expected. Price fluctuation in electricity market, specifically, the German electricity market could be a reference point for the poor frontiers. Figures 5 and 6 depict typical efficient frontiers of the German electricity futures with stocks and bonds inclusive. These graphical displays are a pictorial view of the frontier obtained from the minimisation of the CVaR, which provides accurate measures for capturing the downside risk.

Under most circumstances, poor performing assets have negative influences on the overall portfolio of assets. Therefore, eliminating these poor performing assets such as the year 2008 data and other individual assets from the portfolio produces the graphs in Figures 5 and 6. This provides a better improvement as compared to Figure 3, which represents a typical efficient frontier. It is therefore seen as a significant improvement due to further cleaning of the data. Both the results obtained from French and the German markets prove to have risks well-diversified across the various assets as compared to the earlier analyses. The benefit of expanding the investment opportunity sets depend on the characteristics of assets, which constitute the initial portfolio. Tables 1 to 4 report an extract of some of the estimation results for the optimal weights, risk and expected return of the portfolio from which the efficient frontiers have been obtained. In all cases, the sum of optimal weights equals one.

Overall, some of the above analyses have been carried out using daily returns. However, the robustness of our findings might be questionable because of the noise that is characteristic of high frequency data. Hence, I replicate the analysis using weekly and monthly returns. In this light, I consider weekly data followed by monthly data (not reported). For instance, the efficient frontier obtained using the weekly data is depicted in Figure 6. In sum, the analysis for both the weekly and the monthly data yield a frontier that reflects a typical frontier. Nevertheless, comparing the different frontiers of the weekly and the monthly returns to the daily returns, significant differences are quite evident. Finally, the currently improved data set is employed utilising the three different optimisation techniques to study their superiority. This revealed that the Gaussian assumption plays an essential role in accurately evaluating the risk inherent in the markets under study. Spikes (outliers) present in the data have been trimmed off/removed using the Mardia's multivariate normality test. The Mardia's test is based on the multivariate extension of the third and fourth moments: *skewness* and *kurtosis* respectively. The multivariate skewness is asymptotically distributed as a chi-square random variable for large sample sizes⁵. The optimisation techniques are then applied to the newly extracted data. However, the newly extracted data is still not multivariate

normal since not all individual constituent of the resulting portfolio series follow the normal distribution. Under multivariate normality, if all univariate time series, which are constituent of the portfolio follows the normal distribution, then the portfolio is said to assume multivariate normality, which is not true vice-versa. As a result, the newly extracted data can be seen as a further improvement of the current data in terms of reduction in the number of outliers. There is the likelihood that some components of the data pass the normality test and others do not. Utilising the newly extracted data, Figure 7 is the frontier obtained using the most superior optimisation technique, which is the minimisation of the CVaR under the normality assumption.

Figure 7 Efficient frontier via portfolio optimisation of CVaR under Gaussian assumption using newly extracted German power futures with stocks and bonds (see online version for colours)



Note: This figure shows the efficient frontier obtained with the application of the optimisation of the CVaR under the Gaussian assumption technique using a newly extracted data of weekly German electricity futures with stocks and bonds inclusive (i.e., data with a significant number of outliers trimmed off).

In effect, this approach produces a typical frontier and hence shows that identifying the underlying distribution of a portfolio improves the accuracy of the risk assessment methodology. In conclusion, the estimation method, optimisation of CVaR under the Gaussian assumption proves to be the best estimation technique among the three methods considered. This relates to the fact that knowledge of the underlying distribution of energy portfolio returns improves and yields a more accurate estimation with respect to the extent of risk inherent in the future power markets by capturing the downside risk. Indeed, this actually provides good risk management practices and thereby serves as a good source of information for investors or market players in energy markets.

5 Conclusions

In this paper, three optimisation procedures namely:

- 1 Markowitz's optimisation technique,
- 2 optimisation of conditional value of risk
- 3 optimisation of the CVaR under the Gaussian assumption, have been utilised in order to examine the optimal allocations of electricity futures by investors, who want to hedge against price fluctuations.

The finding of this paper shows that even though utilising the second optimisation approach; optimisation of CVaR only, provides a better estimate than the Markowitz's approach, the last optimisation technique, that is the optimisation of the CVaR under Gaussian assumption provides a best accurate estimate among the three techniques examined. This gives credence to the fact that CVaR is coherent in nature and captures the downside risk. Thus, coupled with a known underlying distribution of the evolution of futures power prices, the third methodology provides accurate risk estimation (risk assessment). These analyses therefore constitute a temporal diversification.

In addition, these estimations revealed that utilising various maturities of the energy futures provide a less risky environment for the investors or market players in the energy future exchanges. It therefore documents numerically and graphically how the optimisation of a coherent and tractable risk measure, the CVaR, can be a useful tool of energy risk management for market players and investors. The comparison of the different models shows that the optimisation of the CVaR with a known underlying distribution provides a very satisfying outcome in that the downside risk inherent in the various market have been captured. This is a confirmation of the assertion of Fianu (2015) even though the author worked in a spatial framework. Note that no single purchased quantity can be identified as optimal, because the problem of optimality is bi-dimensional with the objective of risk and return. I have numerically and graphically shown how to identify the optimal set of portfolios by solving a series of risk minimisation problems and summarising the results in an 'efficient frontier' that displays the Pareto-optimal allocations between the expected return and the CVaR.

The efficient frontier simplifies a complex portfolio management problem: it highlights and clarifies some basic questions: scarcity, efficiency, trade-offs, opportunity cost, and the value of breaking constraints. It also offers a simple way of understanding investment decisions, discovering ways to increase the efficiency of portfolio investments and avoiding investments in regions of diminishing returns. One interesting observation is that the inclusion of risk-free assets (bonds) result in the improvement of the efficient frontiers obtained with respect to energy portfolios. This actually confirms the results of the Tobin (1958), Sharpe (1964) and Lintner (1965). A priori, the tolerance parameter is chosen; thus, the results depend upon the measure of the degree of risk aversion. Quantification of the trade-off between risk and expected return will assist management (in the energy sector) in determining the value of investment for the different set of portfolios. Nevertheless, this is a management problem even in the financial sector.

As intuitively and empirically appealing as these findings appear, they are not without limitations, constraints or shortcomings. For instance, the duration of the data is not quite long and thus an updated data would be useful in providing current relevant information on the extent of risk inherent in the energy future markets under study. On the other hand, the analysis has been restricted to just two energy exchanges. Therefore, the analysis could be extended to cover an up-to-date data on other energy exchanges in future research.

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Notes

- 1 The latter is a power purchasing agreement in which the owner purchases electricity from power plants and pays according to a plan that relates the price to fuel prices.
- 2 See for instance Haldrup and Nielsen (2006), Bosco et al. (2010), Bauwens et al. (2013) and Gianfreda and Grossi (2012) for an overview.
- 3 This paper has been reprinted in 2012 because of its contribution to the literature, see Bar-Lev and Katz (2012) for a reference.
- 4 See Wikipedia (2015) for a brief overview.
- 5 The multivariate normality is checked using the `MardiaTest` function incorporated in the R statistical package called the `MVN` package. The multivariate outliers detection methods present in the `MVN` package are based on the Mahalanobis distance. Employing the Mahalanobis distance methodology involves the following steps: compute robust Mahalanobis distances ($MD(x_i)$), and compute the 97.5% percentile-Q chi-square distribution, see Korkmaz et al. (2014) for an overview.

Abbreviations/nomenclature

| | |
|----------------------|---|
| <i>APX</i> | Netherlands power exchange (APX of Netherlands) |
| CO_2 | carbon dioxide |
| Σ | covariance matrix |
| <i>CVaR</i> | conditional value-at-risk |
| <i>EU</i> | European Union |
| <i>EEEX</i> | European Energy Exchange |
| $CVaR_\alpha(r_p)$ | portfolio CVaR |
| $\sigma_\alpha(r_p)$ | portfolio variance |
| $CVaR_\alpha^N(r_p)$ | portfolio CVaR under Gaussian assumption |
| <i>LPX</i> | Leipzig power exchange |
| \mathcal{E} | mathematical expectation |
| <i>MVP</i> | mean-variance portfolio |
| <i>MVN</i> | multivariate normality |
| MW | megawatts |
| MWh | megawatts hours |
| α | order of risk measure |
| <i>r</i> | returns |
| <i>TCE</i> | tail conditional expectation |
| γ | risk tolerance parameter. |